Bound states in an effective quark model with a nonlocal interaction

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Motivation

- Understanding the complete hadronic spectrum, from light to heavy, is still an open challenge.
- Calculating hadron properties directly from QCD first principles is hindered by persistent technical and conceptual issues.
- Different theoretical methods are required for different meson flavors:
  - Light quark dynamics is constrained by chiral symmetry and QCD low-energy theorems.
  - Heavy quark dynamics is simpler, as their behavior is nearly classical.

Content

- Theoretical construction of the model
- Results

Fundamentals of the model with a separable interaction

Bethe–Salpeter equation in the ladder approximation

$$\Gamma_H(q,P) = -\frac{4}{3}\int \frac{d^4p}{(2\pi)^4} D(q-p)\{\gamma_\alpha S_1(p_1)\Gamma_H(p,P)S_2(p_2)\gamma_\alpha\}$$

with a vertex function

$$\Gamma_H(p,P) = N_H\,\varphi(p^2)\,\gamma_H\,,$$

and the interaction kernel in separable form

$$D(q-p) = D_0\varphi(q^2)\varphi(p^2)\,,$$

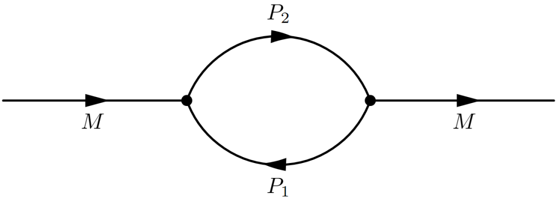
where 4-dimentional form-factor function in Gaussian form

$$\varphi(q^2) = e^{-q^2/\Lambda_H^2},$$

The meson-quark coupling constants N_H are determined by the normalization conditions

$$1 = N_c \frac{P_\mu}{2P^2} \frac{\partial}{\partial P_\mu} \int \frac{d^4 p}{(2\pi)^4} \mathrm{tr}\left\{\Gamma_H(p,P) S_1 \Gamma_H(q,P) S_2\right\},$$

and are defined by the polarization loop



Pseudoscalar mesons

The normalization condition:

$$\begin{aligned} 1 &= -i\frac{N_c N_{ps}^2}{2P^2} P^\mu \int \frac{dp}{(2\pi)^4}\varphi^2(p^2)\cdot \\ &\cdot\{b_1 \mathrm{tr}\,[i\gamma_5 S_1(p_1)\gamma_\mu S_1(p_1) i\gamma_5 S_2(p_2)] + \\ &+ b_2 \mathrm{tr}\,[S_1(p_1) i\gamma_5 S_2(p_2)\gamma_\mu S_2(p_2) i\gamma_5]\}. \end{aligned} \quad (1)$$

Polarization loop:

$$4 = \frac{4}{3}\int \frac{dp}{(2\pi)^4} D_0 \varphi^2(p^2)\, \mathrm{tr}\{\gamma_\alpha S_1(p_1) i\gamma_5 S_2(p_2)\gamma_\alpha i\gamma_5\} \quad (2)$$

For pseudoscalar mesons weak decay constant can be obtained from the matrix element of the axial current in integral form

$$P^\mu f_P = N_c N_P \int \frac{dp}{(2\pi)^4}\varphi(p^2)\mathrm{tr}\{(\mathrm{i}\gamma_5)\mathrm{S}_1(\gamma_\mu\gamma_5)\mathrm{S}_2\}. \quad (3)$$

Vector mesons

The normalization condition:

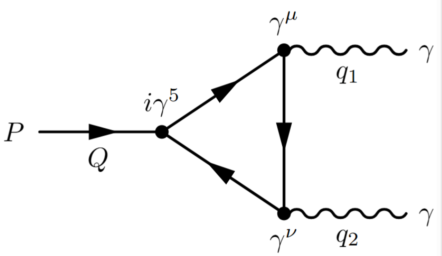
$$
\begin{aligned}
1 \quad = \quad & -i\frac{N_c N_{\rm v}^2}{6P^2} P^{\mu} \int \frac{dp}{(2\pi)^4} \varphi^2(p^2)\Delta_{\rho\sigma} \cdot \\
& \cdot \{ b_1 \mathrm{tr}\left[\gamma_\rho S_1(p_1)\gamma_\mu S_1(p_1)\gamma_\sigma S_2(p_2)\right] + \\
& + b_2 \mathrm{tr}\left[S_1(p_1)\gamma_\rho S_2(p_2)\gamma_\mu S_2(p_2) i\gamma_\sigma\right]\} \qquad (4)
\end{aligned}
$$

Polarization loop:

$$
12 = -\frac{4}{3}\int \frac{dp}{(2\pi)^4} D_0 \varphi^2(p^2)\Delta_{\mu\rho}\Delta_{\mu\nu}\,\mathrm{tr}\{\gamma_\rho\gamma_\alpha S_1(p_1)\gamma_\nu S_2(p_2)\gamma_\alpha\} \qquad (5)
$$

The vector leptonic decay can be written as

$$
g_{\rm v}\Delta_{\mu\nu} \quad = \quad \frac{N_c N_{\rm v}}{M_{\rm v}^2}\int \frac{dp}{(2\pi)^4}\varphi(p^2)\Delta_{\mu\rho}\mathrm{tr}\{\gamma_\rho \mathrm{S}_1 \gamma_\nu \mathrm{S}_2\} \qquad (6)
$$



The diagram describes the $P \to \gamma\gamma$ process, can be written as follows

$$T^{\mu\nu}(q_1, q_2) = N_c N_p \int \frac{dp}{(2\pi)^4} \varphi(p^2)\, \mathrm{tr}\{i\gamma_5 S(p_2)\gamma_\mu S(p_3)\gamma_\nu S(p_1)\}.$$

After taking the trace

$$
\begin{aligned}
T(q_1,q_2) &= i\epsilon_{\mu\nu\alpha\beta}\epsilon_1^{\mu}\epsilon_2^{\nu}q_1^{\alpha}q_2^{\beta}(N_c Q_q^2)\frac{mN_\pi}{4\pi^2}I(Q,q_1,q_2) = \\
&= i\epsilon_{\mu\nu\alpha\beta}\epsilon_1^{\mu}\epsilon_2^{\nu}q_1^{\alpha}q_2^{\beta}(N_c Q_q^2)G_{\pi\gamma\gamma}(Q,q_1,q_2),
\end{aligned}
$$

quark charge for pion $Q_q = (e_u^2 - e_d^2)$, $e_u = 2/3e$, $e_d = -1/3e$.
The two-photon decay coupling constant

$$
g_{\pi\gamma\gamma} = G_{\pi\gamma\gamma}(M_\pi^2,0,0) \simeq \frac{mN_\pi}{4\pi^2\Lambda_\pi^2}I(M_\pi^2,0,0),
$$

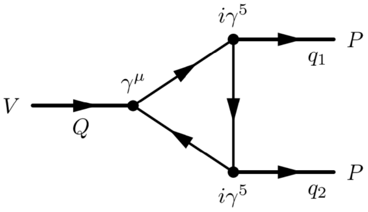
the decay width

$$
\boxed{\Gamma(\pi^0 \to \gamma\gamma) = \frac{M_\pi^3}{64\pi}(4\pi\alpha)^2 g_{\pi\gamma\gamma}^2} \tag{7}
$$

The transition form factor

$$
\boxed{F_{\pi\gamma}(Q^2) = e^2 G_{\pi\gamma\gamma}(M_\pi^2,0,q^2)} \tag{8}
$$

Hadronic decay of mesons



$$T^\mu(q_1,q_2) = N_c G \int \frac{dp}{(2\pi)^4} \mathrm{tr}\{\Gamma_{\rm v} S_i(p_2)\Gamma_{\rm ps} S_i(p_3)\Gamma_{\rm ps} S_i(p_1)\},$$

with $\Gamma_{\rm v} = \Gamma_\rho = \epsilon^\mu(P) N_\rho \varphi_\rho(p^2)\gamma_\mu$, $\Gamma_{\rm ps} = \Gamma_\pi = N_\pi \varphi_\pi(p^2)(i\gamma_5)$.
The amplitude in general form can be split in two terms:

$$T^\mu(q_1,q_2) = \epsilon^\mu(P)\{(q_1-q_2)^\mu f^+(t) + (q_1+q_2)^\mu f^-(t)\},$$

with $t = -(q_1-q_2)^2$, $f^-(t=M_\rho^2) = 0$, $\frac{1}{2} f^+(t = M_\rho^2) = g_{\rho\pi\pi}$

$$\boxed{\Gamma_{\rho\pi\pi} = \frac{1}{6\pi}\frac{k^3}{M_\rho^2} g^2_{\rho\pi\pi}} \tag{9}$$

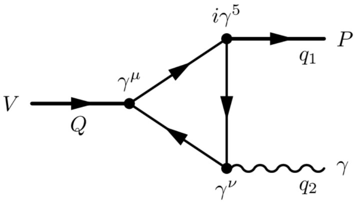


Diagram describe the $V \to P\gamma$ process and gives the matrix element as

$$T^{\mu\nu}(q_1, q_2) = i\epsilon_{\mu\nu\alpha\beta}\epsilon^\mu(P)\epsilon_2^\nu(q_2)q_1^\alpha q_2^\beta\,(N_c Qe)\,\frac{mN_\pi N_\rho}{4\pi^2\Lambda_\pi^2}I(P, q_1, q_2)$$

The decay width is defined as

$$\boxed{\Gamma_{\rho\pi\gamma} = \frac{1}{3}\alpha k^3 {g_{\rho\pi\gamma}}^2} \tag{10}$$

Such integrals appear in the polarization loop

$$I(P^2) \;=\; \int \frac{dp}{\pi^2} F(p^2) \frac{\{1, p^\mu, p^\mu p^\nu, p^\mu p^\nu p^\rho\}}{[p_1^2 + m_1^2]...[p_3^2 + m_3^2]},$$

with the help of the Feynman parametrization all integrals can be presented as

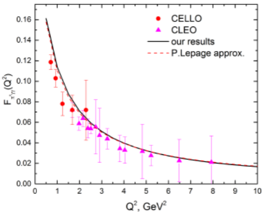
$$I(P^2) \;=\; \int_0^1 d\alpha \int_0^\infty dt \frac{t}{(1+t)^2} [F(z_0)],$$

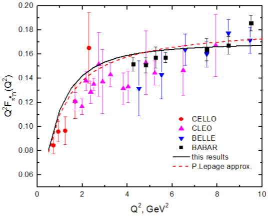
where $z_0 = tD + \frac{t}{1+t} R^2$, $D = \sum \alpha_i (q_i^2 + m_i^2) - R^2$, $R = \sum \alpha_i q_i$. Finaly all vector and scalar integrals are reduced to the form:

$$\boxed{I(a, b, m, n, F) = \int_0^1 \alpha^a (1-\alpha)^b d\alpha \int_0^\infty dt \frac{t^m}{(1+t)^n} [F(z_0)]}$$

Results

Form factor $F_{\pi\gamma}$ as function of the space-like photon momentum Q^2

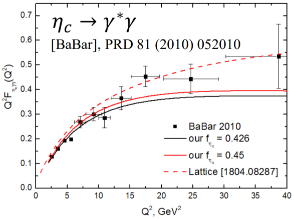


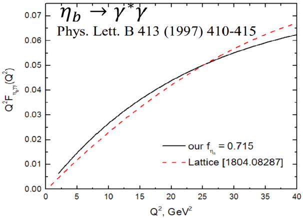


$m_\pi = 0.139$ GeV, $f_\pi = 0.131$ GeV, $m_\rho = 0.77$ GeV, $f_\rho = 0.2$ GeV.
Basic model parameters and constants for light mesons:

$m_{u(d)}$[GeV]	Λ_π[GeV]	N_π	Λ_ρ[GeV]	N_ρ	$\Gamma_{\pi\gamma\gamma}$[eV]	$\Gamma_{\rho\pi\pi}$[KeV]
0.223	1.14	3.724	0.75	3.612	7.76(7.82)	0.151(0.155)

$\Gamma_{\rho\pi\gamma} = 71.8$ KeV, $\Gamma^{(ref)}_{\rho\pi\gamma}$ =59.814 KeV
[arXiv:1009.1681[nuclth]] [arXiv:1704.05439 [hep-lat]]
J. Huston, D. Berg, C. Chandlee, S. Cihangir, T. Ferbel, T, Jensen, C.Nelson Phys. Rev. D, **33**, (1986) 3199–3202





Model parameters and constants for heavy pseudoscalar mesons $\eta_c(\eta_b)$:

	$\Lambda_H[GeV]$	N_H	$M_H[GeV]$	f_H(our)	f_H(refs)
η_c	2.775	3.546	2.985	0.426	0.42
η_b	2.81	8.568	9.39	0.715	0.705

The masses of quarks are fixed: $m_c = 1.6$[GeV], $m_b = 4.77$[GeV]

$\Gamma_{\eta_c\gamma\gamma} = 5.03(4.88 - 6.78)$[KeV], $\Gamma_{\eta_b\gamma\gamma} = 0.18(0.17)$[KeV]

[arXiv:hep-ph/9703364[hep-ph]] [arXiv:1804.08287[hep-ph]]

[arXiv:2305.06231[hep-lat]]

E. S. Ackleh, and T. Barnes, Phys. Rev. D, **45(1)** (1992) 232–240

Heavy vector mesons

Model parameters and constants for heavy vector mesons

	$\Lambda_H[GeV]$	N_H	$M_H[GeV]$	f_H(our)	f_h(refs)
J/ψ	2.03	3.238	3.075	0.435	0.339
Υ	4.22	3.489	9.405	0.705	

The radiative decay widths for heavy mesons

	our	refs
$J/\psi \to \eta_c \gamma$	2.28 KeV	1.83 – 2.49 KeV
$\Upsilon \to \eta_b \gamma$	25.3 eV	5.8 – 45 eV

[arXiv:hep-ph/0601137[hep-ph]]
[arXiv:0805.0252 [hep-ex]]
[arXiv:1208.2855[hep-lat]]
[arXiv:hep-ph/0210381[hep-ph]]
[arXiv:1804.08287 [hep-ph]]

Conclusion

- An approach for describing the properties of light and heavy mesons is developed within an effective quark model with nonlocal interactions. A set of model parameters is obtained for both light and heavy mesons, and the meson-quark couplings are derived in an explicit form.
- All parameters are fixed by observables, namely the meson masses and decay constants.
- The model is applied to calculate physical processes such as the two-photon decays of pseudoscalar mesons and the radiative decays of vector mesons. The results are consistent with experimental data and with the results from other models.

Thank you for your attention