Effects of higher-order QED corrections in e^+e^- annihilation at future collider energies

U. E. Voznaya, A. B. Arbuzov

BLTP JINR

AYSS-2025

27.10.2025

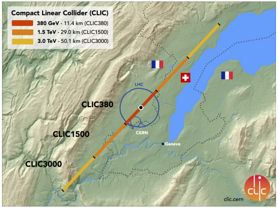
Outline

1. Introduction
2. Future e^+e^--colliders
3. Parton distribution functions approach
4. e^+e^--annihilation
5. Numerical results
6. Factorization
7. Conclusion

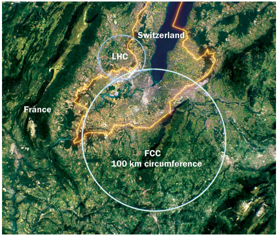
Introduction

- Current and future high-precision experiments require precise theoretical predictions
- Lepton colliders (e^+e^-, $\mu^+\mu^-$) allow experiments with higher precision than hadron ones
- We need calculation of higher-order contributions within perturbation theory (higher-order corrections).
- Calculation of higher-order radiative corrections is a complicated problem
- We need methods to simplify this calculation (e.g. PDF approach)
- Resummation of calculated corrections to improve convergence of perturbation theory series $\rightarrow$ choose a factorization scale

Future e^+e^--colliders



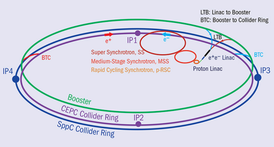
CLIC - start construction ≈2033



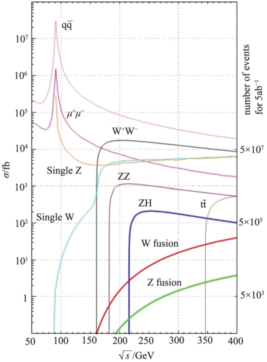
FCC - start construction mid 2030s

$\sqrt{s}$:

- $M_z \approx 91.19$ GeV - Z-peak
- 160 GeV - $W\bar{W}$
- 240 GeV - Higgs vev, max energy of CEPC
- 350, 365 GeV - $t\bar{t}$, energy of CEPC after upgrade, max energy of FCC-ee
- 3 TeV - max energy of CLIC



CEPC - start construction 2026-2030



From Mo et al., CPC 03 (2016) 033001

Parton distribution functions (PDFs) approach

- Parton distribution functions approach allow to calculate only corrections enhanced by the large logarithm:

$$L = \ln \frac{\mu_F^2}{\mu_R^2},$$

μ_F - factorization scale, μ_R - renormalization scale

- Evolution equation of PDFs in QED is analogous to DGLAP equation in QCD

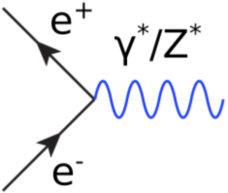
$$D_{ba}(x, \mu^2, \mu_0^2) = \delta(1-x)\delta_{ba} + \sum_{i=e,\bar{e},\gamma} \int\limits_{\mu_0^2}^{\mu^2} \frac{dt\alpha(t)}{2\pi t} \int\limits_x^1 \frac{dy}{y} D_{ia}(y, t, \mu_0) P_{bi}\left(\frac{x}{y}\right)$$

D_{ba}, D_{ia} - parton distribution (fragmentation) functions, P_{bi} - splitting functions

- Equations are solved by iterations (U.V., A.Arbuzov, JPG 2023) with initial conditions defined by subtraction scheme (here $\overline{\mathrm{MS}}$)
- Process independent PDFs are convoluted with functions containing information about the process

e^+e^--annihilation

$e^+e^- \to \gamma^*/Z^*$



LO and NLO ISR corrections to the order $\alpha^5 L^6$ (Ablinger at al., NPB 2020)
We recalculated and corrected some mistakes (A.B. Arbuzov, U.V., PRD 2024, arXiv: 2405.03443 [hep-ph])

$$\frac{d\sigma^{\mathrm{NLO}}_{\bar{e}e}(s')}{ds'} = \sum_{i,j=e,\bar{e},\gamma} \int\limits_{\bar{z}_1}^{1} \int\limits_{\bar{z}_2}^{1} dz_1 dz_2 D^{\mathrm{str}}_{ie}\left(z_1, \frac{\mu_R^2}{\mu_F^2}\right) D^{\mathrm{str}}_{j\bar{e}}\left(z_2, \frac{\mu_R^2}{\mu_F^2}\right) \times$$

$$\times \left(\sigma^{(0)}_{ij}(sz_1z_2) + \bar{\sigma}^{(1)}_{ij}(sz_1z_2) + \mathcal{O}(\alpha^2 L^0)\right) \delta(s' - sz) + \mathcal{O}\left(\frac{\mu_R^2}{\mu_F^2}\right)$$

$$\frac{d\sigma^{\mathrm{NLO}}_{\bar{e}e}(s')}{ds'} = \sigma^{(0)}_{\bar{e}e}\left\{1 + \sum_{i=1}^{\infty}\left(\frac{\alpha}{2\pi}\right)^i \sum_{j=i-1}^{i} c_{ij} L^j + \mathcal{O}(\alpha^i L^{i-2})\right\}$$

e^+e^--annihilation

$$\frac{d\sigma^{\rm NLO}_{\bar{e}e}(s')}{ds'} = \sigma^{(0)}_{\bar{e}e}\left\{1+\sum_{i=1}^{\infty}\left(\frac{\alpha}{2\pi}\right)^i \sum_{j=i-1}^{i} c_{ij}L^j + \mathcal{O}(\alpha^i L^{i-2})\right\}$$

$$h_{ij} = \left(\frac{\alpha}{2\pi}\right)^i L^j c_{ij}$$

$$\sigma_{e^+e^-} = \sum_{i,j}\left(h^{\Delta}_{ij}\sigma^{(0)}(1) + \int\limits_{z_{min}}^{1-\Delta} dz\sigma^{(0)}(z)h^{\theta}_{ij}(z)\right)$$

$$h^{num}_{ij} = h^{\Delta}_{ij}\sigma^{(0)}(1) + \int\limits_{z_{min}}^{1-\Delta} dz\sigma^{(0)}(z)h^{\theta}_{ij}(z)$$

$\Delta = 10^{-7},\ 10^{-8}$

Numerical estimations, %, for $\mu_F^2 = s$

z_{min} is defined by experimental conditions

$\sqrt{s} = 160$ GeV, $z_{min} = 0.5$

h_{11}	h_{10}	h_{22}	h_{21}	h_{33}	h_{32}	h_{44}	h_{43}	h_{55}
				γ				
9.81624	0.26017	-1.28618	0.20722	-0.00845	-0.00714	0.00530	-0.00153	-0.00020
				Pairs				
0	0	0.13182	-0.05573	-0.03105	0.01278	-0.00171	0.00046	-0.00100
				Full				
9.81624	0.26017	-1.15435	0.15149	-0.03949	0.00563	0.00359	-0.00107	-0.00120

$\sqrt{s} = 240$ GeV, $z_{min} = 0.5$

h_{11}	h_{10}	h_{22}	h_{21}	h_{33}	h_{32}	h_{44}	h_{43}	h_{55}
				γ				
3.45872	0.51562	-1.05495	0.14324	0.02506	-0.01176	0.00273	-0.00067	-0.00018
				Pairs				
0	0	0.05956	-0.02900	-0.02628	0.01132	-0.00057	-0.00017	-0.00080
				Full				
3.45872	0.51562	-0.99539	0.11424	-0.00122	-0.00044	0.00216	-0.00084	-0.00098

Numerical estimations, %, for $\mu_F^2 = s$

$\sqrt{s} = 160$ GeV, $z_{min} = 0.1$

h_{11}	h_{10}	h_{22}	h_{21}	h_{33}	h_{32}	h_{44}	h_{43}	h_{55}
				γ				
335.7996	-12.6201	17.1479	0.6467	-1.8223	0.5781	-0.0348	-0.0102	0.0052
				Pairs				
0	0	8.3539	-1.8438	0.0043	-0.0647	-0.0606	0.0357	-0.0922
				Full				
335.7996	-12.6201	25.5019	-1.1971	-1.8182	0.5135	-0.0953	0.0255	-0.0870

$\sqrt{s} = 240$ GeV, $z_{min} = 0.1$

h_{11}	h_{10}	h_{22}	h_{21}	h_{33}	h_{32}	h_{44}	h_{43}	h_{55}
				γ				
324.1463	-11.7621	27.1298	-0.9909	-0.6242	0.6140	-0.0664	0.0149	0.0012
				Pairs				
0	0	25.3337	-1.4772	-0.1401	-0.0207	-0.0269	0.0460	-0.1792
				Full				
324.1463	-11.7621	52.4635	-2.4680	-0.7643	0.5934	-0.0932	0.0609	-0.1780

Factorization scale choice

Redistribution of the corrections:

$\hat{\mu}_F$

exp.

$$\begin{array}{llll} \alpha^1 L & \alpha^1 & & \\ \alpha^2 L^2 & \alpha^2 L & \alpha^2 & \\ \alpha^3 L^3 & \alpha^3 L^2 & \alpha^3 L & \alpha^3 \end{array}$$

Matching equality for $\mathcal{O}(\alpha^2)$:

$$\left(\frac{\alpha}{2\pi}\right)^2 L^2 c_{22} + \left(\frac{\alpha}{2\pi}\right)^2 L c_{21} + \left(\frac{\alpha}{2\pi}\right)^2 c_{20}$$
$$= \left(\frac{\alpha}{2\pi}\right)^2 (L + \Delta L)^2 \hat{c}_{22} + \left(\frac{\alpha}{2\pi}\right)^2 (L + \Delta L) \hat{c}_{21} + \left(\frac{\alpha}{2\pi}\right)^2 \hat{c}_{20}$$

c_{ij} - coefficients for the initial μ_F choice, $\hat{c}_{ij}$ - for the new choice $\hat{\mu}_F$, $\Delta L = \ln(\hat{\mu}_F^2/\mu_F^2)$ is the shift of the large logarithm value.

$$\begin{aligned} \hat{c}_{22} &= c_{22}, \\ \hat{c}_{21} &= c_{21} - 2\Delta L c_{22}, \\ \hat{c}_{20} &= c_{20} - \Delta L c_{21} + (\Delta L)^2 c_{22} \end{aligned}$$

Position of the peak at $z_{min} = 0.1$

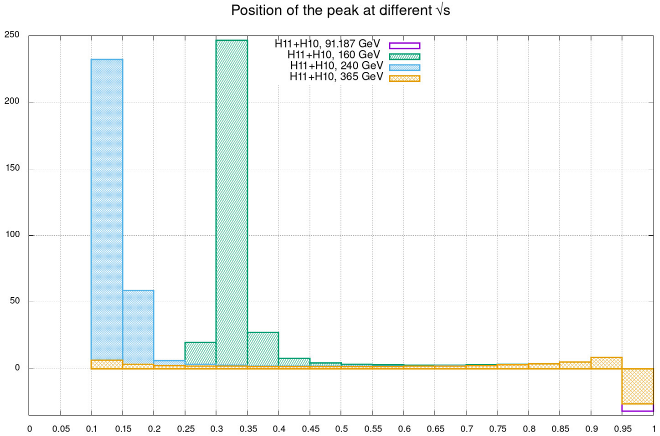


Таблица: Position of Z-peak: $sz = M_z^2$

$\sqrt{s}$	91.1876	160	240	365
Peak, z	1	0.3234	0.1443	0.0624
$h_{\mathbf{11}}$, %	-32.7365	335.7996	324.1463	23.5582

Factorization scale choice, $\mathcal{O}(\alpha^1)$

Compare 3 factorization scales: $\sqrt{s}$, $\sqrt{s/e}$, $\sqrt{sz} \to L$, $L-1$, $L+\ln z$

$$L \leftrightarrow L-1$$

$\mathcal{O}(\alpha^1)$:

$$c_{11} = 2\frac{1+z^2}{1-z}$$

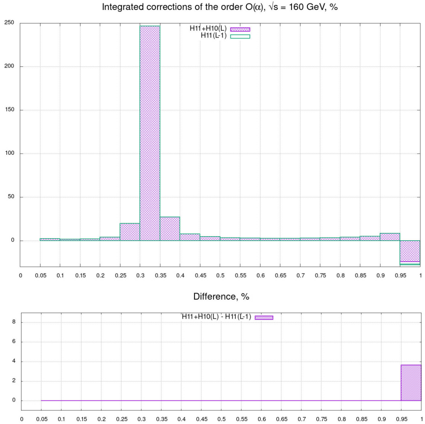
$$c_{10} = -2\frac{1+z^2}{1-z} = -c_{11}$$

$$h_{11}(L) = \frac{\alpha}{2\pi} L c_{11}$$

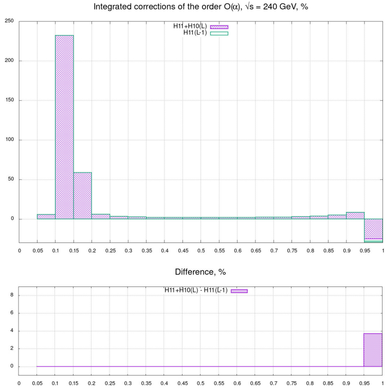
$$h_{11}(L-1) = \frac{\alpha}{2\pi}(L-1)c_{11} =$$

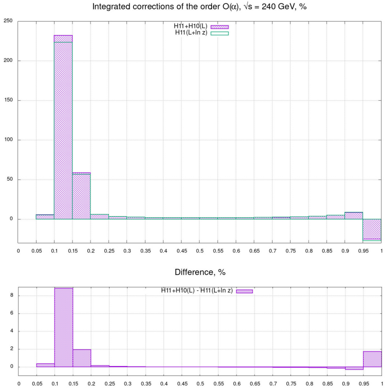
$$= \frac{\alpha}{2\pi} L c_{11} - \frac{\alpha}{2\pi} c_{11} =$$

$$= h_{11}(L) + h_{10}(L)$$

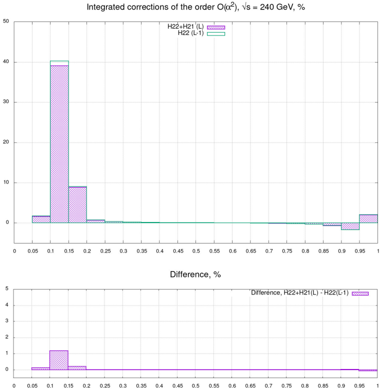


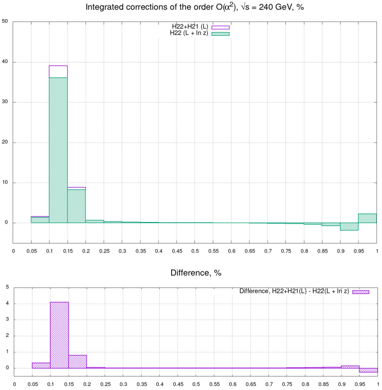
Factorization scale choice, $\mathcal{O}(\alpha^1)$





Factorization scale choice, $\mathcal{O}(\alpha^2)$, NLO





Factorization scale variation $\mu_F \to \mu_F/2,\ 2\mu_F$

Calculated (Δ) and estimated (δ) by variation of factorization scale by factor of 2 corrections of the orders $\mathcal{O}(\alpha^2)$ and $\mathcal{O}(\alpha^3)$

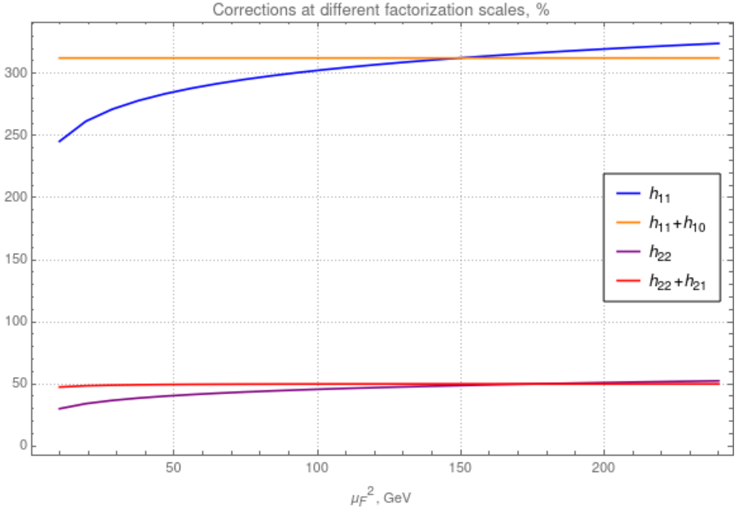
	$\mathcal{O}(\alpha^2)$				$\mathcal{O}(\alpha^3)$	
	LO		NLO		LO	
	Δ_2^{LO}	δ_2^{LO}	Δ_2^{NLO}	δ_2^{NLO}	Δ_3^{LO}	δ_3^{LO}
$\sqrt{s} = M_z$, $z_{min} = 0.1$	0.436	0.524	0.0049	0.0250	0.0250	0.0494
$\sqrt{s} = M_z$, $z_{min} = 0.5$	0.436	0.5246	0.0049	0.0250	0.0250	0.0499
$\sqrt{s} = M_z$, $z_{min} = 0.9$	0.440	0.529	0.0050	0.0252	0.0249	0.0497
$\sqrt{s} = 240$ GeV, $z_{min} = 0.1$	2.468	5.569	0.7907	0.1486	0.5933	0.1218
$\sqrt{s} = 240$ GeV, $z_{min} = 0.5$	0.1142	0.1057	0.0144	0.0061	0.0004	0.0002
$\sqrt{s} = 240$ GeV, $z_{min} = 0.9$	0.073	0.0403	0.0019	0.0039	0.0214	0.0293

$$\Delta_2^{\mathrm{LO}} = h_{21}, \quad \delta_2^{\mathrm{LO}} = \frac{|h_{22} - h_{22}(1/2)| + |h_{22} - h_{22}(2)|}{2}$$

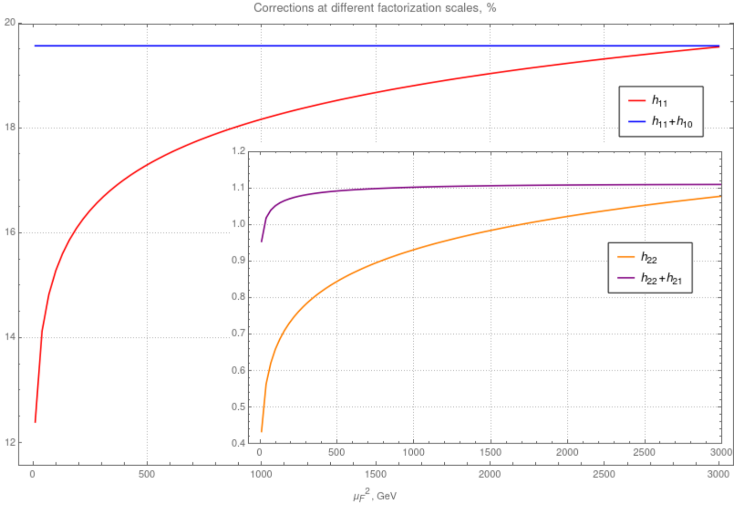
$$\Delta_2^{\mathrm{NLO}} = h_{20}, \quad \delta_2^{\mathrm{NLO}} = \frac{|h_{22} + h_{21} - (h_{22} + h_{21})(1/2)|}{2} + \frac{|h_{22} + h_{21} - (h_{22} + h_{21})(2)|}{2}$$

$$\Delta_3^{\mathrm{LO}} = h_{32}, \quad \delta_3^{\mathrm{LO}} = \frac{|h_{33} - h_{33}(1/2)| + |h_{33} - h_{33}(2)|}{2}$$

$\mathcal{O}(\alpha^1)$ and $\mathcal{O}(\alpha^2)$ corrections, $z_{min} = 0.1$, $\sqrt{s} = 240$ GeV



$\mathcal{O}(\alpha^1)$ and $\mathcal{O}(\alpha^2)$ corrections, $z_{min} = 0.1$, $\sqrt{s} = 3$ TeV



Conclusion

- Numerical estimations of the ISR corrections to e^+e^--annihilation cross section at future collider energies were obtained using PDF approach
- Factorization scale choice in QED is important to improve convergence of perturbation theory series
- Factorization scale change effects depend on energy of the beam and experimental conditions (z_{min})
- Further investigation of different factorization schemes is needed
- We plan to include contributions of other fermions (μ, τ, quarks)

Thank you for your attention!

Backup slides

Numerical estimations, %

Таблица: $\sqrt{s} = M_z$

$Z_{min} = 0.1$								
h_{11}	h_{10}	h_{22}	h_{21}	h_{33}	h_{32}	h_{44}	h_{43}	h_{55}
γ								
-32.73654	2.00167	4.88428	-0.59516	-0.37760	0.07104	0.00344	-0.00185	0.00315
Pairs								
		-0.30571	0.15847	0.08753	-0.04599	0.00157	0.00375	-0.00009
Full								
-32.73654	2.00167	4.57858	-0.43669	-0.29007	0.02505	0.00501	0.00190	0.00306

Таблица: $\sqrt{s} = 365$ GeV

h_{11}	h_{10}	h_{22}	h_{21}	h_{33}	h_{32}	h_{44}	h_{43}	h_{55}
$Z_{min} = 0.1$								
γ								
23.55818	-0.22585	0.53154	0.10899	-0.05905	0.02875	-0.00094	-0.00042	0.00008
Pairs								
0	0	1.30566	-0.11950	-0.02707	0.00843	-0.00378	0.00281	-0.01137
Full								
23.55818	-0.22585	1.83721	-0.01051	-0.08977	0.03718	-0.00472	0.00239	-0.01129

Contributions of different orders

$$\frac{d\sigma_{\bar{e}e}}{ds'} = \sigma^{(0)}\Big[D_{\bar{e}\bar{e}} \otimes D_{ee} \otimes \sigma_{\bar{e}e} + D_{\gamma e} \otimes D_{ee} \otimes \sigma_{e\gamma} + D_{ee} \otimes D_{e\bar{e}} \otimes \sigma_{ee} +$$
$$+D_{\gamma e} \otimes D_{\bar{e}\bar{e}} \otimes \sigma_{\bar{e}\gamma} + D_{\gamma e} \otimes D_{\gamma\bar{e}} \otimes \sigma_{\gamma\gamma} + D_{\gamma e} \otimes D_{e\bar{e}} \otimes \sigma_{e\gamma}$$
$$+D_{\bar{e}e} \otimes D_{\bar{e}\bar{e}} \otimes \sigma_{\bar{e}\bar{e}} + D_{\bar{e}e} \otimes D_{\gamma\bar{e}} \otimes \sigma_{\bar{e}\gamma} + D_{\bar{e}e} \otimes D_{e\bar{e}} \otimes \sigma_{\bar{e}e}\Big]$$

Таблица: Contributions of different orders

i \ j	$\bar{e}$	γ	e
e	$D_{ee}D_{\bar{e}\bar{e}}\sigma_{e\bar{e}}$ LO (1)	$D_{ee}D_{\gamma\bar{e}}\sigma_{e\gamma}$ NLO ($\alpha^2 L$)	$D_{ee}D_{e\bar{e}}\sigma_{ee}$ NNLO ($\alpha^4 L^2$)
γ	$D_{\gamma e}D_{\bar{e}\bar{e}}\sigma_{\gamma\bar{e}}$ NLO ($\alpha^2 L$)	$D_{\gamma e}D_{\gamma\bar{e}}\sigma_{\gamma\gamma}$ NNLO ($\alpha^4 L^2$)	$D_{\gamma e}D_{e\bar{e}}\sigma_{\gamma e}$ NLO ($\alpha^4 L^3$)
$\bar{e}$	$D_{\bar{e}e}D_{\bar{e}\bar{e}}\sigma_{\bar{e}\bar{e}}$ NNLO ($\alpha^4 L^2$)	$D_{\bar{e}e}D_{\gamma\bar{e}}\sigma_{\bar{e}\gamma}$ NLO ($\alpha^4 L^3$)	$D_{\bar{e}e}D_{e\bar{e}}\sigma_{\bar{e}e}$ LO ($\alpha^4 L^4$)

Factorization in NLO

In *Berends(1987), Blumlein(2011)* factorization scale was chosen

$$\mu_F^2 = sz.$$

Then the large log

$$L = \ln(s/m_e^2) + \ln z$$

In the expression for one-loop cross section in *Blumlein(2011)*, the variable $y = z/x$ was exchanged into x:

$$\left[\bar{\delta}^{(1)}_{\bar{e}e}(sx)\right]^* = \frac{\alpha}{\pi}\left\{\left[\frac{1+y^2}{1-y}\right]_+ \ln y + 2(1+y^2)\left[\frac{\ln(1-y)}{1-y}\right]_+ + \right.$$
$$\left. +\delta(1-y)\left(2\zeta_2 - \frac{1}{2}\right)\right\}$$

.

In the approachof *Berends(1987), Blumlein(2011)* this logarithm is integrated. In our calculations $\ln z$ is nt integrated in convolutions (the variable of integration is y):

$$\bar{\delta}^{(1)}_{\bar{e}e}(sx) = \frac{\alpha}{\pi}\left\{\left[\frac{1+y^2}{1-y}\right]_+ (\ln z - \ln y) + 2(1+y^2)\left[\frac{\ln(1-y)}{1-y}\right]_+ + \right.$$
$$\left. +\delta(1-y)\left(2\zeta_2 - \frac{1}{2}\right)\right\}$$