FPGA implementation of shape fitting algorithm for signals with electron-photon and hadron components for reconstruction in the Belle II electromagnetic calorimeter

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Abstract. This paper describes hardware implementation of pulse shape discrimination algorithm into the current detector electronics with no need in major upgrade. According to this algorithm modeling and calculations of signal amplitude, time, photon and hadron contributions into the waveform are presented.

Introduction

Recently applied in Belle II electromagnetic calorimeter (ECL) is a novel approach to discriminate efficiently hadronic and electromagnetic showers using pulse shape discrimination with CsI(Tl) scintillator crystals [1]. Since the start of the Belle II experiment in 2018, ECL data acquisition system continues to operate stably. Electronics of the electromagnetic calorimeter include ShaperDSP modules with Field-Programmable-Gate-Array (FPGA) on board Xilinx Spartan 3 processing the digitized signal from CsI(Tl) counters and perform waveform analysis evaluating the impact of only electromagnetic showers by measuring amplitude and time of the signal. Currently, the hadron identification is made offline and only for a part of events due to the lack of ECL data acquisition system bandwidth. Implementation of this algorithm in current FPGA allows reducing a load on data acquisition system of the detector. Thus, the upgrade of FPGA firmware is ongoing to make the signal reconstruction of scintillation emission both electromagnetic and hadronic showers in the online processing.

Fitting algorithm

The particles in hadronic showers have higher ionization density compared to electromagnetic showers. And in CsI(Tl) crystals higher ionization density results in a scintillation pulse with a faster decay time. This fast scintillation component is therefore called the hadron component in this article. Fitting the signal by sum of electron-photon and hadron components allows separating this hadron fraction. Pulse shape discrimination uses fit algorithm with χ^2 minimization.

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Sampled waveform for digital processing is shown in Figure 1. Points from 0 to 16 are used to evaluate the analog-to-digital converter (ADC) pedestal value, 17 to 31 are signal points.

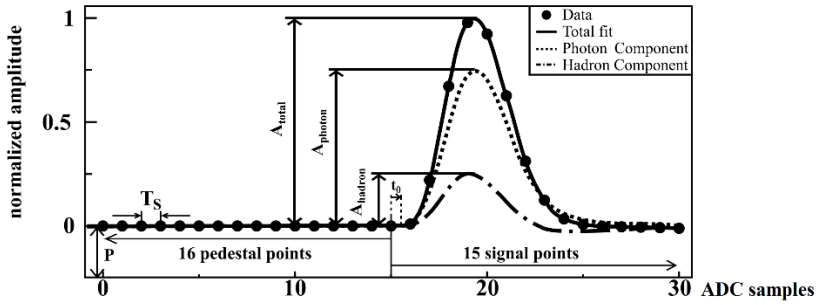

Figure 1. Pulse shape fit with electron-photon and hadron components.

Parallel reconstruction of incoming signal is performed with two fit functions (1) and (2). Function (1) is used to obtain electron-photon contribution to the signal, fit function (2) is used for hadron identification.

$$F(t) = AF_{photon}(t_i - t_0) + P, \tag{1}$$

where fit parameters A – signal amplitude, $F_{photon}\ (t)$ – response function of ECL electronics, t_0 – signal start time, P – ADC pedestal value.

$$F2(t) = A_{photon}F_{photon}(t_i - t_0) + A_{hadron}F_{hadron}(t_i - t_0) + P \tag{2}$$

where $F_{photon}(t_i)$, $F_{hadron}(t_i)$– response functions of photon-like pulse shape and hadron-like pulse shape of ECL electronics, A_{photon}, A_{hadron} – amplitudes of photon and hadron component shown on the Figure 1 and it defines as $A_{photon} = A_1 + A_2$, $A_{hadron} = A_1 - A_2$, $A_1 = (A_{photon} + A_{hadron})/2$, $A_2 = (A_{photon} - A_{hadron})/2$, t_0 – signal start time, P– ADC pedestal value.
Minimization is performed to reconstruct A and t_0 from sampled values y_i [2]:

$$\chi^2 = (y_i - AF(t_i - t_0) - P)S_{ij}^{-1}(y_j - AF(t_j - t_0) - P) \rightarrow min, \tag{3}$$

$$\chi^2 = (y_i - AF2(t_i - t_0) - P)S_{ij}^{-1}(y_j - AF2(t_j - t_0) - P) \rightarrow min, \tag{4}$$

where $S_{ij}^{-1} = \overline{(y_i - \overline{y_i})(y_j - \overline{y_j})}$ is the pre-determined noise covariance matrix. For minimization a time grid with small time step of $\delta t = T_s/96$ is made. The sampling period is $T_s = 567$ ns.

Linearization of the signal samples is performed with a small time shift Δt at the grid points where values of fit functions and its derivatives are pre-calculated and tabulated (F_i^k, F'^k_i). The same is for the tabulated function for equation (2). More details of the current signal reconstruction algorithm are described in [2, 3]. All the equations for hadron component reconstruction are taken from [4].

$$AF(iT_s + k\delta t) \approx AF(iT_s + k\delta t) + A\Delta tF'(iT_s + k\delta t) = AF_i^k + A\Delta tF_i'^k, \qquad (5)$$

$$A_xF_x(t_i - t_0) = A_xF^k_{x\,i} + A_x\Delta tF'^k_{x\,i}, x \in \{photon, hadron\}, , i \in [1,16], k \in [-96,96), \quad (6)$$

After that linearization equation (4) transforms to:

$$\chi^2 = (y_i - A_1F^k_{+i} - A_1\Delta tF'^k_{+i} - A_2\Delta tF'^k_{-i} - A_2F^k_{-i} - P)S^{-1}_{ij}(y_j - A_1F^k_{+j} - A_2F^k_{-j} -$$

$$- A_1\Delta tF'^k_{+j} - A_2\Delta tF'^k_{-i} - P) \to min, \qquad (7)$$

where $F^k_{+i} = (F^k_{photon\,i} + F^k_{hadron\,i})$, $F^k_{-i} = (F^k_{photon\,i} - F^k_{hadron\,i})$. Since a summand $A_2\Delta tF'^k_{-i}$ and $A_2\Delta tF'^k_{-j}$ makes the small contribution to the equation, it can be neglected [4]. Thus 4 parameters are obtained A_1, $A_1\Delta t$, A_2 and P. In this case, χ^2 minimization of equations is equivalent to solving the system of linear equations (8, 9) resulting in amplitude, pedestal and new time value for the signal:

$$\begin{pmatrix} A \\ B \\ P \end{pmatrix} = \begin{pmatrix} \Sigma_{ij}F_i^kS^{-1}_{ij}F_i^k & \Sigma_{ij}F_i^kS^{-1}_{ij}F_i'^k & \Sigma_{ij}F_i^kS^{-1}_{ij} \\ \Sigma_{ij}F_i'^kS^{-1}_{ij}F_i^k & \Sigma_{ij}F_i'^kS^{-1}_{ij}F_i'^k & \Sigma_{ij}F_i'^kS^{-1}_{ij} \\ \Sigma_{ij}S^{-1}_{ij}F_i^k & \Sigma_{ij}S^{-1}_{ij}F_i'^k & \Sigma_{ij}S^{-1}_{ij} \end{pmatrix} \begin{pmatrix} \Sigma_{ij}F_i^kS^{-1}_{ij}y_i \\ \Sigma_{ij}F_i'^kS^{-1}_{ij}y_i \\ \Sigma_{ij}S^{-1}_{ij}y_i \end{pmatrix} = \begin{pmatrix} \Sigma_iFG31^k_iy_i \\ \Sigma_iFG32^k_iy_i \\ \Sigma_iFG33^k_iy_i \end{pmatrix}, \qquad (8)$$

$$\begin{pmatrix} A_1 \\ A_2 \\ B \\ P \end{pmatrix} = \begin{pmatrix} \Sigma_iFG51^k_iy_i \\ \Sigma_iFG52^k_iy_i \\ \Sigma_iFG53^k_iy_i \\ \Sigma_iFG33^k_iy_i \end{pmatrix}. \qquad (9)$$

Trigger time $t_0 = t_{tr}$ is used as the first approximation. For this time the initial grid point number $k = t_{tr}/\delta t$. On the next iteration, new estimated time t_{new} and grid point $k = t_{new}/\delta t$ are obtained. Three iterations are enough for the fit algorithm to converge. The algorithm output data

contain total reconstructed amplitude, amplitude of photon component and a fraction of hadron component into the signal:

$$Hadron\ Fraction = \frac{A_{hadron}}{A_{photon} + A_{hadron}} = \frac{A_{hadron}}{A_{total}}. \quad (10)$$

ECL electronics and FPGA implementation

The analog signals from scintillation counters of Belle II detector go to the input of shaper and digital signal processor module (ShaperDSP). It performs both pulse shaping, digitization and signal reconstruction. Reconstruction results and waveform data are sent to the ECL data acquisition system. Detailed description of the calorimeter electronics is presented in reference [2]. Firmware with algorithm for signal reconstruction is highly parallelized. This structure was inherited from the existing firmware [4]. Implementation of algorithm described in the previous section is based on this firmware written in VHDL.

The input data of the FPGA are arrays of coefficients (8, 9) from double data rate synchronous dynamic random access memory (DDR SDRAM) and ADC samples that are stored in a buffer. The size of ADC buffer is 512 samples for each channel. Fitting procedure in FPGA is initiated by the trigger signal. After that ShaperDSP retrieves the pre-calculated coefficients. Sampled ADC data is multiplied by the coefficients from tables according to current iteration in the algorithm described above. Then it is accumulated in hardware multiply-accumulate block. After that, the evaluation of data quality is made with the calculated χ^2 function and the output firmware data is transferred to the data acquisition system. Reconstructed amplitude is recorded in ADC units and one ADC unit $\approx 0.05\ MeV$.

Modification of existing FPGA project required to add additional tables for hadron fit in ShaperDSP, packaging of reconstructed amplitude, time and hadron intensity in firmware. The bottleneck of implementation on the current FPGA was limited by hardware resources of Spartan 3. So it was decided to use look up tables (LUTs) for multiplication instead of hardware multiply-accumulate block. Therefore, firmware was developed and logic was fully routed.

Test environment

Test environment for developed firmware includes a RTL test bench for Questa Simulator and software written in C++ and emulating the reconstruction algorithm [3]. This is the first step of firmware verification.

Software provides files with ADC data, array of coefficients, threshold values for simulation according to available real events collected on Belle II. Then the software fitting algorithm starts as

well as Questa simulation. Firmware output data and software output data are compared. The results of emulator and firmware match each other.

Conclusion

Implementation of shape fitting algorithm for hadron identification has been done. For now, tests of the developed firmware are carried out on the detector electronics test bench located in KEK.

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