

Lattice study of total momentum and free energy of rotating gluon plasma

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in collaboration with

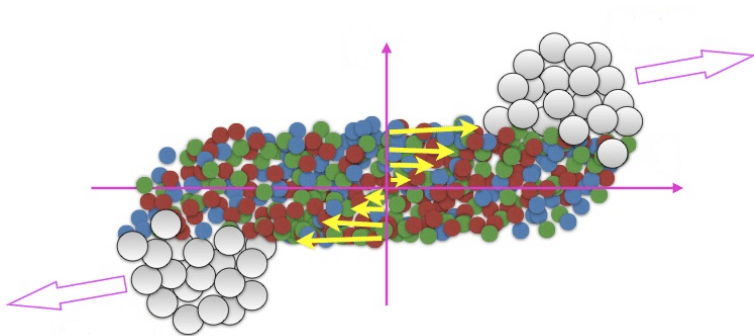
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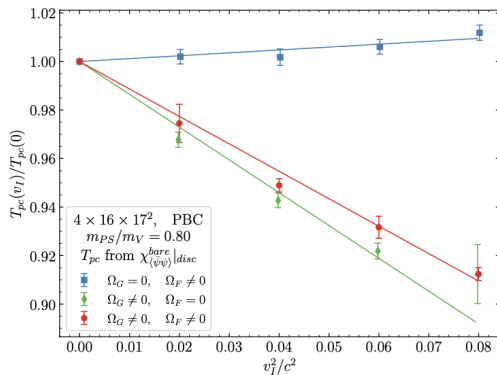
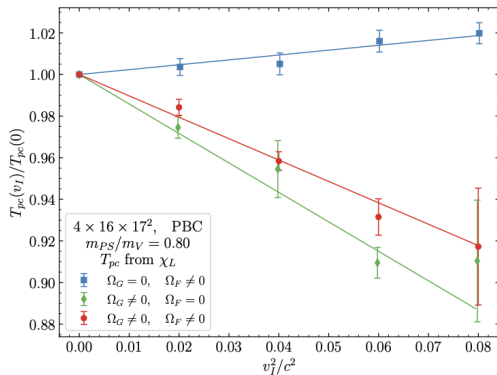
Motivation

- In non-central heavy ion-collisions droplets of QGP with angular momentum $\sim (10^3 - 10^5)\hbar$ might be produced.



Gluodynamics motivation

- The shift in pseudo-critical temperature of rotating QCD is dominated by the influence of rotation on gluons.

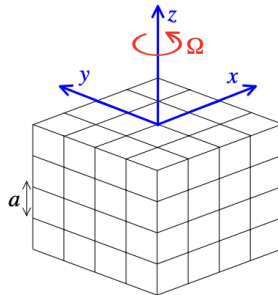


Gludynamics in Minkowski space

$$t = t_{\text{lab}}, \quad r = r_{\text{lab}}, \quad z = z_{\text{lab}}, \quad \varphi = [\varphi_{\text{lab}} - \Omega t]_{2\pi}$$

$$g_{\mu\nu} = \begin{pmatrix} 1 - r^2\Omega^2 & \Omega y & -\Omega x & 0 \\ \Omega y & -1 & 0 & 0 \\ -\Omega x & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

$$\mathcal{L}_G = -\frac{1}{4g_{\text{YM}}^2} g^{\mu\mu'} g^{\nu\nu'} F_{\mu'\nu'}^a F_{\mu\nu}^a$$



Gluodynamics in Euclidean space

After Wick rotation ($t = -i\tau$), action can be rewritten as follows:

$$\mathcal{L}_G = \mathcal{L}_0 - (i\Omega)\mathcal{L}_1 - \frac{\Omega^2}{2}\mathcal{L}_2$$

$$\mathcal{L}_0 = \frac{1}{4g_{\text{YM}}^2} F_{\mu\nu}^a F_{\mu\nu}^a,$$

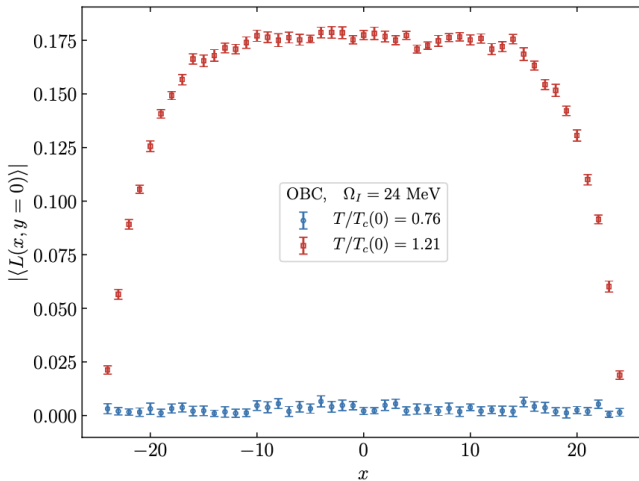
$$\mathcal{L}_1 = \frac{1}{g_{\text{YM}}^2} [-yF_{xy}^a F_{y\tau}^a - yF_{xz}^a F_{z\tau}^a + xF_{yx}^a F_{x\tau}^a + xF_{yz}^a F_{z\tau}^a]$$

$$\mathcal{L}_2 = \frac{1}{g_{\text{YM}}^2} [r^2(F_{xy}^a)^2 + x^2(F_{yz}^a)^2 + y^2(F_{xz}^a)^2 - 2xyF_{xz}^a F_{yz}^a]$$

To avoid the sign problem ($P \propto \exp(-S_E)$) for Monte-Carlo simulations, one can:

- Either perform analytical continuation $\Omega_I = -i\Omega$
- Or calculate expansion coefficients at $\Omega = 0$

Boundary conditions



Local thermalization approximation

For $SU(3)$ gluodynamics the phase transition is of the first order.

$$\xi \ll L_x, L_y, L_z$$

Then in the vicinity of the $(x, y) = (x_0, y_0)$ point:

$$\mathcal{L}_G \approx \mathcal{L}_G^{(loc)} \equiv \mathcal{L}_0^{(loc)} - (i\Omega)\mathcal{L}_1^{(loc)} - \frac{\Omega^2}{2}\mathcal{L}_2^{(loc)},$$

where

$$\begin{aligned}\mathcal{L}_0^{(loc)} &= \frac{1}{4g_{\text{YM}}^2} F_{\mu\nu}^a F_{\mu\nu}^a, \\ \mathcal{L}_1^{(loc)} &= \frac{1}{g_{\text{YM}}^2} \left[-y_0 F_{xy}^a F_{y\tau}^a - y_0 F_{xz}^a F_{z\tau}^a + x_0 F_{yx}^a F_{x\tau}^a + x_0 F_{yz}^a F_{z\tau}^a \right], \\ \mathcal{L}_2^{(loc)} &= \frac{1}{g_{\text{YM}}^2} \left[r_0^2 (F_{xy}^a)^2 + x_0^2 (F_{yz}^a)^2 + y_0^2 (F_{xz}^a)^2 - 2x_0 y_0 F_{xz}^a F_{yz}^a \right].\end{aligned}$$

Angular momentum density

$$J_{i,\text{lab}} = \frac{1}{2} \epsilon_{ijk} \int d^3 \bar{x} \left(\bar{x}^j T_{\text{lab}}^{0k} - \bar{x}^k T_{\text{lab}}^{0j} \right)$$

$$T_{\text{lab}}^{\mu\nu} = -\frac{1}{g_{YM}^2} \left(\bar{F}_{\mu\alpha}^a \bar{F}^{a,\alpha}_{\nu} - \frac{1}{4} g_{\mu\nu} \bar{F}_{\alpha\beta}^a \bar{F}^{a,\alpha\beta} \right)$$

$$J_{\text{lab}} \equiv J_{z,\text{lab}} = -\frac{1}{g_{YM}^2} \int d^3 \bar{x} \left(-\bar{x} \bar{F}_{yx}^a \bar{F}_{xt}^a - \bar{x} \bar{F}_{yz}^a \bar{F}_{zt}^a + \bar{y} \bar{F}_{xy}^a \bar{F}_{yt}^a + \bar{y} \bar{F}_{xz}^a \bar{F}_{zt}^a \right)$$

$$\bar{F}_{\mu\nu} = \frac{\partial x^\alpha}{\partial \bar{x}^\mu} \frac{\partial x^\beta}{\partial \bar{x}^\nu} F_{\alpha\beta}$$

$$J \equiv J_z = \int d^3 x (\mathcal{L}_1 + 2\Omega \mathcal{L}_2)$$

Angular momentum density

$$R = L_s/2 \quad \text{— characteristic size.}$$

$$J = \Omega \cdot I = \Omega \cdot V R^2 \left(i_2 + i_4 (\Omega R)^2 + i_6 (\Omega R)^4 + \dots \right)$$

$$\frac{j_I(T)}{\Omega_I R^2 T^4} = -\frac{i_2(T)}{T^4} + \frac{i_4(T)}{T^4} v_I^2 + \dots \quad (\text{where } v_I = \Omega_I R)$$

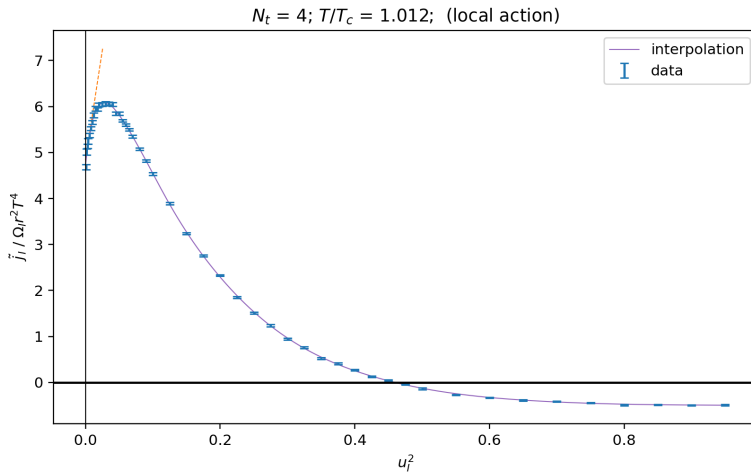
$$\frac{\tilde{j}_I(T)}{\Omega_I r^2 T^4} = -\frac{\tilde{i}_2(T)}{T^4} + \frac{\tilde{i}_4(T)}{T^4} u_I^2 + \dots \quad (\text{where } u_I = \Omega_I r)$$

$$j(T, R, \Omega) = \frac{J}{V} = \frac{1}{V} \int_V d^3x \tilde{j}(T, r, \Omega)$$

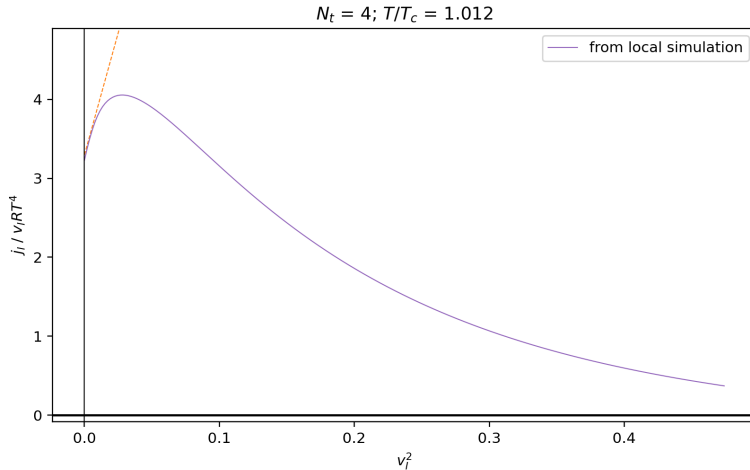
$$i_n = \tilde{i}_n \alpha_n$$

$$\alpha_n = \frac{1}{V R^n} \int_V d^3x r^n$$

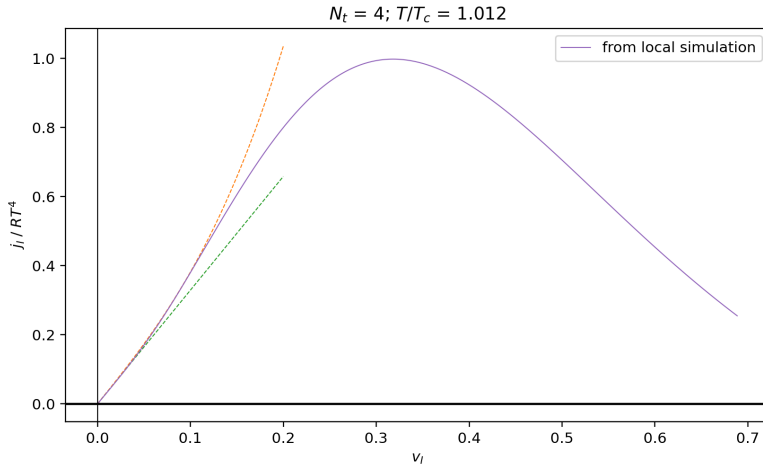
Local thermalization



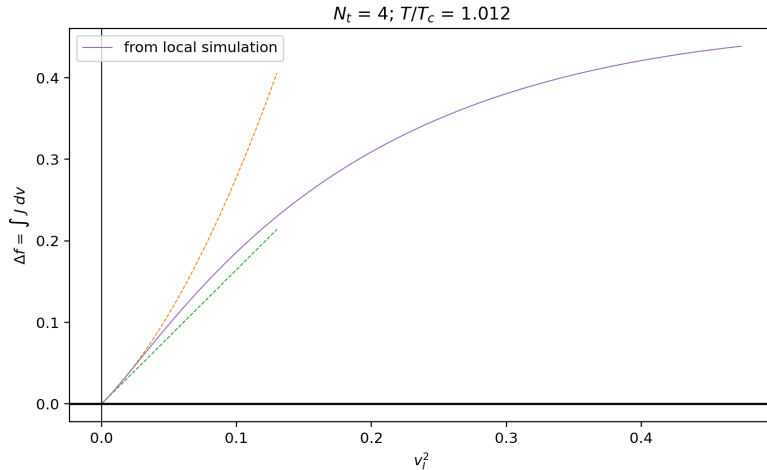
Local thermalization



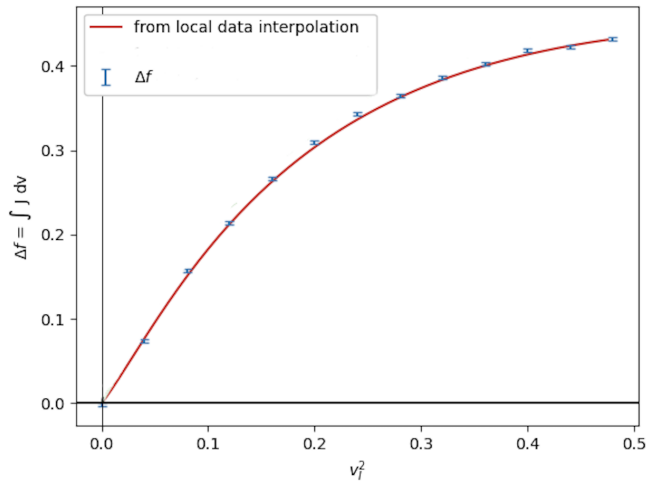
Local thermalization



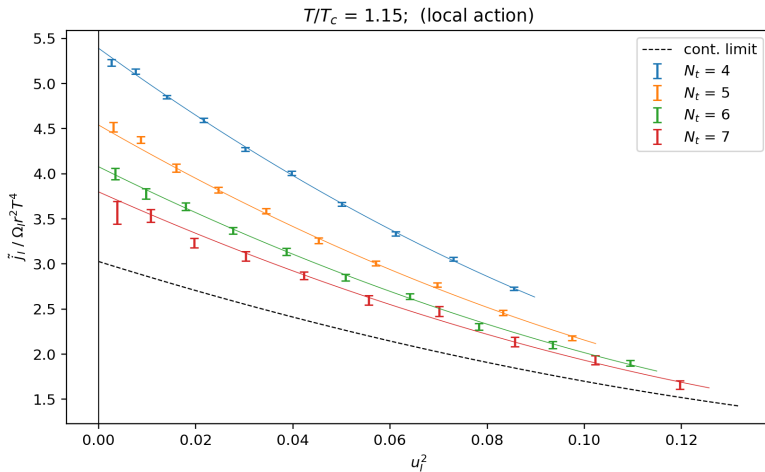
Local thermalization



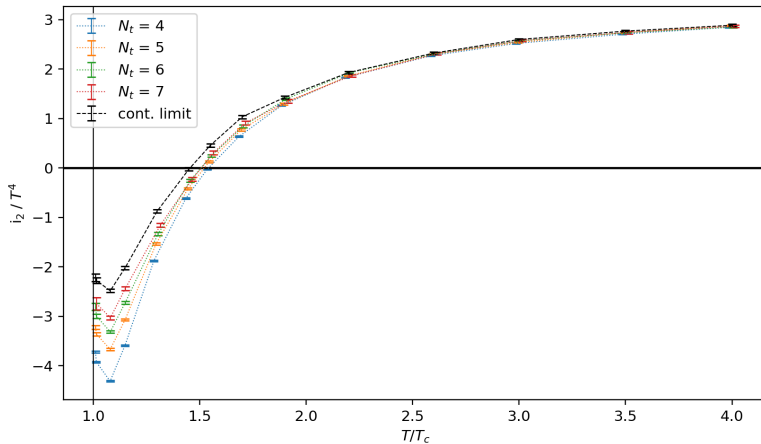
Local thermalization



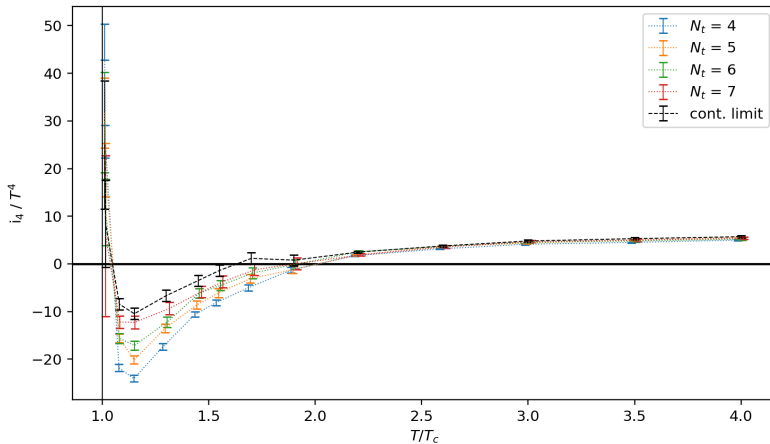
Local thermalization



Local thermalization



Local thermalization

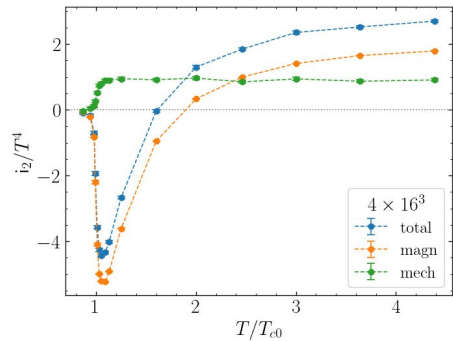


Magnetic gluon condensate

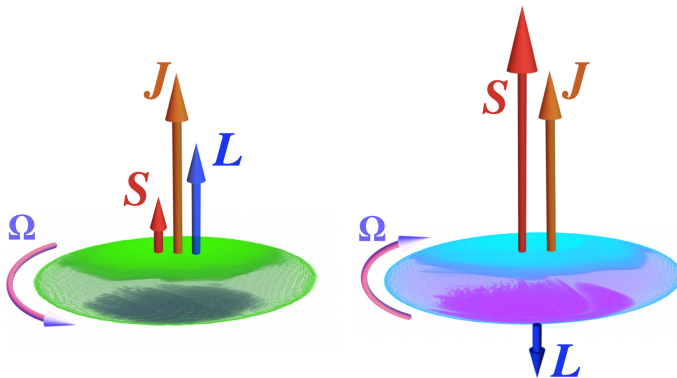
In previous works it was shown that one can split i_2 into the following parts:

$$i_2 = i_2^{\text{mech}} + i_2^{\text{magn}},$$
$$i_2^{\text{mech}} = \frac{1}{T}(\langle J^2 \rangle - \langle J \rangle^2) \geq 0,$$
$$i_2^{\text{magn}} = \frac{\alpha}{3} R^2 \langle (G_{\text{magn}})^2 \rangle.$$

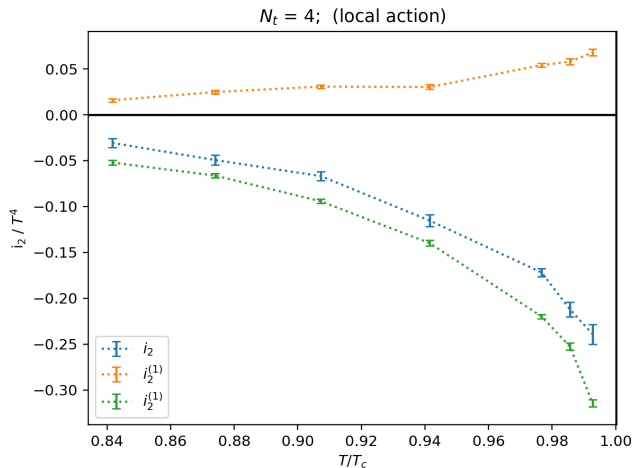
Then magnetic gluon condensate G_{magn} is solely responsible for negative sign of i_2 .



Negative Barnett effect



Negative Barnett effect at $T < T_c$



Decomposition of i_4

$$i_4 = i_4^{\text{mech}} + i_4^{\text{mix}} + i_4^{\text{magn}}$$

$$i_4^{\text{mech}} = \frac{T}{6VR^4} (\langle S_1^4 \rangle - 3\langle S_1^2 \rangle^2)$$

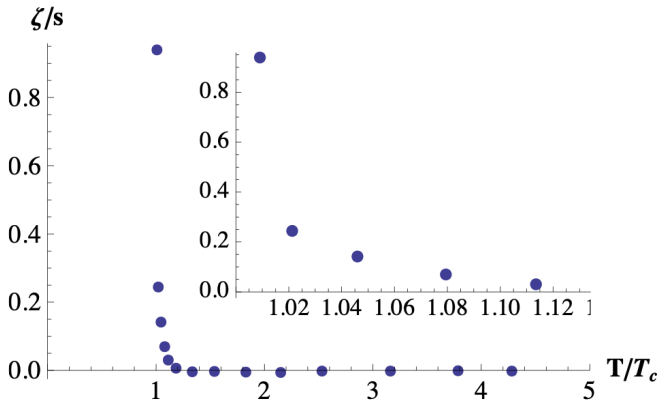
$$i_4^{\text{mix}} = \frac{T}{VR^4} (2\langle S_1^2 \rangle \langle S_2 \rangle - 2\langle S_2 S_1^2 \rangle)$$

$$i_4^{\text{magn}} = \frac{T}{VR^4} (2\langle S_2^2 \rangle - 2\langle S_2 \rangle^2)$$

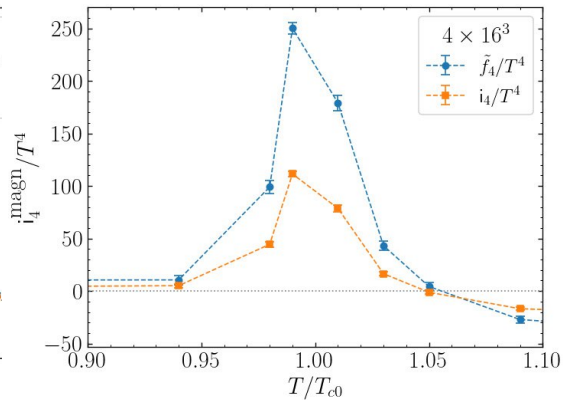
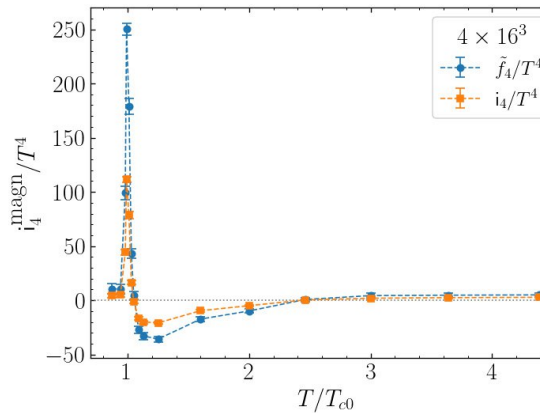
$$\tilde{f}_4^{\text{magn}} = -2\beta^2 \left. \frac{\partial \langle ((H_x^a)^2 + (H_z^a)^2) \rangle}{\partial \tilde{\beta}} \right|_{\tilde{\beta}=\beta}$$

$$\zeta = \frac{1}{9\omega_0} \left[- \left(\frac{\beta(g)}{4g} \right)^2 \frac{\partial \langle (E_i^a)^2 + (H_i^a)^2 \rangle}{\partial \beta} + 16|\epsilon_v| \right]$$

Bulk viscosity



Behaviour near T_c



Summary

- Local thermalization approximation holds for large enough volumes;
- The moment of inertia seems to be negative at $T < T_c$. The negative Barnett effect might occur;
- The i_4 coefficient seems to change sign at $T \sim 1.03(2) * T_c$ and $T \sim 1.70(12) * T_c$;
- The i_4^{magn} is related to the magnetic part of the bulk viscosity.

Summary

Thanks for attention!