Inflation in Scalar-Coupled BF Gravity

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Outline

Motivation

Emergent Gravity

- Gravity can emerge from gauge theories with symmetry breaking (from $SO(3,2)$ or a larger group to the Lorentz $\to SO(3,1)$ one)
- MacDowell-Mansouri (1977): Explicit symmetry breaking yields $R - \Lambda$
- Wilczek's improvement: Spontaneous symmetry breaking with scalar field
- **Problem**: These models lack an **inflaton field**

Cosmological Challenges

- ΛCDM model lacks inflationary source
- DESI data: Dark energy more complex than cosmological constant
- Need extended gravity theories with inflationary degrees of freedom

Our Approach

Key Idea

- Start with **topological** theory
- Introduce **constraints** to move on the mass shell of general relativity
- Apply the **disformal mixing** with a scalar field ϕ
- Generate **scalar-tensor gravity** with inflationary potential
- Obtain theories beyond $R - \Lambda$ models

Advantages

- Natural emergence of scalar field couplings
- Connection to string theory compactifications
- Generates wide class of inflationary models

BF Theory

Basic Setup

For principal G-bundle over n-dimensional manifold M:

- Connection 1-form $A \in \Omega^1(M, \mathfrak{g})$
- Curvature 2-form $F = dA + A \wedge A \in \Omega^2(M, \mathfrak{g})$
- Lagrange multiplier $(d-2)$-form $B \in \Omega^{d-2}(M, \mathfrak{g})$

BF Action

$$S_{BF} = \int_M \mathrm{tr}(B \wedge F) = \int_M B^{ab} \wedge F^{cd} \epsilon_{abcd} \tag{1}$$

Equations of Motion

$$d_A B = 0, \quad F = 0. \tag{2}$$

The theory is **purely topological** — it has no propagating degrees of freedom

Gravity as Constrained BF Theory

Adding Constraints

To obtain GR, impose constraints on B:

$$S = \int \mathrm{tr}(B \wedge F) + \frac{1}{2}\theta_{abcd} B^{ab} \wedge B^{cd} + \mu H(\theta) \tag{3}$$

where θ_{abcd} and μ are Lagrange multipliers.

Constraints Solution is

$$B = \alpha e \wedge e + \beta \star (e \wedge e) \tag{4}$$

where $e \in \Omega^1(M, \mathfrak{so}(3,1))$ is the tetrad field.

Resulting Action

$$S = \int e^a \wedge e^b \wedge R^{cd} \epsilon_{abcd} + \frac{1}{\gamma} \int e^a \wedge e^b \wedge R_{ab} \tag{5}$$

Einstein-Hilbert-Palatini action with Holst term

MacDowell-Mansouri Theory

Extended Gauge Group

- Reductive Cartan geometry $\mathfrak{g} = \mathfrak{so}(3,1) \oplus \mathbb{R}^{3,1}$ with connection $\mathcal{A} = (\omega, \frac{1}{\sqrt{l}} e)$

Curvature Decomposition

$$\mathcal{F} = (\hat{\mathcal{F}}, T), \qquad \hat{\mathcal{F}}^{ab} = R^{ab} + \frac{1}{l} e^a \wedge e^b, \quad T^a = \frac{1}{\sqrt{l}} d^\omega e^a \tag{6}$$

MacDowell-Mansouri Action

$$S_{MM} = \frac{1}{64\pi G} \int \mathrm{tr}(\hat{\mathcal{F}} \wedge \star \hat{\mathcal{F}}) \tag{7}$$

Expanding gives Einstein-Hilbert action with cosmological (Λ) and topological (GB) terms:

$$S_{MM} = \frac{1}{32\pi G} \int \left(R^{ab} \wedge e^c \wedge e^d - \Lambda e^a \wedge e^b \wedge e^c \wedge e^d + R^{ab} \wedge R^{cd} \right) \epsilon_{abcd} \tag{8}$$

BF Formulation for MacDowell-Mansouri

Deformed BF Action

$$S = \int B^{AB} \wedge \mathcal{F}_{AB} - \frac{\alpha}{4} B^{AB} \wedge B^{CD} \epsilon_{ABCDE} v^E \tag{9}$$

with fixed $SO(3,2)$ vector $v^I = (0,0,0,0,\alpha/2)$

Field Decomposition

Using $A = (a, 5)$ decomposition:

$$S = \int B^{ab} \wedge \hat{\mathcal{F}}_{ab} + B^{a5} \wedge T^a - \frac{\alpha}{4} B^{ab} \wedge B^{cd} \epsilon_{abcd} \tag{10}$$

Equations and Recovery

- Variation w.r.t B^{a5}: $T^a = 0$ (torsion-free condition)
- Variation w.r.t B^{ab}: $\hat{\mathcal{F}}_{ab} = \frac{\alpha}{2} \epsilon_{abcd} B^{cd}$
- Substitution yields original MacDowell-Mansouri action

Disformal Mixing: Basic Example

Key Idea

Modify **already constrained** (that is, on-shell) fields B and F:

- Make simplest choice: $B = e \wedge e$, $F = R$
- Introduce the scalar field ϕ and its kinetic term $X = -\frac{1}{2}(\nabla\phi)^2$
- Apply some **scalar field mixing** to B and F

Simple Disformal Mixing

$$\begin{aligned} B \mapsto \tilde{B} = B + f(\phi) \star F = e \wedge e + f(\phi) \star R \\ F \mapsto \tilde{F} = F + X \star B = R + X \star (e \wedge e) \end{aligned} \tag{11}$$

The Action Transforms

$$S = \int \mathrm{tr}\left(B \wedge F + C_a T^a\right) \to \int \mathrm{tr}\left(B_{\text{on-sh}} \wedge F_{\text{on-sh}}\right) \to \int \mathrm{tr}\left(\tilde{B} \wedge \tilde{F}\right) \tag{12}$$

Resulting Scalar-Tensor Theory

Transformed Action

$$S = \int \left[(e^a \wedge e^b + f(\phi) R^{ab}) \wedge (R^{cd} + X e^c \wedge e^d) \right] \epsilon_{abcd} \tag{13}$$

In Metric Form

$$S = \int d^4x \sqrt{-g} \left[R + f(\phi)\mathcal{G} - \frac{1}{2}(\nabla\phi)^2 - \frac{1}{2}(\nabla\phi)^2 f(\phi) R \right] \tag{14}$$

where $\mathcal{G} = R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}$ is Gauss-Bonnet term.

Features

- Scalar Gauss-Bonnet gravity
- Non-minimal kinetic coupling between scalar field and curvature
- Contains source for inflationary expansion

General Disformal Mixing

General Transformation

$$\hat{B} = \Psi_1(\phi, X)B + \Phi_1(\phi, X) \star F$$
$$\hat{F} = \Phi_2(\phi, X)F + \Psi_2(\phi, X) \star B \tag{15}$$

Transformed Action

$$S = \int \mathrm{tr}\left(\Phi_1\Phi_2 F \wedge \star F + \Psi_1\Phi_2 B \wedge F + \Phi_1\Psi_2 \star B \wedge \star F + \Psi_1\Psi_2 B \wedge \star B\right) \tag{16}$$

For Constrained BF Theory ($B = e \wedge e,\ F = R$)

$$S = \int d^4x\sqrt{-g}\left[\Phi_1\Phi_2\mathcal{G} + (\Phi_1\Psi_2 + \Psi_1\Phi_2)R + \Psi_1\Psi_2\right] \tag{17}$$

For Macdowell-Mansouri ($B = \star F$)

$$S = \int d^4x\sqrt{-g}[f(\phi, X)(R - \Lambda + \mathcal{G})] \tag{18}$$

Gauge Freedom

Rescaling Invariance

Action is invariant under ($k(\phi, X) \neq 0$):

$$\Phi_1 \mapsto k(\phi, X)\Phi_1, \quad \Psi_1 \mapsto k(\phi, X)\Psi_1$$
$$\Phi_2 \mapsto \frac{\Phi_2}{k(\phi, X)}, \quad \Psi_2 \mapsto \frac{\Psi_2}{k(\phi, X)} \tag{19}$$

Physical (Gauge-Invariant) Combinations

$$\xi = \Phi_1\Psi_2 + \Psi_1\Phi_2, \quad \eta = \Phi_1\Phi_2, \quad \zeta = \Psi_1\Psi_2 \tag{20}$$

General Scalar-Tensor Action

$$S_{ST} = \int d^4x\sqrt{-g}\,[\xi(\phi, X)R + \eta(\phi, X)\mathcal{G} + \zeta(\phi, X)] \tag{21}$$

Physical content is entirely encoded in invariant triplet $\{\xi, \eta, \zeta\}$

Some Examples

Brans-Dicke Theory

$$S_{BD} = \int d^4x\sqrt{-g}\left(\phi R + \frac{X}{\phi} - V(\phi)\right) \tag{22}$$

Disformal mixing functions:

$$\Psi_1 = 1, \quad \Psi_2 = \frac{X}{\phi} - V(\phi), \quad \Phi_1 = 0, \quad \Phi_2 = \phi \tag{23}$$

Nonminimal Gauss-Bonnet

$$S_{GB} = \int d^4x\sqrt{-g}\,(R - \xi(\phi)\mathcal{G} - \omega(\phi)X - V(\phi)) \tag{24}$$

Disformal Mixing Functions:

$$\begin{aligned} \Psi_1 &= -1, & \Psi_2 &= \omega(\phi)X + V(\phi) \\ \Phi_1 &= \frac{1 \pm \sqrt{1 - 4\xi(\phi)\Psi_2}}{\Psi_2}, & \Phi_2 &= -\frac{\xi(\phi)}{\Phi_1} \end{aligned} \tag{25}$$

Additional Generated Theories

Non-minimal Coupling

$$S = \int d^4x\sqrt{-g}\left(-\frac{M^2 + K(\phi)}{2}R + X - V(\phi)\right) \tag{26}$$

Mixing: $\Psi_1 = 1$, $\Psi_2 = X - V(\phi)$, $\Phi_1 = 0$, $\Phi_2 = -\frac{M^2+K(\phi)}{2}$

Dirac-Born-Infeld with Gauss-Bonnet term

$$S_{DBI} = \int d^4x\sqrt{-g}\left[R - (f(\phi))^{-1}\sqrt{1 - f(\phi)X} + \alpha(\phi)\mathcal{G}\right] \tag{27}$$

Mixing: $\Psi_1 = \frac{1\pm\sqrt{1-4\alpha(\phi)\zeta}}{2\alpha(\phi)}$, $\Psi_2 = 1 - \alpha(\phi)\Psi_1$, $\Phi_1 = 1$, $\Phi_2 = \alpha(\phi)$

Encompasses:

- Inflationary scenarios
- Cosmological bounce solutions
- Dark energy models

Ad Hoc? Effective Formulation!

General Effective Action

$$S = \frac{1}{3} \int \mathrm{tr} \left(\hat{B} \wedge \hat{F} - \lambda_i U^i(B, F, \hat{B}, \hat{F}) + \mu_i V^i(\phi, X, \lambda_i) \right) \tag{28}$$

With Coupling Potentials

$$\begin{aligned} U^1 &= \hat{B} \wedge F, \quad U^2 = \hat{B} \wedge \star B \\ U^3 &= B \wedge \hat{F}, \quad U^4 = \star F \wedge \hat{F} \end{aligned} \tag{29}$$

And the Scalar Field Coupling

$$V^i(\phi, X, \lambda_i) = \lambda_i - f_i(\phi, X) \tag{30}$$

The equations of motion automatically lead to the disformal mixing.

$$\begin{aligned} S_{\text{on-shell}} &= \int \mathrm{tr} \left(\hat{B} \wedge \hat{F} \right), \\ \hat{B} &= f_1(\phi, X) B + f_2(\phi, X) \star F, \\ \hat{F} &= f_3(\phi, X) F + f_4(\phi, X) \star B \end{aligned} \tag{31}$$

Connection to String Theory

Natural Scalar Fields from String Theory

- **Dilaton**: Couples via $e^{-\phi}\mathcal{G}$
- **Axions**: Linear coupling $\alpha(\phi) R \wedge R$ (Green-Schwarz terms)
- **k-essence**: Non-standard kinetic terms $\mathcal{L} = p(\phi, X)$
- **Moduli fields:** ???

Top-Down Derivation

- Functions $f_i(\phi, X)$ parametrize effective Lagrangian couplings, emerging from integrating out internal dimensions
- Expected: Specific compactification $\rightarrow$ Specific $f_i(\phi, X)$
- Phenomenology $\rightarrow$ Constraints on compactification

Summary of Results

Conclusions

Developed mechanism for inflation from topological BF theory

Introduced disformal mixing generating scalar-tensor gravity

Obtained wide class of inflationary models (Brans-Dicke, Gauss-Bonnet, DBI)

Physical Interpretation

Early Universe: Topological symmetry breaking $\to$ local degrees of freedom

Scalar field interactions $\to$ effective deformation

Result: Inflation is triggered, cosmological constant made dynamical

Future Directions

Study cosmological implications (bounce scenarios, dark energy)

Construct BF-theory/string correspondence

Generate Horndeski theory

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Thank you!

Questions?