Relativistic description of tetraquarks with two heavy quarks

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Introduction

- "Ordinary" hadrons:
  - baryons qqq,
  - mesons $\mathrm{q\bar{q}}$.
- Exotic hadrons:
  - tetraquarks $\mathrm{qq\bar{q}\bar{q}}$,
  - pentaquarks $\mathrm{qqqq\bar{q}}$, etc.
- Searches for heavy tetraquarks are conducted on the Large Hadron Collider (LHC) by the LHCb, ATLAS and CMS Collaborations.
- First experimental data on doubly charmed tetraquarks are already available ($\mathrm{T_{cc}}(3875)^+$).

Model description I

- Quark content:
  - $\mathrm{Q}, \mathrm{Q}' = \mathrm{c}, \mathrm{b},\ \mathrm{Q} \neq \mathrm{Q}'$.
  - $\mathrm{s};\ \mathrm{q} = \mathrm{u}, \mathrm{d}$.
- Doubly heavy tetraquarks:
  - "Heavy+light":
    - Double strange tetraquarks:
      $\mathrm{QQ}\overline{\mathrm{ss}} \quad (+\mathrm{c.c.})$,
      $\mathrm{QQ}'\overline{\mathrm{ss}} \quad (+\mathrm{c.c.})$.
    - Single strange tetraquarks:
      $\mathrm{QQ}\overline{\mathrm{sq}} \quad (+\mathrm{c.c.})$,
      $\mathrm{QQ}'\overline{\mathrm{sq}} \quad (+\mathrm{c.c.})$.
    - Non–strange tetraquarks:
      $\mathrm{QQ}\overline{\mathrm{qq}} \quad (+\mathrm{c.c.})$,
      $\mathrm{QQ}'\overline{\mathrm{qq}} \quad (+\mathrm{c.c.})$.

Model description II

- "Heavy–light + heavy–light":
  - Hidden strange tetraquarks:
    $\mathrm{Qs}\overline{\mathrm{Q}}\overline{\mathrm{s}}$ $(+\mathrm{c.c.})$,
    $\mathrm{Qs}\overline{\mathrm{Q}}'\overline{\mathrm{s}}$ $(+\mathrm{c.c.})$.
  - Single strange tetraquarks:
    $\mathrm{Qs}\overline{\mathrm{Q}}\overline{\mathrm{q}}$ $(+\mathrm{c.c.})$,
    $\mathrm{Qs}\overline{\mathrm{Q}}'\overline{\mathrm{q}}$ $(+\mathrm{c.c.})$.
  - Non–strange tetraquarks:
    $\mathrm{Qq}\overline{\mathrm{Q}}\overline{\mathrm{q}}$ $(+\mathrm{c.c.})$,
    $\mathrm{Qq}\overline{\mathrm{Q}}'\overline{\mathrm{q}}$ $(+\mathrm{c.c.})$.

Model description III

- Doubly charmed tetraquarks:
  - double strange:

$$\mathrm{cc\bar{s}\bar{s}} \quad (+\mathrm{c.c.}),$$

  - strange:

$$\mathrm{cc\bar{s}\bar{q}} \quad (+\mathrm{c.c.}),$$

  - nonstrange:

$$\mathrm{cc\bar{q}\bar{q}} \quad (+\mathrm{c.c.}).$$

Model description IV

- Charmed bottom tetraquarks:
  - double strange:

$$\mathrm{cb\bar{s}\bar{s}} \quad (+\mathrm{c.c.}),$$

  - strange:

$$\mathrm{cb\bar{s}\bar{q}} \quad (+\mathrm{c.c.}),$$

  - nonstrange:

$$\mathrm{cb\bar{q}\bar{q}} \quad (+\mathrm{c.c.}).$$

Model description V

- Doubly bottom tetraquarks:
  - double strange:

$$\mathrm{bb}\overline{\mathrm{ss}} \quad (+\mathrm{c.c.}),$$

  - strange:

$$\mathrm{bb}\overline{\mathrm{sq}} \quad (+\mathrm{c.c.}),$$

  - nonstrange:

$$\mathrm{bb}\overline{\mathrm{qq}} \quad (+\mathrm{c.c.}).$$

Model description VI

- Diquark–antidiquark bound state:
  - doubly heavy:
    - "Heavy+light": $\{(\mathrm{Q}_1\mathrm{Q}_2)-(\overline{\mathrm{q}}_3\overline{\mathrm{q}}_4)\}$,
    - "Heavy–light + heavy–light": $\{(\mathrm{Q}_1\mathrm{q}_2)-(\overline{\mathrm{Q}}_3\overline{\mathrm{q}}_4)\}$.
- Diquarks under the consideration:
  - nonpoint–like (the internal structure is taken into account),
  - ground state $(1\mathrm{S})$,
  - color-antitriplet $(\overline{3}_c)$,
  - all masses and form factors of diquarks were calculated earlier during analyzing the properties of baryons.

Model description VII

- Ground state diquark spin:
  - $J = 0$ — scalar (S),
  - $J = 1$ — axialvector (A).
- Allowed diquark states:
  - only axialvector (A):
    - QQ,
    - ss.
  - both axialvector and scalar (A, S):
    - $\mathrm{QQ}^{\prime}$,
    - Qs,
    - Qq,
    - sq,
    - qq.

Model description VIII

- Tetraquark's possible configurations:
  - Doubly charmed tetraquarks:
    - $\mathrm{cc\bar{s}\bar{s}}$ $(+\mathrm{c.c.})$ — $\mathrm{A\bar{A}}$.
    - $\mathrm{cc\bar{s}\bar{q}}$ $(+\mathrm{c.c.})$ — $\mathrm{A\bar{A}}$, $\mathrm{A\bar{S}}$ $(\mathrm{S\bar{A}})$.
    - $\mathrm{cc\bar{q}\bar{q}}$ $(+\mathrm{c.c.})$ — $\mathrm{A\bar{A}}$, $\mathrm{A\bar{S}}$ $(\mathrm{S\bar{A}})$.
  - Charmed bottom tetraquarks:
    - $\mathrm{cb\bar{s}\bar{s}}$ $(+\mathrm{c.c.})$ — $\mathrm{A\bar{A}}$, $\mathrm{S\bar{A}}$.
    - $\mathrm{cb\bar{s}\bar{q}}$ $(+\mathrm{c.c.})$ — $\mathrm{A\bar{A}}$, $\mathrm{A\bar{S}}$, $\mathrm{S\bar{A}}$, $\mathrm{S\bar{S}}$.
    - $\mathrm{cb\bar{q}\bar{q}}$ $(+\mathrm{c.c.})$ — $\mathrm{A\bar{A}}$, $\mathrm{A\bar{S}}$, $\mathrm{S\bar{A}}$, $\mathrm{S\bar{S}}$.
  - Doubly bottom tetraquarks:
    - $\mathrm{bb\bar{s}\bar{s}}$ $(+\mathrm{c.c.})$ — $\mathrm{A\bar{A}}$.
    - $\mathrm{bb\bar{s}\bar{q}}$ $(+\mathrm{c.c.})$ — $\mathrm{A\bar{A}}$, $\mathrm{A\bar{S}}$ $(\mathrm{S\bar{A}})$.
    - $\mathrm{bb\bar{q}\bar{q}}$ $(+\mathrm{c.c.})$ — $\mathrm{A\bar{A}}$, $\mathrm{A\bar{S}}$ $(\mathrm{S\bar{A}})$.

Relativistic quark model I

- Relativistic Schrödinger-type quasipotential equation:

$$\left(\frac{b^2(M)}{2\mu_{\mathrm{R}}(M)} - \frac{\mathbf{p}^2}{2\mu_{\mathrm{R}}(M)}\right)\Psi_{\mathrm{d,T}}(\mathbf{p}) = \int \frac{d^3q}{(2\pi)^3}\, V(\mathbf{p},\mathbf{q};M)\Psi_{\mathrm{d,T}}(\mathbf{q})$$

$$\mu_{\mathrm{R}} = \frac{E_1 E_2}{E_1 + E_2} = \frac{M^4 - (m_1^2 - m_2^2)^2}{4M^3}$$

$$b^2(M) = \frac{[M^2 - (m_1 + m_2)^2][M^2 - (m_1 - m_2)^2]}{4M^2}$$

Relativistic quark model II

- All parameters of the model (including the constituent masses of quarks) are fixed from previous studies of the properties of mesons and baryons.
- Diquark–antidiquark interaction quasipotential:

$$V(\mathbf{p},\mathbf{q};M) = \frac{\langle d(\mathcal{P})|\, J_\mu \,|d(\mathcal{Q})\rangle}{2\sqrt{E_{\mathrm{d}}}\sqrt{E_{\mathrm{d}}}}\,\frac{4}{3}\alpha_{\mathrm{s}} D^{\mu\nu}(\mathbf{k})\,\frac{\langle d'(\mathcal{P}')|\, J_\nu \,|d'(\mathcal{Q}')\rangle}{2\sqrt{E_{\mathrm{d}'}}\sqrt{E_{\mathrm{d}'}}}$$

$$+\Psi^*_{\mathrm{d}}(\mathcal{P})\Psi^*_{\mathrm{d}'}(\mathcal{P}')[J_{\mathrm{d};\mu}J^{\mu}_{\mathrm{d}'}\,V^{\mathrm{vect.}}_{\mathrm{conf.}}(\mathbf{k}) + V^{\mathrm{scal.}}_{\mathrm{conf.}}(\mathbf{k})]\Psi_{\mathrm{d}}(\mathcal{Q})\Psi_{\mathrm{d}'}(\mathcal{Q}')$$

Results I

◇ Doubly charmed tetraquarks:

Table 1: Masses $M_{\mathrm{QQ}\overline{\mathrm{qq}}'}$ of the ground (1S) states and radial and orbital (1P, 2S, 1D, 2P, 3S) excitations of doubly charmed tetraquarks ($\mathrm{cc}\overline{\mathrm{ss}}$, $\mathrm{cc}\overline{\mathrm{sq}}$, $\mathrm{cc}\overline{\mathrm{qq}}$).

$\mathrm{d}\overline{\mathrm{d}}'$	nL	$\mathrm{n_r}$	L	S	$\mathrm{J^P}$	$\mathrm{M_{cc\overline{ss}}}$	$\mathrm{M_{cc\overline{sq}}}$	$\mathrm{M_{cc\overline{qq}}}$
$\mathrm{A}\overline{\mathrm{A}}$	1S	0	0	0	0^+	4308	4161	3983
				1	1^+	4330	4188	4017
				2	2^+	4368	4233	4074
	1P	0	1	1	0^-	4726	4603	4460
				0	1^-	4701	4574	4423
				1		4734	4613	4472
				2		4754	4636	4500
				1	2^-	4716	4591	4443
				2		4753	4635	4498
				2	3^-	4738	4617	4474
	2S	1	0	0	0^+	5013	4897	4760
				1	1^+	5015	4899	4764
				2	2^+	5016	4901	4768
	1D	0	2	2	0^+	5132	5029	4915
				1	1^+	5111	5004	4881
				2		5137	5035	4923
				0	2^+	5082	4968	4832
				1		5119	5013	4893
				2		5146	5047	4939
				1	3^+	5092	4980	4846
				2		5131	5028	4911
				2	4^+	5105	4995	4864
	2P	1	1	1	0^-	5367	5265	5148
				0	1^-	5325	5215	5085
				1		5366	5265	5149
				2		5394	5297	5188
				1	2^-	5332	5223	5095
				2		5377	5277	5163
				2	3^-	5343	5236	5111
	3S	2	0	0	0^+	5663	5567	5456
				1	1^+	5660	5565	5454
				2	2^+	5653	5556	5445

Results II

Table 1: table continues.

$\mathbf{d\bar{d}'}$	nL	$\mathbf{n_r}$	L	S	$\mathbf{J^P}$	$\mathbf{M_{cc\bar{s}\bar{s}}}$	$\mathbf{M_{cc\bar{s}\bar{q}}}$	$\mathbf{M_{cc\bar{q}\bar{q}}}$
$A\bar{S}$	1S	0	0	1	1^+		4093	3869
	1P	0	1		0^-		4493	4291
					1^-		4500	4299
					2^-		4514	4316
	2S	1	0		1^+		4801	4622
	1D	0	2		1^+		4900	4721
					2^+		4909	4733
					3^+		4923	4750
	2P	1	1		0^-		5162	5000
					1^-		5162	5000
					2^-		5162	5000
	3S	2	0		1^+		5479	5332

Results III

◇ Charmed bottom tetraquarks:

Table 2: Masses $M_{QQ'\overline{qq}'}$ of the ground (1S) states of charmed bottom tetraquarks ($cb\bar{s}\bar{s}$, $cb\bar{s}\bar{q}$, $cb\bar{q}\bar{q}$).

$d\bar{d}'$	nL	n_r	L	S	J^P	$M_{cb\bar{s}\bar{s}}$	$M_{cb\bar{s}\bar{q}}$	$M_{cb\bar{q}\bar{q}}$
$A\bar{A}$	1S	0	0	0	0^+	7634	7499	7337
				1	1^+	7645	7511	7353
				2	2^+	7665	7535	7381
$A\bar{S}$				1	1^+		7410	7198
$S\bar{A}$						7649	7516	7361
$S\bar{S}$				0	0^+		7403	7191

◇ Doubly bottom tetraquarks:

Table 3: Masses $M_{QQ\overline{qq}'}$ of the ground (1S) states of doubly bottom tetraquarks ($bb\bar{s}\bar{s}$, $bb\bar{s}\bar{q}$, $bb\bar{q}\bar{q}$).

$d\bar{d}'$	nL	n_r	L	S	J^P	$M_{bb\bar{s}\bar{s}}$	$M_{bb\bar{s}\bar{q}}$	$M_{bb\bar{q}\bar{q}}$
$A\bar{A}$	1S	0	0	0	0^+	10896	10763	10606
				1	1^+	10904	10773	10618
				2	2^+	10919	10790	10638
$A\bar{S}$				1	1^+		10670	10461

Analysis I

- If energetically possible, the tetraquark will fall-apart into a meson pair through the quark rearrangement.

$$\Delta = M_{\text{tetraquark}} - M^{\text{lowest}}_{\text{threshold}}$$

- If $\Delta < 0$, state is stable against fall-apart strong decays.
- The smaller $\Delta > 0$, the narrower is the state.

Analysis II

- Most states lie well above thresholds with $\Delta > 100$ MeV.
- Some states lie above thresholds with $50 < \Delta < 100$ MeV.
- Several states lie slightly above thresholds with $0 < \Delta < 50$ MeV.
- A number of states lie below thresholds with $\Delta < 0$.

◇ The most promising to be stable states:

- doubly charmed tetraquarks:

Table 4: Ground and radially and orbitally excited states of doubly charmed tetraquarks ($cc\bar{s}\bar{s}$, $cc\bar{s}\bar{q}$, $cc\bar{q}\bar{q}$), which lie slightly above or below the meson–meson fall-apart strong decay thresholds.

$QQ\bar{q}\bar{q}'$	$d\bar{d}'$	nL	S	J^P	M	$M_{thr.}$	Δ	meson pair
$cc\bar{s}\bar{s}$	$A\bar{A}$	1P	2	3^-	4738	4681	57	$D_s^{*\pm}\ D_{s2}^*(2573)$
$cc\bar{s}\bar{q}$	$A\bar{A}$	1P	2	3^-	4617	4573	44	$D_2^*(2460)\ D_s^{*\pm}$
	$A\bar{S}$	1P	1	2^-	4514	4429	85	$D_2^*(2460)\ D_s^{\pm}$
$cc\bar{q}\bar{q}$	$A\bar{A}$	1S	2	2^+	4074	4014 4017 4021	60 57 53	$D^*(2007)^0\ D^*(2007)^0$ $D^*(2007)^0\ D^*(2010)^{\pm}$ $D^*(2010)^{\pm}\ D^*(2010)^{\pm}$
		1P	2	3^-	4474	4468 4471	6 3	$D^*(2007)^0\ D_2^*(2460)$ $D^*(2010)^{\pm}\ D_2^*(2460)$
		1D	2	4^+	4864	4770	94	$D^*(2007)^0\ D_3^*(2750)$
	$A\bar{S}$	1S	1	1^+	3869	3875	-6	$D^0\ D^*(2010)^{\pm}$
		1P		0^-	4291	4208	83	$D^0\ D_0^*(2300)$
				1^-	4299	4282	17	$D^{\pm}\ D_1(2430)^0$
				2^-	4316	4326	-10	$D^0\ D_2^*(2460)$

Experimental data I

- In 2022 the LHCb Collaboration discovered doubly charmed tetraquark state $\mathrm{T_{cc}}(3875)^+$.
- Our calculations agree with these results:

Table 5: Experimentally observed doubly charmed tetraquark state and our candidate for its interpretation.

$\mathrm{T_{cc}}(3875)^+$	experiment (PDG)	our calculations
quark content	$\mathrm{cc\bar{u}\bar{d}}$	$\mathrm{cc\bar{q}\bar{q}}$
spin-parity	1^+	
mass, MeV	3874.74 ± 0.10	3869 ± 50

- New experimental data are expected in the near future, including regions and mass sectors of our interest.

Conclusion I

- Masses of ground and radially and orbitally excited states of doubly charmed tetraquarks were calculated.
- Masses of ground states of charmed bottom and doubly bottom tetraquarks were calculated.
- The finite size of a diquark was taken into account.
- Diquarks and antidiquarks were considered to interact as a whole.

Conclusion II

- Doubly charmed tetraquark states which are the most convenient for the experimental detection were identified.
- Our calculations agrees with the recent experimental results.

Publications

◇ Previous publications related to the topic:

- Masses of the $\mathrm{QQ\overline{Q}\overline{Q}}$ tetraquarks in the relativistic diquark–antidiquark picture, Physical Review D, 2020, vol. 102, №11, p. 114030;
- Heavy Tetraquarks in the Relativistic Quark Model, Universe, 2021, vol. 7, №4, p. 94;
- Fully Heavy Tetraquark Spectroscopy in the Relativistic Quark Model, Symmetry, 2022, vol. 14, №12, p. 2504;
- Relativistic description of asymmetric fully heavy tetraquarks in the diquark–antidiquark model, The European Physical Journal A, 2024, vol. 60, №96;
- Relativistic Description of Asymmetric Fully Heavy Tetraquarks, Physics of Particles and Nuclei Letters, 2024, vol. 21, №4, p. 597–600;
- Relativistic Description of Fully Heavy Tetraquark Spectroscopy, Moscow University Physics Bulletin, 2024, vol. 79, suppl. 1, p. S170–S173;
- Masses of Ground States of Triply Heavy Tetraquarks, Physics of Particles and Nuclei, 2025, vol. 56, №2, p. 330–334;
- Triply Heavy Tetraquark Spectroscopy, Physics of Atomic Nuclei, 2025, vol. 88, №1, p. 124–129;
- Relativistic Description of Triply Heavy Tetraquarks, Physics of Particles and Nuclei Letters, 2025, vol. 22, №5, p. 985–988;

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