Momentum spread measurement method for the space charge-dominated ion beams

29th International Scientific Conference of Young Scientists and Specialists (AYSS-2025)

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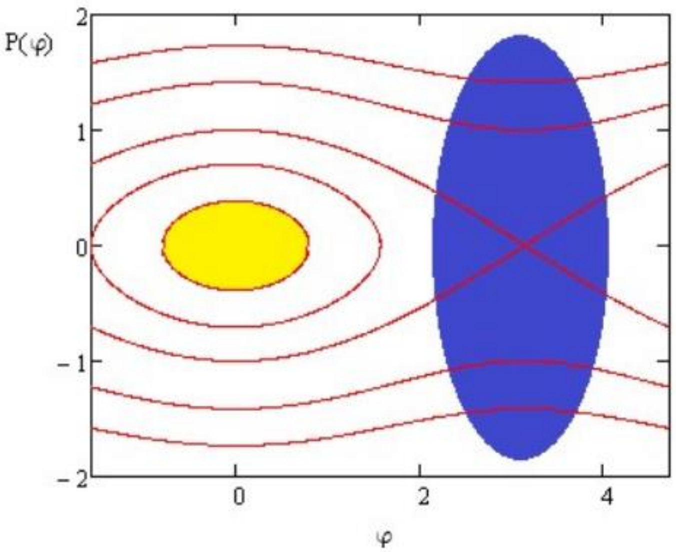
Why do we need to know momentum spread?

- **Intensity** of beam is **crucial** for colliders at each stage;
- NICA Booster synchrotron [1]:
  - accumulates ions (up to 10x);
  - accelerates beam (3.2 MeV/n → 578 MeV/n);
  - extracts beam to Nuclotron;



Scheme of repeated single-lap injection [2]

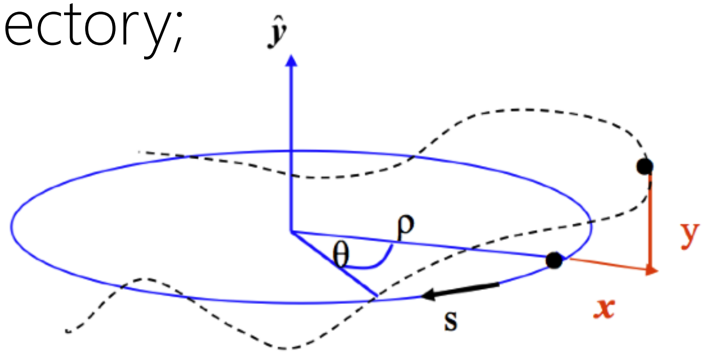
- Beam fills in available phase space;
- **No accumulation** using only magnetic optics – we need **dissipative force**;
- Cooling exploits such forces, **reduces volume** of beam **in phase space**;

- Injection → Cooling→Phase space reduced →Repeat;
- Low momentum spread – enough space for new injection

[1] NICA Booster: a new-generation superconducting synchrotron. DOI: 10.3367/UFNe.2021.12.039138
[2] Acceleration in Booster and Nuclotron, V. Lebedev, September, 2023

Particle distribution evolution

- Coordinate frame moves along ideal synchronous particle trajectory;
- Longitudinal coordinate s w.r.t. synchronous particle
  OR phase ϕ w.r.t. synchronous particle[3]



- Particle distribution function $f = f(s, v, t) \rightarrow f(\phi, \delta, t)$;
- $\delta = \frac{\Delta p}{p}$ is relative momentum spread of particle;
- σ_δ is RMS momentum spread;
- Fokker-Planck equation shows evolution of particle distribution function [4]
- **Haissinski equation is its stationary solution[4]**

linear density →

$$f(\phi) = f_0 \cdot \exp\left(-\frac{U(\phi)}{kT_\parallel}\right) = f_0 \cdot \exp\left(-\frac{U_{RF}(\phi)+U_{SC}(\phi)}{kT_\parallel}\right)$$

f_0 is normalization factor ($\int_{-\pi}^{\pi} f(\phi)\, d\phi = 1$); $kT_\parallel$ is longitudinal beam temperature (we determine it in terms of σ_δ); $U(\phi)$ is the potential part of longitudinal beam Hamiltonian

[3] Beam Dynamics and Beam Losses - https://cds.cern.ch/record/2206741/plots
[4] Haïssinski J. Exact longitudinal equilibrium distribution of stored electrons in the presence of self-fields // Il Nuovo Cimento B. – 1973. – Vol. 18. – № 1. – P. 72-82

Longitudinal beam Hamiltonian with space charge

$$H(\delta,\phi) = \frac{1}{2}h\omega_0\eta\delta^2 + \frac{ZeV_{RF}\omega_0}{2\pi\beta^2 E}(1-\cos\phi) + U_{SC}(\phi)$$

Kinetic term — Potential term, RF bunching — Potential term, space charge

$$U_{SC}(\phi) = \frac{Ze \cdot h^2\omega_0 N_0}{V_{RF}} \cdot \left(\frac{Z_\parallel}{n}\right) \cdot f(\phi)$$

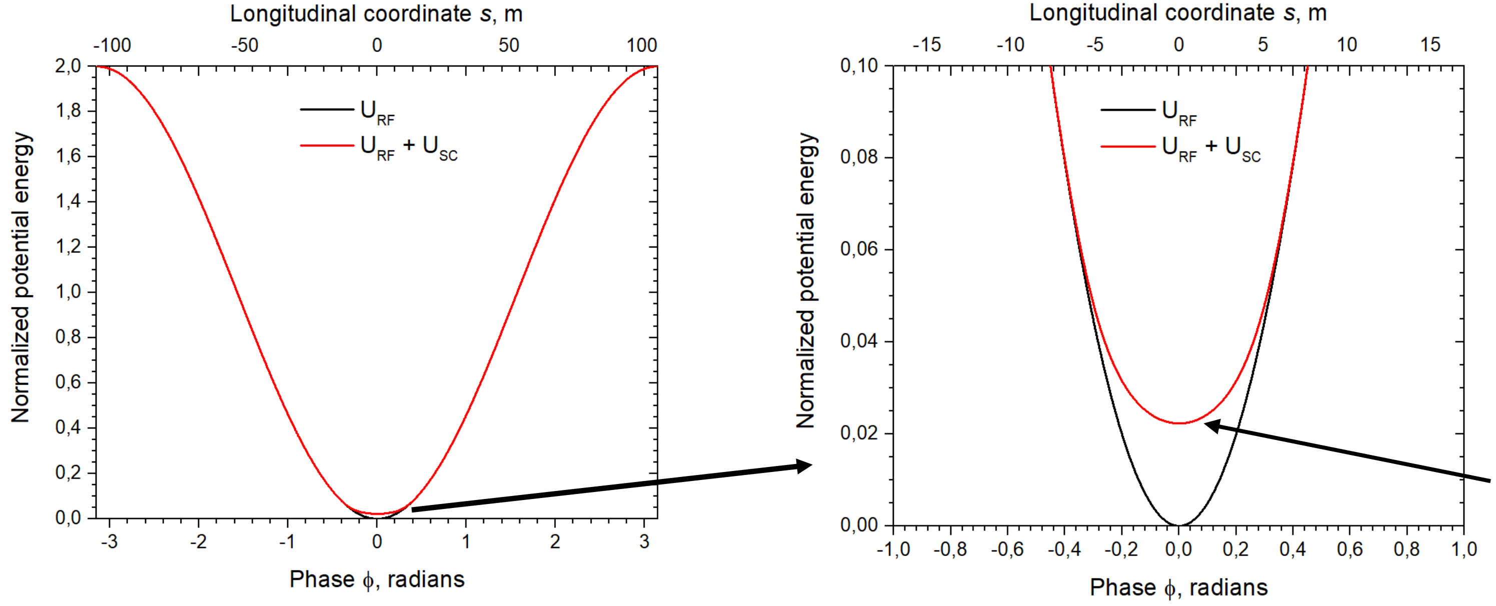
$$\phi = \frac{2\pi hs}{C_a}$$

Now we construct normalized Hamiltonian:

$$H(\delta,\phi) = \frac{1}{2}\zeta^2 + (1-\cos\phi) + \kappa \cdot f(\phi)$$

$\kappa = \frac{Ze \cdot h^2\omega_0 N_0}{V_{RF}} \cdot \left(\frac{Z_\parallel}{n}\right)$ determines space charge

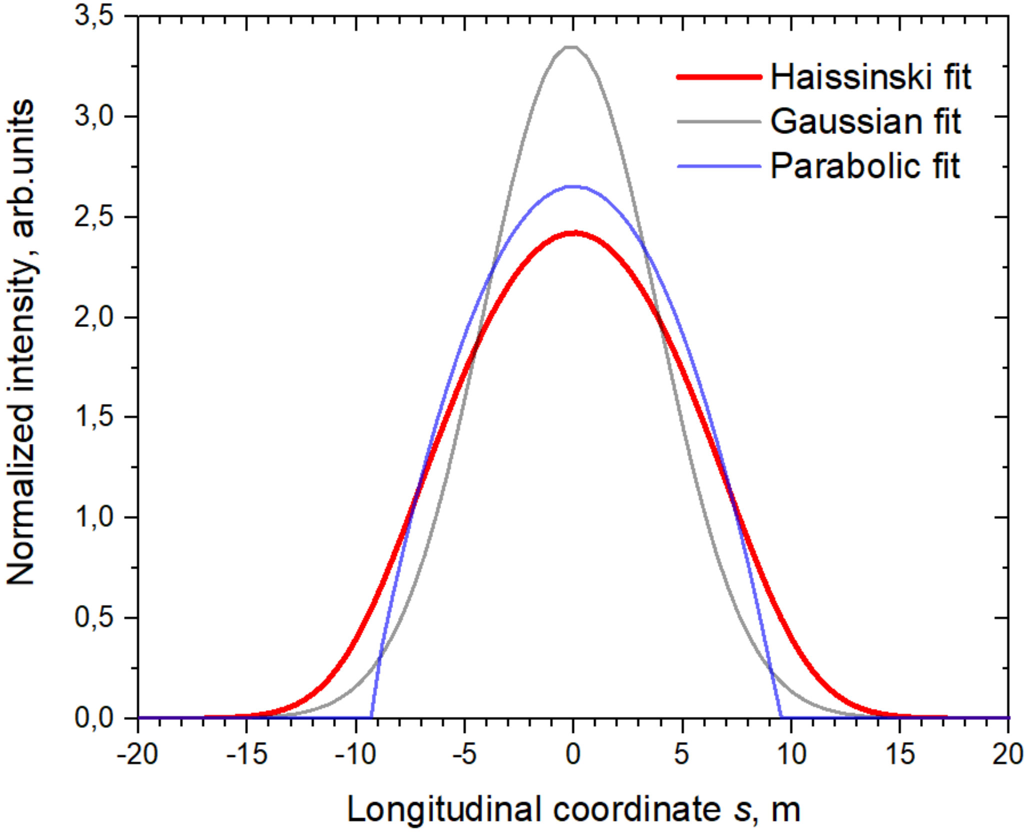
$\zeta = \frac{h\omega_0\eta\delta}{\omega_s} = \frac{h\eta\delta}{\nu_s}$ is conjugate momentum



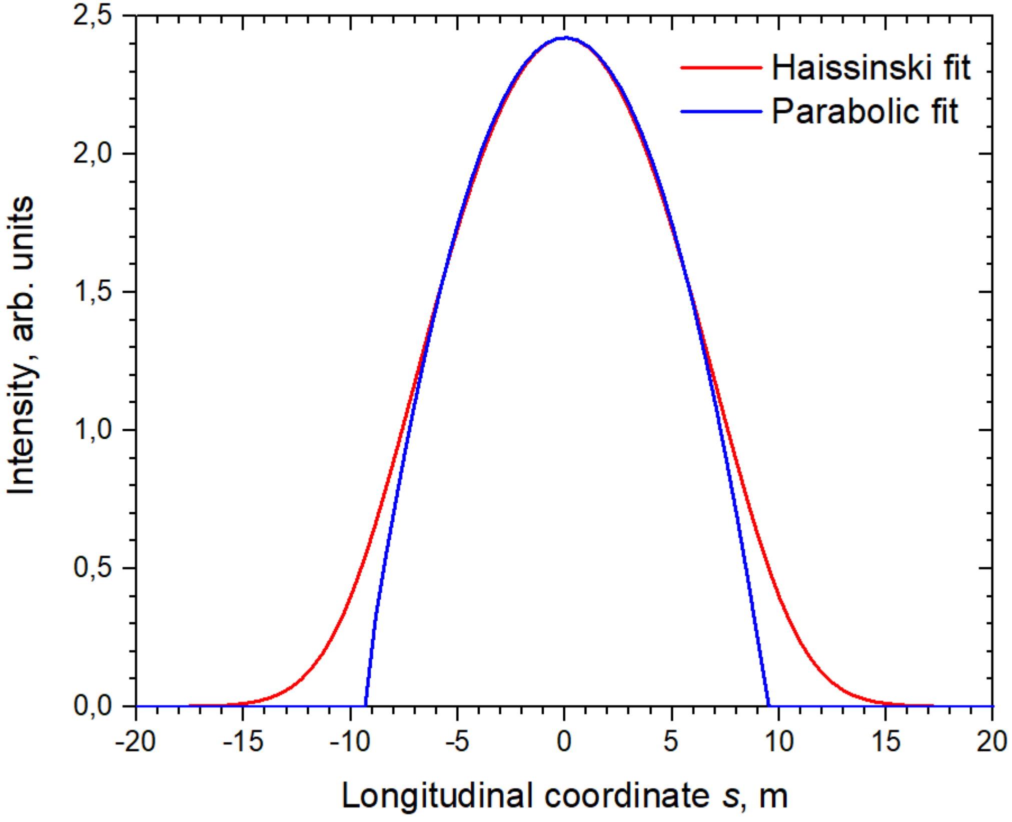
$Z = 26$, e is elementary charge, $\omega_0 = 2\pi f_0$ is circulation frequency, N_0 is total number of particles, V_{RF} is RF voltage amplitude, $h = 1$ is RF harmonic, $\eta = 0.94$ is slip factor, ω_s is synchrotron oscillations frequency, $\frac{Z_\parallel}{n}$ is effective impedance

Space charge results in potential well **flattening**

Equilibrium bunch distributions



Haissinski fit:
$\sigma_\delta = 5.6 \cdot 10^{-5}$;
$Z_\parallel/n = 4560\ \Omega$
Gaussian fit:
$\sigma_\delta = 5.6 \cdot 10^{-5}$;
$Z_\parallel/n = 0$;
Parabolic fit:
$\sigma_\delta = 0$;
$Z_\parallel/n = 4560\ \Omega$;



Distorted potential well → bunch shape changes.

Zero space charge (not distorted well): **Gaussian**, $\sigma_\delta = 2\pi \frac{\sigma_l}{C_a}\sqrt{\frac{ZeV_{RF}h}{2\pi\beta^2 E_s \cdot \eta}}$ [5] (σ_l is bunch length, m) – grey line;

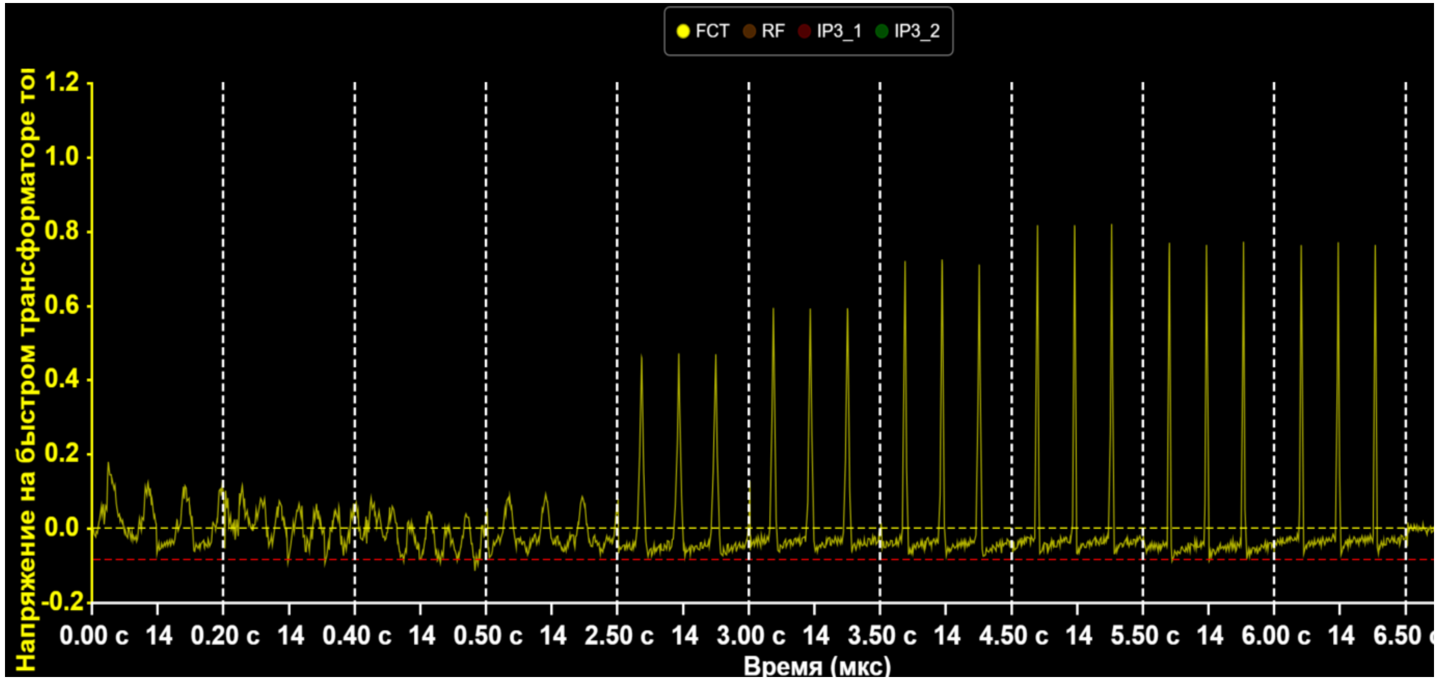
Cooling ON: distribution tends to **parabolic with Gaussian tails**, $\sigma_\delta \downarrow$, impedance $Z_\parallel/n \uparrow$ - red line;

Limiting case: $\sigma_\delta = 0,\ Z_\parallel/n \neq 0$ - pure **parabola** - blue line.

[5] M.Bryzgunov et al. First Experiments on Electron Cooling of Ion Beam in the NICA Booster. doi: 10.1134/S1547477124700122

Longitudinal beam distribution diagnostics

Beam fast current transformer (FCT)
manufactured by Bergoz [6]



FCT signal, e-cooling is on

Distribution over time → over longitudinal coordinate → over phase

$$f(t) \rightarrow f(s) = \frac{\omega_0 f(t)}{R} \rightarrow f(\phi) = \omega_0 f(t)$$

$R = C_a/2\pi$

Approximate FCT data with Haissinski equation:
vary $\left(\frac{Z_\parallel}{n}\right)$ and $\boldsymbol{\sigma_\delta}$ to get best fit. Goal: find $\boldsymbol{\sigma_\delta}$ of beam.

[6] Bergoz FCT: Fast Non-Destructive pulsed or CW beams observation https://www.bergoz.com/products/fct/

Accelerator parameters

NICA accelerator complex Booster synchrotron
Parameters in 29.06.2025

- Circumference $C_a = 210.96$ m;
- Circulating ions $^{124}Xe^{26+}$ (Target ion charge number $Z = 26$);
- Injection energy 3.2 MeV/nucleon → $\beta = 0.0826; \gamma = 1.003$;
- Beam circulation frequency at injection energy $\frac{\omega_0}{2\pi} = 117550$ Hz;
- RF amplitude $V_{RF} = 40$ V, harmonic $h = 1$;
- Slip factor $|\eta| = 0.94$;
- Measured synchrotron oscillations frequency $\frac{\omega_s}{2\pi} = 51$ Hz is the same as calculated for given V_{RF};
- Number of particles in bunch is calculated via integration of FCT signal with gain coefficient;
- Electron cooling was turned on with electron beam current $I_e = 15$ mA;
- Electron gun cathode voltage $U_c = 1773$ V;
- Longitudinal cooling time is $\tau_{cool} \approx 70$ ms.

Haissinski equation construction algorithm

Haissinski equation is **transcendent**.

f_0 from normalization: $\int_{-\pi}^{\pi} f(\phi)d\phi = 1$.

$$f(\phi) = f_0 \cdot \exp\left(-\frac{(1-\cos\phi)+\frac{Ze\cdot h^2\omega_0 N_0}{V_{RF}}\cdot\left(\frac{Z_\parallel}{n}\right)\cdot f(\phi)}{(\sigma_\delta h|\eta|/\nu_s)^2}\right)$$

$\phi = \frac{2\pi hs}{C_a}$;

$\nu_s = \frac{\omega_s}{\omega_0}$;

Use method of iterations. Initial guess is Gaussian with σ_δ from bunch length:

$$f^0(\phi) = f_0 \cdot \exp\left(-\frac{(1-\cos\phi)}{(\sigma_\delta h|\eta|/\nu_s)^2}\right)$$

Each iteration adds to function the following exponential term:

$$f^a(\phi) = \exp\left(-\frac{(1-\cos\phi)+\frac{Ze\cdot h^2\omega_0 N_0}{V_{RF}}\cdot\left(\frac{Z_\parallel}{n}\right)\cdot f(\phi)}{(\sigma_\delta h|\eta|/\nu_s)^2}\right),$$

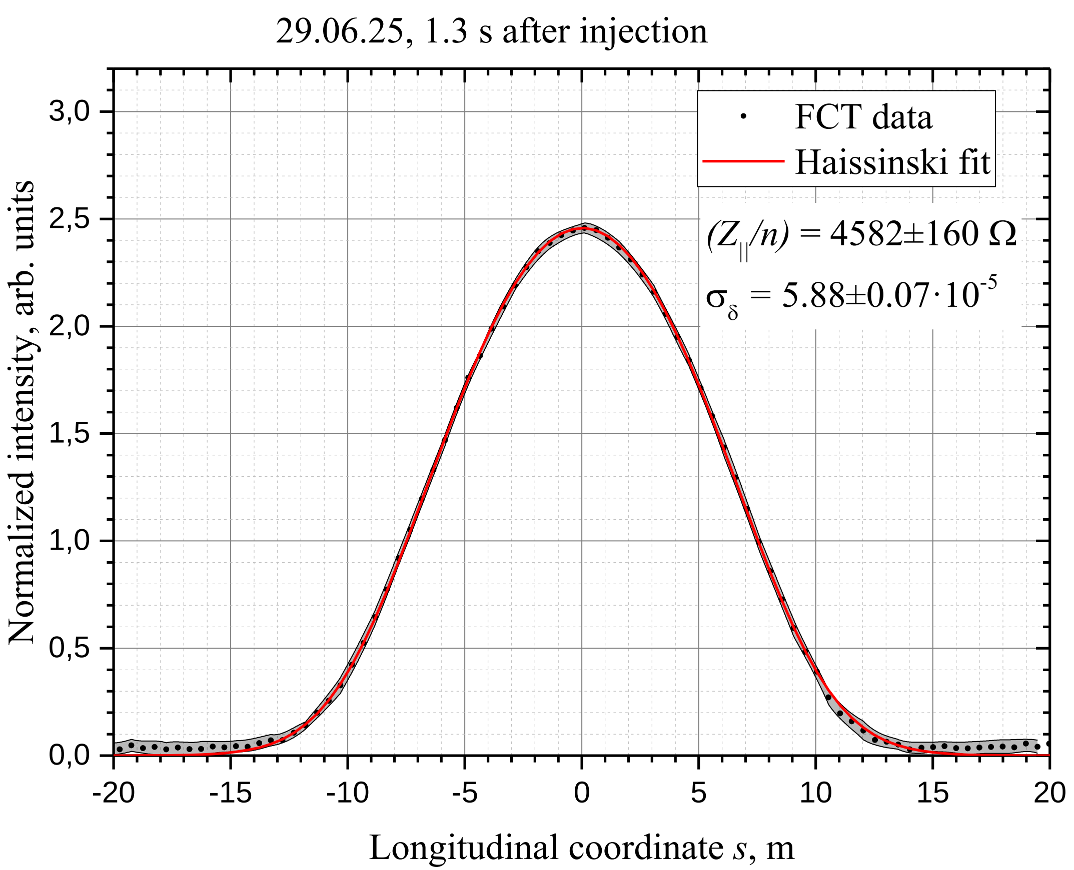
so after i-th iteration the value of j-th distribution point is:

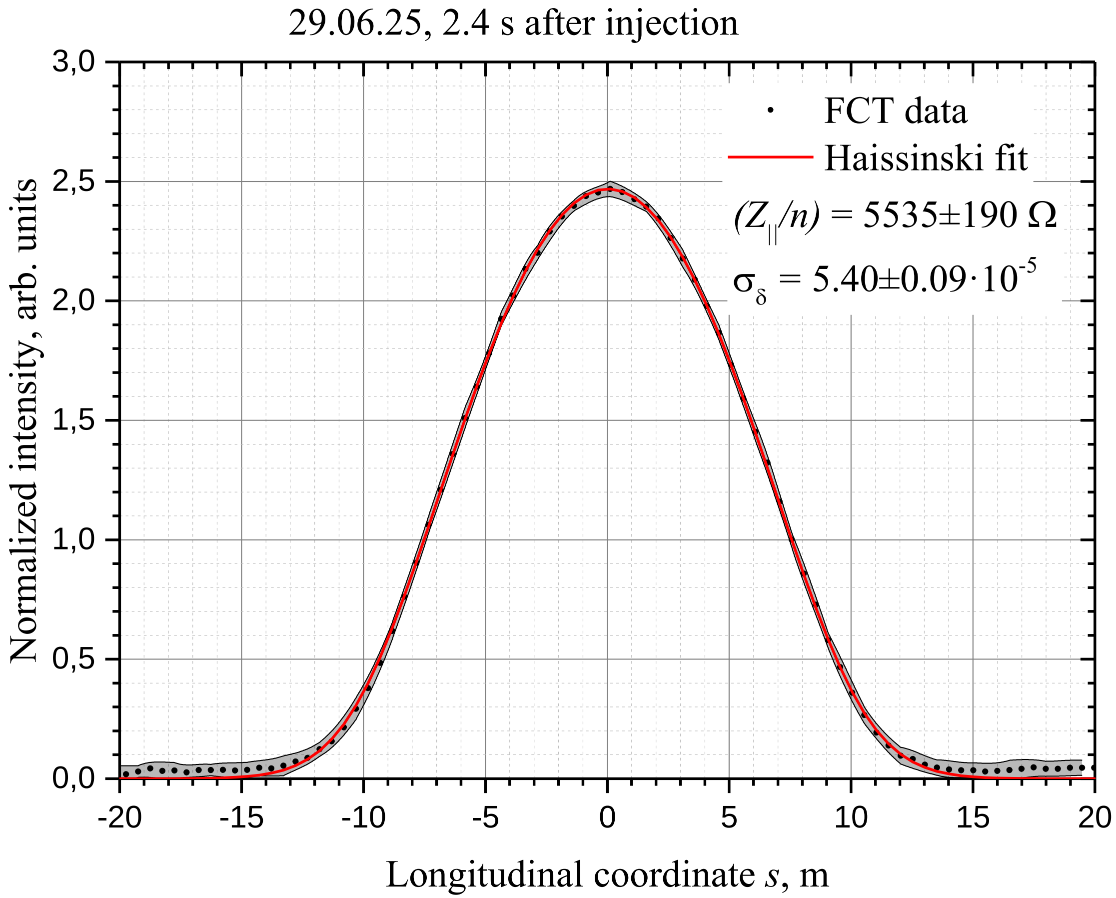
$$f_j^i(\phi) = f_j^{i-1}(\phi)\cdot(1-\alpha) + \alpha\cdot f_j^a(\phi), 0 < \alpha < 1$$

Converges after 50-100 iterations

Then we run over $\left[\left(\frac{Z_\parallel}{n}\right); \sigma_\delta\right]$ grid to find distribution that fits data with least squares algorithm

Approximation results

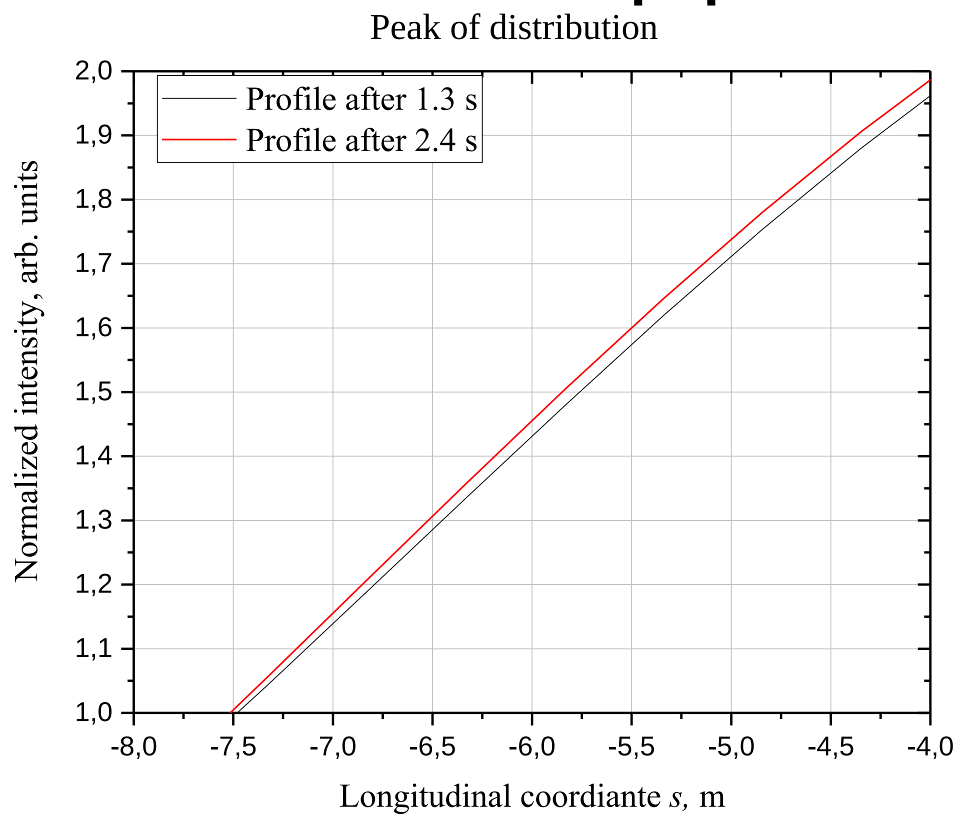


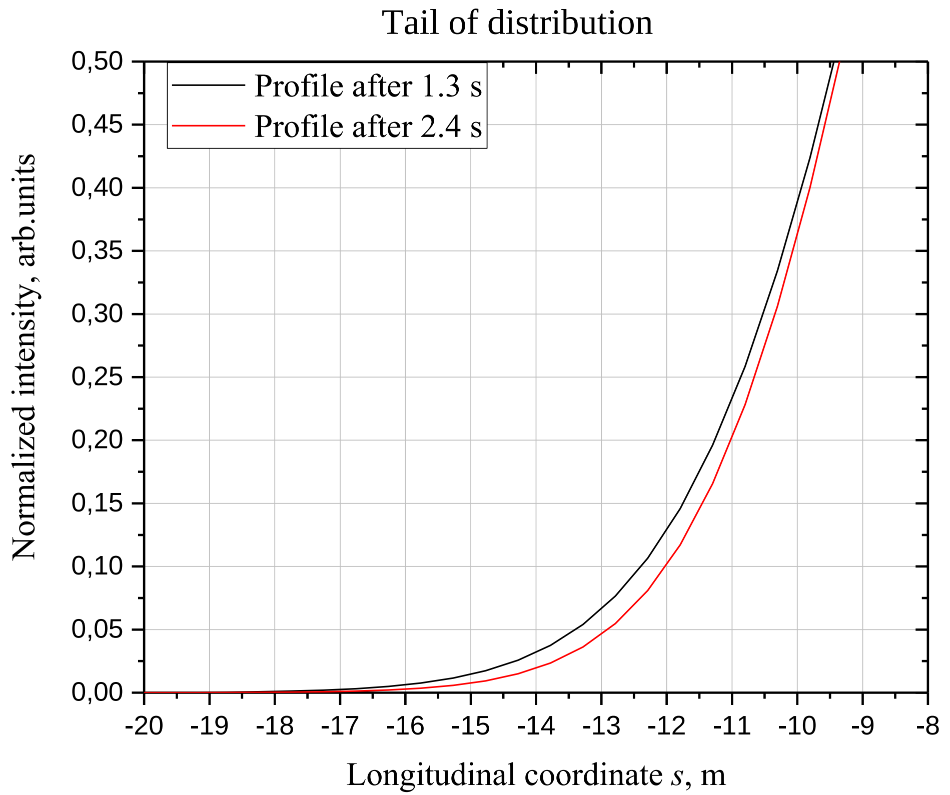


Comparison of two longitudinal beam profiles (avgd. 20 turns) with cooling ON, difference between shots is 1.1 s.

- σ_δ decreases;
- Effective impedance grows due to transverse beam size shrinkage.

Approximation results





Active cooling → increasing space charge force and impedance:

Broadening near peak → parabolic-like distribution;

Tails are shrinking as momentum spread decreases.

Summary

- For beams with negligible space charge (Gaussian): σ_δ can be found from bunch length σ_l as $\sigma_\delta = 2\pi \frac{\sigma_l}{C_a}\sqrt{\frac{ZeV_{RF}h}{2\pi\beta^2 E_s \cdot \eta}}$
- One cannot use bunch length to find σ_δ for space-charge dominated beam due to lengthening;
- σ_δ measurement for space charge dominated beams with Haissinski fit;
- Effective impedance can be found as well;

Future plans:

- Analysis of ⊥ beam size evolution with data from profile monitor;
- Analysis of effective impedance on ⊥ beam size;
- Restoration of longitudinal phase space density;
- Haissinski fit with barrier-like RF voltage.

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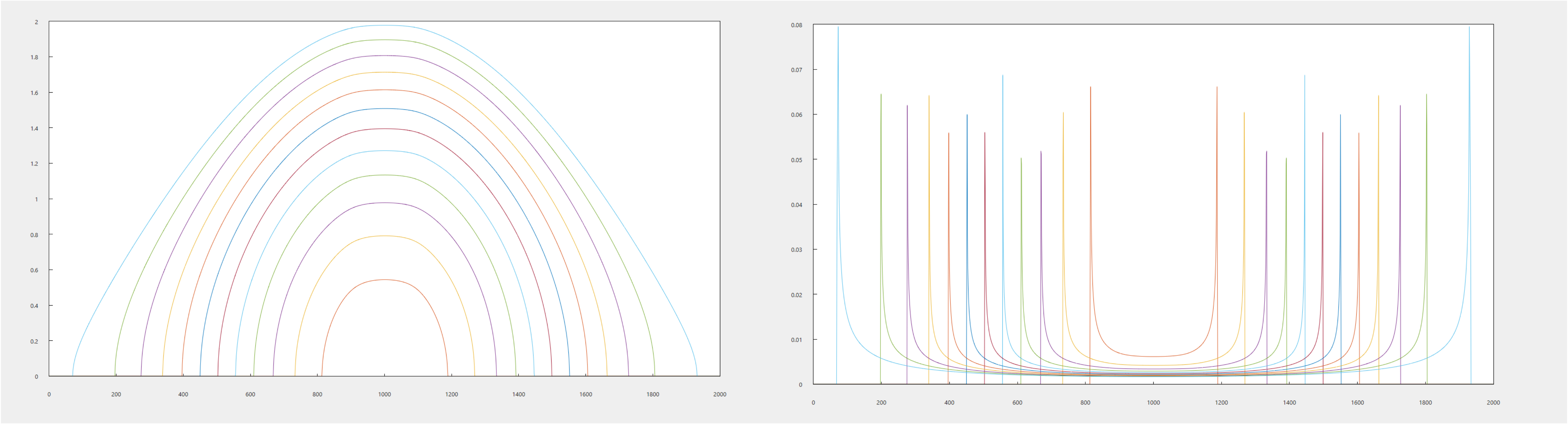
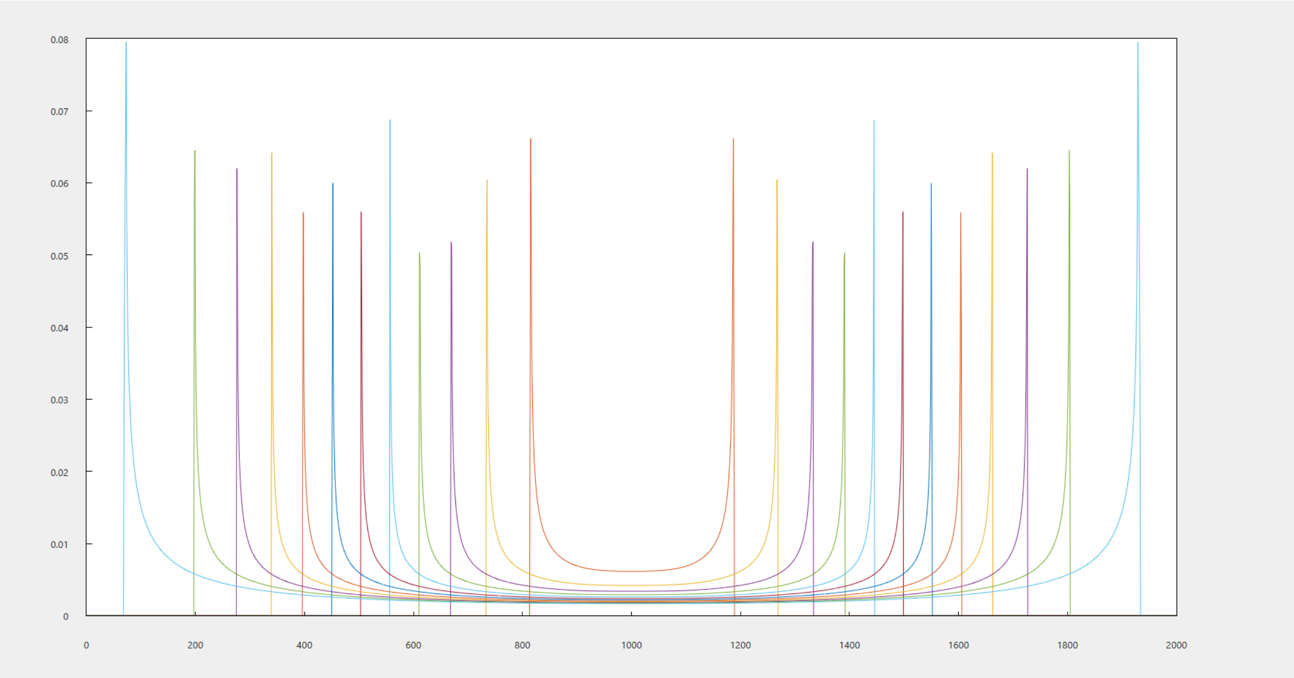
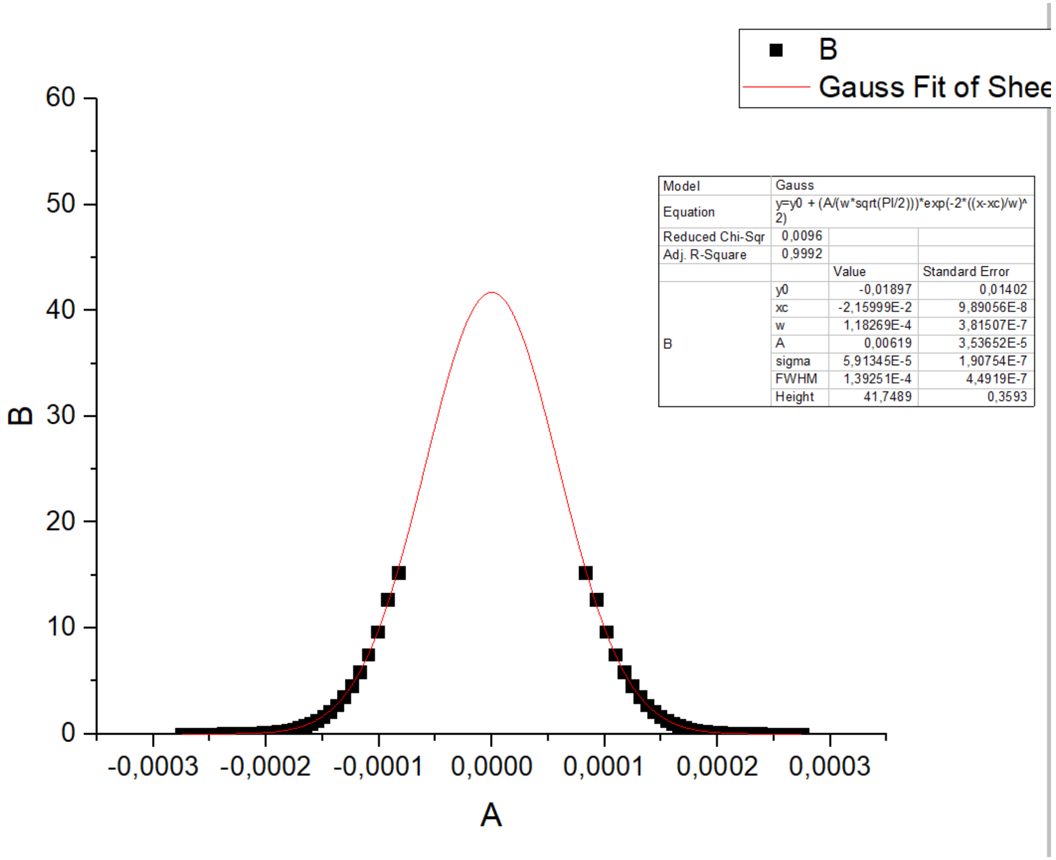
Potential well flattening due to SC



Left – Separatrix with phase space ellipses for space charge + RF Hamiltonian;
Right – Functions of difference between phase space ellipses; these ones are used to restore distribution

Momentum spread distribution restoration

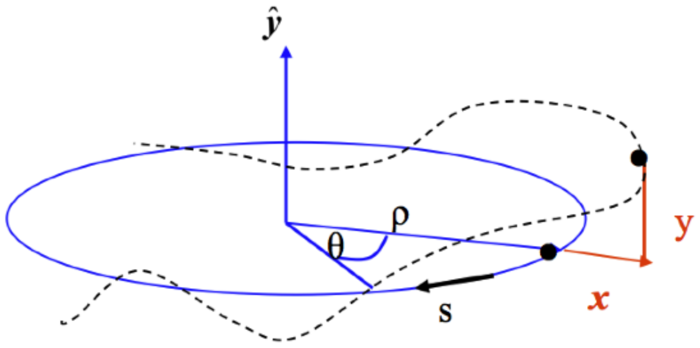




Model	Gauss		
Equation	y=y0 + (A/(w*sqrt(PI/2)))*exp(-2*((x-xc)/w)^2)		
Reduced Chi-Sqr	0,0096		
Adj. R-Square	0,9992		
		Value	Standard Error
B	y0	-0,01897	0,01402
	xc	-2,15999E-2	9,89056E-8
	w	1,18269E-4	3,81507E-7
	A	0,00619	3,53652E-5
	sigma	5,91345E-5	1,90754E-7
	FWHM	1,39251E-4	4,4919E-7
	Height	41,7489	0,3593

Phase space ellipses were used to get weights restoring phase space density distribution;
Momentum spread distribution is a projection of phase space density distribution

Fokker-Planck equation



Fokker-Planck equation

$$\frac{\partial f}{\partial t} = \sum_{m,n} \frac{\partial}{\partial v_m}\left(-f\frac{F_m}{M} + \frac{\partial(fD_{mn})}{\partial v_n}\right); (m,n \to \{x,y,s\})$$

Cooling force $F = \frac{M_i\langle\Delta v\rangle}{\Delta t}$;

Diffusion power $D_{mn} = \frac{\langle\Delta v_m \Delta v_n\rangle}{\Delta t}$

Haissinski equation: $f(s) = f_0 \cdot \exp\left(-\frac{U(s)}{kT_\parallel}\right)$;

$kT_\parallel \to \left(\frac{\sigma_\delta h|\eta|}{\nu_s}\right)^2$;

$$U(s) = ZeV_{RF}\left(1 - \cos\frac{hs}{R}\right) + (Zeh)^2\omega_0\gamma^2 N_0 \cdot \left(\frac{Z_\parallel}{n}\right) \cdot f(s);$$

$$f(s) = f_0 \cdot \exp\left(-\frac{\left(1-\cos\frac{hs}{R}\right) + \frac{Ze\cdot h^2\omega_0\gamma^2 N_0}{V_{RF}} \cdot \left(\frac{Z_\parallel}{n}\right) \cdot f(s)}{(\sigma_\delta h|\eta|/\omega_s)^2}\right)$$

Measured beam correction

- Digital filter compensating BTF was applied