# Electron-positron pair creation in asymmetric collisions of heavy nuclei

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Previously developed two-center approach for calculations of electron-positron pair creation in collisions of two identical heavy nuclei is extended to nonsymmetric collisions where the colliding nuclei differ from each other. The time-dependent Dirac equation for the positron wave function is solved numerically with the help of the generalization pseudospectral method in modified prolate spheroidal coordinates and Crank-Nicolson algorithm for time propagation. For the case of the asymmetric $U^{92+} - Cm^{96+}$ collisions, we calculate total probabilities of electron–positron pair creation, as well as angle-integrated and angle-resolved energy distributions of the emitted positrons. Characteristic signatures of the supercritical regime are found in total probabilities and energy distributions. The angular distributions show high degree of isotropy.

## Introduction

The search for experimental opportunities to observe nonperturbative effects of strong-field quantum electrodynamics remains one of the principal challenges of modern physics. Direct experimental observation of such effects is hindered by the high critical field strength, on the order of $10^{16}\,\mathrm{V/cm}$. Although rapid advances in high-power laser technologies have brought the peak field intensities closer to this limit, it is still not possible to create truly supercritical fields in laboratory settings.

An alternative route toward exploring the strong-field regime involves slow collisions of heavy nuclei. When the combined nuclear charge $Z = Z_1 + Z_2$ exceeds the critical value $Z_{\mathrm{cr}} \approx 173$, at internuclear distances smaller than the critical value $R_{\mathrm{cr}}$ the lowest bound quasimolecular state dives into the negative-energy continuum, opening up the channel of spontaneous electron–positron pair creation [1]. The main challenge for experimental observation of spontaneous pair creation in heavy ion collisions is strong masking of this channel by the dynamical pair creation, caused by the time-dependent electromagnetic field of the moving nuclei.

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In the works of St. Petersburg theoretical research group (see, e.g., [2–4] and references therein) it was shown that signatures of spontaneous pair creation can be observed in collisions with fixed minimal internuclear distance $R_{\min}$ and varying scaled collision energy $\varepsilon = E/E_0$ ($\varepsilon \geq 1$), where $E$ is the collision energy and $E_0$ is the energy of the head-on collision. In supercritical collisions, the total electron-positron pair creation probability increases as $\varepsilon \to 1$. This indicates an increasing contribution of the spontaneous pair creation channel since dynamical channel contribution diminishes due to slower nuclear motion.

In our previous works (see [4] and references therein), a two-center approach was developed to solve the time-dependent Dirac equation in prolate spheroidal coordinates for symmetric collisions of identical nuclei. The present work extends the previous studies by generalizing the method to asymmetric collisions. Motivated by the planned collision experiments at the NICA facility (JINR), in the present paper we calculate the total pair creation probabilities, as well as angle-integrated and angle-resolved energy spectra for $U^{92+}$–$Cm^{96+}$ collisions. The $U^{92+}$–$Cm^{96+}$ collisions were previously studied within the monopole approximation [2]. In the present work, the angular and energy distributions of emitted positrons are calculated by solving the two-center time-dependent Dirac equation using the generalized pseudospectral method in prolate spheroidal coordinates. Atomic units ($\hbar = |e| = m_e = 1$) are used throughout the paper.

## Methods

The theoretical and computational methods of the present study extend the approach applied for symmetric collisions in our previous work (see, e.g., [3, 4]). Here we only briefly outline the key points of the methods, more details will be published elsewhere. The collision of two different bare nuclei is analyzed within the Dirac hole picture framework for positrons. In this approach, the lower Dirac continuum and discrete states form the filled positron vacuum (Dirac sea). Electron–positron pair creation occurs either through interaction with an external time-dependent field or spontaneously when a positron vacuum state crosses into the upper continuum. The dynamics is governed by the time-dependent Dirac equation (TDDE) for a positron in the field of two moving nuclei:

$$i\frac{\partial}{\partial t}\Psi(\mathbf{r},t) = \left[c(\boldsymbol{\alpha}\cdot\mathbf{p}) + c^2\beta + U_n(|\mathbf{r}-\mathbf{a}(t)|) + U_n(|\mathbf{r}+\mathbf{a}(t)|)\right]\Psi(\mathbf{r},t), \quad (1)$$

where $\Psi(\mathbf{r},t)$ is the four-component positron wave function, $\boldsymbol{\alpha}$ and $\beta$ are the Dirac matrices, $c$ is the speed of light, and the vector $\mathbf{a}(t)$ is directed along the instantaneous internuclear axis with its length equal to one half of the internuclear distance. The potential $U_n(r)$ in Eq. (1) is spherically symmetric and represents the static electric potential of the nucleus within the extended nucleus model. We perform calculations in the rotating frame of reference using

modified prolate spheroidal coordinates $(\lambda, \eta, \phi)$ (see, e.g., [3]) which are well suited for two-center quantum systems. The time-dependent Dirac equation is solved numerically using the generalized pseudospectral method in modified prolate spheroidal coordinates. Time propagation is performed with the Crank-Nicolson scheme for 16 highest-lying discrete vacuum positron states. At the end of the time propagation the unbound positron wave packet is projected onto the relativistic plane waves to obtain the energy–angle distributions of outgoing positrons.

## Results

In the present work, the total pair creation probabilities, as well as angle-integrated and angle-resolved energy distributions of outgoing positrons are calculated for $U^{92+}$–$Cm^{96+}$ collisions with $R_{\min} = 17.5$ fm at scaled collision energies $\varepsilon = 1.0$, 1.1 and 1.2. In Table 1 the total pair creation probabilities are presented. The data in Table 1 were also confirmed by the calculations using a totally different method, namely, the one-center multipole expansion of the nuclear potential (for the details of this method see, e.g., [5]). As can be seen from the table, the total probability of electron–positron pair production decreases with increasing collision energy. This behavior is governed by the high contribution of the spontaneous pair-creation mechanism in supercritical collisions: the lower the scaled collision energy, the larger the time spent in the supercritical regime where the spontaneous mechanism contributes. Therefore, the total electron-positron pair creation probability is increased.

*Table* 1. The total probabilities of pair creation for $U^{92+}$–$Cm^{96+}$ collisions with $R_{\min} = 17.5$ fm at $\varepsilon = 1.0$, 1.1 and 1.2.

| $\varepsilon$ | Total probability |
|---|---|
| 1.0 | $2.236\times10^{-2}$ |
| 1.1 | $2.162\times10^{-2}$ |
| 1.2 | $2.102\times10^{-2}$ |

Fig. 1 shows angle-integrated energy distributions of emitted positrons. The energy distributions further support the supercritical signatures found for the total probabilities: the peak magnitude of the spectra decreases with increasing scaled collision energy.

In Fig. 2 the angle-resolved energy distributions of the emitted positrons are shown in the collision plane $z$-$x$ for the head-on collision ($\varepsilon = 1.0$). The pattern in Fig. 2 exhibits a high degree of isotropy. This behavior can be attributed to the fact that electron–positron pair creation occurs at extremely small internuclear distances, where the quasimolecular configuration formed by the two colliding nuclei can be treated as nearly spherically symmetric.

In the supercritical regime, the angular patterns display qualitatively similar behavior, with significant differences arising primarily in the intensity of

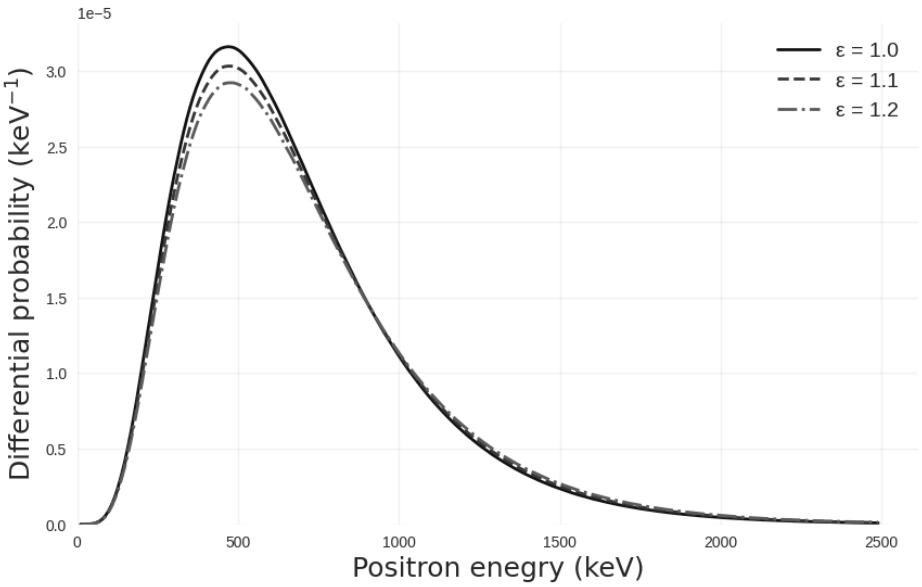


Fig. 1. Angle-integrated energy distribution of emitted positron for $U^{92+}$–$Cm^{96+}$ collisions with $R_{\min} = 17.5$ fm at $\varepsilon = 1.0$, 1.1 and 1.2.

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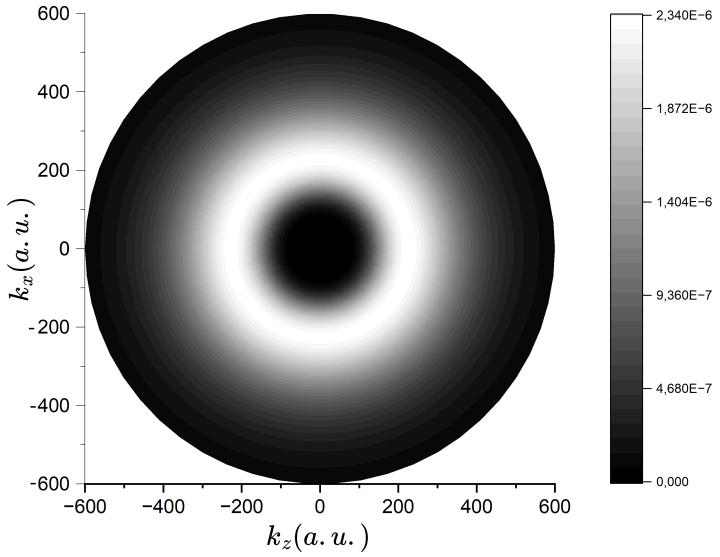


Fig. 2. Pseudocolor plot of positron energy and emission angle distribution in the $k_z$–$k_x$ momentum plane for $U^{92+}$–$Cm^{96+}$ collisions at $R_{\min} = 17.5$ fm and $\varepsilon = 1.0$. The color scale represents the positron emission probability density in units of keV$^{-1}$.

the maximum, with higher intensities for lower $\varepsilon$. This observation is consistent with the signatures found for the total probabilities as well as for the angle-integrated energy spectra.

## Conclusion

In the present work, the two-center approach for calculations of electron-positron pair creation in collisions of heavy nuclei is extended to nonsymmetric collisions where the colliding nuclei differ from each other. The total probabilities, as well as angle-integrated and angle-resolved energy distributions of emitted positrons for the supercritical $U^{92+}$–$Cm^{96+}$ collisions have been calculated. Our results show clear signatures of the supercritical regime in agreement with those found in our previous works. The angle-resolved energy distribution show no significant anisotropy for the studied collisions.

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## Conflict of interest

Authors declare that they have no conflicts of interest.

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