MEASUREMENT OF MOMENTUM SPREAD IN SPACE CHARGE-DOMINATED ION BEAMS

D. Chumakov,* V. Lebedev

Joint Institute for Nuclear Research, Dubna, 141980, Russia,

**e-mail:* chumadan@jinr.ru

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Аннотация. Поле пространственного заряда интенсивных нерелятивистских ионных пучков оказывает влияние на их динамику, искажая форму потенциальной ямы, заданной группирующим ВЧ напряжением и, соответственно, воздействует на продольную плотность пучка. В равновесном состоянии фазовая плотность пучка определяется равенством мощности силы электронного охлаждения и мощности нагрева пучка за счет внутрипучкового рассеяния. Форма продольных фазовых траекторий такого пучка искажена полем пространственного заряда. В этой статье рассмотрено использование уравнения Хайссинского для нахождения равновесного импульсного разброса сгруппированного ионного пучка в применении к бустерному синхротрону ускорительного комплекса NICA.

Abstract. The space charge of intense non-relativistic ion beam affects its dynamics. For the longitudinal degree of freedom, the space charge force distorts the RF potential well and, consequently, affects the longitudinal beam density. In equilibrium the beam phase space density is set by balance between electron cooling force and beam heating due to intrabeam scattering. The shape of separatrix and phase space trajectories for such beam are modified by its self-field, which needs to be accounted in computation of the beam longitudinal density and momentum spread. In this paper, we consider usage of Haissinski equation for finding equilibrium momentum spread of bunched ion beam in application to Booster synchrotron of NICA accelerator complex.

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INTRODUCTION

The Booster synchrotron of NICA accelerator complex is designed to accumulate and accelerate up to 10^9 heavy ions to an energy of 600 MeV/u [1]. During test runs in 2025-2026, $^{124}Xe^{26+}$ ions were used for accumulation in Booster. Since longitudinal cooling is much faster that the transverse one, the preferrable accumulation scheme is in the longitudinal plane with multiple single-lap injections [2]. Electron cooling of ion beam decreases its emittance and, thus, increases its phase space density. That results in an increase of space charge field and subsequent distortion of the longitudinal beam density profile. An ability to reconstruct the phase space density and the beam momentum spread is of special importance for optimization of beam accumulation.

PARTICLE HAMILTONIAN WITH THE SPACE CHARGE

The Hamiltonian of particle longitudinal motion in presence of bunching voltage V is [3]:

$$H(\delta,\phi)=\frac{1}{2}h\omega_0\eta\delta^2+\frac{\omega_0 ZeV}{2\pi\beta^2 E_S}[\cos\phi_s-\cos\phi+(\phi_s-\phi)\sin\phi_s], \quad (1)$$

where $\delta = \frac{p - p_s}{p_s}$ is the relative difference of particle momentum p w.r.t. synchronous particle momentum p_s, ϕ is the particle phase, h is the bunching voltage harmonic, $\omega_0 = 2\pi \frac{\beta c}{C_a}$ is the particle revolution frequency, η is accelerator slip factor, Z is the ion charge number, $e = 1.6 \cdot 10^{-19}$ C is the elementary charge (SI), C_a is the synchrotron circumference, $\beta = v / c$ is the ratio of particle speed to speed of light, E_s is the synchronous particle total energy, $\phi_s = 0$ is the synchronous phase of bunching voltage (zero for accumulation in Booster).

It is convenient to transit to normalized Hamiltonian as presented in [4] with the addition of space charge potential term. For the normalization one should use the conjugate momentum $\zeta = \frac{h\eta\delta}{\nu_s}$, where $\nu_s = \sqrt{\frac{ZeVh\eta}{2\pi\beta^2 E_s}}$ is the synchrotron tune. The normalized Hamiltonian without space charge term is derived in details in [4], and has following form:

$$H(\delta, \phi) = \frac{\zeta^2(\delta)}{2} + [\cos\phi_s - \cos\phi + (\phi_s - \phi)\sin\phi_s]. \quad (2)$$

Detailed derivation of beam longitudinal space charge field and corresponding potential is outlined in [5]. The space charge driven potential part of Hamiltonian is:

$$U(\phi) = (Ze)^2 \omega_0 h^2 N_p \cdot \frac{\mathcal{Z}_L}{n} \cdot \rho(\phi), \quad (3)$$

where N_p is the total number of beam particles, $\frac{\mathcal{Z}_L}{n} = \mathrm{Im}[\frac{\mathcal{Z}_L}{n}]_{SC} = \frac{\mathcal{Z}_0 g_0}{\beta\gamma^2}$ is the imaginary part of longitudinal impedance due to beam space charge field [5], g_0 is the factor dependent on the shape of the beam and the beam chamber, $\mathcal{Z}_0 = \frac{1}{\varepsilon_0 c} = 377\ \Omega$ is the impedance of free space (SI), $\varepsilon_0 = 8.85 \cdot 10^{-12}$ F/m is the permittivity of free space (SI), $\rho(\phi)$ is the beam profile with the normalization:

$$\int_{-\pi}^{\pi} \rho(\phi) d\phi = 1. \quad (4)$$

For the elliptic vacuum chamber of NICA Booster with width $w = 128$ mm, height $h = 65$ mm and beam with radius $\sigma = 4.9$ mm, $g_0 = 0.783 + \ln\frac{h/2}{1.5\sigma} = 2.27$ was calculated in parallel plates approximation, while for round beam chamber with radius $b = (w + h)/4 = 48.5$ mm and same

beam, $g_0 = 0.25 + \ln\frac{b}{\sigma} = 2.54$. To be used in normalized Hamiltonian, Eq. (3) must be divided by ZeV. Resulting Hamiltonian for longitudinal motion of particle under the influence of RF bunching and space charge with $\kappa = \kappa(g_0) = \frac{Ze\omega_0 h^2 N_p}{V} \cdot \frac{\mathcal{Z}_L}{n}(g_0)$ is:

$$H(\delta,\phi) = \frac{\zeta^2(\delta)}{2} + [\cos\phi_s - \cos\phi + (\phi_s - \phi)\sin\phi_s] + \kappa\rho(\phi). \quad (5)$$

HAISSINSKI FIT FOR BEAM PROFILES

Use of electron, stochastic or laser cooling of ion beams allows to achieve equilibrium phase space density of beam $\psi(\phi,\delta,t)$ with its explicit form presented as the solution of Vlasov-Fokker-Planck equation [6]:

$$\frac{\partial\psi}{\partial t} + \frac{\partial\phi}{\partial t}\frac{\partial\psi}{\partial\phi} + \frac{\partial\delta}{\partial t}\frac{\partial\psi}{\partial\delta} = -\frac{\partial}{\partial\delta}\left(F_{EC}(\delta)\right)\psi + \frac{1}{2}\frac{\partial}{\partial\delta}\left(D(\delta,\phi)\frac{\partial\psi}{\partial\delta}\right), \quad (6)$$

where $F_{EC}(\delta)$ is the cooling force, $D(\delta,\phi)$ is the diffusion power.

In equilibrium, the phase space density function $\psi(\phi,\delta)$ as well as its projections $\rho(\phi)$, $\lambda(\delta)$ are stationary, i.e., do not depend on time. Solution of Eq. (6) implies knowledge of cooling force and diffusion which are not known with sufficient accuracy. However, the core of momentum distribution is quite close to the Gaussian one. Therefore, neglecting the distribution tails, we can decompose the particle distribution into the product of space and momentum distributions, $\psi(\phi,\delta) = \rho(\phi)\lambda(\delta)$, assuming the momentum distribution is Gaussian. In this case the spatial part of the stationary solution of VFP equation (6) is described by the Haissinski equation [7, 8]:

$$\rho(\phi) = \rho_0 \exp[-\frac{U(\phi)}{kT_L}], \quad (7)$$

where ρ_0 is found from the normalization condition (4), $U(\phi)$ is the potential part of longitudinal motion Hamiltonian (5) and $kT_L = \zeta^2(\sigma_\delta) = \left(\frac{h\eta\sigma_\delta}{\nu_s}\right)^2$ is the longitudinal beam temperature expressed via conjugate momentum dependent on rms relative momentum spread σ_δ.

During the beam cooling process, its phase space density is increased, accompanied with the potential well and separatrix flattening due to growing beam space charge field. This distortion, in turn, changes the shape of longitudinal beam profile, making it wider. In the absence of space charge and with linear RF force the equilibrium beam profile is gaussian [9].

For the limiting case of zero beam temperature and linear RF, i.e. sufficiently short bunch, the space charge field makes the beam profile to be parabolic [9] with total bunch length:

$$l_b = C_a \cdot \left(\frac{3 \cdot Ze \cdot N_p \cdot g_0}{8\pi^2 \varepsilon_0 C_a \cdot h \cdot V} \right)^{1/3} . \qquad (8)$$

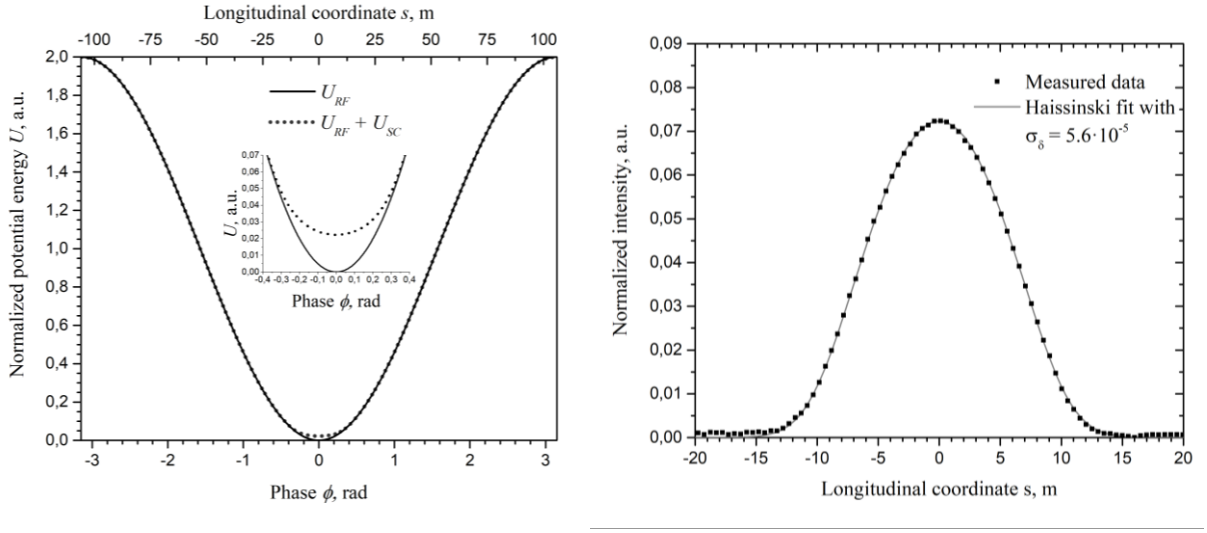


Fig. 1. Left panel: shape of longitudinal potential well with space charge of beam (dotted line) and without it (black). Right panel: measured longitudinal beam profile with electron cooling acting for 2 s (points) and corresponding Haissinski fit with eq. (7) (solid line).

Longitudinal beam profiles are measured in Booster using fast current transformer (FCT) manufactured by Bergoz Instrumentation [10, 11]. Measured longitudinal beam profiles are then averaged per 20 consecutive turns in order to reduce noise.

To compute the density distribution for the condition of Booster beam measurements we solve the Haissinski equation (7) using the method of successive approximations. We set initial guess function to be Gaussian-like Haissinski equation without space charge effect $(\kappa = 0)$ $\rho(\phi) = \rho_0 \exp\left(-\frac{1-\cos\phi}{\zeta^2(\sigma_\delta)} \right)$. At every iteration, the correction $f(\phi) = \exp\left(-\frac{1-\cos\phi + \kappa\rho(\phi)}{\zeta^2(\sigma_\delta)} \right)$ with weight $0 < \alpha < 1$ is added in such way that for every *i*-th iteration, the profile is $\rho^i(\phi) = \rho^{i-1}(\phi) \cdot (1-\alpha) + \alpha \cdot f(\phi)$. Fit is typically converged within 100 iterations. The fitting of Haissinski equation to measured beam profile is performed via least squares method for variable parameters $\left[\sigma_\delta, \kappa(g_0) \right]$. As one can see in the right panel of Fig. 1, there is excellent coincidence between measurements and computations. There are only two fitting parameters: rms momentum spread equal to $\sigma_\delta = 5.6 \cdot 10^{-5}$ and correction to beam emittance due to space

charge field in form of geometric factor equal to $g_0 = 2.23$, which matches theoretical estimate presented above with relative error of 2%.

SUMMARY

In this paper the method of momentum spread determination for equilibrium ion beams is presented. This method fits well for cooled ion beams in equilibrium state, and it can be used to find both the beam momentum spread and the effective impedance influencing longitudinal beam profile. For single injection of $2.1 \cdot 10^7$ $^{124}Xe^{26+}$ ions after 2 s of cooling, the relative momentum spread is set by balance of the cooling force and the intrabeam scattering diffusion of ions in bunch and is equal to $\sigma_\delta = 5.6 \cdot 10^{-5}$.

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