Angular and enegry distributions of positron created in supercritical colissions of heavy nuclei

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Contents

Introduction

- The study investigates electron–positron pair creation in collisions of asymmetric nuclei U–Cm (Z=92, Z=96).
- Calculations are performed for various impact energies while keeping the minimal internuclear distance fixed.
- The total nuclear charge exceeds the critical value, causing the $1s$ level to dive below $-mc^2$ (into the negative-energy continuum).
- This leads to a yet unobserved spontaneous mechanism of electron–positron pair production.

Theoretical Background

- Atomic units ($\hbar = |e| = m_e = 1$) are used throughout the paper unless specified otherwise.
- Solving the Dirac equation for positron states in the field of colliding nuclei:

$$i\frac{\partial}{\partial t}\Psi(\mathrm{r}, t) = H\Psi(\mathrm{r}, t),$$

- where $\Psi(\mathrm{r}, t)$ is the four-component spinor wave function, and the Hamiltonian H has the form:

$$H = c(\boldsymbol{\alpha} \cdot \mathrm{p}) + c^2\beta + U_n(|\mathrm{r} - \mathrm{a}(t)|) + U_n(|\mathrm{r} + \mathrm{a}(t)|).$$

- U_n represents the nuclear potential of uranium and curium.
- the vector $\vec{a}(t)$ is directed along the internuclear axis with its length equal to one half of the internuclear distance

Methods

- The time-dependent Dirac equation is solved in two main steps:
  1. First, the stationary Dirac equation is solved using the Generalized Pseudospectral (GPS) method to obtain the initial bound state wave function.
  2. Then, the time evolution of the wave function is computed using the Crank–Nicolson algorithm:

$$\left(1+\frac{i\Delta t}{2}H\left(t+\frac{\Delta t}{2}\right)\right)\psi(t+\Delta t)=\left(1-\frac{i\Delta t}{2}H\left(t+\frac{\Delta t}{2}\right)\right)\psi(t)$$

The spectrum of emitted positrons is obtained by projecting the final positron wave packet onto relativistic plane waves. Momentum direction: (ϑ_k, ϕ_k), energy–momentum relation:

$$k = \frac{1}{c}\sqrt{E_k(E_k + 2c^2)}.$$

Differential probabilities for spin states $1/2$ and $-1/2$:

$$\frac{dP^{(i)}}{dE_k\, d\Omega} = \frac{1}{c}\sqrt{E_k(E_k + 2c^2)}(E_k + c^2)\left|\langle \Psi_{\mathrm{k}}^{(i)} \mid \Psi^{(c)} \rangle\right|^2, \quad i = 1, 2$$

$\Psi_k^{(i)}$ — unbound wave packet in the positive-energy continuum.
$\Psi^{(c)}$ — the final solution of the time-dependent Dirac equation, projected onto the positive-energy continuum.

Before the results

Below we show the energy–angle distributions of emitted positrons:

- These distributions were computed for different collision energy parameters. The following notation is used:

$$E = \varepsilon E_0, \quad \text{where } E_0 = 5.9\ \mathrm{MeV}$$

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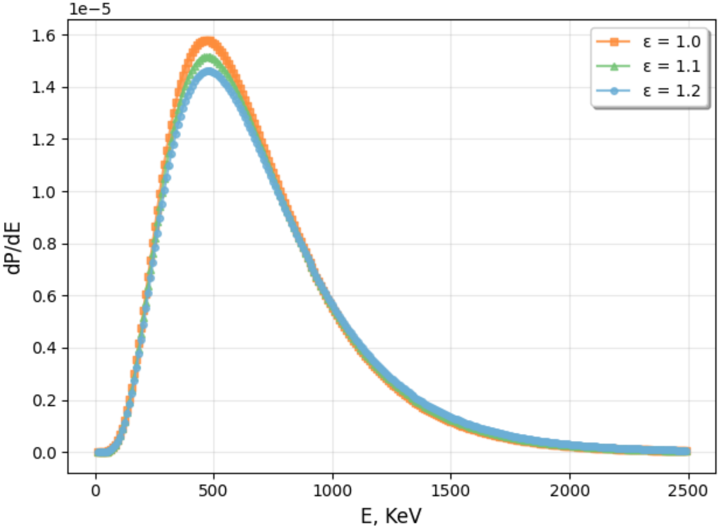
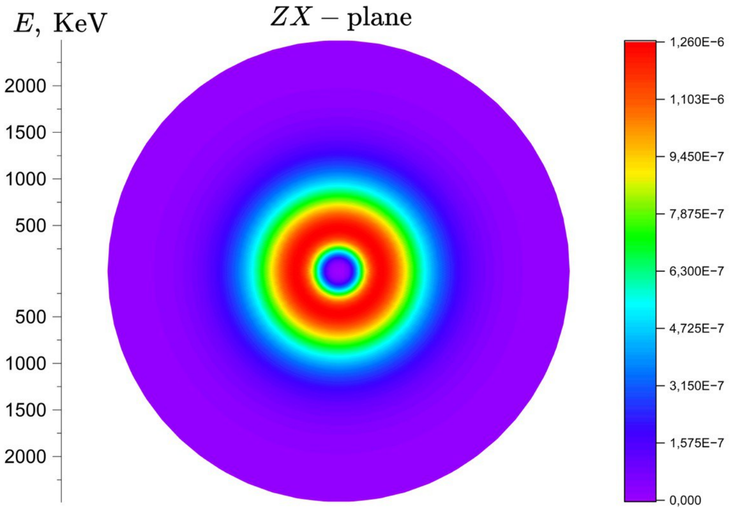
- The total probability of positron creation in a collision is given by the integral over the differential distribution.

Energy Distribution of the Emitted Positron



Energy-Angle Distribution of the Emitted Positron



Analysis

- The obtained distributions show that higher collision energies correspond to probability peaks at lower positron energies.
- This inverse ordering was predicted by our group within the nonperturbative framework, beyond the applicability of perturbative methods.
- Experimental observation of this effect would indicate a spontaneous pair–production mechanism.