Simulation of the dynamics of an annular system of parallel Josephson junctions under the influence of external electromagnetic radiation

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27-31 Oktober, Dubna

Main Tasks

Within the framework of the joint project of MLIT and BLTP an ecosystem is being developed for the tasks of studying a system based on Josephson junctions using Python in the form of Jupyter Book is being developed.

The following tasks were solved:

1. Development of algorithm for calculation of current-voltage characteristics of Annular system of parallel Josephson junctions under the external radiation and amplitude dependence of Shapiro step width.
2. Realization of parallel calculation of amplitude dependence of Shapiro step width.

Ecosystem ML/DL/HPC of HybriLIT platform

Component for educational purposes

JupyterLab Server [**https://studhub1.jinr.ru**]
[**https://studhub2.jinr.ru**]

- ❑ 2x Intel Xeon E5-2680 v3 (12 Cores @ 2.5 GHz)
- ❑ 256 GB RAM

Component for computational tasks

Server with NVIDIA Volta [**https://jhub1.jinr.ru**]
[**https://jhub2.jinr.ru**]

- ❑ 2x Intel Xeon Gold 6148 (20 Cores @ 2.4 GHz)
- ❑ 4x **NVIDIA Volta V100** SXM2 32 GB HBM2
- ❑ 512 GB RAM

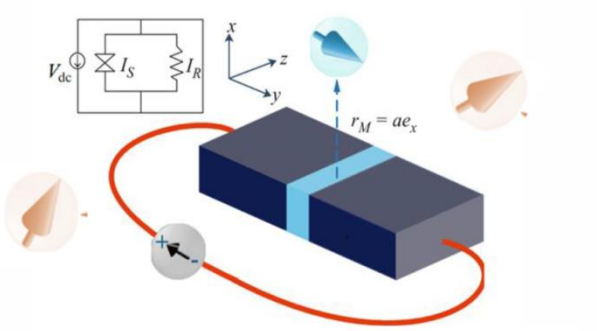
Component for scientific projects

Server with NVIDIA Ampere [**https://ampere03.jinr.ru**]
[**https://ampere05.jinr.ru**]

- ❑ 2x AMD EPYC 7763 (64 Cores @ 2.4 GHz)
- ❑ 8x **NVIDIA Ampere A100** SXM4 80 GB HBM2
- ❑ 2 TB RAM

Research of systems based on Josephson junctions

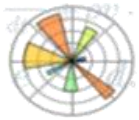
Symbolic Computing Unit



$$\gamma_{m_i} = = -\frac{\mu_0}{2\Phi_0}\int d\mathbf{r}_i \frac{\mathbf{M}_i \times \mathbf{r}_i}{r^3}$$

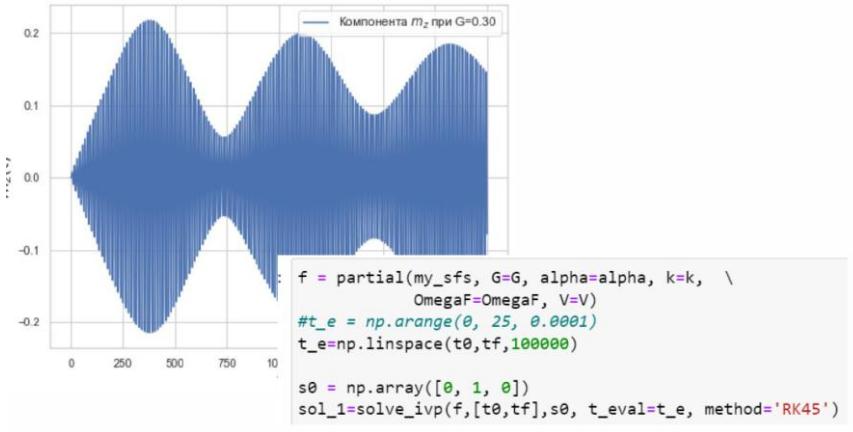
$$B_{12}(r_{12}, m_1) = \frac{\mu_0}{4\pi}\left(\frac{3(m_1 \cdot \hat{r})\hat{r}}{b^5} - \frac{m_1}{b^3}\right)$$

SymPy is a Python library for symbolic mathematics.



Matplotlib is a main library for building graphs, diagrams in Python

Calculation, modeling and analysis unit



SciPy is an open-source software for mathematics, science and engineering.

Acceleration of multiparameter calculations

Joblib is a set of tools to provide lightweight pipelining in Python

Numba is an open source JIT compiler that translates a subset of Python and NumPy code into fast machine code.

Various possibilities of Jupyter Book

jupyter {book}

Welcome to HLIT Jupyter Book

Python-инструментарий для моделирования динамики джозефсоновского перехода под воздействием внешнего излучения

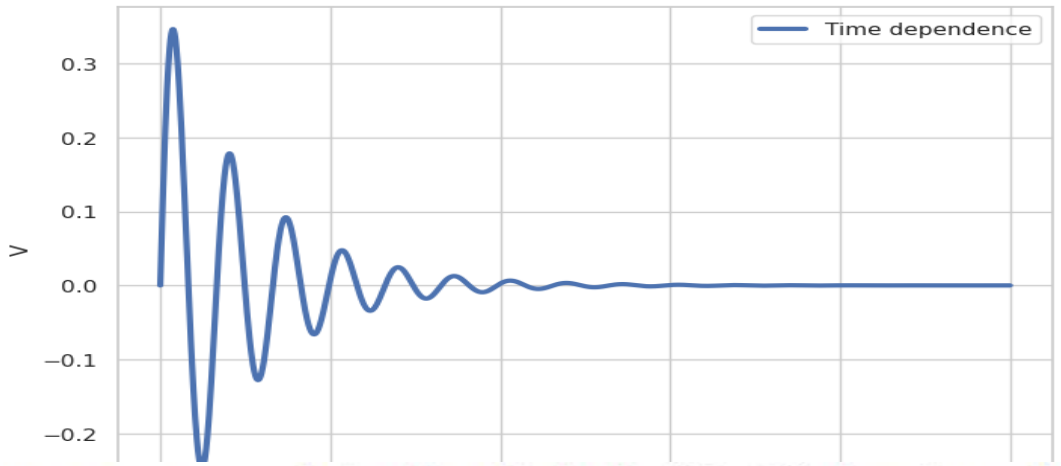
Python toolkit for simulation of the Josephson junction dynamics under the influence of external electromagnetic

jupyterhub

We draw a time dependence $V(t)$

.ipynb
.pdf

```
fig = plt.figure(figsize=(8, 6))
plt.plot(time_array, Vtime, label='Time dependence', linewidth=3.0)
plt.xlabel('Time', size=12)
plt.ylabel('V', size=12)
plt.legend(loc='upper right')
plt.show()
```



Stationary Josephson effect. When a current passes below the critical value ($I < I_c$), there is no voltage in the junction and a superconducting current flows through the junction. This current is proportional to the sine of the phase difference of the order parameters of the superconductor layers:

$$I_s = I_c \sin \varphi. \quad (1)$$

Non-stationary Josephson effect. When the current increases above the critical value ($I > I_c$), an AC voltage appears in the JJ, which is proportional to the time derivative of the phase difference

$$V = \frac{\hbar}{2e} \frac{d\varphi}{dt}. \quad (2)$$

KEY FEATURES

Jupyter Book is an **open-source** project for creating interactive and publishable online books, documentation, and blogs based on computational materials.

Jupyter Book supports .ipynb, Markdown, and reStructuredText files, allowing you to combine text with **code, equations, images, and interactive elements**.

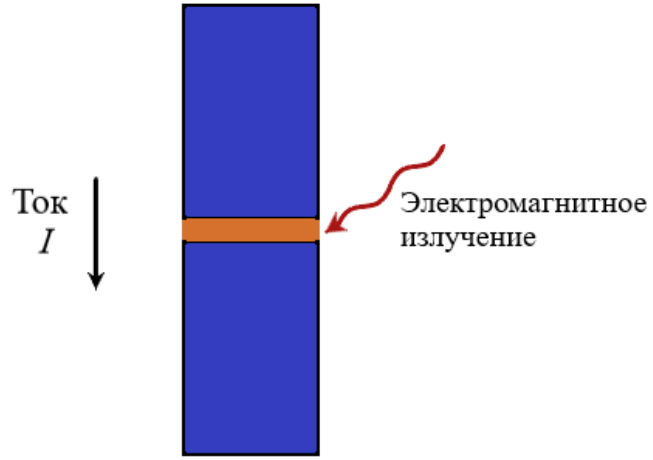
Code in Jupyter Notebook can be executed while building the book, and the results (output, plots) will be embedded in the book.

Generation of various types of materials. We can get the book as a single or multi-page website, and we can **export it to PDF.**

Development of an electronic recourse Jupyter Book for conducting research in Python
http://studhub.jinr.ru:8080/books

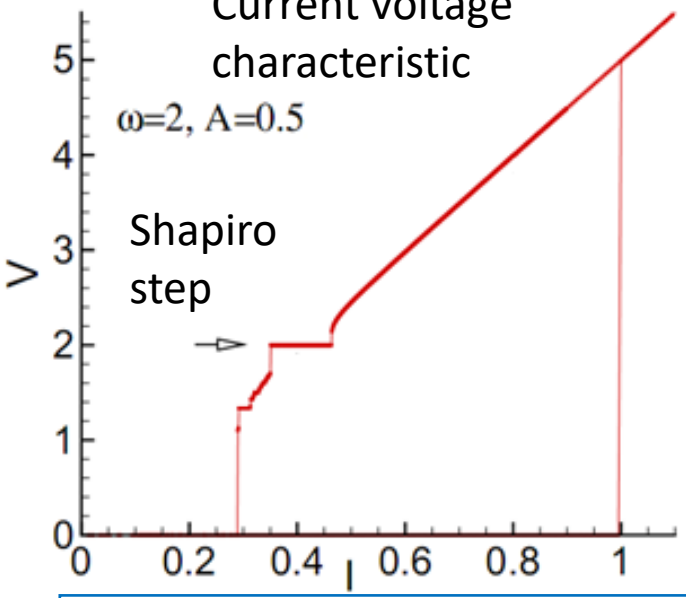
The development is carried out on the basis of the ecosystem ML/DL/HPC heterogeneous platform HybriLIT

The influence of external radiation on the dynamics of the Josephson junction



Under the influence of external radiation, in case of the Josephson frequency is a multiple of the radiation frequency ($n\,\omega_j = k\,\omega$), a constant voltage step appears on the current-voltage characteristic (CVC) of the Josephson junction. This step is called a **Shapiro step**.

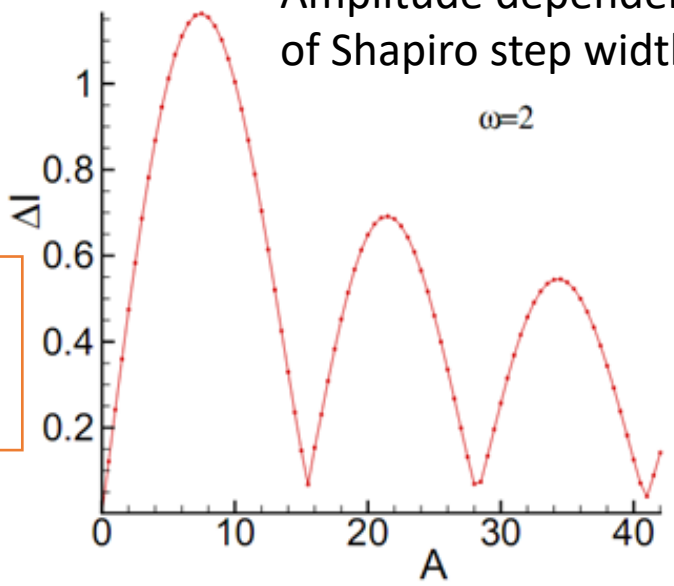
Current voltage characteristic



The width of the Shapiro step depends on the amplitude and frequency of the radiation.

The Shapiro step has practical interest, for example, a voltage standard was implemented on its basis

Amplitude dependence of Shapiro step width



K. K Likharev, Introduction to Dynamics of Josephson junctions» — Moscow, «Nauka», 1985

Algorithm for calculating the current-voltage characteristic

We set the values of the parameters

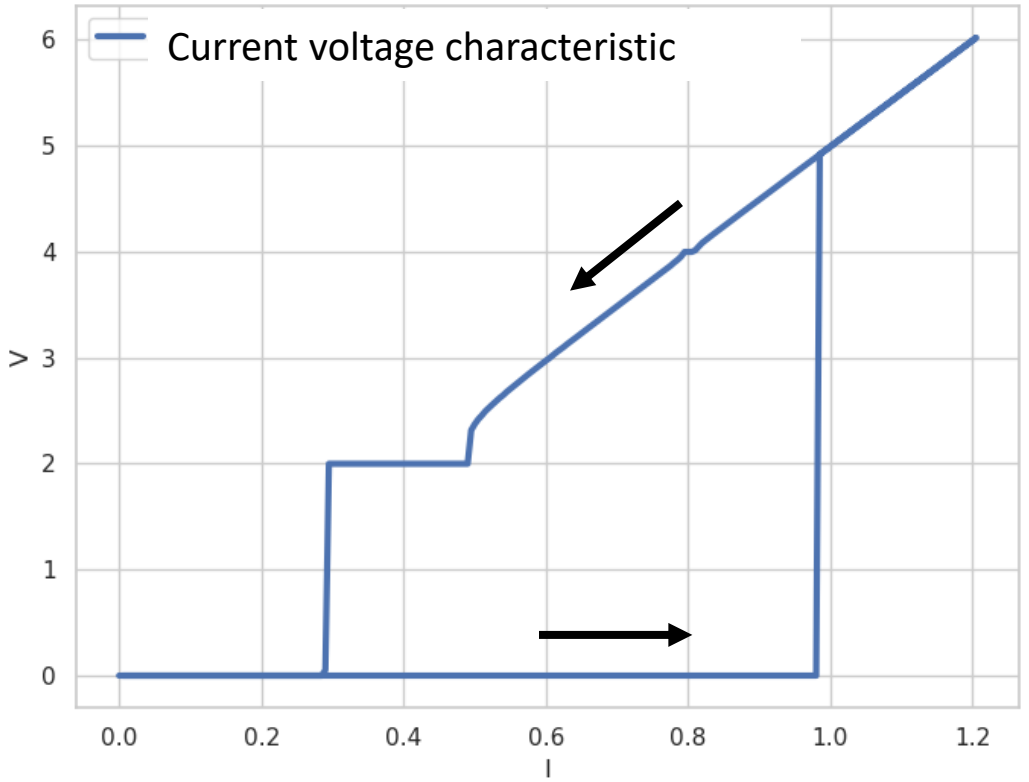
We set the current cycle from I = 0 to Imax and back to zero

We numerically solve the Cauchy problem for fixed I and obtain V(t), φ(t), u(t)

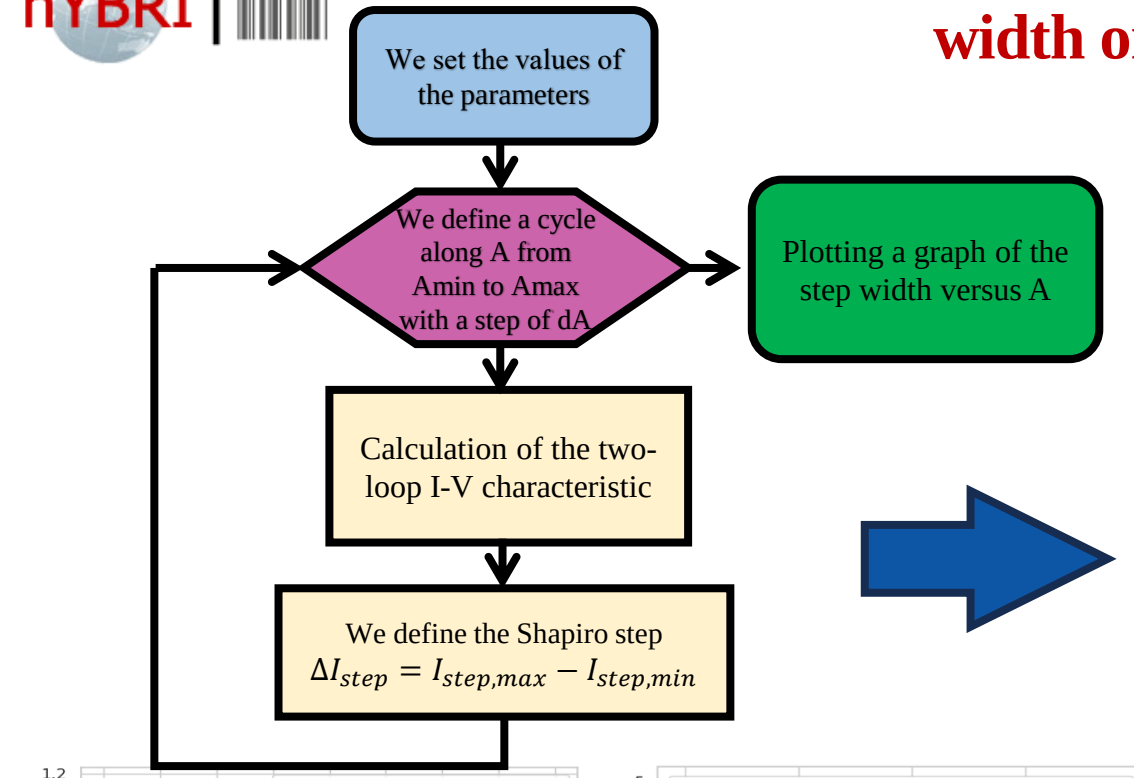
Calculate the average voltage

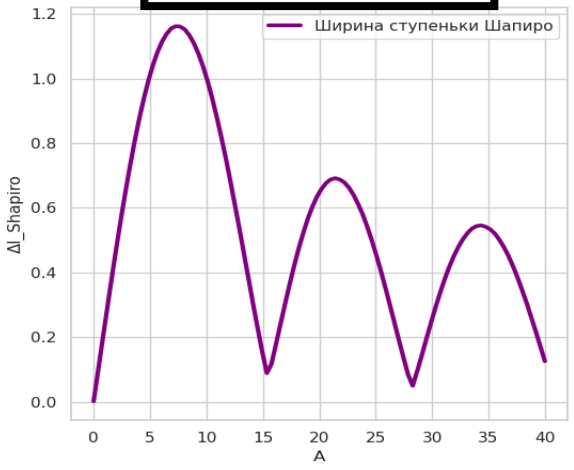
$$\bar{V} = \frac{1}{T_{max} - T_{min}} \int_{T_{min}}^{T_{max}} V(t)dt$$

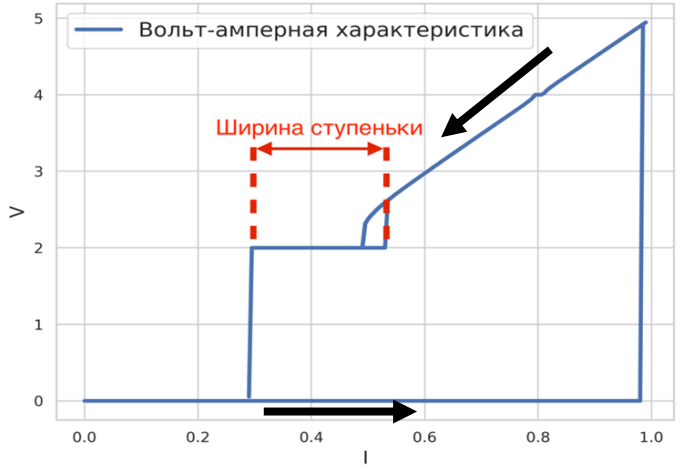
Plotting a current-voltage characteristic graph

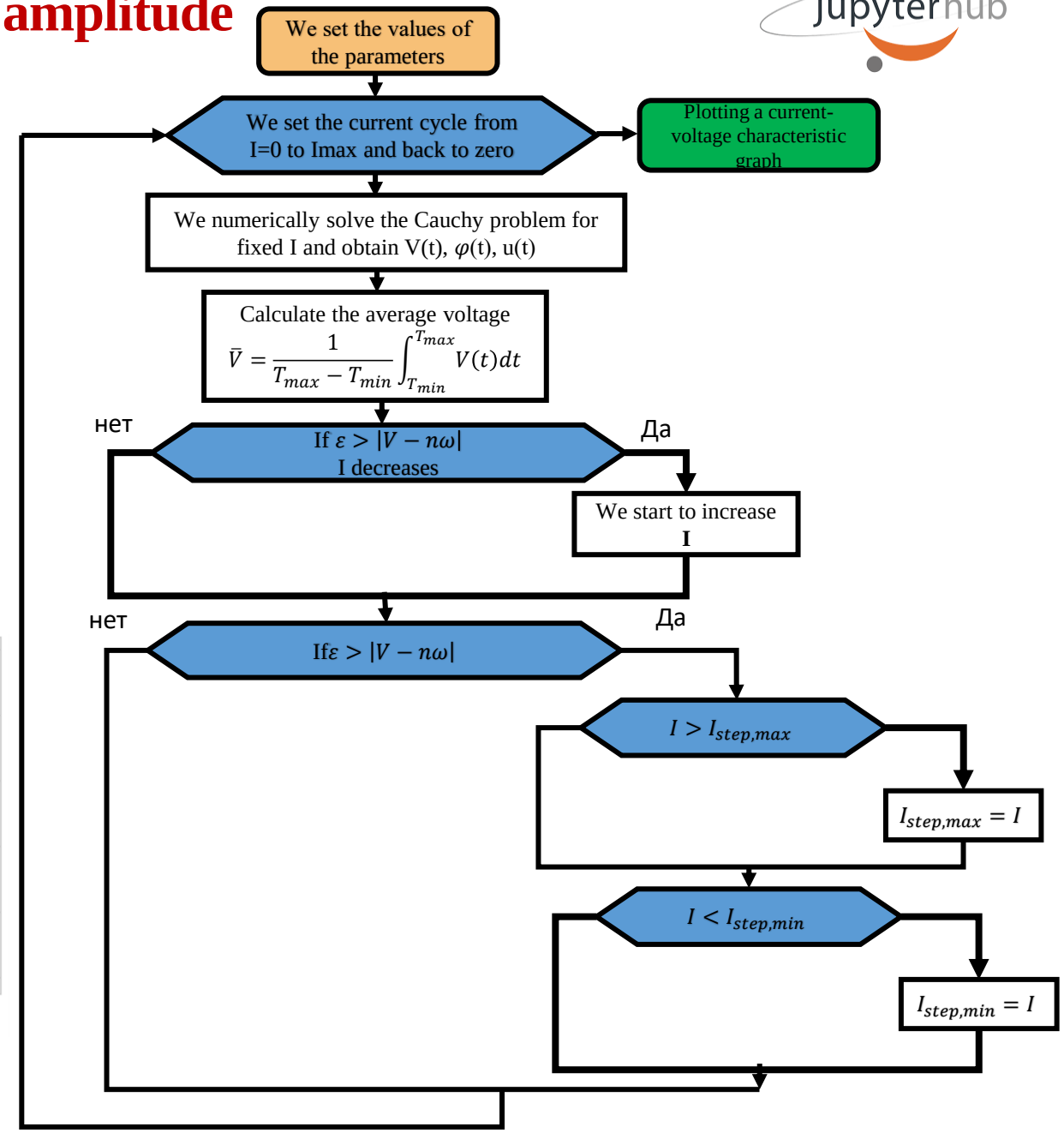


Algorithm for calculating the dependence of the Shapiro step width on the amplitude



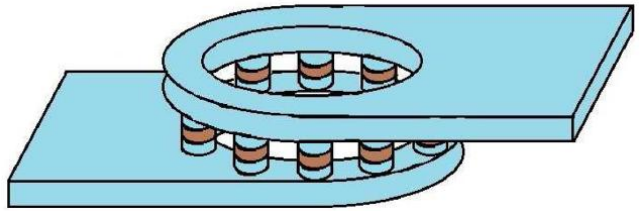




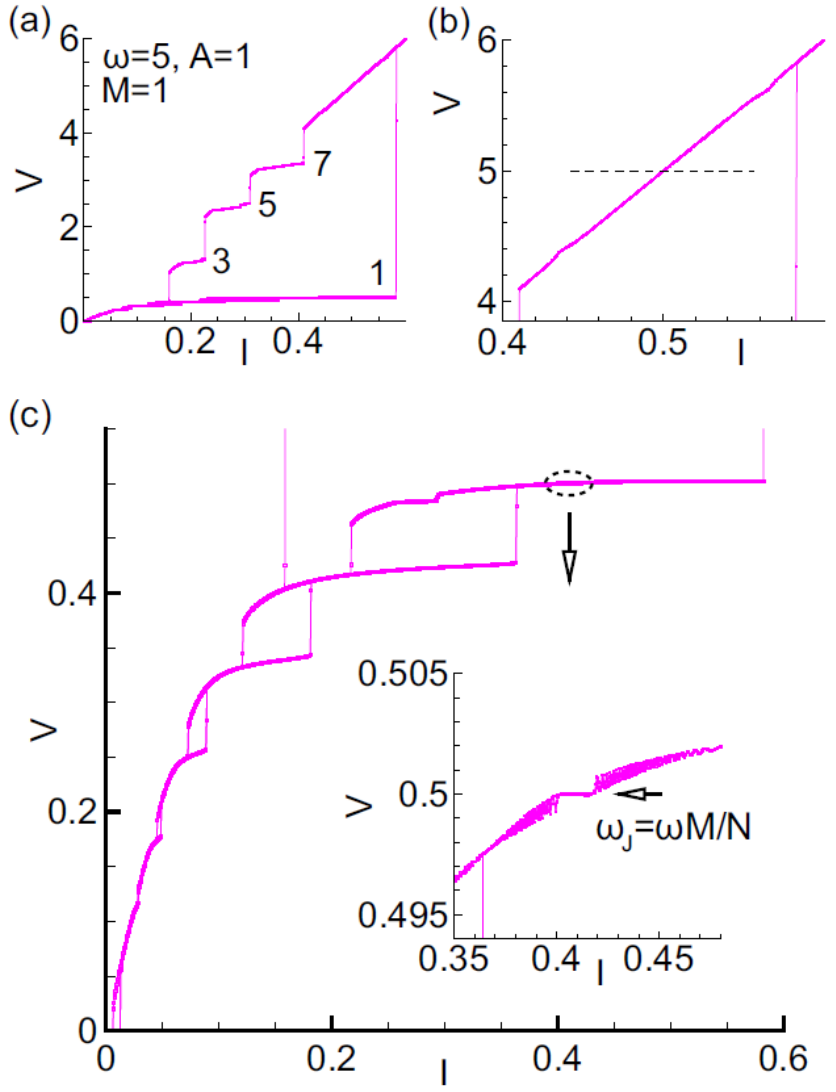


A.R. Rahmonova, I.R. Rahmonova, O.I. Streltsova, M.I.Zuev, Physics of Particles and Nuclei, Vol. 55, pp. 528-531 (2024).

Annular system of parallel JJs under the external electromagnetic radiation



Schematic view Annular system of parallel JJ. It can be described with the Frenkel-Kontorova model



The current-voltage characteristic of a annular system of parallel Josephson junctions under the influence of external radiation was studied. The authors demonstrated that the constant voltage step (Shapiro) arises as a result of **frequency locking of fluxon rotation by external radiation**, which differs from the usual case in which **frequency locking of Josephson oscillations by external radiation** occurs.

Until now, the dependence of the step width on the amplitude in this case has not been investigated.

I. R. Rahmonov, J. Teki´c, P. Mali, A. Irie, and Yu. M. Shukrinov, Phys. Rev. B 101, 024512 (2020)

Statement of the problem

Develop an algorithm for finding the step width from the amplitude and speed up the calculation process by parallel implementation

Frenkel-Kontorova model

$$\frac{d^2\varphi_i}{dt^2} - \frac{\varphi_{i+1} - 2\varphi_i + \varphi_{i-1}}{a^2} + sin\varphi_i + \beta\frac{d\varphi_i}{dt} = I + A\,sin(\omega t)$$

φ_i - phase difference
a - distance between JJs
β - dissipation parameter
I - external current
A - Amplitude of external radiation
ω - frequency of external radiation

$i = 1, 2, \ldots, N$, N – number of Junction

Initial condition

$$t = 0, \varphi_i = 0, \frac{d\varphi_i}{dt} = 0$$

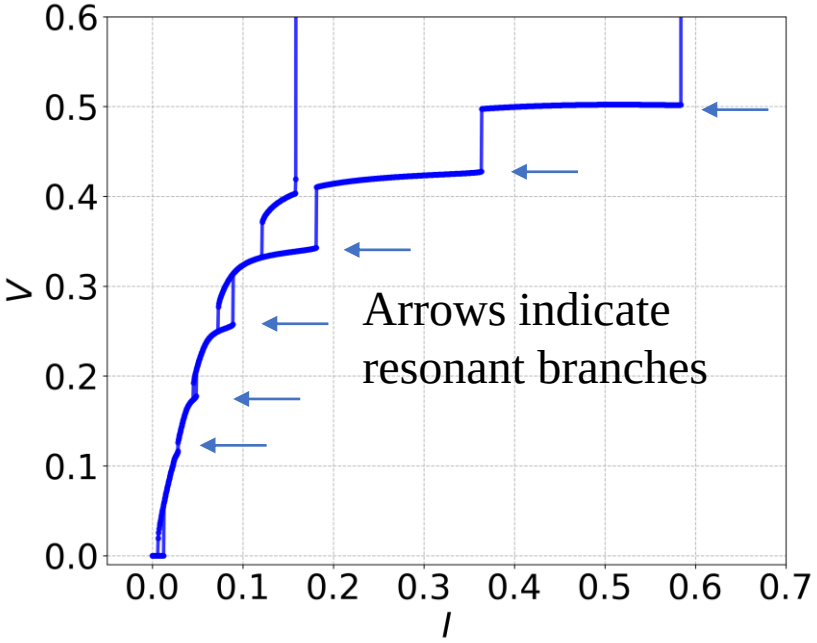
Boundary condition

$$\varphi_{N+1} = \varphi_1 + 2\pi M$$
$$\varphi_0 = \varphi_N - 2\pi M$$

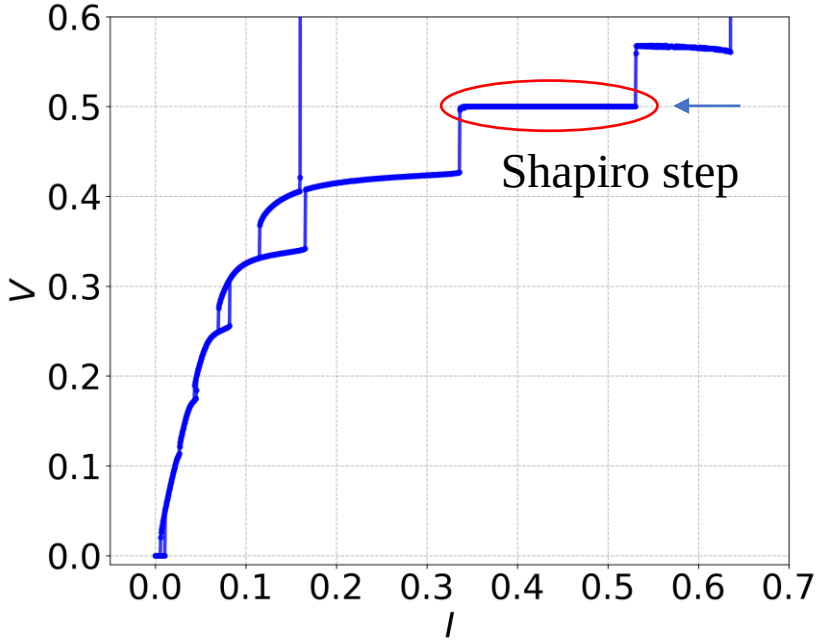
M – number of introduced fluxon

I. R. Rahmonov, J. Teki´c, P. Mali, A. Irie, and Yu. M. Shukrinov, Phys. Rev. B 101, 024512 (2020)

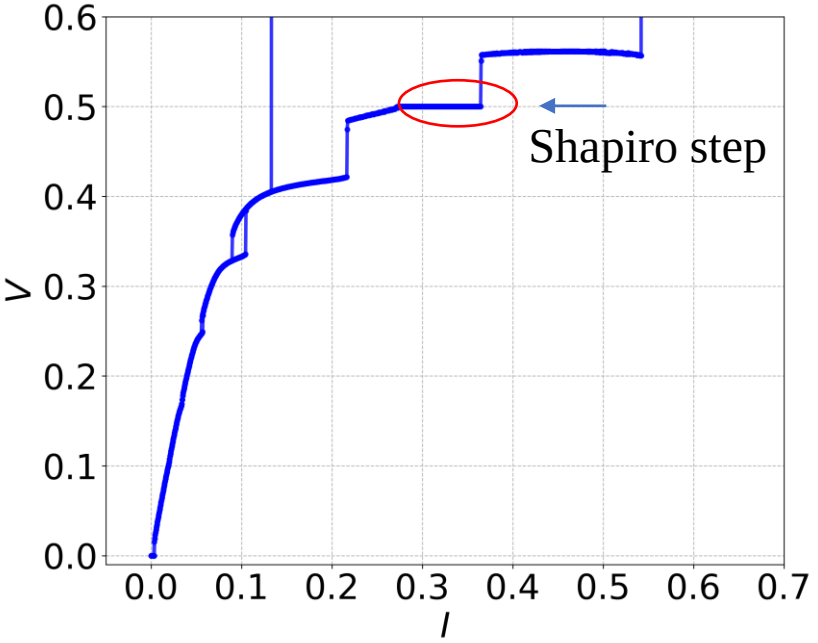
The influence of the amplitude of external radiation on the current-voltage characteristics of a annular system



Current-voltage characteristic at A=0, no step



Current-voltage characteristic at A=10



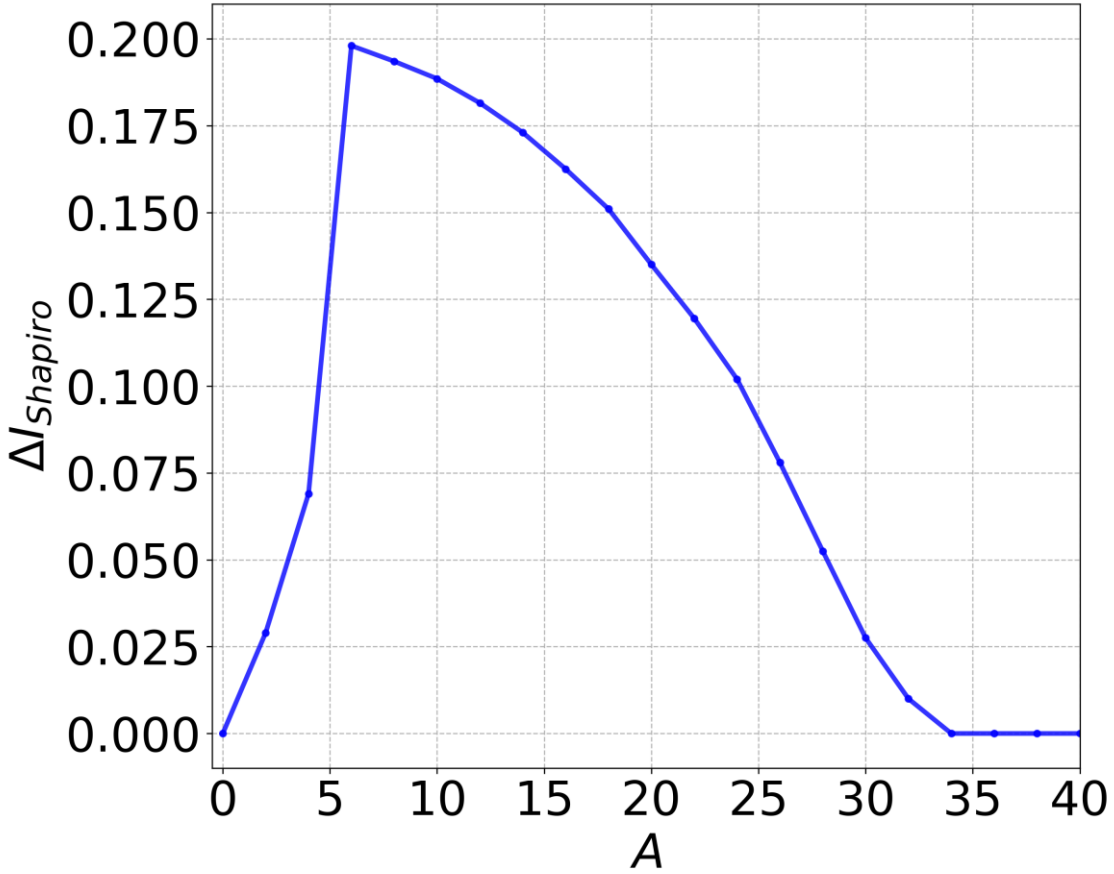
Current-voltage characteristic at A=25

In this current voltage characteristics we can see resonance branches and in cases A=10 and A=25 we can see also Shapiro steps which are highlighted with red ellipse.

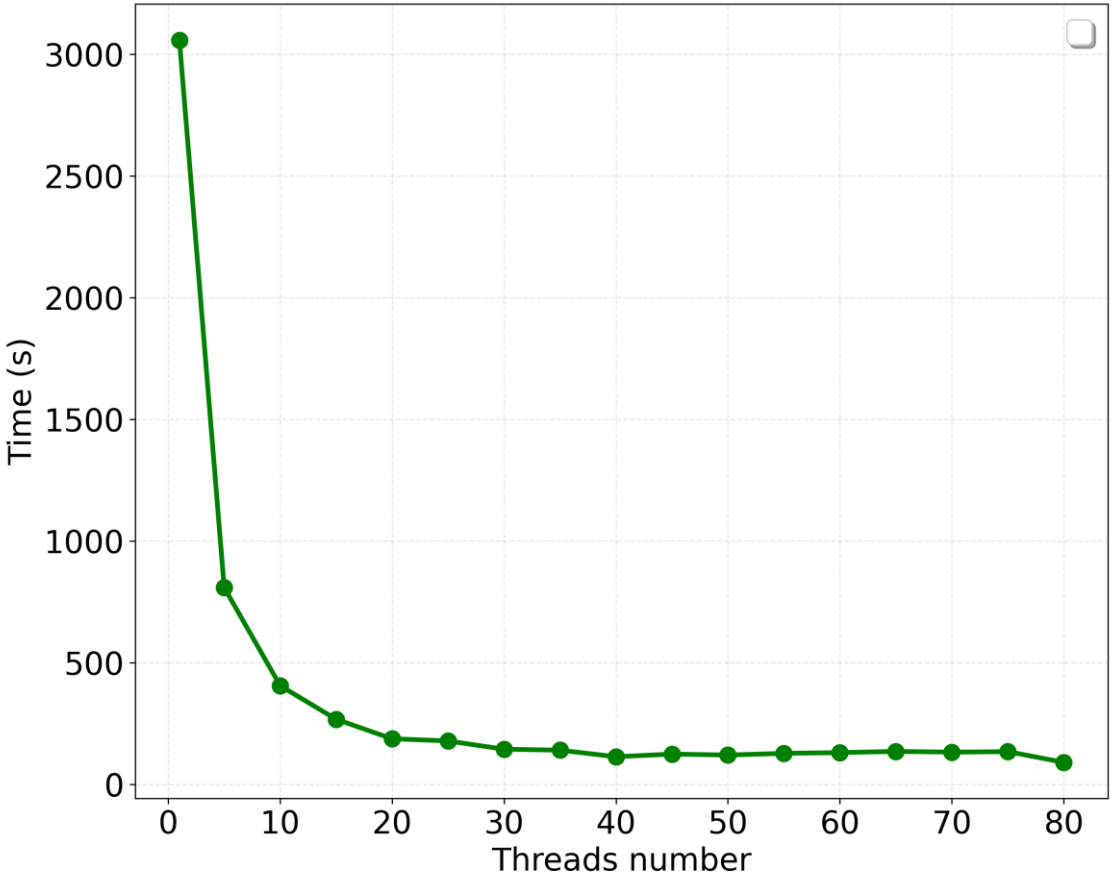
The computations were performed on the ML/DL/HPC component of the HybriLIT heterogeneous platform ecosystem with the following specifications: 2x Intel Xeon Gold 6148 (2.4 GHz, 20 cores/40 threads), 512 GB of DDR4 RAM.

CPU parallelization for multiprocessor computation was performed using the **numba.prange** function.

Dependence of the Shapiro step width on the amplitude and acceleration of calculations



Dependence of the Shapiro step width on the external radiation amplitude at a frequency of $\omega = 5$



Calculation time as a function of the number of threads. Acceleration achieved by **34 times**

Conclusion

- A toolkit for modeling the physical properties of a annular system of parallel Josephson junctions under the influence of an external electromagnetic radiation has been developed.

- Algorithms for calculating the current-voltage characteristic of the considered system and dependence of Shapiro step width on amplitude of external radiation have been developed and realized on the Python programming language.

- The software realization of the developed algorithms is performed using the Numba library function, including the parallelization mechanism when calculating the amplitude dependence of the Shapiro step width.

Welcome to HLIT Jupyter Book

Инструментарий для моделирования гибридных наноструктур сверхпроводник/магнетик

Задача 1. Инструментарий для моделирования ступеньки Шапиро на вольт-амперной характеристике джозефсоновского перехода

В работе рассмотрена задача о моделировании динамики джозефсоновского перехода под воздействием внешнего излучения. С использованием Python в среде Jupyter book разработаны алгоритмы для вычисления вольтамперной характеристики джозефсоновского перехода под воздействием внешнего излучения и вычисления зависимости ширины ступеньки Шапиро от амплитуды. Реализован параллельный алгоритм для рассматриваемой задачи с использованием библиотеки Joblib и проанализирована эффективность параллельных вычислений.

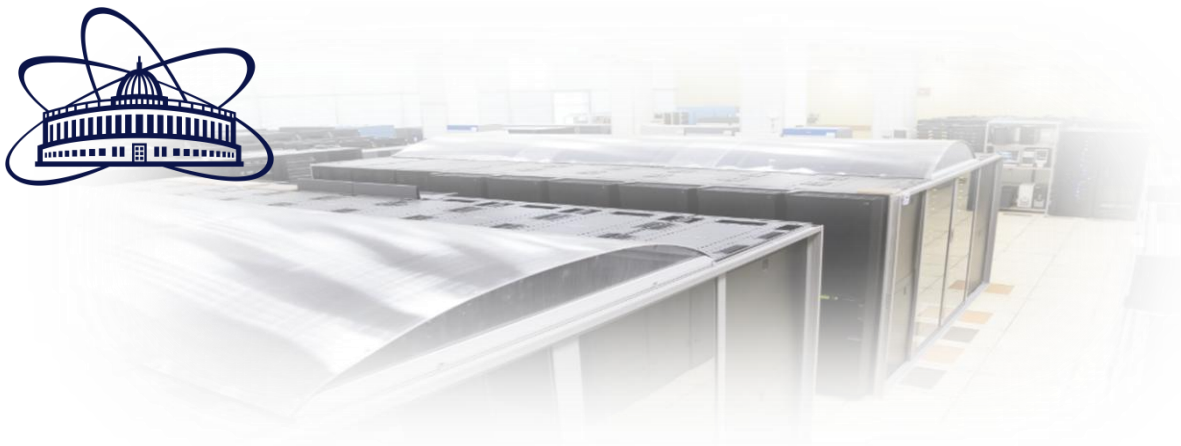
Работа поддержана грантом РНФ № 22-71-10022.

Проект *«Математическое моделирование сверхпроводящих наноструктур с магнетиком для исследования возможностей контроля намагниченности и магнитных возбуждений с использованием высокопроизводительных вычислительных систем»*

Toolkit for modeling superconductor/magnetic hybrid nanostructures

Task 1. Toolkit for modeling of the Shapiro step on the current-voltage characteristic of a Josephson junction

The problem of modeling of the Josephson junction dynamics under the influence of external radiation is considered. Using Python in the Jupyter book environment, algorithms of calculation of the Josephson junction current-voltage characteristic under the influence of external radiation and calculation of the amplitude dependence of the Shapiro step width have been developed. Using the Joblib library a parallel algorithm of considered problem was realized and the efficiency of parallel computing was analyzed.



A toolkit based on Python libraries and the Jupyter ecosystem for solving of scientific and applied problems

jupyter {book}

http://studhub.jinr.ru:8080/books

http://studhub.jinr.ru:8080/jjbook/intro.html