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Hypersphercal Function Method for Calculating Binding Energies of Three-Body Systems

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This work presents a study of three-body quantum systems using the hyperspherical harmonics method combined with a finite-difference numerical method. The primary goal was to master the procedure for deriving, solving, and analyzing the coupled hyperradial Schrödinger equations using model coordinate potentials.

The hyperspherical harmonics method is chosen for its remarkable simplicity and versatility, particularly for systems involving a combination of Coulomb and short-range potentials. It offers a powerful framework that can be generalized to systems with a larger number of interacting particles. This approach is especially valuable when the exact form of the pairwise potentials is unknown, as it allows for the efficient determination of potential parameters that bind a system at the edge of its stability, thereby helping to interpret or correct ambiguous experimental data.

The study begins with a theoretical derivation of the system of coupled differential equations for three- and four-body systems in the hyperspherical basis. For numerical implementation, a finite-difference scheme was developed to discretize the hyperradial equations. The binding energies are then determined as the eigenvalues for which the determinant of the resulting block matrix equals zero, signifying the existence of a non-trivial bound-state solution.

The numerical code was rigorously validated through benchmark tests. First, it was applied to model systems of two and three coupled differential equations, with results showing excellent agreement with known analytical solutions (involving trigonometric and Bessel functions). Second, a physically meaningful test confirmed the correct reproduction of the dependence of the three-body binding energy on the binding energy of its two-body subsystem.

The method was then applied to calculate the binding energies of the triton (nnp) and helium-3 (npp) nuclei. A simple Gaussian potential was used to model the nucleon-nucleon interaction. For the npp system, the Coulomb repulsion between protons was included using a screened potential. The calculated binding energy for the triton is 8.6 MeV, which is in good agreement with the experimental value of 8.48 MeV. For helium-3, the energy is 8.7 MeV without the Coulomb potential and 7.7 MeV with it, closely matching the experimental value of 7.72 MeV. The 1.0 MeV difference highlights the significant and quantitatively correct contribution of the Coulomb force, consistent with findings in the existing literature.

The convergence of the solution with respect to the number of included hyperspherical harmonics (K) and the density of the coordinate grid was also analyzed. It was shown that higher precision for systems with larger K requires a finer computational grid.

In conclusion, the hyperspherical harmonics method, coupled with a robust finite-difference method, is confirmed as an effective, accurate, and computationally accessible tool for studying bound states in few-body quantum systems. The developed framework provides a solid foundation for future theoretical studies aimed at complementing and refining experimental data on the properties of light nuclei and cross-sections of nuclear reactions.

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