29th International Scientific Conference of Young Scientists and Specialists (AYSS-2025)

Monte Carlo Enhancement of Z'-boson Sensitivity at Future e^+e^- Colliders

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27 October to 31 October 2025
JINR, Dubna, Russia

What is an Z'-boson?

- *Z' is a hypothetical heavy neutral boson that is predicted by many SM extensions (GUTs e.g. $SO(10)$, SUSY, SuperStrings and other theories with an extra $U(1)$ gauge group);*
- *Current lower bounds on the Z' mass for the SSM, LR and E_6 models from LHC and Tevatron:*

Z' Model	$M_{Z'}$, TeV	Source, 95 % C.L.
Z'_{SSM}, Same as SM	5.15	Direct pp search (LHC)
Z'_{LR}, $SU(2)_L \times SU(2)_R \times U(1)$	0.62	Direct $p\bar{p}$ search (Tevatron)
Z'_{LR}, $SU(2)_L \times SU(2)_R \times U(1)$	1.16	Electroweak fit
Z'_{χ}, $SO(10) \to SU(5) \times U(1)_{\chi}$	4.80	Direct pp search (LHC)
Z'_{ψ}, $E_6 \to SO(10) \times U(1)_{\psi}$	4.56	Direct pp search (LHC)
Z'_{χ}, $E_6 \to SU(3) \times SU(2) \times U(1) \times U(1)_{\eta}$	3.90	Direct pp search (LHC)

- *Constraints on the Z-Z' mixing angle, $\theta_{Z-Z'} \lesssim (1-3)\cdot 10^{-3}$, come from precision measurements at the Z-pole in LEP and SLC. The allowed values are so small that their effect on the cross section is orders of magnitude smaller than the dominant contribution from direct virtual Z' exchange. Therefore, Z-Z' mixing effects are neglected in this work.*

European Strategy update :
" The Physics Briefing Book for the 2026 update of the European Strategy for Particle Physics (ESPP) was released on 2 October. Following the recommendations of the ESPP 2020 update, it prioritises the need for an e^+e^- collider dedicated to precision Higgs boson studies and, in the longer term, the Multi-TeV collider. "

Setting constraints on Z'-boson characteristics is part of the scientific programs of both ILC and CLIC.

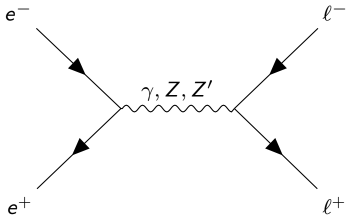
Key features:

- *Energy flexibility and high energies up to the TeV scale:*
  - *ILC: 0.25 TeV $\rightarrow$ 0.5 TeV $\rightarrow$ 1 TeV (from 20 to 31 km);*
  - *CLIC: 0.38 TeV $\rightarrow$ 1.5 TeV $\rightarrow$ 3 TeV (from 11 to 50 km);*
- *Clean e^+e^- collision environment ($\mathscr{L} \sim \mathrm{ab}^{-1}$);*
- *The initial polarization option:*
  - *ILC: $P_{e^-} = \pm 80\,\%$, $P_{e^+} = \pm 20 - 30\,\%$;*
  - *CLIC: $P_{e^-} = \pm 80\,\%$, $P_{e^+} = 0\,\%$.*

The purpose of the study is to develop a methodology to improve the constraints on the Z' at ILC and CLIC.

At ILC and CLIC, Z' can only be studied indirectly. We derive constraints by assuming Z' contributions don't exceed SM uncertainties. Essentially, we study the " ideal " case where no deviation is observed.

The study focuses on the process $e^+e^- \to \ell^+\ell^-$, where $\ell \neq e$.



Feynman diagram for the process $e^+e^- \to \ell^+\ell^-$.

Methodology outline :

- *Introduction of parameters $\Delta Q_{1,2}$ characterizing deviations from the SM cross section, which enter linearly;*
- *Quasi-experiment based on cross section: Generation of data (events $\mathcal{N}$) using MC for the random variable $z \equiv \cos\theta$ (θ is the scattering angle);*
- *Binning of events $\mathcal{N}_k$ (We sort these generated events into bins);*
- *Fitting of a single MC: Determination of optimal $\Delta Q_{1,2}$ and their errors;*
- *To avoid random MC fluctuations, we repeat steps 2-4 thirty times and calculate the weighted average [1];*
- *The final step is to " translate (convert) " the constraints on $\Delta Q_{1,2}$ into constraints on the Z', where:*
  - *The Z' width is explicitly included (usually neglected);*
  - *Interference effects of Z' are taken into account.*

[1] Cowan G. Statistical Data Analysis. — Oxford: Clarendon Press, 1998 (2011). — 352 p.

The differential cross section and its deviation from SM predictions:

$$\frac{d\sigma^{SM+Z'}}{dz}(P_{e^-}, P_{e^+}) = C(P_{e^-}, P_{e^+}) \cdot \left[(1-z)^2 Q_1^{SM+Z'} + (1+z)^2 Q_2^{SM+Z'}\right] , \quad (1)$$

$$\Delta \frac{d\sigma}{dz}(P_{e^-}, P_{e^+}) = C(P_{e^-}, P_{e^+}) \cdot \left[(1-z)^2 \Delta Q_1 + (1+z)^2 \Delta Q_2\right] , \quad (2)$$

where $C(P_{e^-}, P_{e^+}) \equiv \frac{\alpha_{\rm em}\pi}{8s}(1 - P_{e^-} P_{e^+})$.

The total cross section and its deviation from SM predictions:

$$\sigma^{SM+Z'}_{[z_{min}, z_{max}]}(P_{e^-}, P_{e^+}) = \int_{z_{min}}^{z_{max}} \left(\frac{d\sigma^{SM+Z'}}{dz}(P_{e^-}, P_{e^+}) \right) dz \simeq$$

$$\simeq C(P_{e^-}, P_{e^+}) \cdot \left[(Q_1^{SM+Z'} - Q_2^{SM+Z'})(z_{min} - z_{max})(z_{min} + z_{max}) - \right.$$

$$\left. -(Q_1^{SM+Z'} + Q_2^{SM+Z'})(z_{min} - z_{max}) - \frac{1}{3}(Q_1^{SM+Z'} + Q_2^{SM+Z'})(z_{min}^3 - z_{max}^3)\right] , \quad (3)$$

$$\Delta\sigma_{[z_{min}, z_{max}]}(P_{e^-}, P_{e^+}) = \int_{z_{min}}^{z_{max}} \left(\Delta \frac{d\sigma}{dz}(P_{e^-}, P_{e^+}) \right) dz \simeq$$

$$\simeq C(P_{e^-}, P_{e^+}) \cdot \left[(\Delta Q_1 - \Delta Q_2)(z_{min} - z_{max})(z_{min} + z_{max}) - \right.$$

$$\left. -(\Delta Q_1 + \Delta Q_2)(z_{min} - z_{max}) - \frac{1}{3}(\Delta Q_1 + \Delta Q_2)(z_{min}^3 - z_{max}^3)\right] . \quad (4)$$

Introduced Parameters Characterizing the Deviation

Generalized deviation parameters:

$$\Delta Q_1\left(p_{\text{eff}}^-, p_{\text{eff}}^+\right) = Q_1^{SM+Z'} - Q_1^{SM} = p_{\text{eff}}^- \Delta q_{\mathrm{LR}} + p_{\text{eff}}^+ \Delta q_{\mathrm{RL}} \ , \tag{5}$$

$$\Delta Q_2\left(p_{\text{eff}}^-, p_{\text{eff}}^+\right) = Q_2^{SM+Z'} - Q_2^{SM} = p_{\text{eff}}^- \Delta q_{\mathrm{LL}} + p_{\text{eff}}^+ \Delta q_{\mathrm{RR}} \ , \tag{6}$$

$$P_{\text{eff}} = (P_{e^-} - P_{e^+})/(1 - P_{e^-} P_{e^+}) \ , \ p_{\text{eff}}^{\pm} = 1 \pm P_{\text{eff}} \ . \tag{7}$$

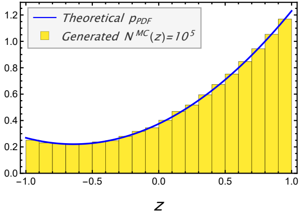
Helicity deviation parameters:

$$\Delta q_{\lambda_e \lambda_\ell} = |q_{\lambda_e \lambda_\ell}^{SM+Z'}|^2 - |q_{\lambda_e \lambda_\ell}^{SM}|^2 \ , \tag{8}$$

$$q_{\lambda_e \lambda_\ell}^{SM+Z'} = \sum_{V=\gamma, Z, Z'} g_{V,e}^{\lambda_e} g_{V,\ell}^{\lambda_\ell} \cdot \frac{s}{s - M_V^2 + iM_V \Gamma_V} \ , \ \lambda_{e,\ell} \equiv \mathrm{L/R} \ . \tag{9}$$

MC Simulation

- *Data generation and verification:*
  1. *Probability density function based on differential cross section:* $p_{PDF}(z) = \frac{\mathrm{d}\sigma^{SM}}{\mathrm{d}z} / \sigma^{SM}_{[z_{min}, z_{max}]}$ $\left(\int_{z_{min}}^{z_{max}} p_{PDF}(z)\mathrm{d}z = 1\right)$;
  2. *Generator creation based on* $p_{PDF}(z)$ *from* z_{min} *to* z_{max} *(Wolfram 14.3);*
  3. *Visual comparison of quasi-experimental histogram and theoretical* $p_{PDF}(z)$*:*



- *ILC and CLIC quasi-experiment:*
  1. *Generation of theoretically expected SM events:*

$$\mathcal{N}^{MC} = \mathcal{N}^{SM} = \mathscr{L} \cdot \sigma^{SM}_{[z_{min}, z_{max}]} , \tag{10}$$

$$\mathcal{N}^{MC}_{ILC} \approx 6 \times 10^5 \ (z \in [-0.9, 0.9], n_z = 19) , \ \mathcal{N}^{MC}_{CLIC} \approx 5 \times 10^4 \ (z \in [-1, 1], n_z = 20) , \tag{11}$$

where $\mathscr{L}$ *is the integrated luminosity that accounts for the final lepton detection efficiency and the collider running time for different polarization configurations;*

  2. *Binning of events* $\mathcal{N}^{MC}_k$ *(We sort* $\mathcal{N}^{MC}$ *into bins);*
  3. *Bin error calculation as* $\sqrt{\mathcal{N}^{MC}_k}$.

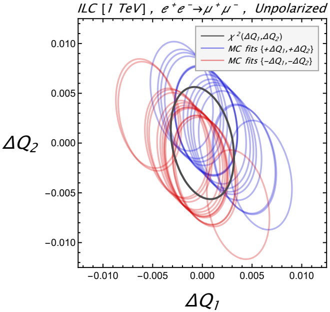
Fitting Procedure

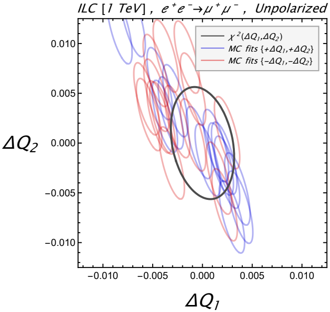
1) Differential method:

$$\mathcal{N}^{MC}(z)/\Delta z = \mathscr{L}\left(\frac{\mathrm{d}\sigma^{SM}}{\mathrm{d}z} + \frac{\mathrm{d}\sigma^{Z'}}{\mathrm{d}z}\right) . \quad (12)$$

2) Integral (Cumulative) method:

$$\mathcal{N}^{MC}(z) = \mathscr{L}\int_{z_{min}}^{z}\left(\frac{\mathrm{d}\sigma^{SM}}{\mathrm{d}z} + \frac{\mathrm{d}\sigma^{Z'}}{\mathrm{d}z}\right)\mathrm{d}z . \quad (13)$$

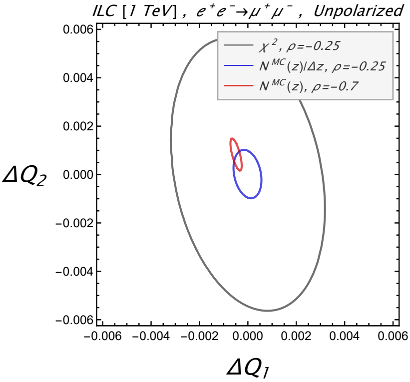


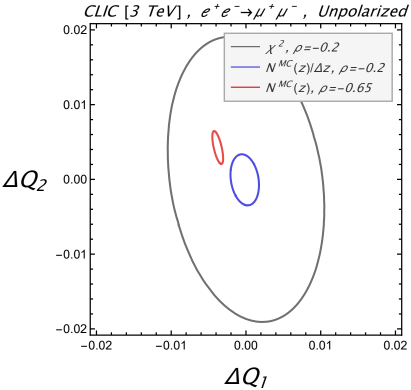


$$\chi^2(\Delta Q_{1,2}) = \sum_k \left[\frac{\Delta\mathcal{N}_k(\Delta Q_{1,2})}{\delta\mathcal{N}_k^{SM}}\right]^2 , \quad \Delta\mathcal{N}_k(\Delta Q_{1,2}) = \mathcal{N}_k^{SM+Z'}(Q_{1,2}^{SM+Z'}) - \mathcal{N}_k^{SM}(Q_{1,2}^{SM}) . \quad (14)$$

The cumulative method gives us the best precision, but at the cost of greater correlation ($\rho \sim -0.25 \to -0.7$).

We run 30 MC and their subsequent fittings ΔQ_1 and ΔQ_2, then calculate their weighted average.





We can conclude that random fluctuations in a single MC were suppressed, resulting in more precise confidence regions and, therefore, increased sensitivity of the methodology.

Impact of Methodology on Z' Mass

Lower bounds on the Z' mass in TeV at 95% C.L. for Z' models, extracted from the process $e^+e^- \to \mu^+\mu^-$ assuming width $\Gamma_{Z'} = \xi_{Z'} \cdot M_{Z'}$ ($\xi_{Z'} = \xi_Z = \Gamma_Z/M_Z \approx 2.7\%$).

1 – Traditional χ^2,
2 – Averaging 30 MC fittings of $\mathcal{N}^{MC}(z)/\Delta z$,
3 – Averaging 30 MC fittings of $\mathcal{N}^{MC}(z)$.

ILC [1 TeV],
$a = \{-0.8, 0.2\}$ и $b = \{0.8, -0.2\}$:

Модель Z'	$M_{Z'}$, 1	$M_{Z'}$, 2	$M_{Z'}$, 3
SSM	14.4	20	24.8
χ	13.8	20.9	25.9
ψ	7.2	9	11.2
η	8	11.3	13.7
LR	10.9	15.9	19.4
ALR	19.8	26.9	33.3
LH	9.7	15.9	19.8
$U(1)_X^{[-2,1]}$	15.8	22.9	28
$U(1)_X^{[-1,1]}$	6.6	9.9	12
$U(1)_X^{[1,1]}$	12.2	16.4	20
$U(1)_X^{[2,1]}$	21.4	29.5	36

CLIC [3 TeV],
$a = \{-0.8, 0\}$ и $b = \{0.8, 0\}$:

Модель Z'	$M_{Z'}$, 1	$M_{Z'}$, 2	$M_{Z'}$, 3
SSM	25	36.3	45.5
χ	24.7	37.9	47.5
ψ	12.3	16.5	20.6
η	12.1	14.8	18.1
LR	17.32	20.6	25.4
ALR	34.3	48.7	61
LH	17.8	28.9	36.2
$U(1)_X^{[-2,1]}$	22.9	29.6	36.6
$U(1)_X^{[-1,1]}$	9.6	13	15.9
$U(1)_X^{[1,1]}$	18.6	21.3	26.2
$U(1)_X^{[2,1]}$	32.2	38	46.9

- *The cumulative method is the most effective;*
- *However, the differential method also achieves substantial sensitivity improvement.*

Conclusion

- *Developed and successfully tested a procedure to enhance sensitivity for indirect Z'-boson searches at ILC and CLIC;*
- *Demonstrated the effectiveness of ensemble averaging over 30 MC, which suppresses random MC fluctuations and yields more stringent and statistically robust constraints;*
- *At the cost of higher correlation, the cumulative integral function provides tighter confidence regions;*
- *Obtained preliminary lower bounds for Z' mass based on quasi-experiments for a wide class of models at ILC and CLIC.*

Thank You For Your Attention!