Relativistic description of tetraquarks with two heavy quarks

Релятивистское описание тетракварков с двумя тяжёлыми кварками

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Массы тетракварков, содержащих два тяжёлых кварка ($QQ'\bar{q}\bar{q}'$), вычислены в рамках релятивистской кварковой модели, основанной на квазипотенциальном подходе в КХД и дикварк–антидикварковой картины тетракварков. Релятивистское квазипотенциальное уравнение учитывает все зависящие и не зависящие от спина релятивистские поправки и внутреннюю структуру дикварков. Вычисленные массы тетракварков сравниваются с порогами сильных распадов на пары мезонов. Определены состояния, которые могут наблюдаться как узкие резонансы в других каналах распадов.

Masses of tetraquarks with two heavy quarks ($QQ'\bar{q}\bar{q}'$) are calculated within the relativistic quark model, based on the quasipotential approach in QCD and diquark–antidiquark picture of tetraquarks. The relativistic quasipotential equation takes into account all spin–dependent and spin–independent relativistic corrections and the internal structure of the diquarks. The calculated masses of such tetraquarks are compared with the strong fall–apart decay thresholds into a pair of mesons. The states that could be observed as narrow resonances in other decay modes are determined.

PACS: 12.39.Ki, 14.40.Lb, 14.40.Nd, 14.40.Rt

1. Introduction

The existence of hadrons as various combinations of quarks and antiquarks has been proposed within the quark model in the 1960s. While three–quark baryons and quark–antiquark mesons were abundantly observed since the very beginning, the convincing evidence of other compositions was absent

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for a long time. However, 40 years later the first exotic hadrons (tetraquarks and pentaquarks) were finally discovered [1].

A doubly heavy tetraquark is a bound system of two heavy (charm, bottom, but not top, since t-quark decays too fast) and two light (up, down, strange) flavors of quarks and antiquarks. All possible flavor compositions can be divided into 2 categories: two heavy flavor quarks/antiquarks + two light flavor antiquarks/quarks and heavy and light flavor quarks + heavy and light flavor antiquarks. In this study we consider only the first category, explicitly exotic: doubly charmed, charmed bottom and doubly bottom. Calculations of the masses of the ground [2, 3] and excited [4, 5] states of all compositions of fully heavy tetraquarks have been done previously.

2. Relativistic Diquark–Antidiquark Model of Tetraquarks

The doubly heavy tetraquark is considered within the diquark–antidiquark picture as a bound state of the (in our case, heavy) diquark $\mathrm{QQ}^{'} \equiv \mathrm{d}$ and the (in our case, light) antidiquark $\overline{\mathrm{qq}}^{'} \equiv \overline{\mathrm{d}}^{'}$ $(\mathrm{Q}, \mathrm{Q}^{'} = \mathrm{c}, \mathrm{b};\ \mathrm{q}, \mathrm{q}^{'} = \mathrm{u}, \mathrm{d}, \mathrm{s})$. Note that the diquark model gives a good agreement of the calculations (for example, baryon masses) with experimental data [6] and, thus, is widely used in the hadron spectroscopy. Another popular approach for the description of four–quark bound states is the meson–meson molecular model. We consider such a picture to be less probable for the heavy tetraquarks in particular since the binding due to the meson exchange would not be sufficient.

The estimates show that heavy quark velocities can reach up to half the speed of light, therefore heavy tetraquarks should be treated relativistically. We use the Relativistic Quark Model based on the quasipotential approach which has successfully been applied for calculating the mass spectra of ordinary three-quark baryons [6] and quark–antiquark mesons [7]. In this framework the mass of a tetraquark state (T) with a diquark–antidiquarks interaction quasipotential $V(\mathbf{p}, \mathbf{q}; M)$ is the solution of the relativistic Schrödinger-type quasipotential equation [6, 7]:

$$\left(\frac{b^2(M)}{2\mu_{\mathrm{R}}(M)} - \frac{\mathbf{p}^2}{2\mu_{\mathrm{R}}(M)}\right)\Psi_{\mathrm{T}}(\mathbf{p}) = \int \frac{d^3q}{(2\pi)^3}\, V(\mathbf{p}, \mathbf{q}; M)\Psi_{\mathrm{T}}(\mathbf{q}). \qquad (1)$$

Here $\mathbf{p}$ is a vector of the relative momentum; M is the mass of the bound state; μ_{R} is the relativistic reduced mass of the constituents, $b^2(M)$ is the on-mass-shell relative momentum in the center-of-mass system squared; $\Psi_{\mathrm{T}}(\mathbf{p})$ is the tetraquark wave functions.

The effective diquark–antidiquark interaction is the sum of the usual one–gluon exchange term (dominates at short distances) with the mixture of the long–range vector and scalar linear confining potentials (dominant at long distances), where the vector confining potential vertex contains the additional Pauli term. We also take into account the diquark internal structure treating constituent diquarks as non-pointlike objects (which interact with each other as a whole) by employing diquark–gluon form factors. The details of calculations can be found in Sec. 3 of Ref. [4].

3. Mass Spectra and Analysis

Tetraquarks have a very rich spectroscopy even within the diquark–antidiquark model. However, most of the predicted states can't be observed in experiments due to the fast fall–apart strong decays into a meson pair through the quark rearrangement. In order to determine the most relatively stable states we put a number of constraints. First, we consider diquarks only in the color–triplet state, as the color–sextet diquarks have the repulsive internal interaction potential which makes such bound state unlikely to exist (note that the inclusion of quark–antiquark cross interactions may stabilize this color configuration). Second, we consider only the ground state diquarks, since excitations within the diquark and/or antidiquark increase the overlap between them, thus, enhancing the probability of the decay into a meson pair. The calculated mass spectra of the doubly heavy tetraquarks are presented in Table 1. We roughly estimate the uncertainties to be around $20-50$ MeV from the comparison of meson and baryon mass spectra calculated in our model with experimental data.

Furthermore, in order to analyze the possibility of the experimental observation for each state it is useful to compare the calculated tetraquark masses with the corresponding decay thresholds into a pair of mesons composed from the same quarks and antiquarks as the initial tetraquark. The fall–apart strong decay is the main channel if it is energetically possible and allowed by the quantum numbers. If this process is forbidden, the main channels are either quark–antiquark annihilation into gluons (OZI–suppressed) or radiative decays. These tetraquark states can be observed experimentally in other decay channels as narrow resonances. Thus our main goal is the identification of such states.

The analysis shows that for almost all of the considered states of doubly heavy tetraquarks there is a possibility of a fall–apart strong decay into a pair of mesons. However, a few states (most of which are first orbital excitations) lie below such heavy–light+heavy–light meson pair thresholds. Thus, these states are the main candidates for the experimental observation.

Our results are in agreement with the lattice QCD calculations for the masses of doubly charmed [8] and doubly bottom [9] tetraquarks with $I(J^P) = 0(1^+)$. The other popular approach is the meson–meson molecular model [10].

In 2021 LHCb Collaboration observed doubly charmed charged tetraquark state $T_{cc}(3875)^+$ [11]. Its quark content is determined to be $cc\bar{u}\bar{d}$, with spin–parity $J^P = 1^+$ and mass $M = 3874.74 \pm 0.10$ MeV. Our predictions for the ground state of the $A\bar{S}$–configuration of the $cc\bar{q}\bar{q}$ tetraquark with the proper spin–parity and mass $M = 3869 \pm 50$ MeV agrees with these data.

Table 1: Masses $M_{\mathrm{QQ}'\overline{\mathrm{qq}}'}$ of the ground and excited states of doubly heavy tetraquarks. Here A, S are the axialvector and scalar diquarks and antidiquarks, respectively, n_r is the radial quantum number, S is the total diquark–antidiquark spin. All masses are given in MeV.

$\mathrm{d}\overline{\mathrm{d}}'$	nL	n_r	L	S	J^P	Q, Q$'$	$M_{\mathrm{QQ}'\overline{\mathrm{ss}}}$	$M_{\mathrm{QQ}'\overline{\mathrm{sq}}'}$	$M_{\mathrm{QQ}'\overline{\mathrm{qq}}'}$
A$\overline{\mathrm{A}}$	1S	0	0	0	0^+	Q = c, Q$'$ = c	4308	4161	3983
				1	1^+		4330	4188	4017
				2	2^+		4368	4233	4074
	1P	0	1	1	0^-		4726	4603	4460
				0	1^-		4701	4574	4423
				1			4734	4613	4472
				2			4754	4636	4500
				1	2^-		4716	4591	4443
				2			4753	4635	4498
				2	3^-		4738	4617	4474
	2S	1	0	0	0^+		5013	4897	4760
				1	1^+		5015	4899	4764
				2	2^+		5016	4901	4768
	1D	0	2	2	0^+		5132	5029	4915
				1	1^+		5111	5004	4881
				2			5137	5035	4923
				0	2^+		5082	4968	4832
				1			5119	5013	4893
				2			5146	5047	4939
				1	3^+		5092	4980	4846
				2			5131	5028	4911
				2	4^+		5105	4995	4864
	2P	1	1	1	0^-		5367	5265	5148
				0	1^-		5325	5215	5085
				1			5366	5265	5149
				2			5394	5297	5188
				1	2^-		5332	5223	5095
				2			5377	5277	5163
				2	3^-		5343	5236	5111
	3S	2	0	0	0^+		5663	5567	5456
				1	1^+		5660	5565	5454
				2	2^+		5653	5556	5445
A$\overline{\mathrm{S}}$	1S	0	0	1	1^+			4093	3869
	1P	0	1		0^-			4493	4291
					1^-			4500	4299
					2^-			4514	4316
	2S	1	0		1^+			4801	4622
	1D	0	2		1^+			4900	4721
					2^+			4909	4733
					3^+			4923	4750
	2P	1	1		0^-			5162	5000
					1^-			5162	5000
					2^-			5162	5000
	3S	2	0		1^+			5479	5332
A$\overline{\mathrm{A}}$	1S	0	0	0	0^+	Q = c, Q$'$ = b	7634	7499	7337
				1	1^+		7645	7511	7353
				2	2^+		7665	7535	7381
A$\overline{\mathrm{S}}$				1	1^+			7410	7198
S$\overline{\mathrm{A}}$							7649	7516	7361
S$\overline{\mathrm{S}}$				0	0^+			7403	7191
A$\overline{\mathrm{A}}$				0	0^+	Q = b, Q$'$ = b	10896	10763	10606
				1	1^+		10904	10773	10618
				2	2^+		10919	10790	10638
A$\overline{\mathrm{S}}$				1	1^+			10670	10461

4. Conclusion

We calculated mass spectra of the doubly heavy tetraquarks within the framework of the relativistic quark model: ground states for all flavor com-

positions and additionally orbitally and radially excited states for the doubly charmed states. The important feature of our calculations is the consistent account of the relativistic effects and the finite size of the diquark.

The calculated tetraquark masses are compared with the thresholds of the fall–apart strong decays into a pair of two heavy–light mesons. Most of the tetraquark states lie significantly above such thresholds. However, a few tetraquark states are found to lie slightly above or even below such thresholds. As a result, these states are most likely to be observed as narrow resonances in other decay modes. Additionally, our results agree with the LHCb data on the doubly charmed tetraquark $T_{cc}(3875)^+$. More experimental data on exotic states are awaited.

Funding

The work of Elena M. Savchenko was supported in part by the Foundation for the Advancement of Theoretical Physics and Mathematics "BASIS" grant number 22-2-10-3-1.

Conflict of interest

The authors declare no conflict of interest.

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