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JOINT INSTITUTE FOR NUCLEAR RESEARCH
ОБЪЕДИНЁННЫЙ ИНСТИТУТ ЯДЕРНЫХ ИССЛЕДОВАНИЙ

BLTP Bogoliubov Laboratory of Theoretical Physics

EMPIRICAL SYSTEMATICS FOR SPONTANEOUS FISSION AND ALPHA-DECAY HALF-LIVES OF HEAVY AND SUPERHEAVY NUCLEI

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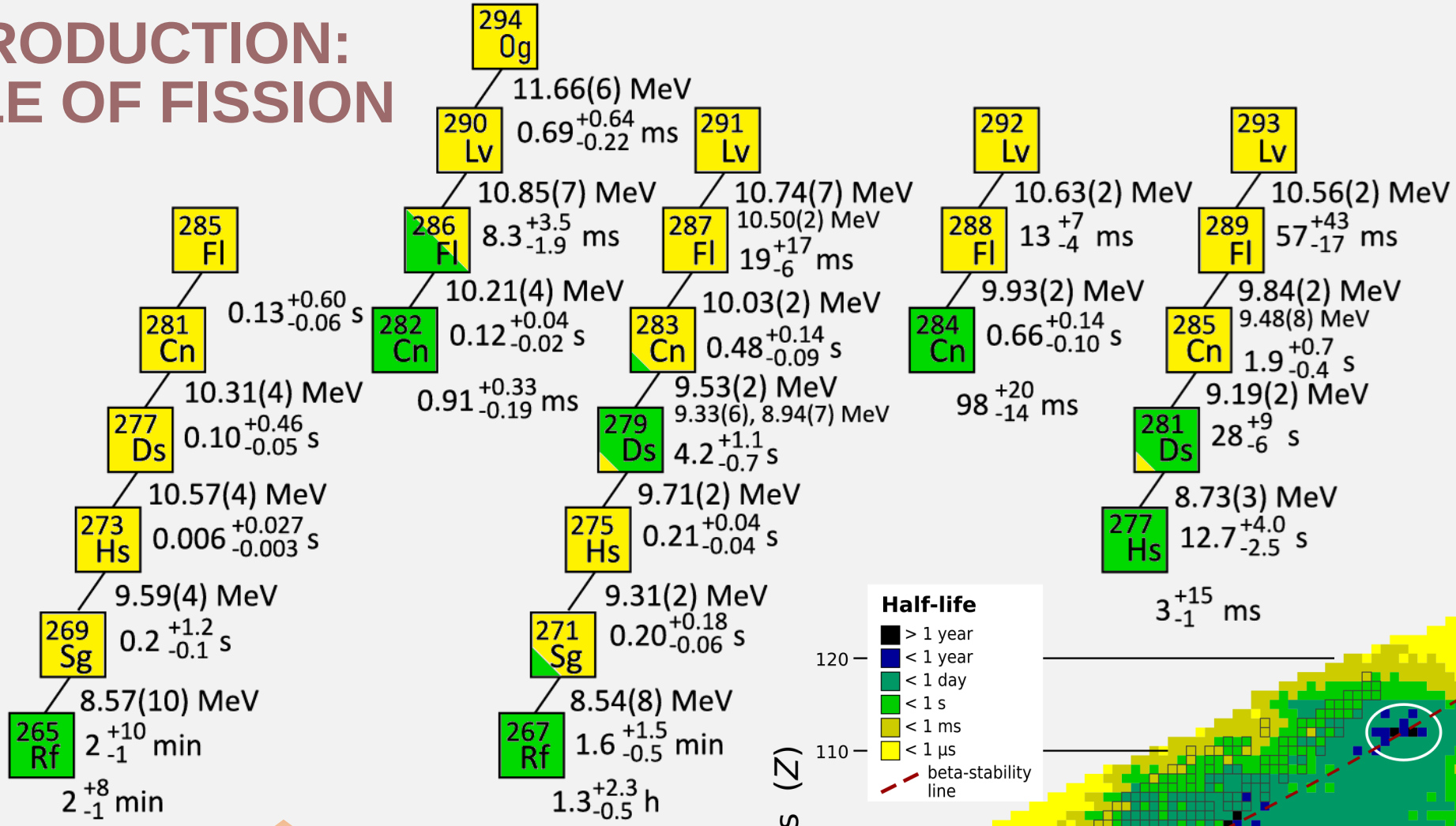
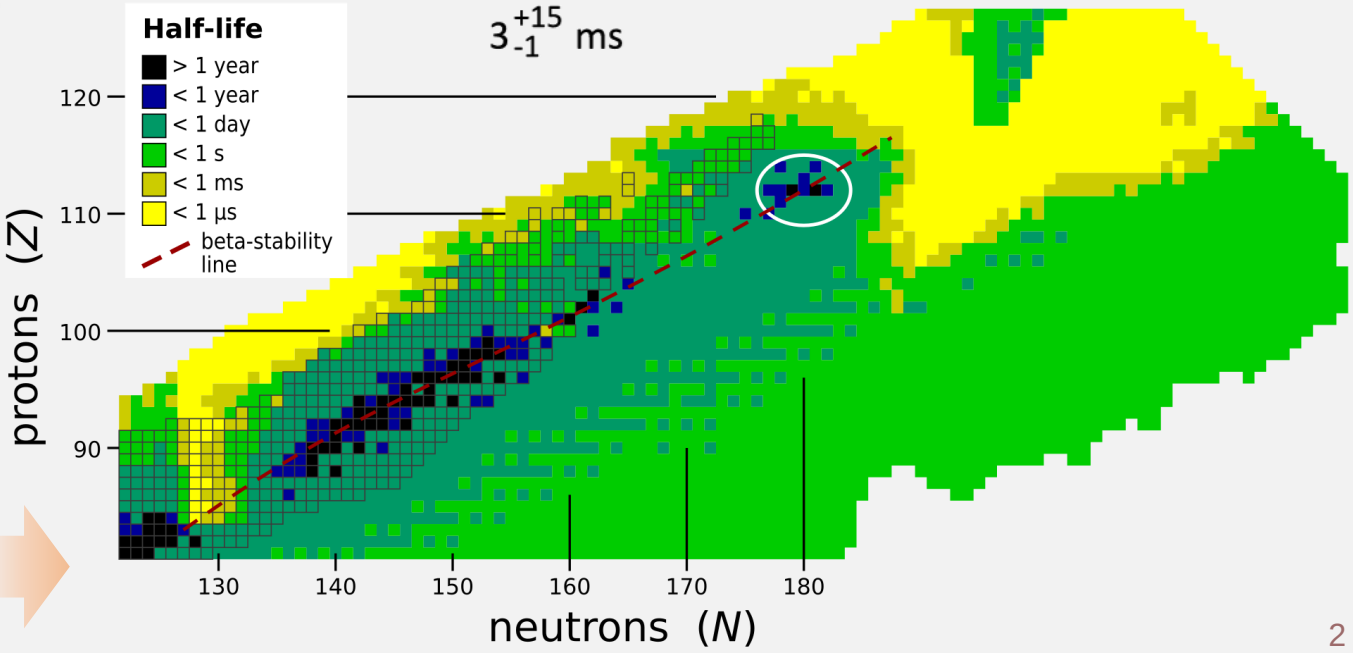
Dubna, 24–30 May 2026

INTRODUCTION: ROLE OF FISSION



Summary decay properties of the isotopes of even-Z elements synthesized in the reactions of ^{48}Ca with ^{238}U, ^{242,244}Pu, ^{245,248}Cm, and ^{249}Cf target nuclei.

Predicted half-lives of superheavy nuclides. The expected location of the island of stability around $Z = 112$ is circled.



THEORETICAL MODELS

MICROSCOPIC APPROACHES

- Fundamental approaches
- Mean Field (MF) method, Hartree-Fock method
- Relativistic Mean Field (RMF) method

MICROSCOPIC-MACROSCOPIC APPROACHES

- FRDM (Finite Range Droplet Model)
- FRLDM (Finite Range Liquide Droplet Model)
- WS (Weizsäcker-Skyrme)
- WS+RBF (WS + Radial Basis Functions)

PHENOMENOLOGICAL APPROACHES

- Local Mass Relations (LMR) method

PROBLEMS

complexity of the fission process

uncertainty in the height and shape of the fission barrier

analysis of inter-nucleon interaction potentials

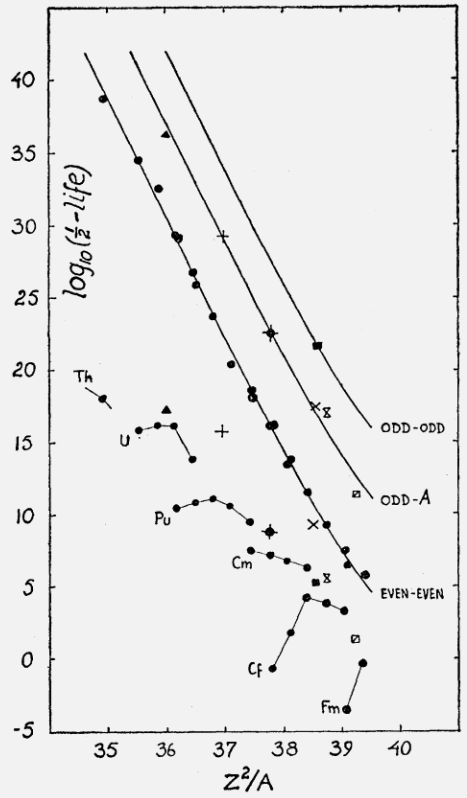
distribution of forces in a multi-nucleon system

difficulty of calculations

models do not always have sufficient accuracy

REVIEW OF FORMULAS FOR $T_{1/2}^{\mathrm{SF}}$

[Świątecki, 1955] [Dorn, 1961] [Viola & Seaborg, 1966] [Ren & Xu, 2005] [Xu & Ren, 2005] [Xu *at al.*, 2008] [Santhosh *at al.*, 2010] [Karpov *at al.*, 2012] [Zdeb *et al.*, 2014] [Bao *et al.*, 2015] [Santhosh & Nithya, 2016] [Yuan *et al.*, 2024] [Babu *et al.*, 2024] [Xie *at al.*, 2025]



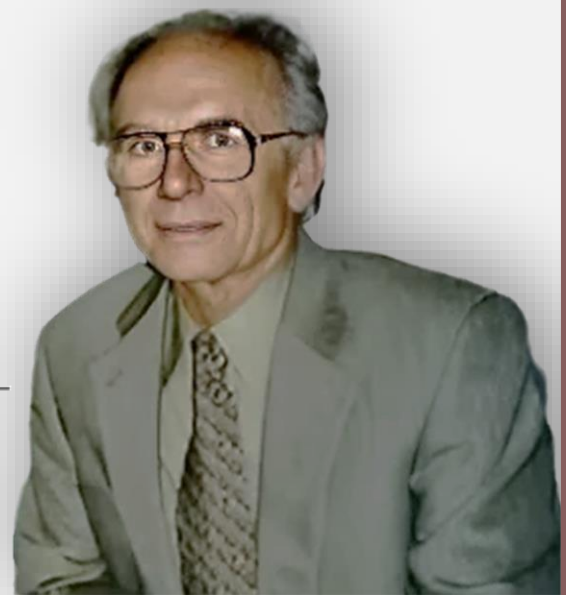
$$\log_{10} T_{1/2}^{\mathrm{SF}}\ (\text{years}) = -7.8\theta + 0.35\theta^2 + 0.073\theta^3 - (5-\theta)\delta M + \begin{cases} 18.2, & \text{even-even}, \\ 24.8, & \text{odd-}A, \\ 29.7, & \text{odd-odd}, \end{cases}$$

$$\theta = (Z^2/A) - 37.5,$$

$$\delta M = M_{\mathrm{exp}} - M_{\mathrm{ref}},$$

$$M_{\mathrm{ref}} = 1000A - 8.3557A + 19.120A^{2/3} + 076278\frac{Z^2}{A^{1/3}} + 25.444\frac{(N-Z)^2}{A} + 0.420(N-Z).$$

Władysław J. Świątecki
(1926 – 2009)



AIM:

TO EXPLORE A **NOVEL APPROACH TO THE SYSTEMATICS OF SPONTANEOUS FISSION HALF-LIVES**

OBJECTIVES:

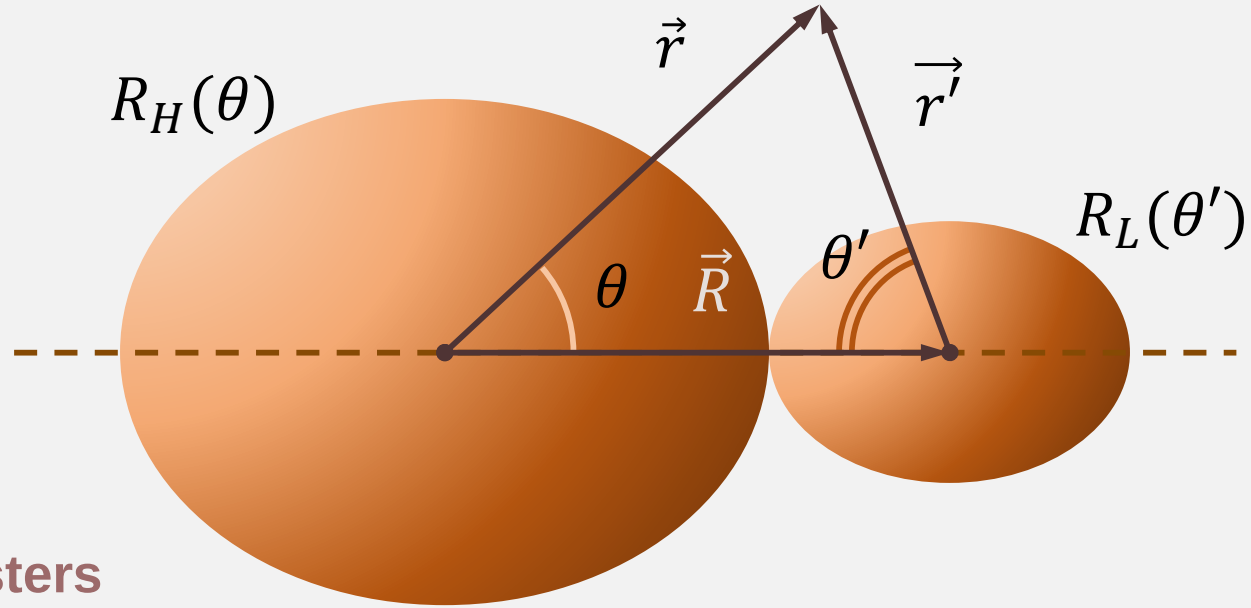
REVIEW OF FORMULAS FOR THE SPONTANEOUS FISSION HALF-LIFE OF HEAVY AND SUPERHEAVY NUCLEI

EXAMINATION OF THE SYSTEMATICS OF SPONTANEOUS FISSION HALF-LIVES

DEVELOPMENT OF AN **EMPIRICAL FORMULA** FOR THE SPONTANEOUS FISSION HALF-LIFE OF HEAVY AND SUPERHEAVY NUCLEI

OBTAINING ESTIMATES OF SPONTANEOUS FISSION HALF-LIVES FOR EXPERIMENTALLY UNKNOWN NUCLEI

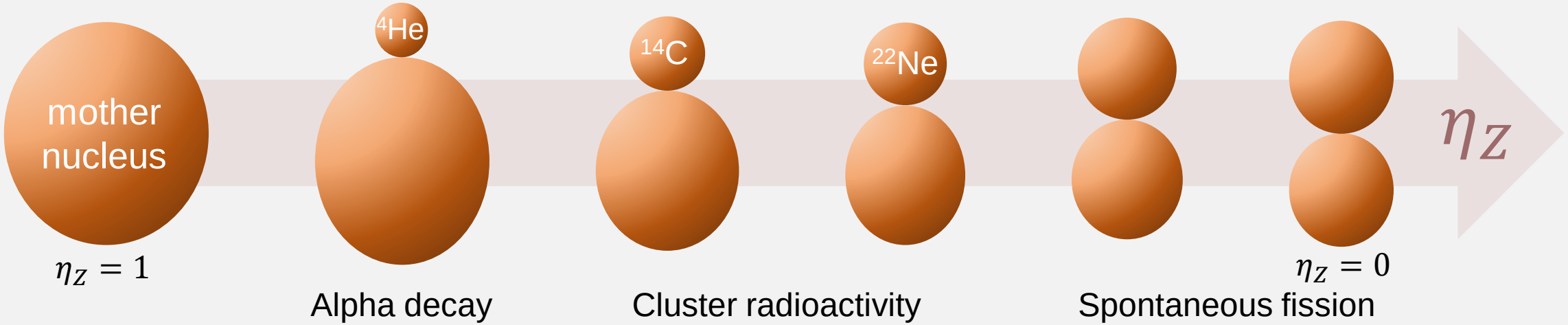
DNS



Schematic representation of the DNS in the case of axially symmetric mutual arrangement of clusters

Characteristics of DNS:

- relative **distance $\vec{R}$ between clusters**
- **Euler angles**
- **charge asymmetry** $\eta_Z = \frac{Z_H - Z_L}{Z_H + Z_L}$, Z_H and Z_L are charge numbers of the heavy and light clusters



[Adamian, G., Antonenko, N., & Scheid, W. (2012). Clustering effects within the dinuclear model. *Clusters in Nuclei, Vol. 2*, 165-227.]

CORRELATION BETWEEN α-DECAY AND SF HALF-LIVES

$$\log_{10}\frac{T^{\mathrm{SF}}_{1/2}}{T_{\mathrm{c}}} = f\log_{10}\frac{T^{\alpha}_{1/2}}{T_{\mathrm{c}}} + c$$

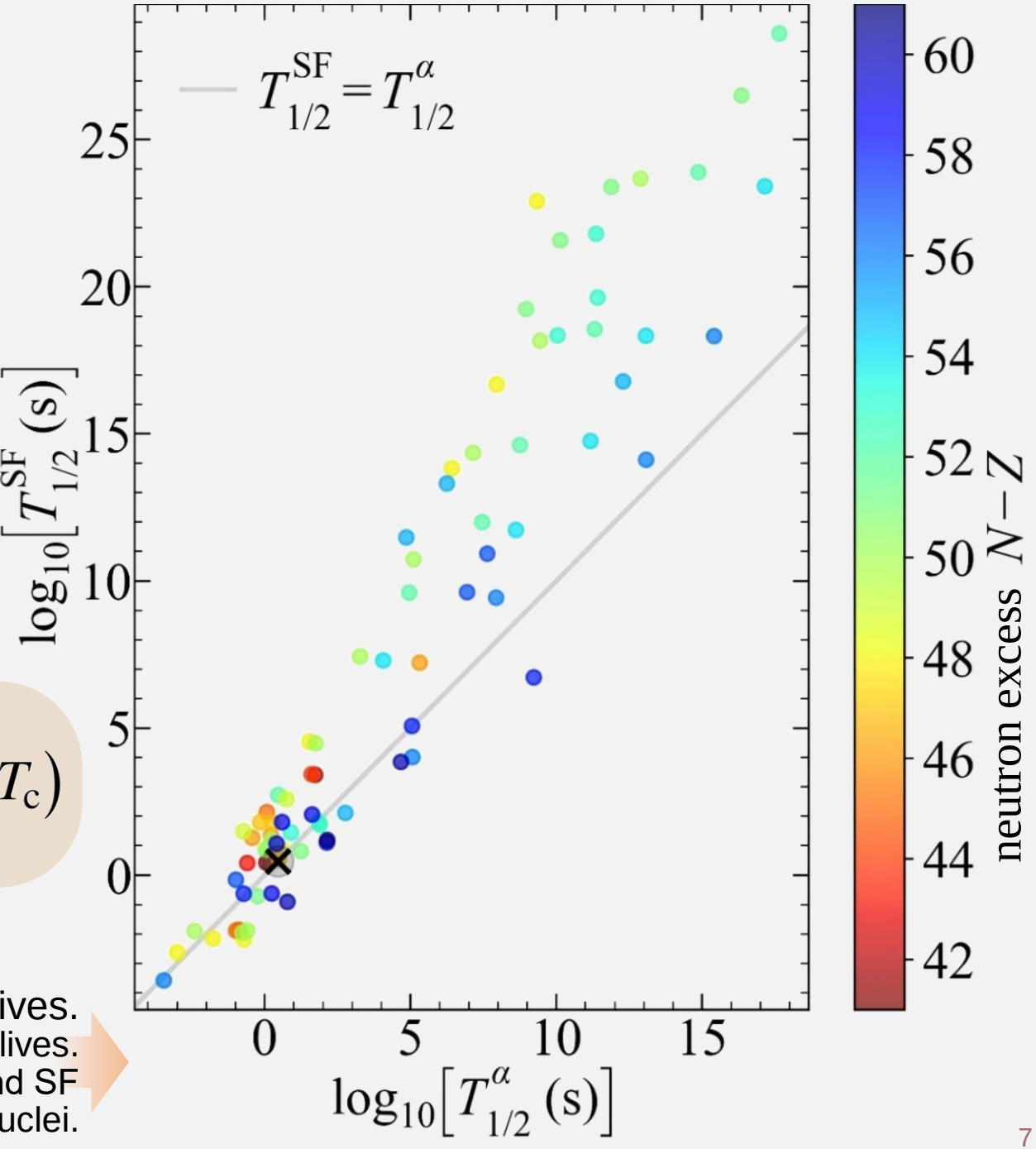
$$c = 0$$

point of strongest competition between α-decay and SF:

$$T^{\mathrm{SF}}_{1/2} \approx T^{\alpha}_{1/2} \approx T_{\mathrm{c}}$$

$$T_{\mathrm{c}} \approx 1\ \mathrm{s}$$

$$f = \frac{\log_{10}\left(T^{\mathrm{SF}}_{1/2}/T_{\mathrm{c}}\right)}{\log_{10}\left(T^{\alpha}_{1/2}/T_{\mathrm{c}}\right)} \equiv \log_{T^{\alpha}_{1/2}/T_{\mathrm{c}}}\left(T^{\mathrm{SF}}_{1/2}/T_{\mathrm{c}}\right)$$



Experimental α-decay and SF half-lives.
The gray diagonal line indicates equal half-lives.
The black cross marks the half-life value T_{c} where α decay and SF tend to become comparable for known nuclei.

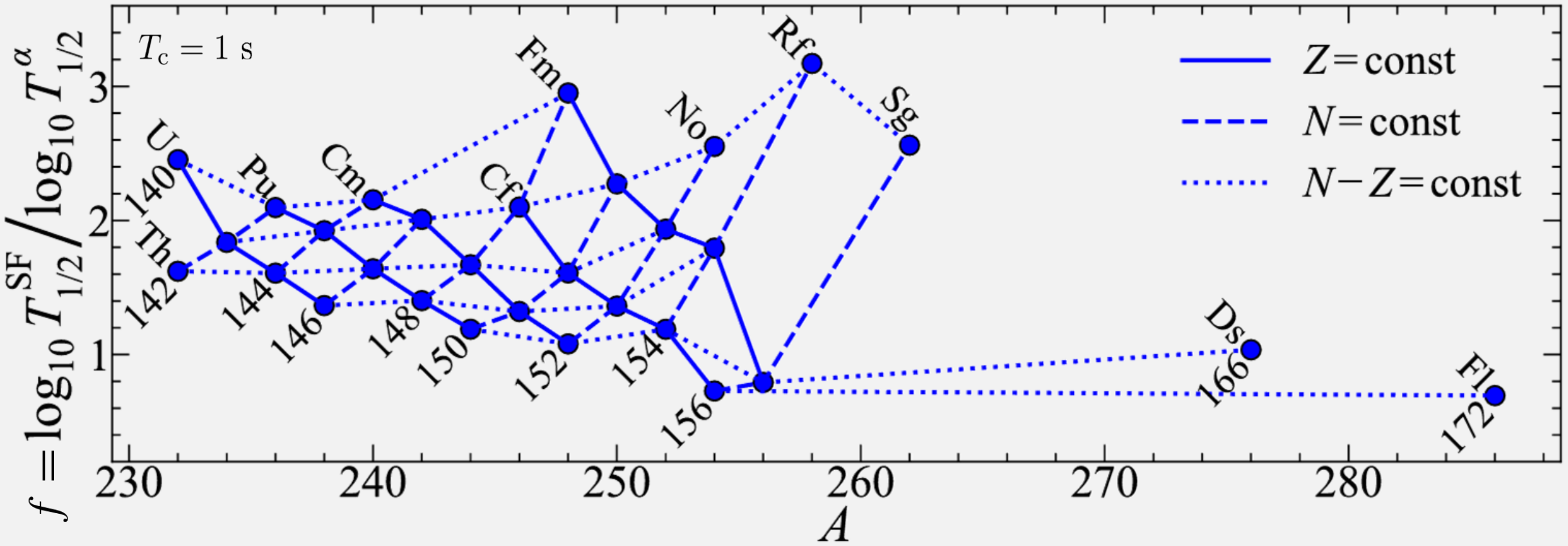
CORRELATION BETWEEN α-DECAY AND SF HALF-LIVES

Grid-like structure:

- at fixed Z, f decreases with increasing A or N
- at fixed N, f increases with increasing A or Z
- at fixed $N-Z$, f remains constant

$$f = f(N-Z)$$

$$f = c_0 + c_1(N-Z)$$



The ratio of experimental logarithmic SF and α-decay half-lives of even-even isotopes with $90 \le Z \le 114$ against the mass number A.

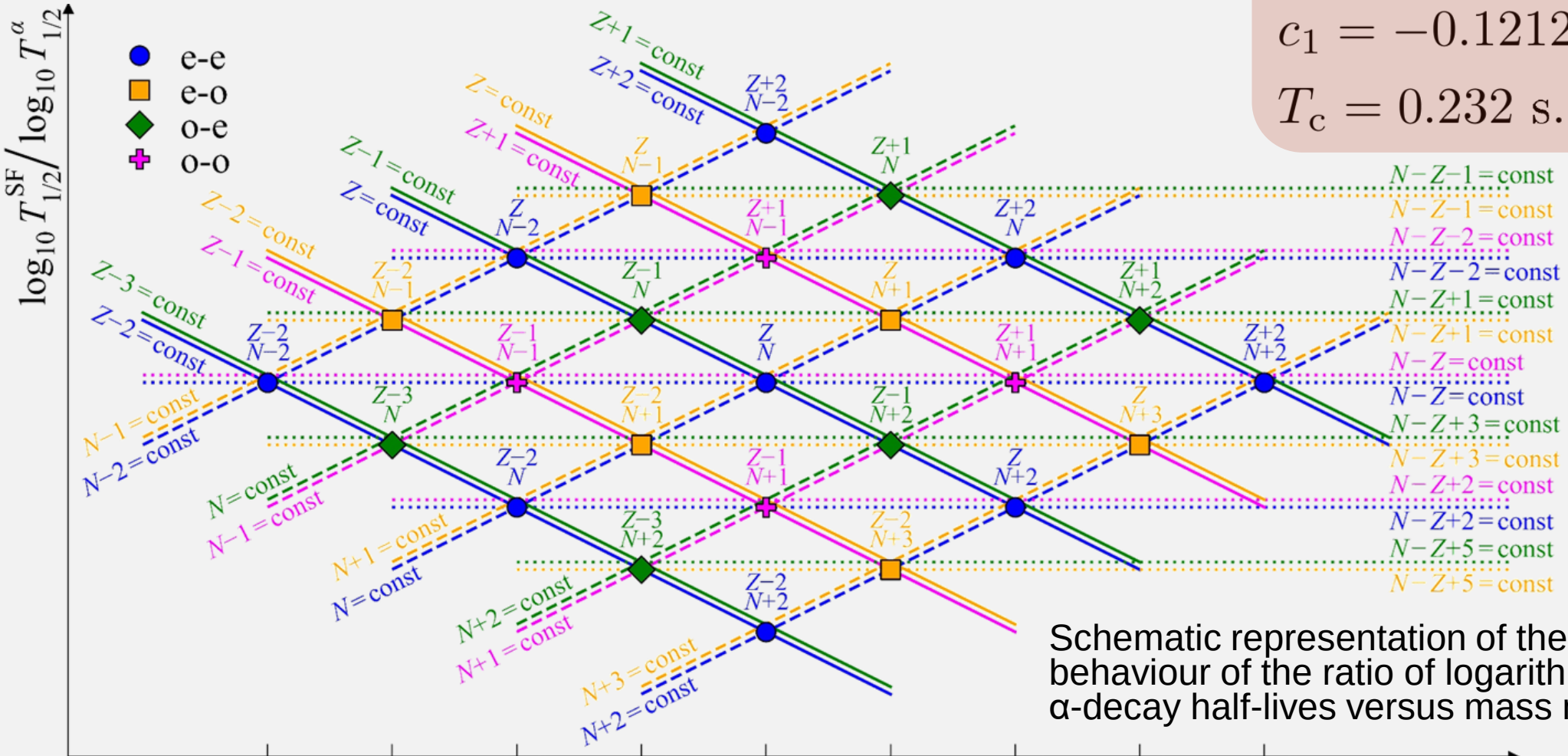
NEW EMPIRICAL FORMULA RELATING HALF-LIVES

$$\frac{T^{\mathrm{SF}}_{1/2}}{T_{\mathrm{c}}} = \left(\frac{T^{\alpha}_{1/2}}{T_{\mathrm{c}}}\right)^{c_0 + c_1\left[N - Z + (-1)^{N-Z} - 1\right]}$$

$$c_0 = 7.912357,$$

$$c_1 = -0.1212846,$$

$$T_{\mathrm{c}} = 0.232 \text{ s}.$$



Schematic representation of the systematic behaviour of the ratio of logarithmic SF and α-decay half-lives versus mass number A.

[Moiseev, Antonenko, & Adamian, 2026]

NEW EMPIRICAL FORMULA RELATING HALF-LIVES

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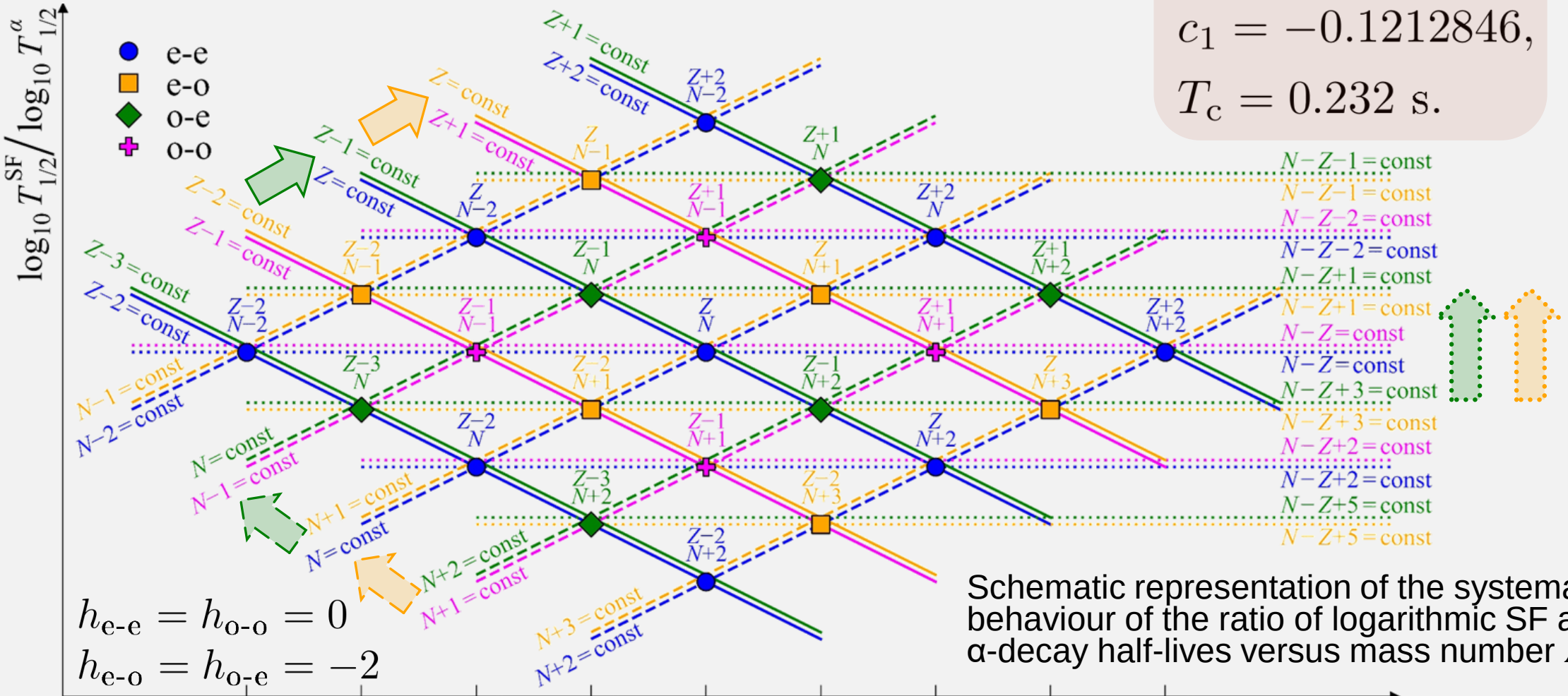
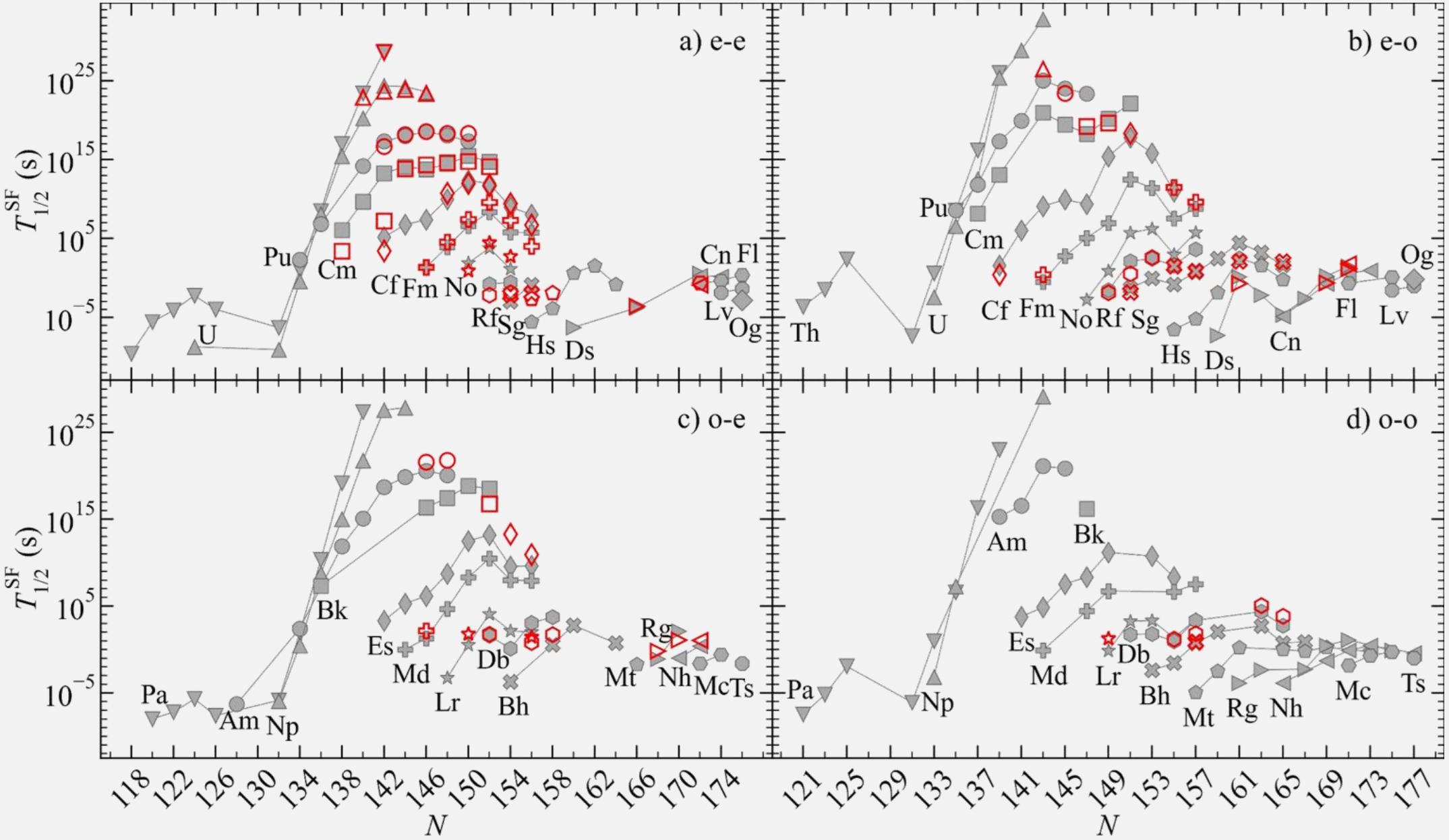


Schematic representation of the systematic behaviour of the ratio of logarithmic SF and α-decay half-lives versus mass number A.

[Moiseev, Antonenko, & Adamian, 2026]



$n_{\mathrm{nucl}} = 78$
$\delta_{\mathrm{rms}} = 1.16$
$\langle\delta\rangle = 0.94$
$\delta_{\mathrm{max}} = 2.95$

$\delta < 1$:
62% *nuclei*



$\delta < 2$:
90% *nuclei*



$\delta > 4$:
4 *nuclei:*
^{235}U, ^{238}Cm,
^{253}Fm, ^{253}Es

Calculated SF half-lives half-lives (filled gray symbols connected by lines) of even-even (a), even-odd (b), odd-even (c) and odd-odd (d) nuclei in comparison with the experimental data [Kondev, 2021] (open red symbols).

^{253}Lr

[Kondev, 2021]

Nuclide	Mass excess (keV)		Excitation Energy (keV)			Half-life	J^π	Decay modes and intensities (%)
^{253}Lr	88520	160			*	632 ms 46	$(7/2^-)$	α=90 10;SF=1.0 6;β^+ ?
^{253}Lr^m	88560#	190#	30#	100#	*	1.32 s 0.14	$(1/2^-)$	α=90 10;SF=12 3;β^+ ?;

Isomer assignment:

* when the available experimental information is insufficient to unambiguously determine which state is the ground state and which one is the excited isomer

	$T_{1/2}^{\text{SF, calc.}}$	$T_{1/2}^{\text{SF, exp.}}$	$\delta = \log_{10} T_{1/2}^{\text{SF, calc.}} - \log_{10} T_{1/2}^{\text{SF, exp.}}$
^{253}Lr	63 s	3.5 s	−1.26
^{253}Lr^m	21 s	11 s	0.28

Viola-Seaborg (VS) semi-empirical relationship
[Viola & Seaborg, 1966]

$$\log_{10} T_{1/2}^{\alpha} = \frac{(aZ+b)}{\sqrt{Q_\alpha}} + (cZ+d) + h_i$$

Formula relating half-lives
[Moiseev, Antonenko, & Adamian, 2026]

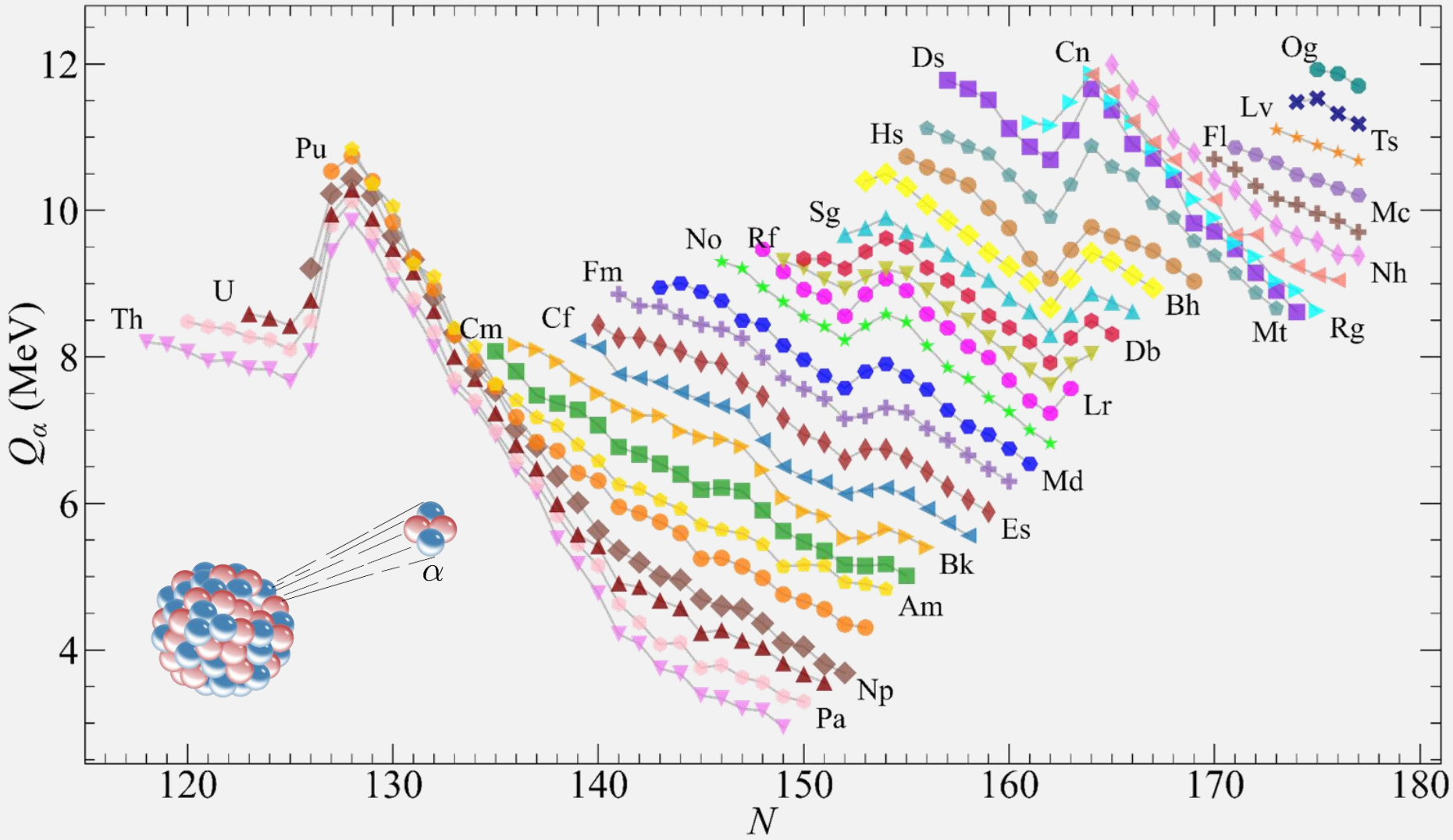
$$\frac{T_{1/2}^{\mathrm{SF}}}{T_{\mathrm{c}}} = \left(\frac{T_{1/2}^{\alpha}}{T_{\mathrm{c}}}\right)^{c_0 + c_1\left[N - Z + (-1)^{N-Z} - 1\right]}$$

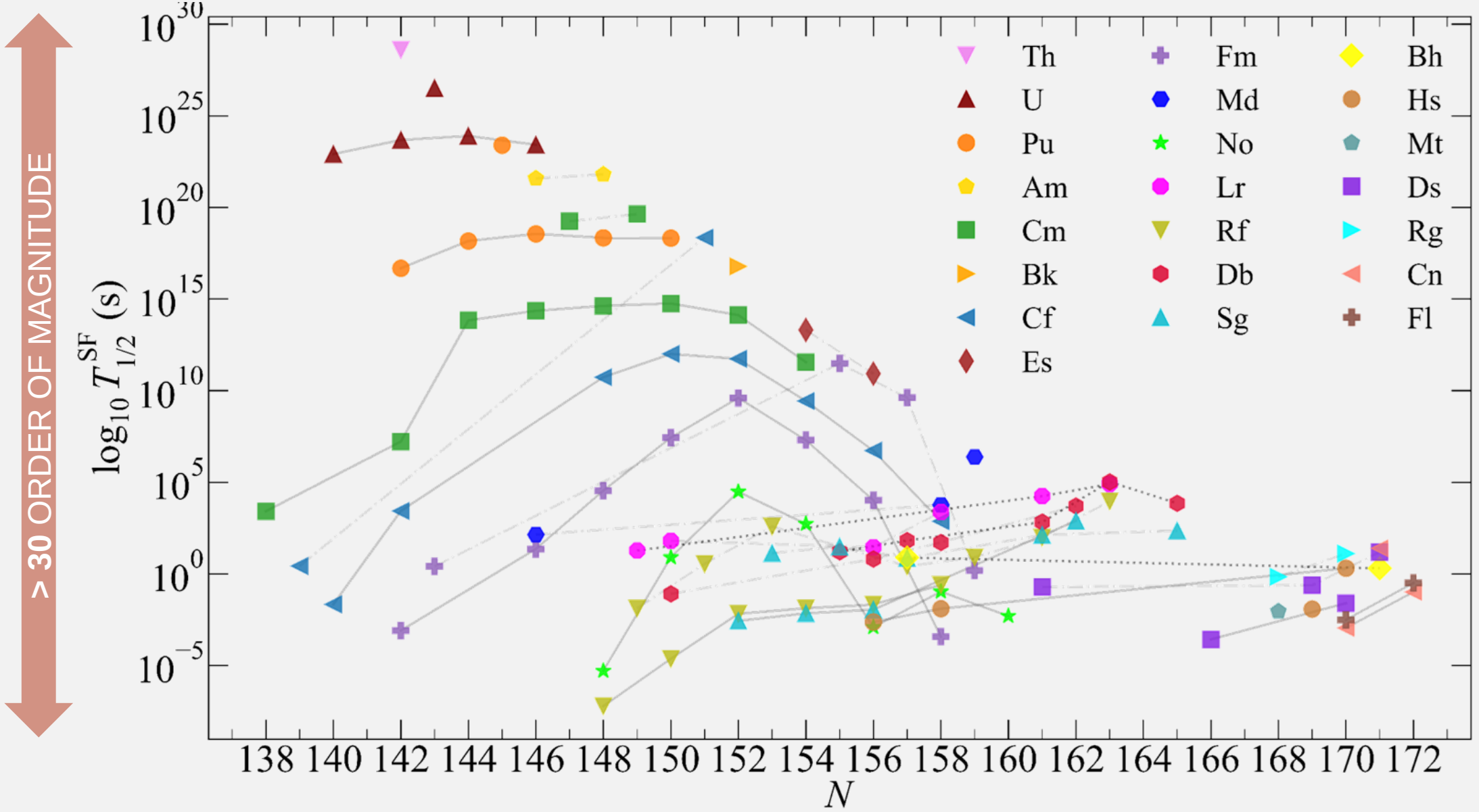
$$\Bigg\} \; T_{1/2}^{\mathrm{SF}}\left(Q_\alpha, N-Z\right)$$

$$Q_\alpha(Z,A) = M(Z,A) - M(Z-2,A-4) - M(^4\mathrm{He})$$

$A > 200$

$Q_\alpha = (4 - 11)$ MeV



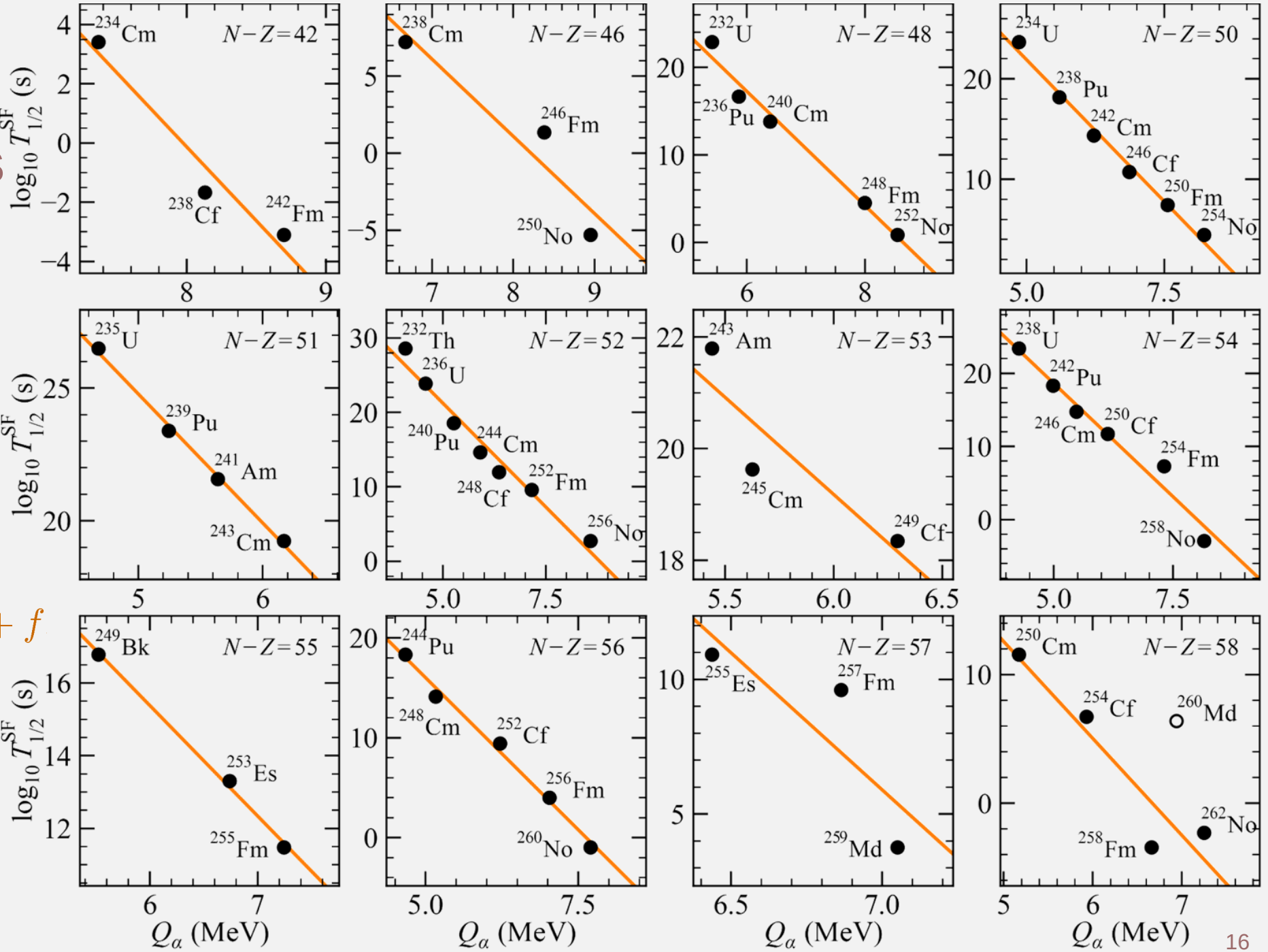


Spontaneous fission half-lives $T_{1/2}^{\mathrm{SF}}$ for nuclei with $90 \leq Z \leq 114$ [Wang, 2021]. Solid lines are even-even nuclei, dashed-dotted lines are even-odd and odd-even nuclei, and dotted lines are odd-odd nuclei.

PECULIARITIES OF DEPENDENCIES OF $T_{1/2}^{\mathrm{SF}}$ ON Q_α

Decimal logarithm of the experimental $T_{1/2}^{\mathrm{SF}}$ as a function of Q_α for fixed $N - Z$ values for nuclei with $90 \leq Z \leq 102$ (black round symbols) and their approximation (orange lines) by

$$\log_{10} T_{1/2}^{\mathrm{SF}}(Q_\alpha) = c_Q Q_\alpha + f$$



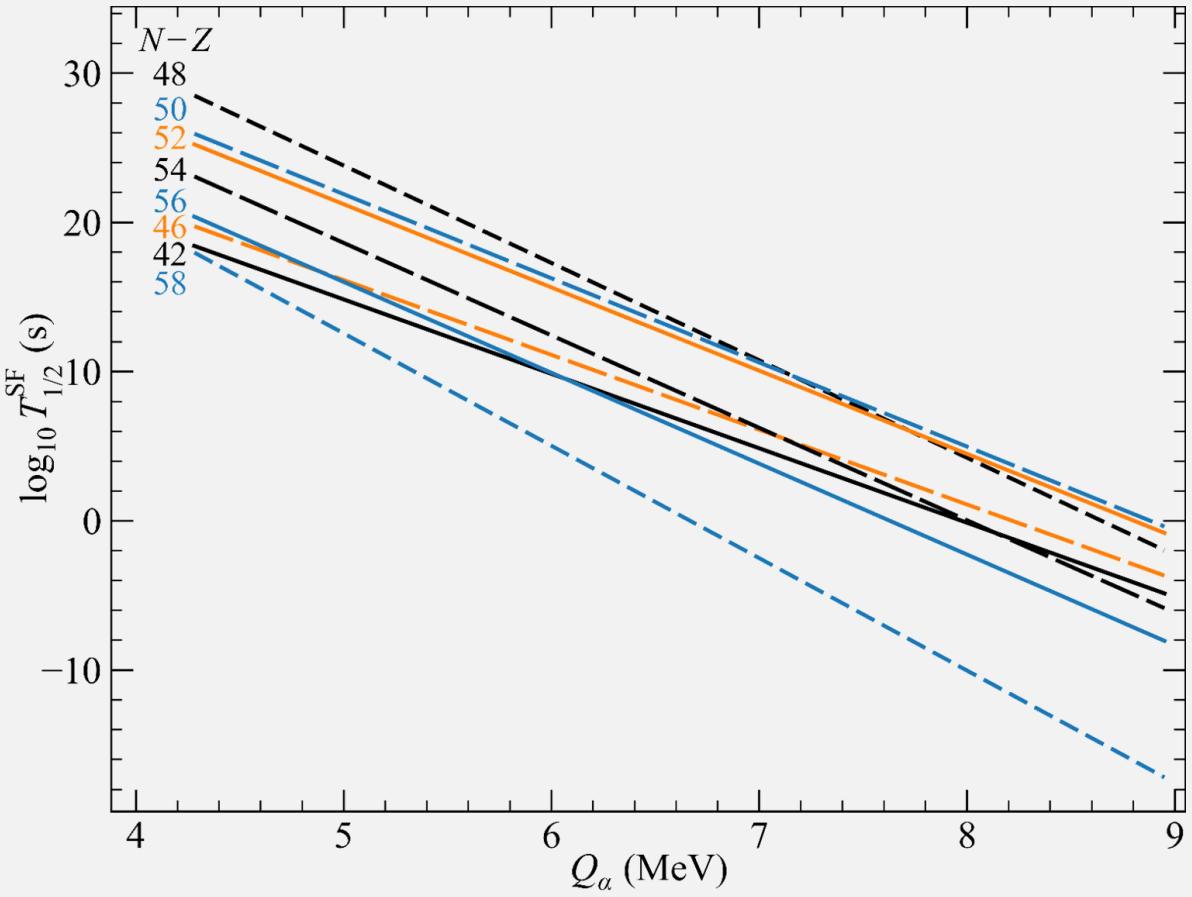
$$\log_{10} T^{\mathrm{SF}}_{1/2}(Q_\alpha) = c_Q Q_\alpha + f$$

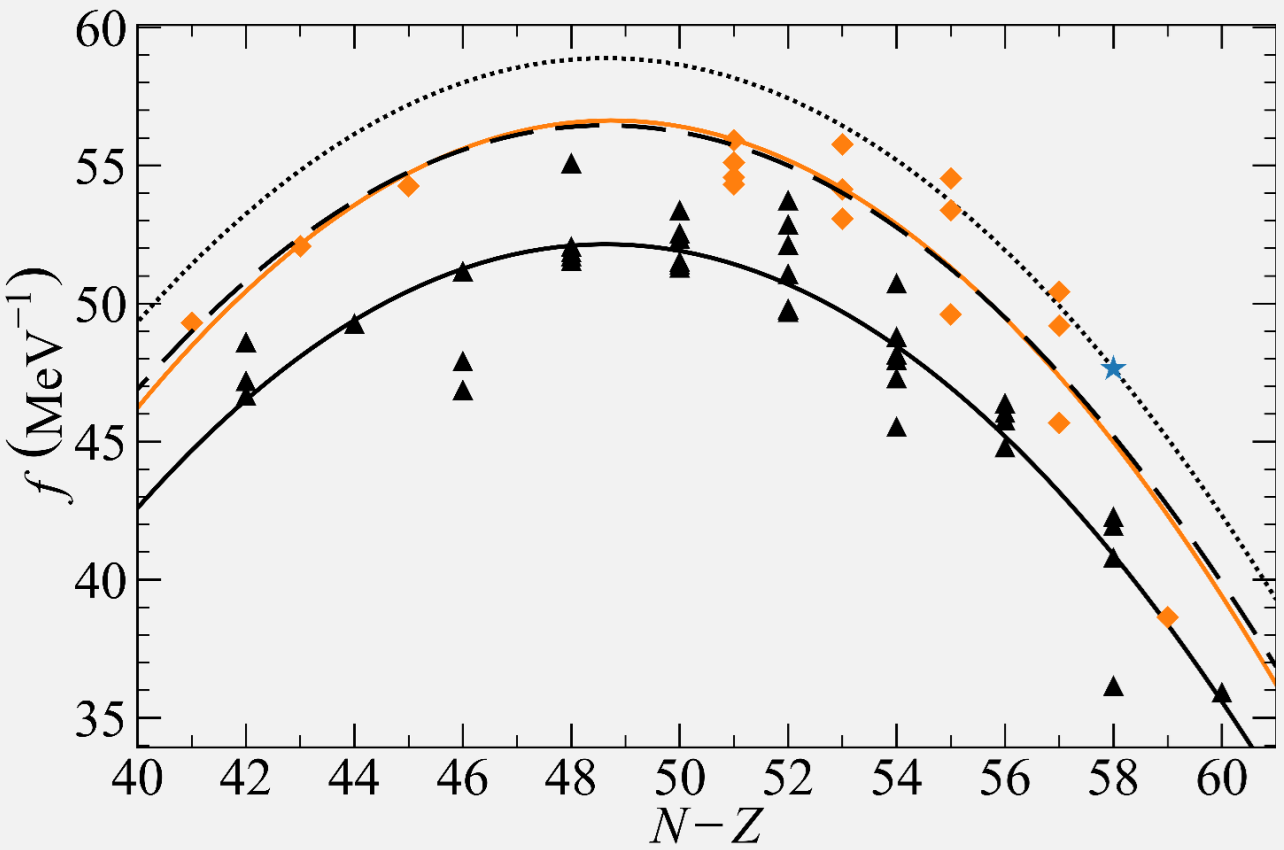
$$c_Q = c_Q^{\mathrm{mean}} = -5.946$$

$$f = f(A,Z) \Rightarrow f = f(N-Z)$$

$$f(N-Z) = \log_{10} T^{\mathrm{SF}}_{1/2} - c_Q^{\mathrm{mean}} Q_\alpha$$

$$f(N-Z) = c_0 + c_1(N-Z) + c_2(N-Z)^2 + c_{\mathrm{shift}}^{\text{o-}A/\text{o-o}}$$





Comparison of approximating lines

NEW EMPIRICAL FORMULA FOR SPONTANEOUS FISSION HALF-LIVE

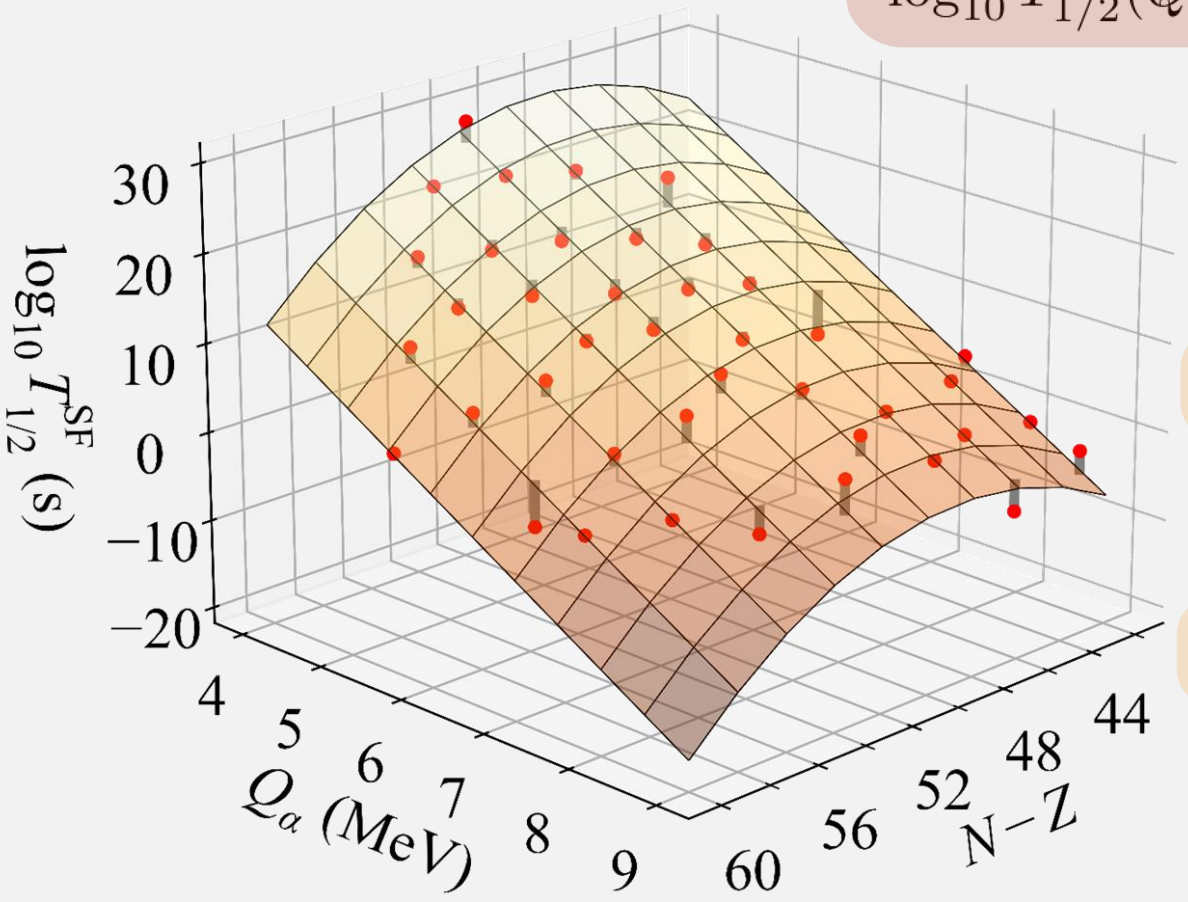
$$\log_{10} T_{1/2}^{\mathrm{SF}}(Q_\alpha, N-Z) = c_0 + c_1(N-Z) + c_2(N-Z)^2 + c_Q Q_\alpha + \begin{cases} 0, & \text{for even-even,} \\ h, & \text{for odd-}A\text{,} \\ 1.575h, & \text{for odd-odd.} \end{cases}$$

$90 \leqslant Z \leqslant 102$:

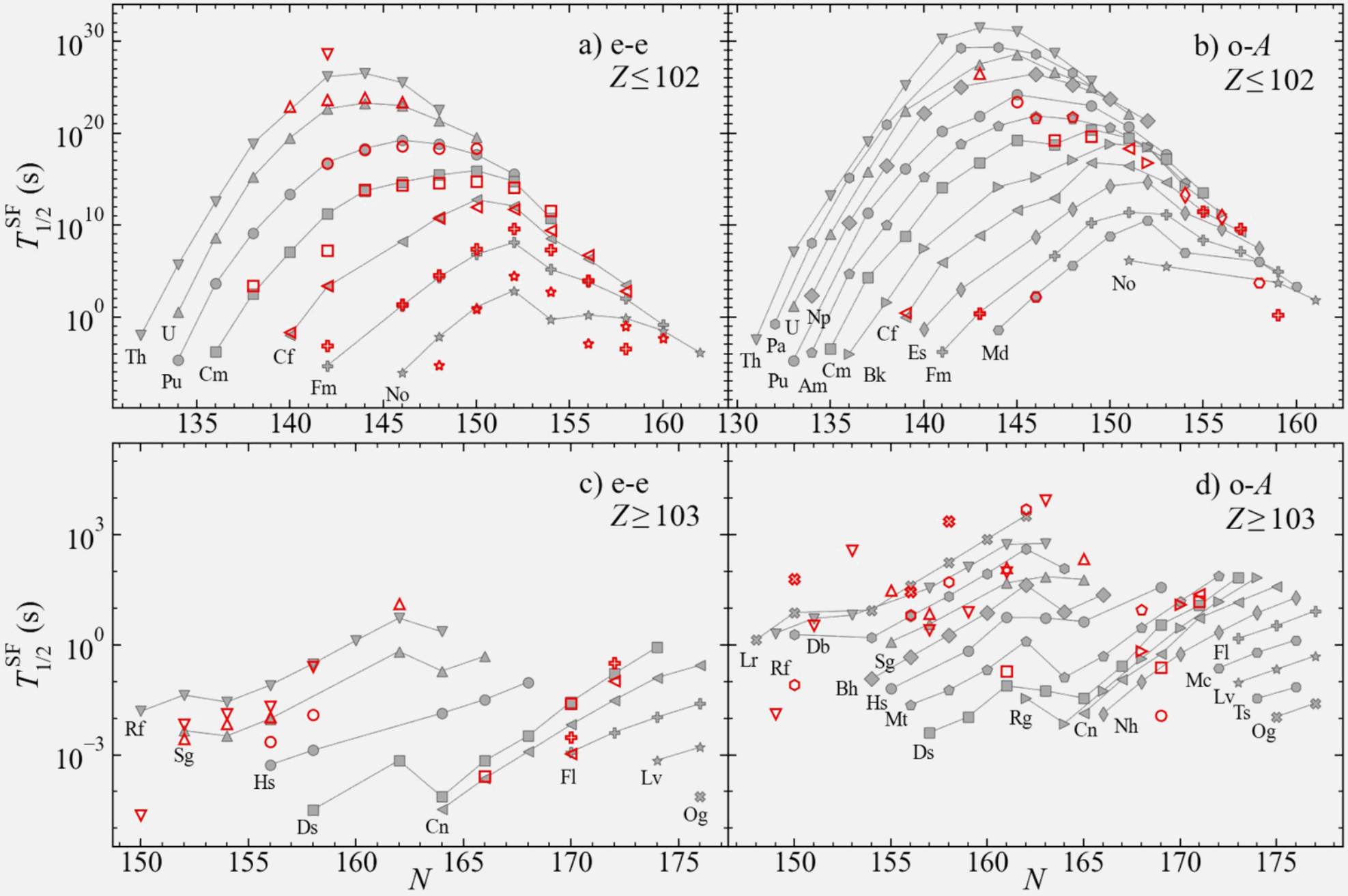
$c_0 = -244.845, \quad c_1 = 12.2843, \quad c_2 = -0.12658,$
$c_Q = -6.1023, \quad h = 4.309,$

$Z \geqslant 103$:

$c_0 = -0.414, \quad c_1 = 0.1379, \quad c_2 = 0,$
$c_Q = -0.9097, \quad h = 2.327.$



Decimal logarithm of the spontaneous fission half-life of even-even isotopes with atomic numbers $90 \leq Z \leq 102$, plotted as a function of Q_α and $N - Z$. The surface is based on calculations using proposed formula. Experimental values [Kondev, 2021] are shown as red circles. Gray segments visualize the deviations of the experimental data from the calculated surface.



$n_{\mathrm{nucl}} = 111$
$\delta_{\mathrm{rms}} = 1.43$
$\langle \delta \rangle = 1.00$
$\delta_{\mathrm{max}} = 4.62$

$\delta < 1$:
60% *nuclei*



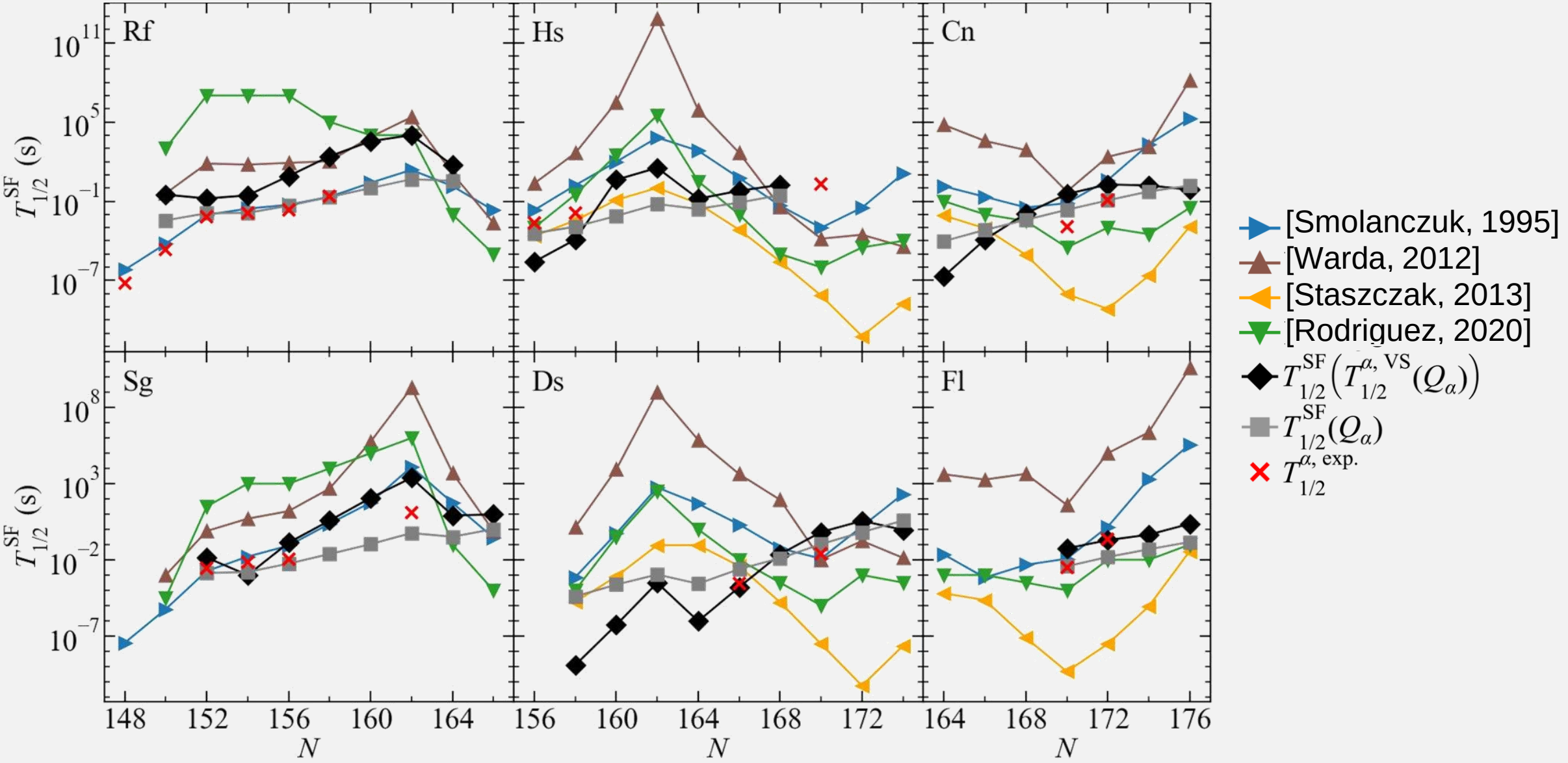
$\delta < 2$:
86% *nuclei*



$\delta > 4$:
3 *nuclei:*
^{238}Cm, ^{258}Fm, ^{277}Hs

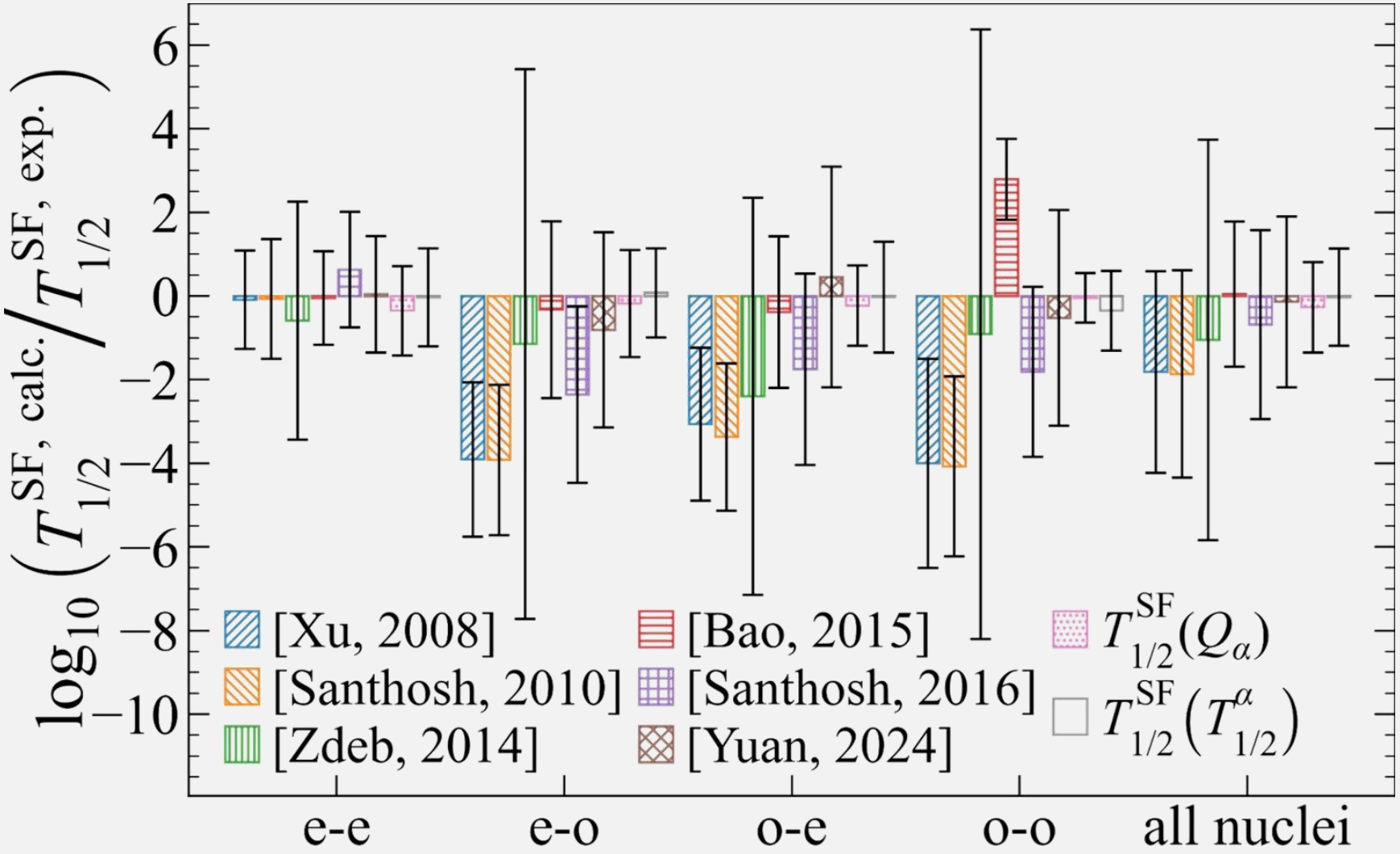
Calculated spontaneous fission half-lives of even-even nuclei with charge numbers $90 \leq Z \leq 102$ (solid gray symbols connected by lines) in comparison to the experimental values [Kondev, 2021] (open red symbols).

COMPARISON WITH THEORETICAL MODEL PREDICTIONS



Calculated SF half-lives of even-even superheavy nuclei (black dimonds and gray squares), compared with theoretical predictions (triangles) and experimental values (red crosses). Q_α values are taken from [Wang, 2021].

COMPARISON WITH OTHER EMPIRICAL FORMULAS



$$\langle\delta\rangle = \frac{1}{n_{\mathrm{nucl}}}\sum_{i=1}^{n_{\mathrm{nucl}}}\frac{T_{1/2}^{\mathrm{SF,\ calc}}}{T_{1/2}^{\mathrm{SF,\ exp}}}$$

$$\delta_{\mathrm{rms}} = \left[\frac{1}{n_{\mathrm{nucl}}}\sum_{i=1}^{n_{\mathrm{nucl}}}\left(\log_{10}\frac{T_{1/2}^{\mathrm{SF,\ calc}}}{T_{1/2}^{\mathrm{SF,\ exp}}}\right)^2\right]^{1/2}$$

Mean values of the decimal logarithms of the ratio of calculated-to-experimental SF half-lives. Experimental data are taken from [Kondev, 2021]. The error bars represent the standard deviations.

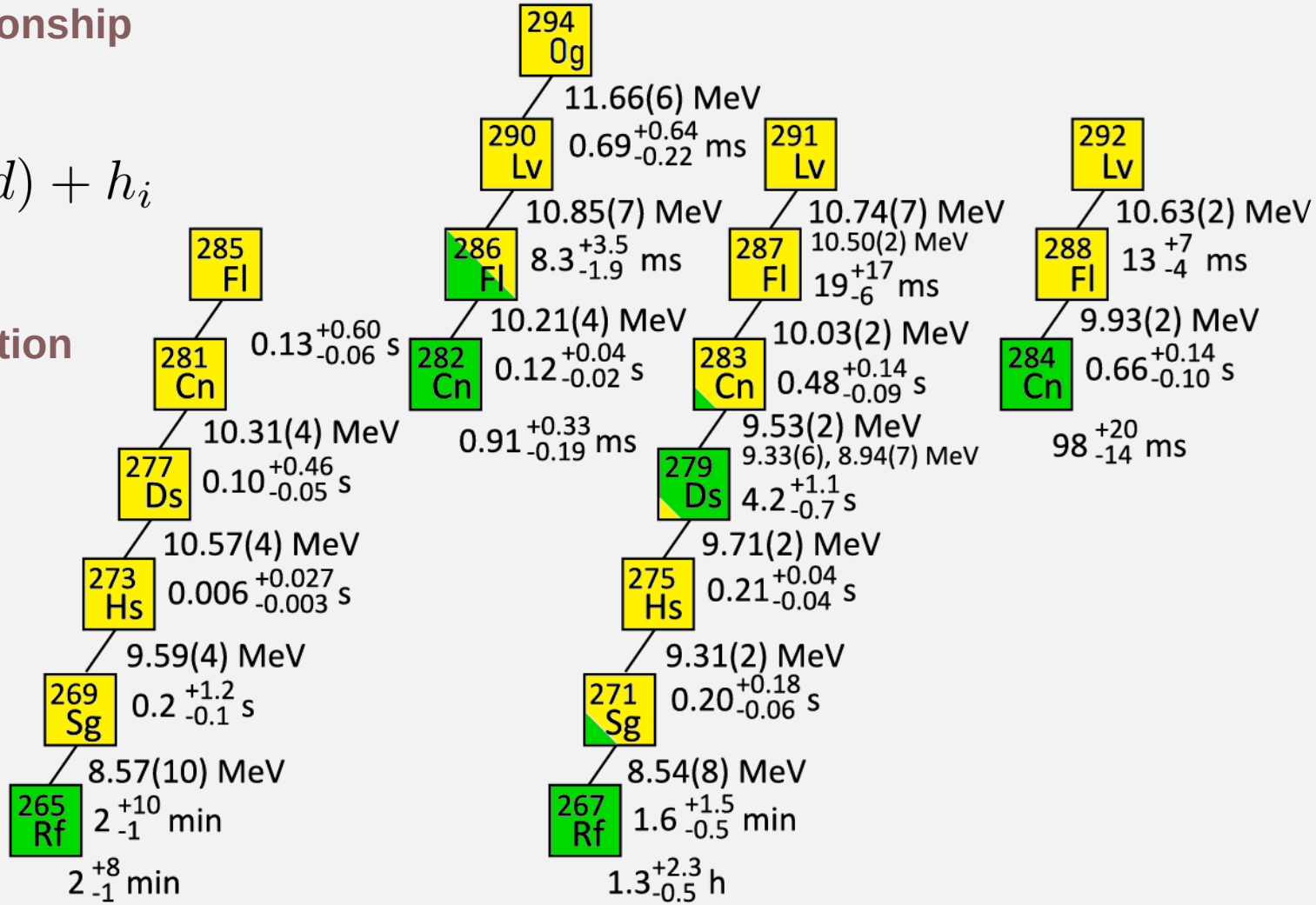
COMPETITION BETWEEN SF AND α-DECAY

Viola-Seaborg (VS) semi-empirical relationship
[Viola & Seaborg, 1966]

$$\log_{10} T_{1/2}^{\alpha} = \frac{(aZ+b)}{\sqrt{Q_\alpha}} + (cZ+d) + h_i$$

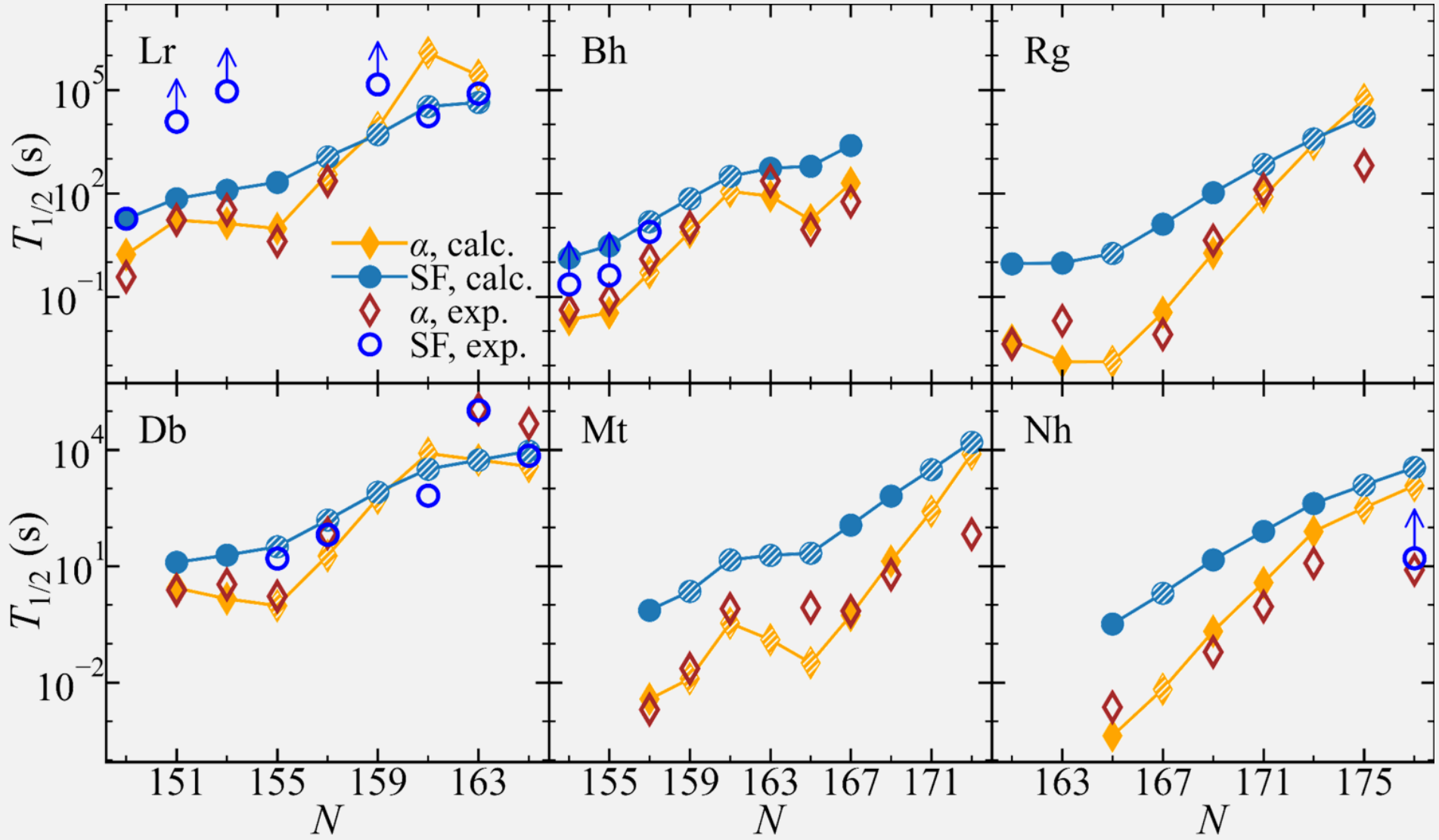
Parkhomenko-Sobichevsky parametrization
[Parkhomenko, 2005]

$$a = 1.3892, b = 13.862,$$
$$c = -0.1086, d = -41.458,$$
$$h_i = \begin{cases} 0, & \text{even-even}, \\ 0.437, & \text{odd-even}, \\ 0.641, & \text{even-odd}, \\ 1.024, & \text{odd-odd} \end{cases}$$



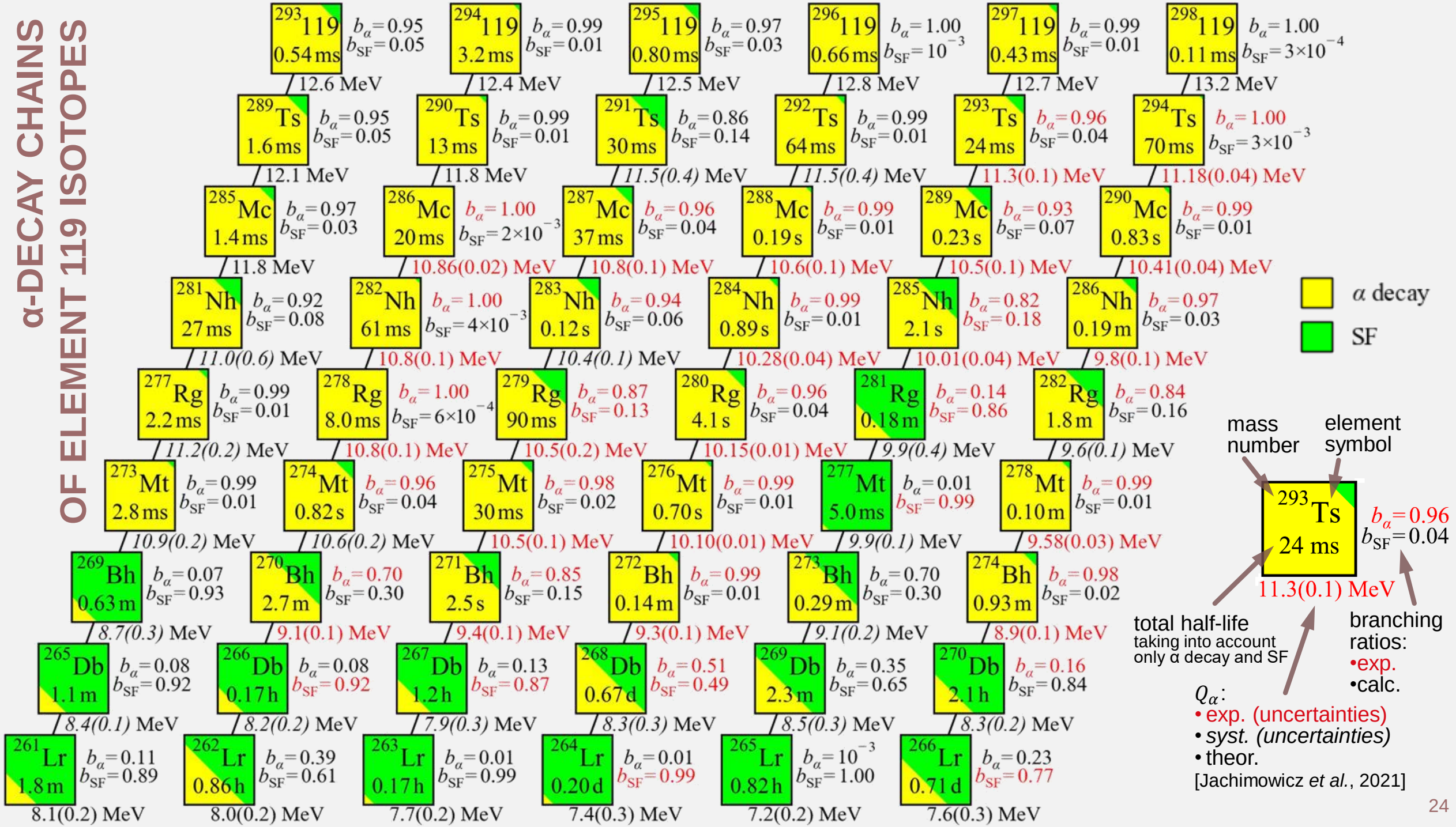
α-decay chains for even-Z elements synthesized in complete fusion reactions ^{48}Ca + ^{238}U, $^{242, 244}$Pu, $^{245, 248}$Cm, ^{249}Cf.

COMPETITION BETWEEN SF AND α-DECAY

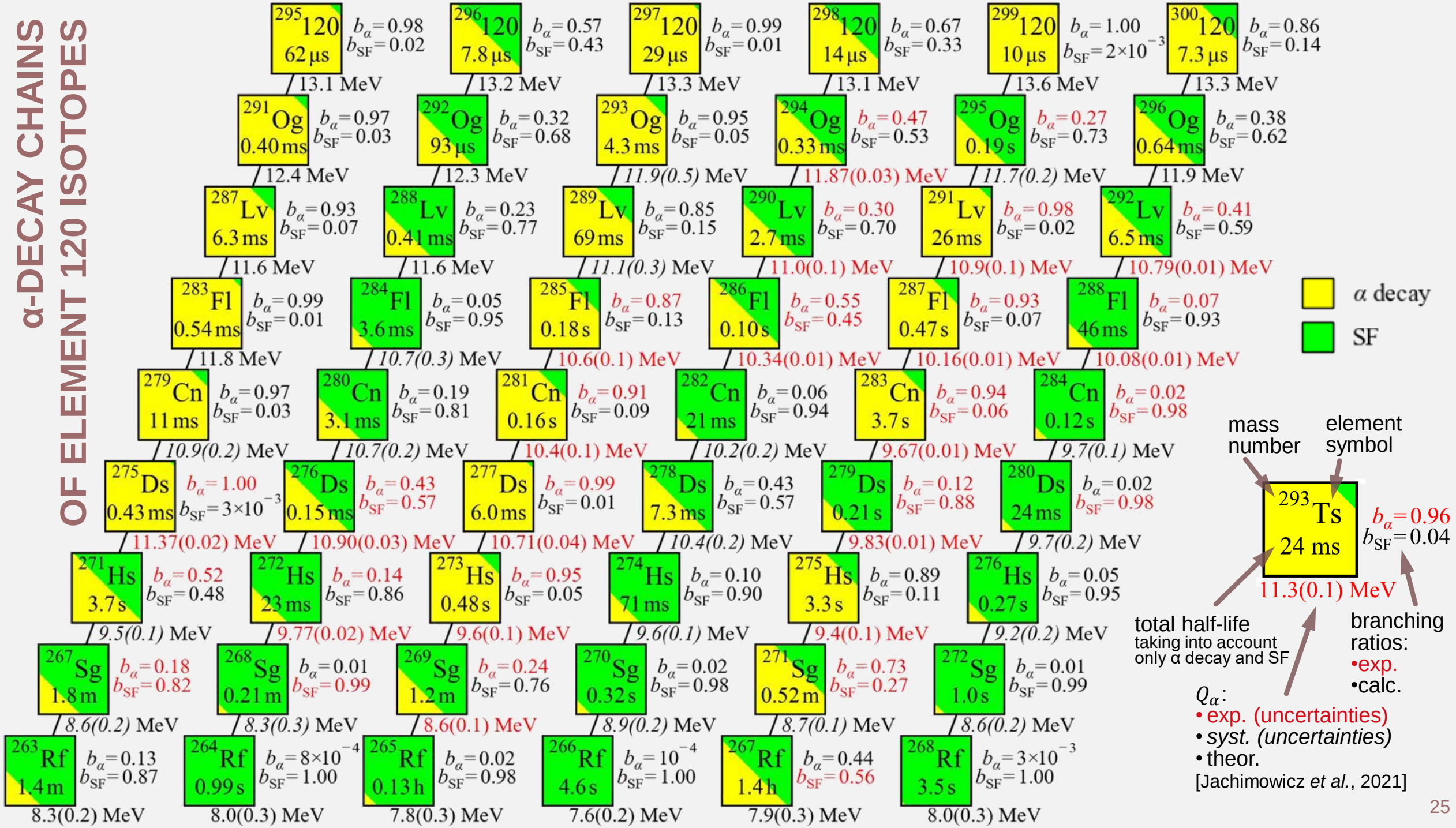


Calculated α-decay (solid diamonds) and spontaneous fission (solid circles) half-lives of **odd-odd** SHN. Solid (hatched) symbols represent calculations using experimental (systematic) Q_α values [Wang, 2021]. Experimental α-decay (open diamonds) and spontaneous fission (open circles) half-lives are taken from [Kondev, 2021]. The down (up) arrow indicates only the upper (lower) limit of the half-life of the corresponding decay mode.

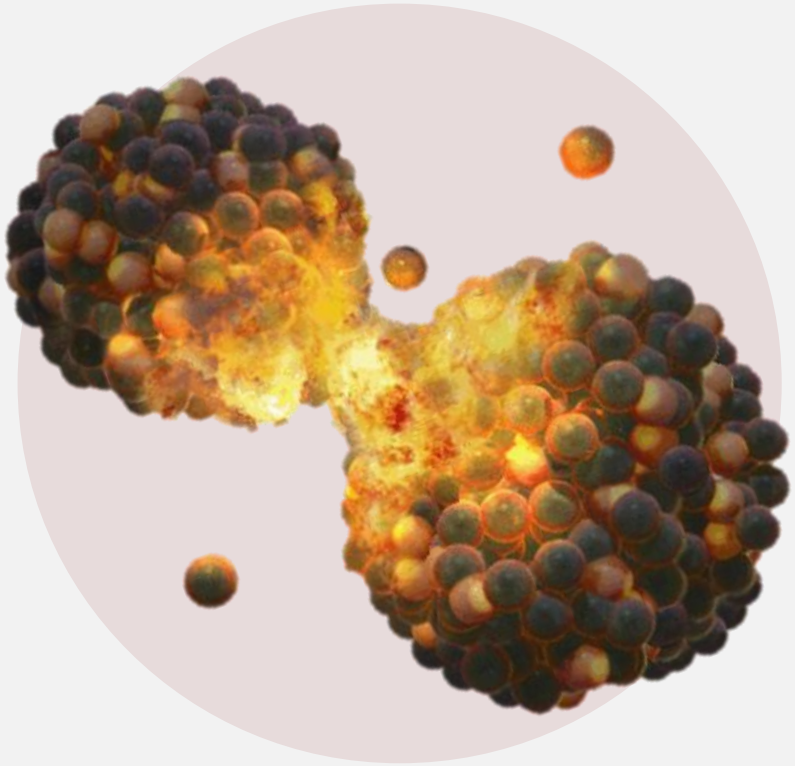
α-DECAY CHAINS OF ELEMENT 119 ISOTOPES



α-DECAY CHAINS OF ELEMENT 120 ISOTOPES



CONCLUSIONS



influence of neutron excess on systematics of **SF half-lives** of actinides and SHN was revealed;

systematic **relationship between SF and α-decay half-lives** was established;

three-parameter **formula linking** $T_{1/2}^{\mathrm{SF}}$ **to** $T_{1/2}^{\alpha}$ **and** $N-Z$ was proposed, unified for all parity types;

linear dependence of the decimal **logarithm of SF half-life** on **energy release of α decay** at fixed values of neutron excess was found;

semi-empirical formula was proposed **for the SF half-lives**, depending on Q_α and $N-Z$;

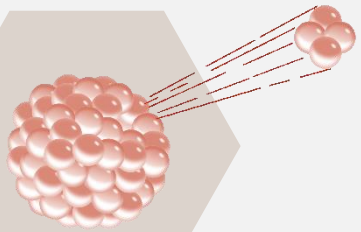
comparison with **theoretical models** demonstrated that the formula **accounts for shell effects** and predicts a **slower increase in SF half-lives** with increasing neutron number **in Cn** and **Fl**;

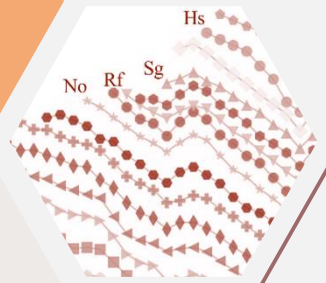
competition between α decay and SF was analyzed along α-decay chains of elements 119 and 120; promising **candidates for experimental identification** were identified;

discovered systematics underscores the importance of α-cluster formation and confirms the fundamental **connection between SF and α decay**, suggesting these processes as diff**erent manifestations of nuclear fission**

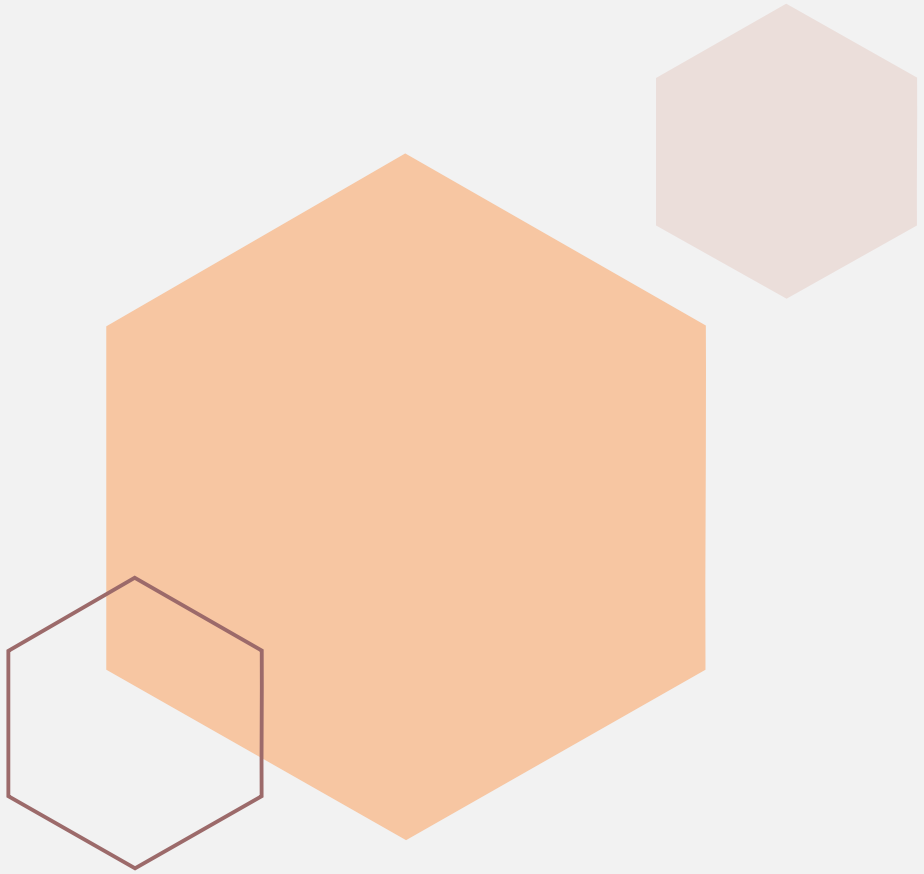






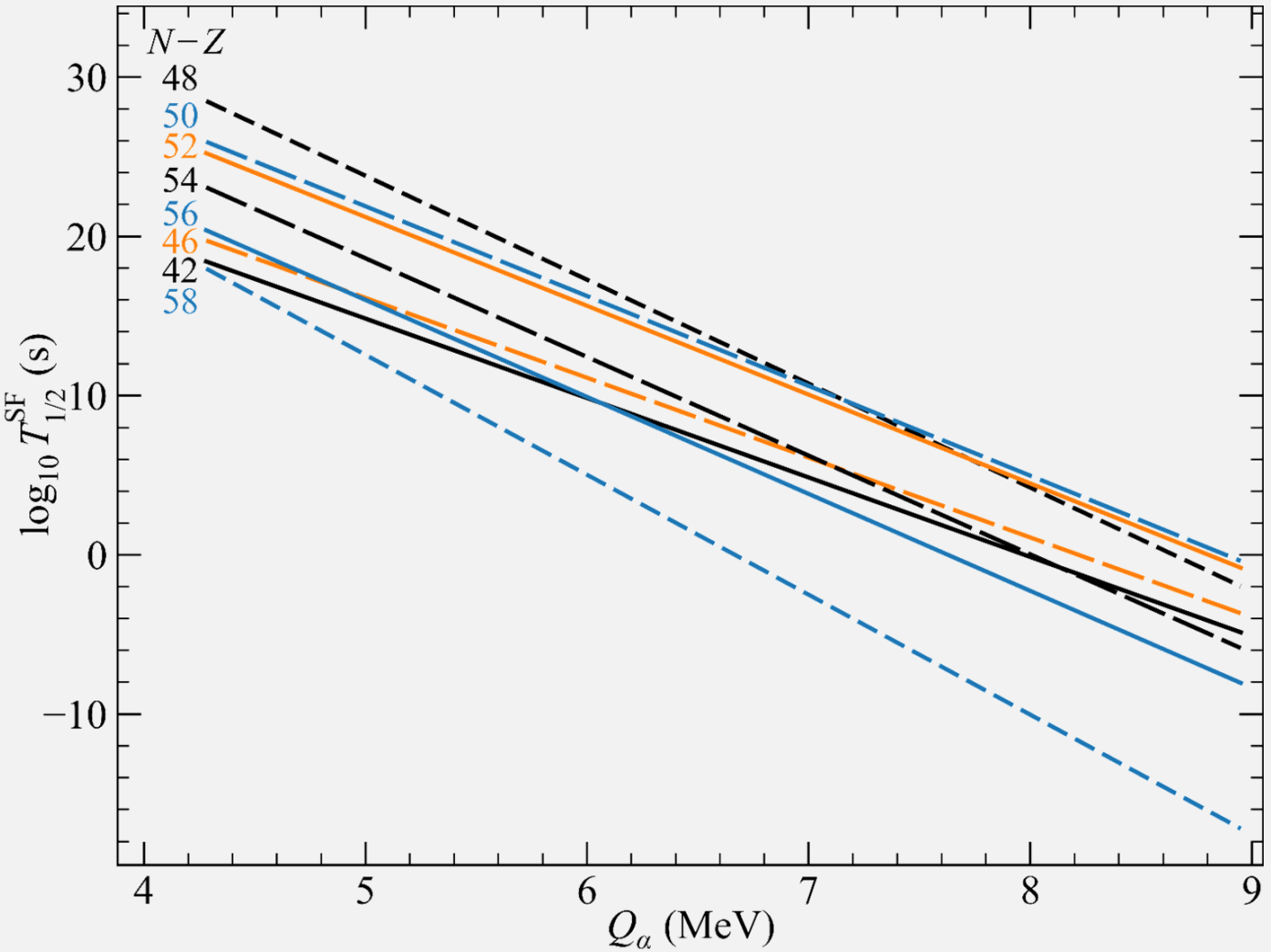


THANK YOU
FOR ATTENTION



BACKUP SLIDES

Comparison of approximating lines



Values of slope and intercept obtained by linear approximation of experimental points

$N-Z$	c_Q	f
42	−4.989	39.78
46	−5.014	41.21
48	−6.528	56.45
50	−5.640	50.09
51	−4.859	49.07
52	−5.577	49.10
53	−3.475	40.03
54	−6.198	49.62
55	−3.039	33.61
56	−6.090	46.46
57	−10.1791	77.15
58	−7.533	50.23

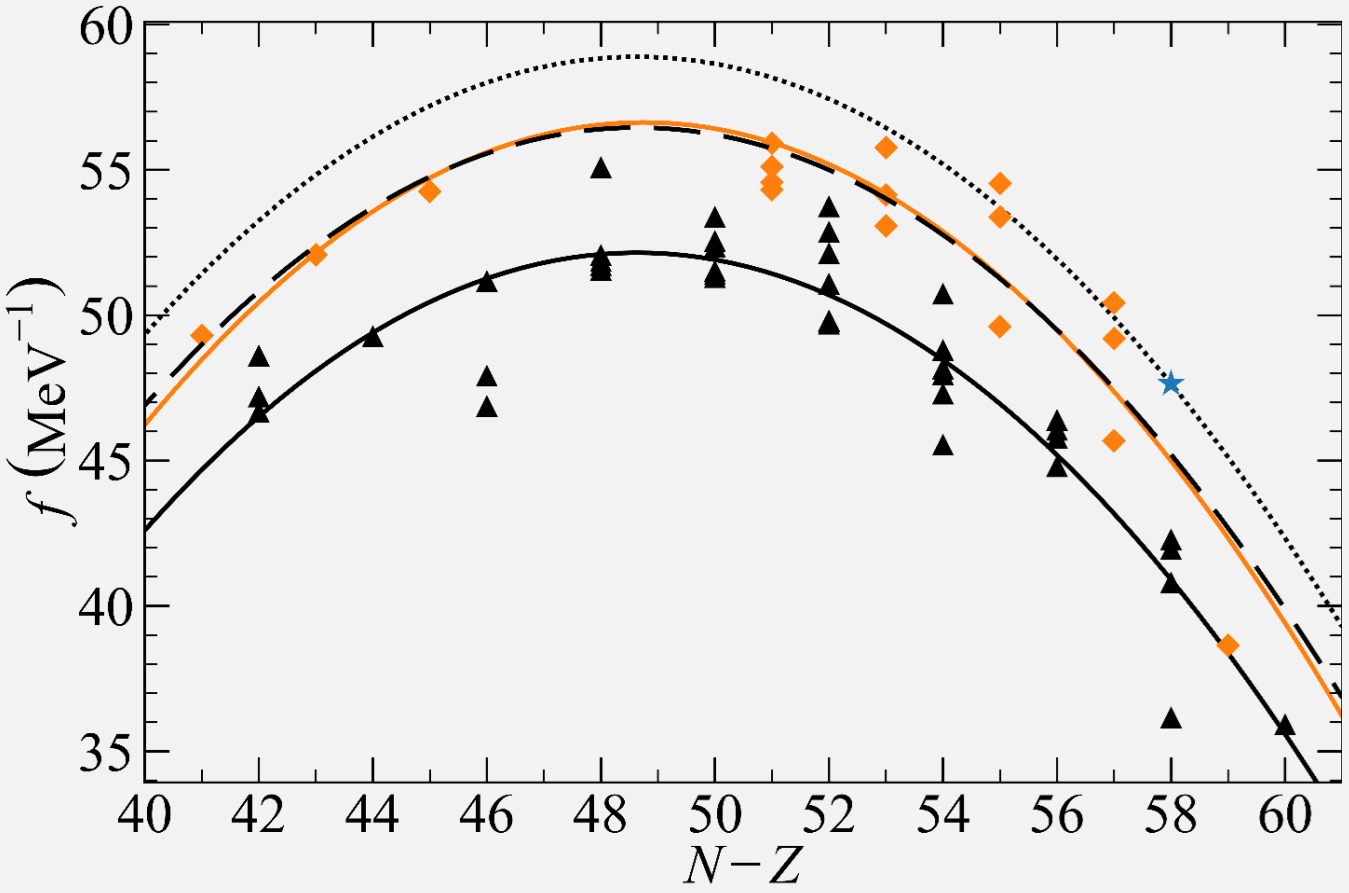
$$\log_{10} T_{1/2}^{\mathrm{SF}}(Q_\alpha) = c_Q Q_\alpha + f$$

$$c_Q = c_Q^{\mathrm{mean}} = -5.946 \qquad f = f(A,Z) \Rightarrow f = f(N-Z)$$

$$f(N-Z) = \log_{10} T^{\mathrm{SF}}_{1/2} - c_Q^{\mathrm{mean}} Q_\alpha$$

$$c_Q^{\mathrm{mean}} = -5.946$$

$$f(N-Z) = c_0 + c_1(N-Z) + c_2(N-Z)^2 + c_{\mathrm{shift}}^{\text{o-}A/\text{o-o}}$$



even-even: ▲

$n_{\mathrm{nucl}} = 41$

$c_0 = -250.93$, $c_1 = 12.464$, $c_2 = -0.1281$

$\mathrm{rms} = 1.65$

odd-A: ◆

$n_{\mathrm{nucl}} = 17$

$c_0 = -266.44$, $c_1 = 13.255$, $c_2 = -0.1360$

$\mathrm{rms}_{\mathrm{fit}} = 1.82$

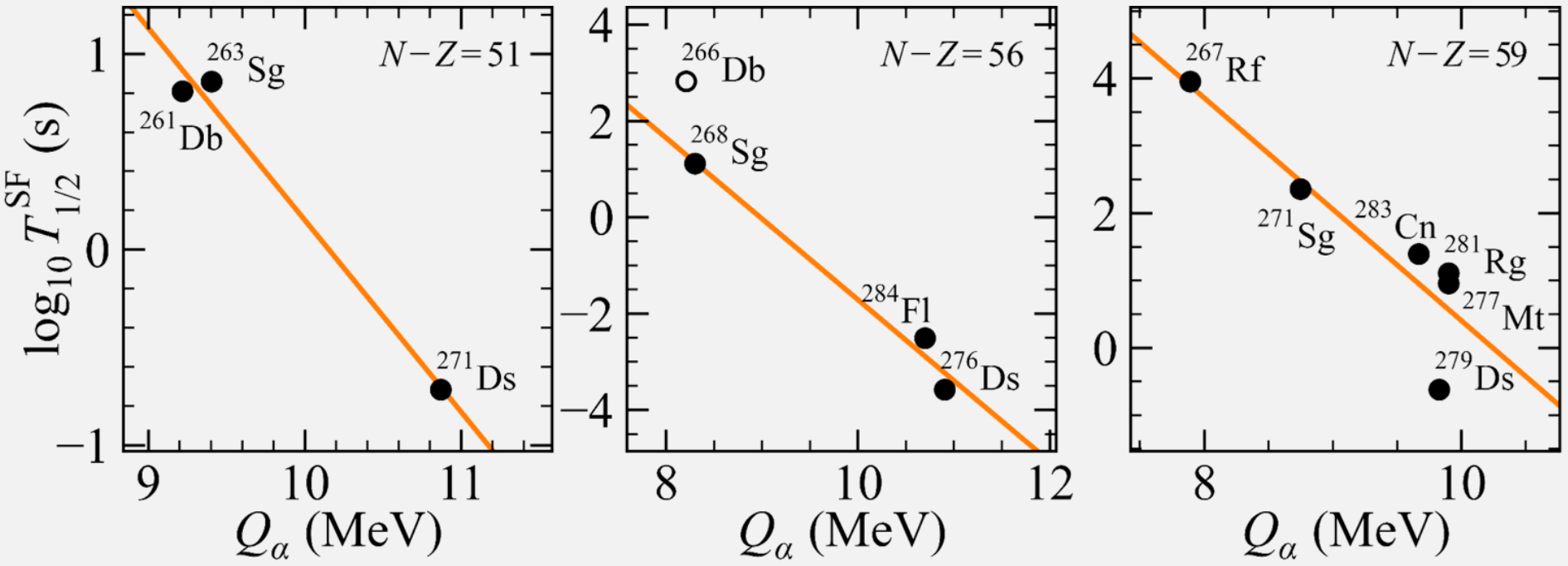
$c_{\mathrm{shift}} = 4.30$

$\mathrm{rms}_{\mathrm{shift}} = 1.84$

odd-odd: ★

$n_{\mathrm{nucl}} = 1$

$c_{\mathrm{shift}} = 6.74$



Decimal logarithm of the experimental $T_{1/2}^{\mathrm{SF}}$ as a function of Q_α for fixed values of the neutron excess $N-Z$ for nuclei with $103 \leq Z \leq 118$ (**black round symbols**) and their approximation (orange lines) by

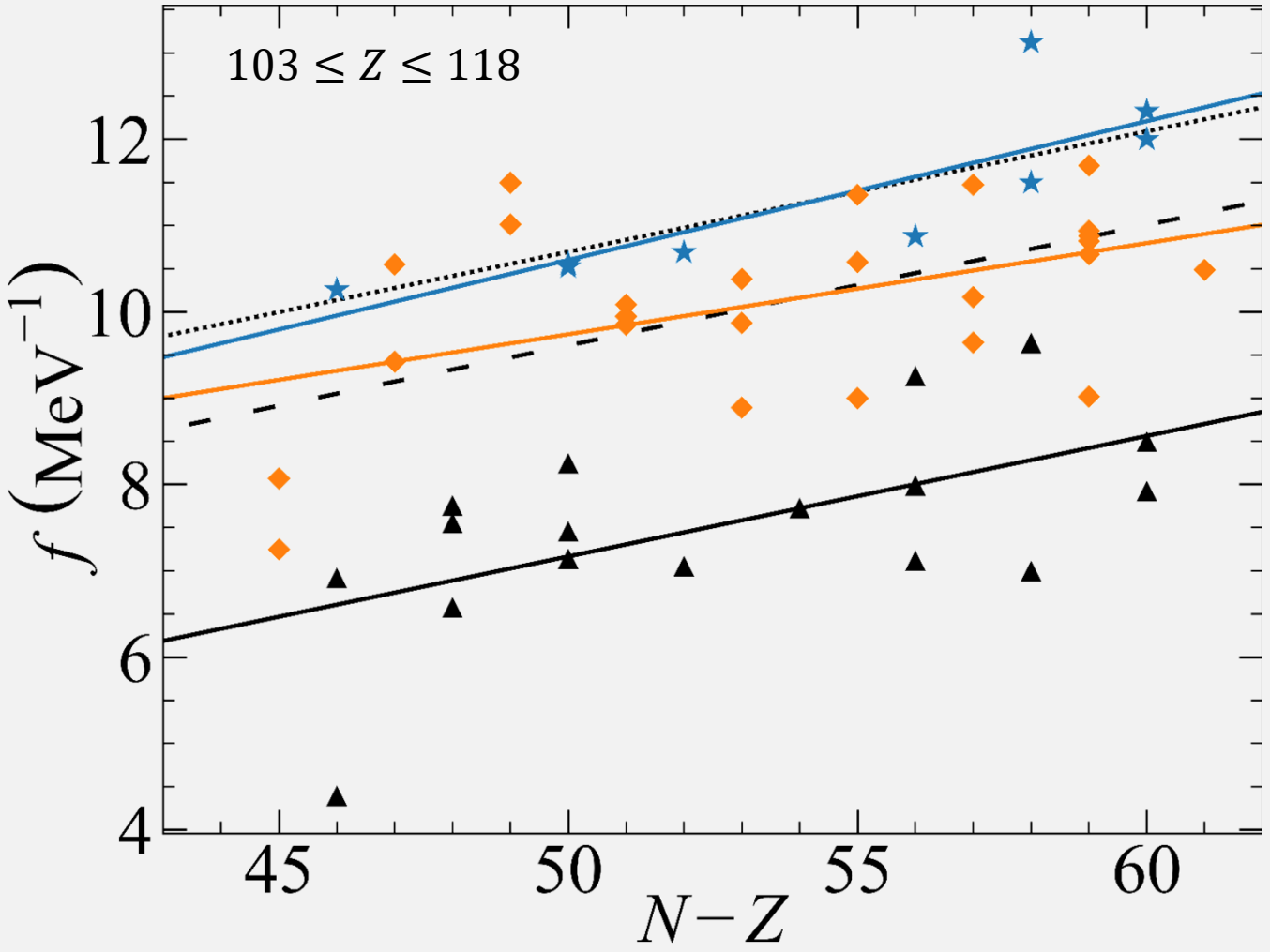
$$\log_{10} T_{1/2}^{\mathrm{SF}}(Q_\alpha) = c_Q Q_\alpha + f$$

Values of slope and intercept obtained by linear approximation of experimental points

$N-Z$	c_Q	f
51	−0.98	9.96
56	−1.68	15.09
59	−1.65	16.93

$$f(N-Z) = \log_{10} T^{\mathrm{SF}}_{1/2} - c_Q^{\mathrm{mean}} Q_\alpha$$

$$f(N-Z) = c_0 + c_1 (N-Z)$$



$c_Q^{\mathrm{mean}} = -0.981$

even-even: ▲

$n_{\mathrm{nucl}} = 17$

$c_0 = 0.19,\ c_1 = 0.140$

$\mathrm{rms} = 0.91$

odd-A: ◆

$n_{\mathrm{nucl}} = 25$

$c_0 = 4.46,\ c_1 = 0.106$

$\mathrm{rms}_{\mathrm{fit}} = 0.94$

$c_{\mathrm{shift}} = 2.44$

$\mathrm{rms}_{\mathrm{shift}} = 0.95$

odd-odd: ★

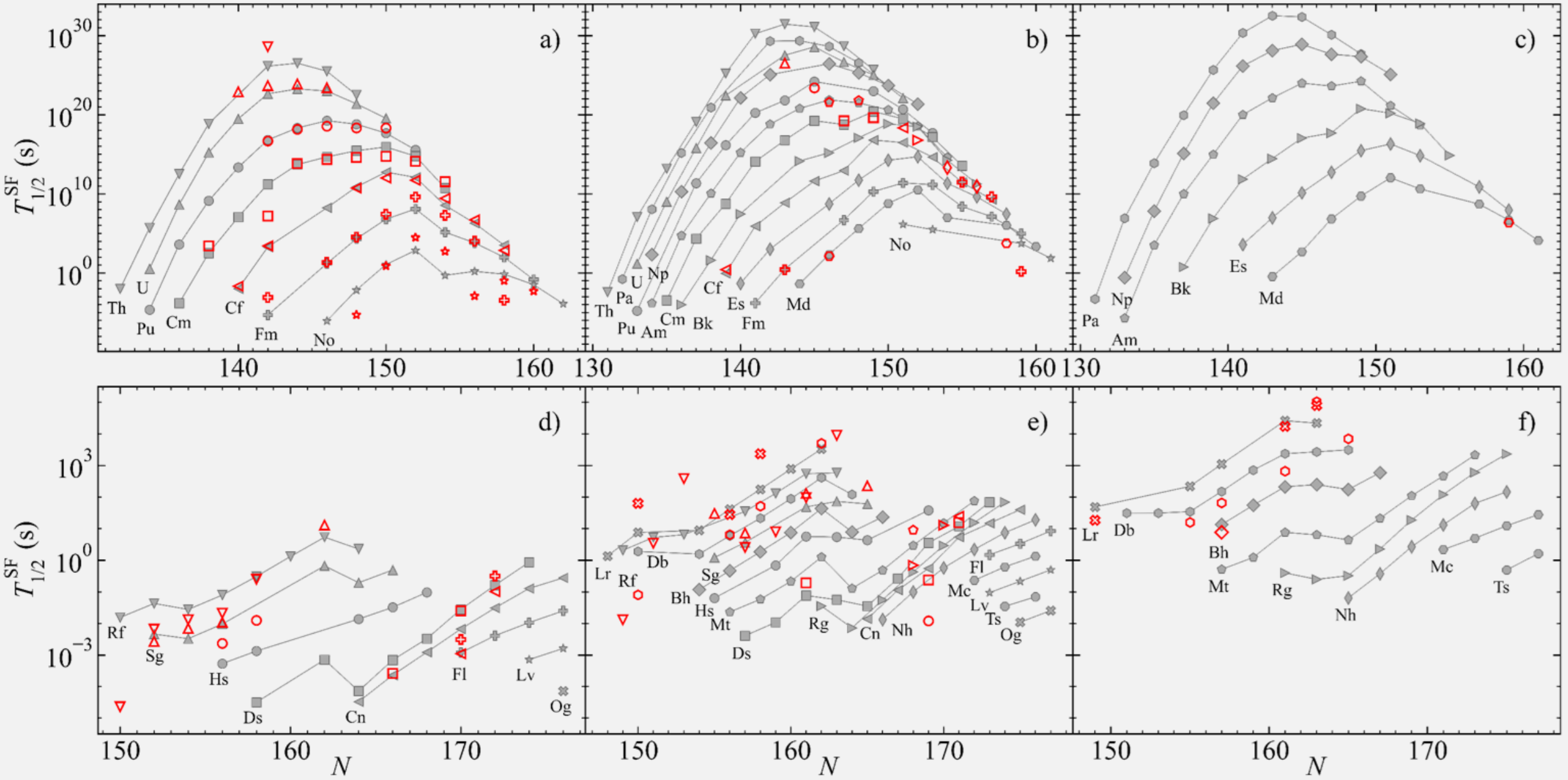
$n_{\mathrm{nucl}} = 9$

$c_0 = 2.56,\ c_1 = 0.161$

$\mathrm{rms}_{\mathrm{fit}} = 0.51$

$c_{\mathrm{shift}} = 3.53$

$\mathrm{rms}_{\mathrm{shift}} = 0.5$



Half-lives of spontaneous fission (filled gray symbols connected by lines) of **even-even** (a,d), **odd-A** (b,e) and **odd-odd** (c,f) nuclei with $90 \leq Z \geq 102$ (top line) and $Z \geq 103$ (bottom line) calculated using the proposed semi-empirical formula in comparison with the experimental data (open red symbols).

AVERAGE VALUES

Root mean square deviation (rms) of calculated values of decimal logarithms of spontaneous fission half-lives from experimental values:

$$\delta_{\mathrm{rms}} = \left\{ \frac{1}{n} \sum_{i=1}^{n} \left[\log_{10} T_{1/2}^{SF} \left((Q_\alpha)_i , N_i - Z_i \right) - \log_{10} \left(T_{1/2}^{SF\ exp} \right)_i \right]^2 \right\}^{1/2}$$

n is the number of cores used to estimate the standard deviation.

Average absolute deviation:

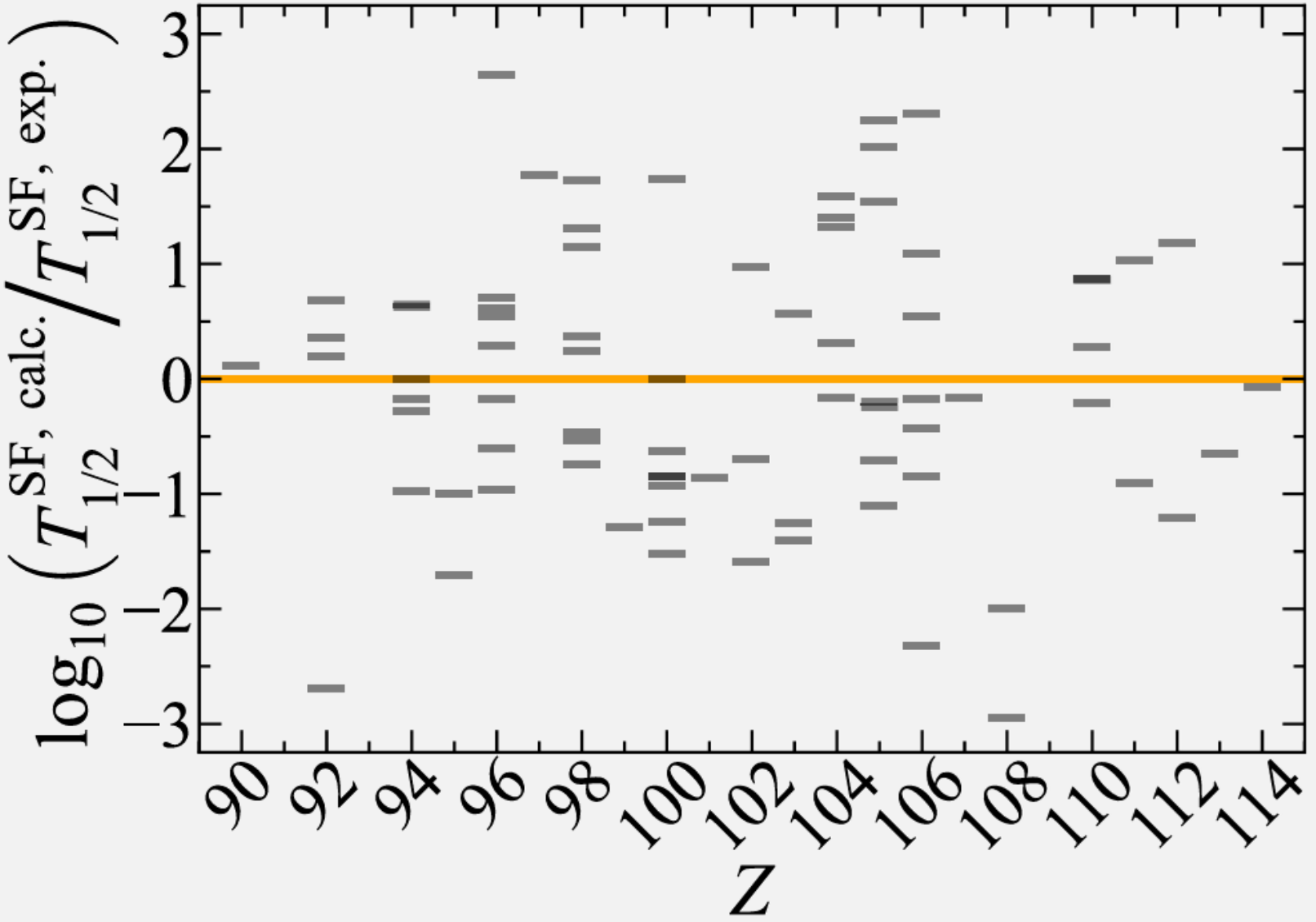
$$\langle \delta \rangle = \frac{1}{n} \sum_{i=1}^{n} \left| \log_{10} T_{1/2}^{SF} \left((Q_\alpha)_i , N_i - Z_i \right) - \log_{10} \left(T_{1/2}^{SF\ exp} \right)_i \right| = \frac{1}{n} \sum_{i=1}^{n} \left| \log_{10} \frac{\left(T_{1/2}^{SF\ \mathrm{calc.}} \right)_i}{\left(T_{1/2}^{SF\ \mathrm{exp.}} \right)_i} \right|$$

Maximum absolute deviation:

$$\delta_{\max} = \max_i \left| \log_{10} T_{1/2}^{SF} \left((Q_\alpha)_i , N_i - Z_i \right) - \log_{10} \left(T_{1/2}^{SF\ exp} \right)_i \right|$$

Mean absolute deviation $\langle\delta\rangle$, rms deviation δ_{rms}, and maximum absolute deviation $\delta_{\max}$ of logarithmic spontaneouse fission half-lives calculated with proposed formula $T_{1/2}^{\mathrm{SF}}\left(T_{1/2}^{\alpha}\right)$ and the experimental data from Refs. [Kondev, 2021] for different nuclide regions; n is the number of experimentally studied nuclei in each region.

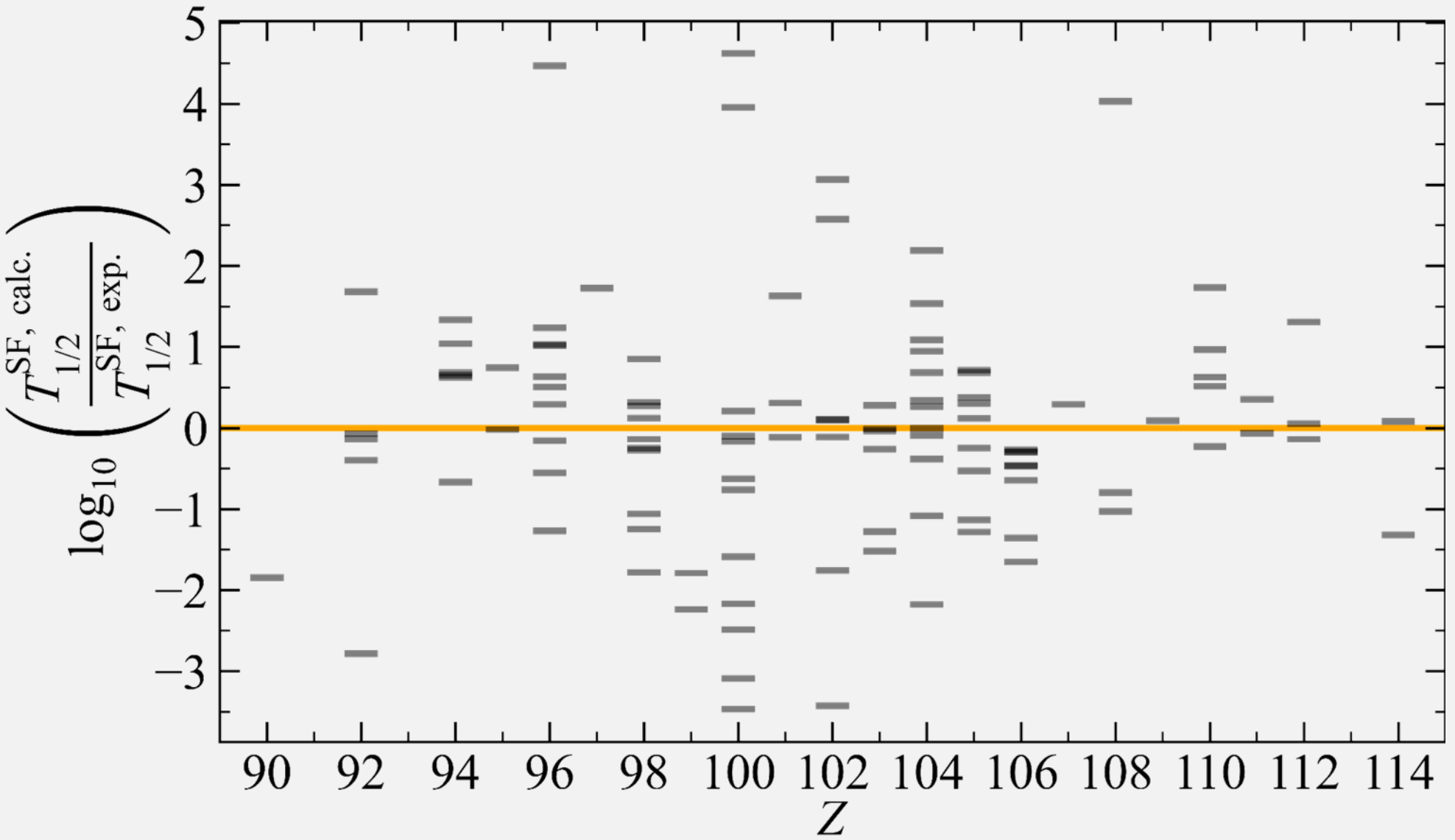
Nuclei	Z	n	$\langle\delta\rangle$	δ_{rms}	$\delta_{\max}$
e-e	$90 \leqslant Z \leqslant 114$	40	0.91	1.17	2.95
e-o	$94 \leqslant Z \leqslant 112$	19	0.88	1.07	2.33
o-e	$95 \leqslant Z \leqslant 113$	13	1.19	1.33	2.25
o-o	$103 \leqslant Z \leqslant 107$	6	0.86	1.02	1.54
all	$90 \leqslant Z \leqslant 114$	78	0.94	1.16	2.95



Deviations of predictions according to formula $T_{1/2}^{\mathrm{SF}}\left(T_{1/2}^{\alpha}\right)$ from the experimental SF half-life values [Kondev, 2021] against charge number of the fissioning nuclei.

Mean absolute deviation $\langle\delta\rangle$, rms deviation δ_{rms}, and maximum absolute deviation $\delta_{\max}$ of logarithmic spontaneouse fission half-lives calculated with proposed formula $T_{1/2}^{\mathrm{SF}}(Q_\alpha)$ and the experimental data from Refs. [Kondev, 2021] for different nuclide regions; n is the number of experimentally studied nuclei in each region.

Nuclei	Z	n	$\langle\delta\rangle$	δ_{rms}	$\delta_{\max}$
e-e		41	1.14	1.64	4.62
o-A	$Z \leq 102$	17	1.49	1.89	3.95
o-o		1	0.11	0.11	0.11
e-e		17	0.69	0.90	2.19
o-A	$Z \geq 103$	26	0.90	1.24	4.03
o-o		9	0.40	0.54	1.28
e-e		58	1.01	1.46	4.62
o-A	$\forall\ Z$	43	1.13	1.53	4.03
o-o		10	0.37	0.51	1.28
all	$\forall\ Z$	111	1.00	1.43	4.62



$n_{\mathrm{nucl}} = 111$
$\delta_{\mathrm{rms}} = 1.43$
$\langle\delta\rangle = 1.00$
$\delta_{\mathrm{max}} = 4.62$

$\delta < 1$:
60% *nuclei*



$\delta < 2$:
86% *nuclei*



$\delta > 4$:
3 *nuclei:*
^{238}Cm, ^{258}Fm, ^{277}Hs

Deviations of predictions of formula $T_{1/2}^{\mathrm{SF}}(Q_\alpha)$ from the experimental half-life values [Kondev, 2021] against charge number of parent nuclei.

COMPARISON WITH ANOTHER EMPIRICAL FORMULA

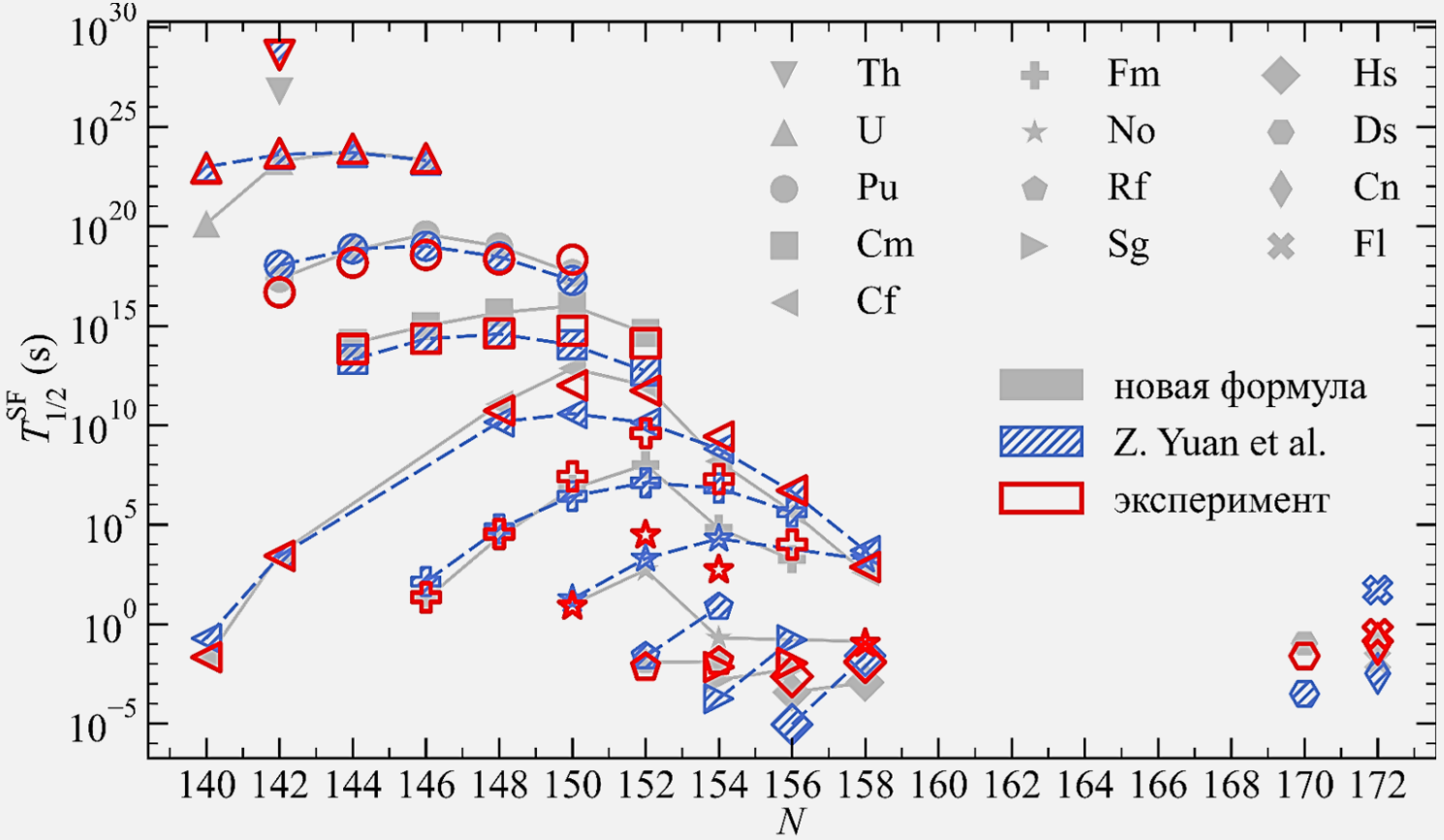
Z. Yuan, D. Bai, Zh. Wang, and Zh. Ren, Improved formulas of spontaneous fission half-lives for heavy and superheavy nuclei, 2024

$$\log_{10} T_{1/2}^{\mathrm{SF}}\ (\mathrm{s}) = 28.60 + c_1 \frac{Z-90}{A} + c_2 \frac{(Z-90)^2}{A} + c_3 \frac{|Z-90|^{1/2}\,(N-Z-52)^2}{A} +$$

$$+ c_4 \frac{(Z-90)(N-152)^2}{A} + c_5 \frac{(Z-90)^2(N-162)}{A} + c_6 \frac{|Z-90|^{5/2}\,|N-162|^{3/2}}{A} + h$$

$c_1 = -646.834955,\ c_2 = 10.058292,\ c_3 = -11.472012,\ c_4 = 0.699861,$

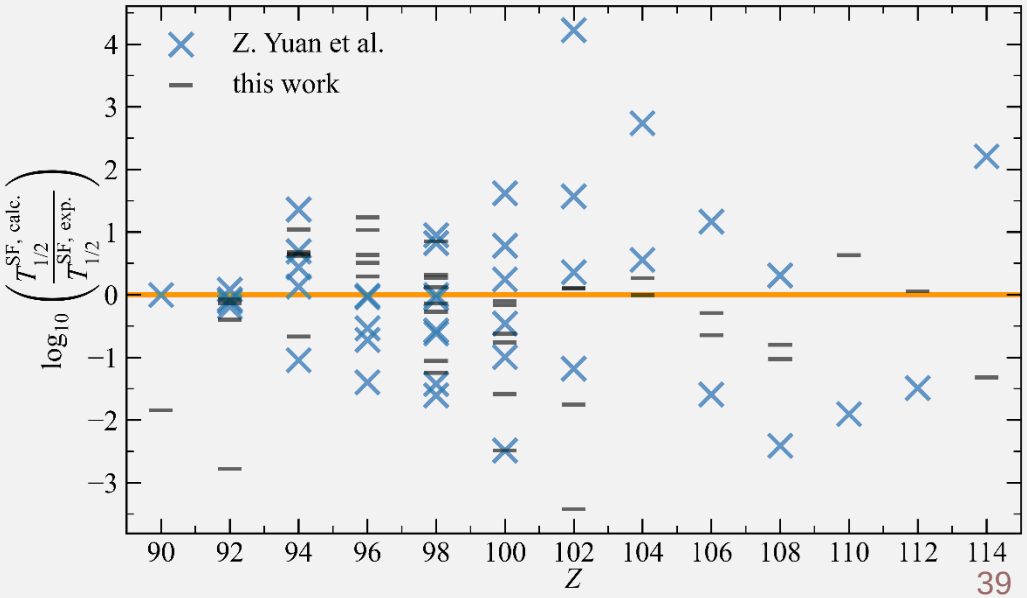
$c_5 = -0.209631,\ c_6 = -0.016230,\ h = 0$ (e-e), $h = 3.128822$ (o-A, o-o)

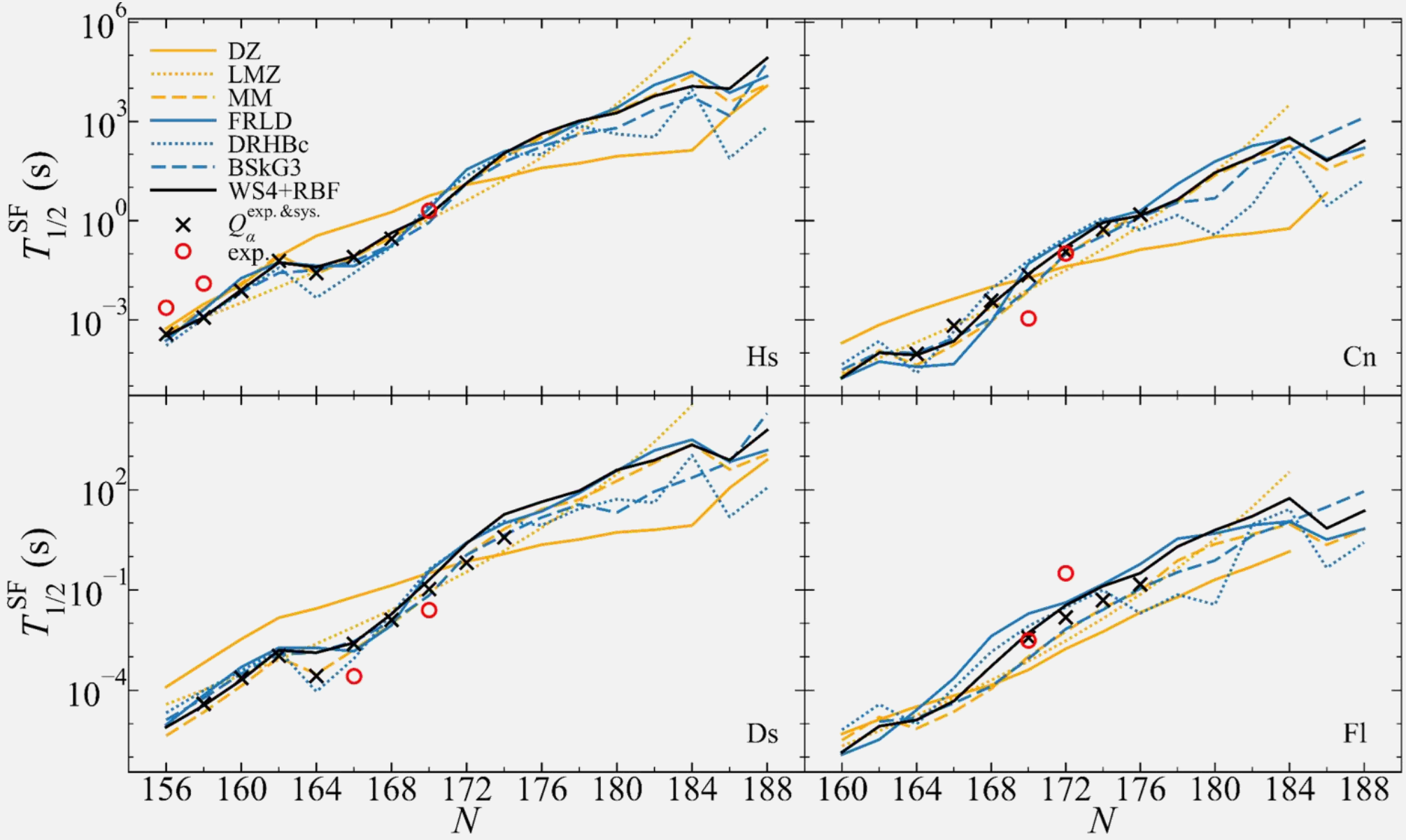


$n_{\mathrm{nucl}} = 42$

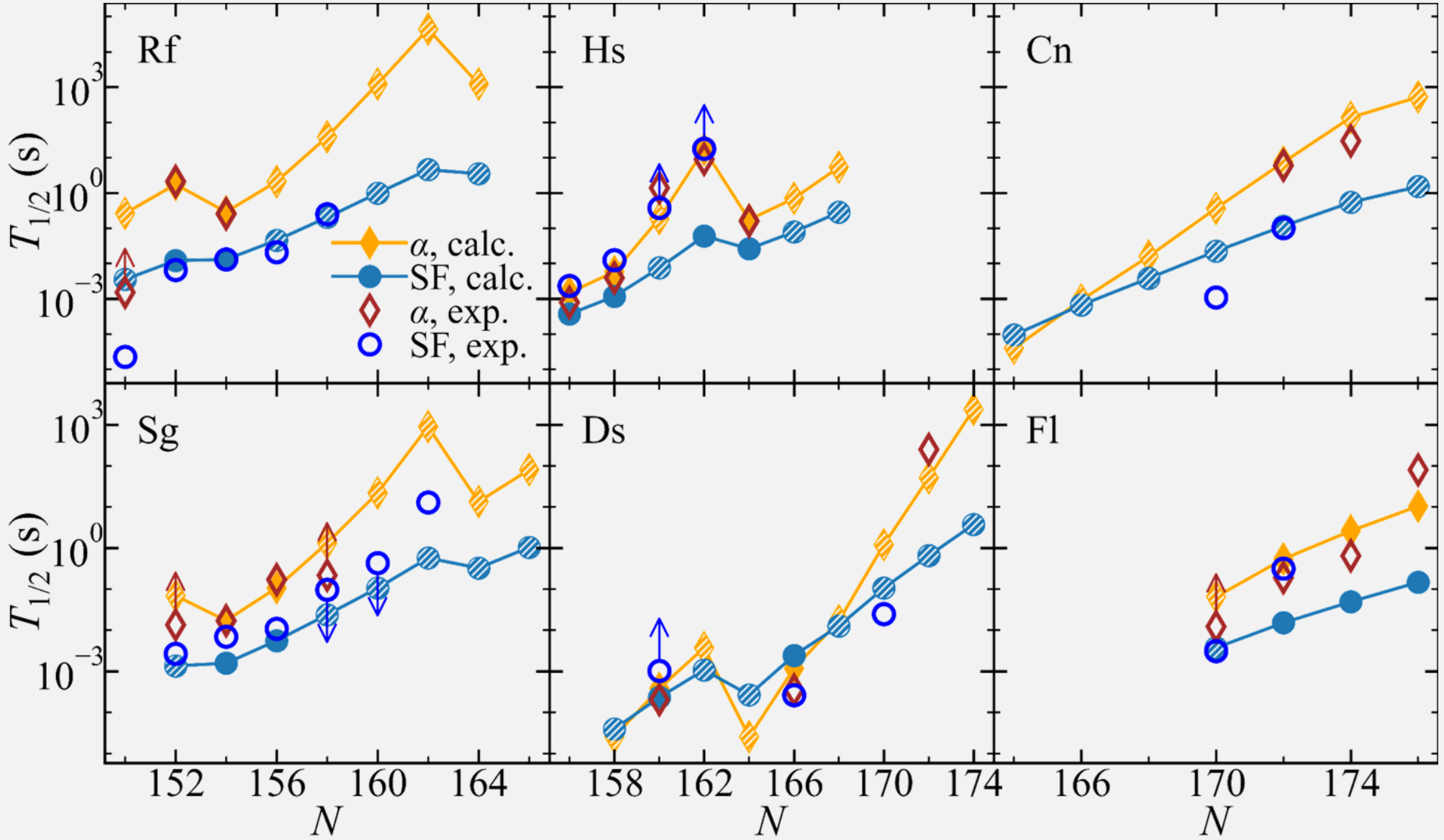
[Yuan *et al.*, 2024]
$\delta_{\mathrm{rms}} = 1.33$
$\langle\delta\rangle = 0.98$

this work
$\delta_{\mathrm{rms}} = 1.09$
$\langle\delta\rangle = 0.79$

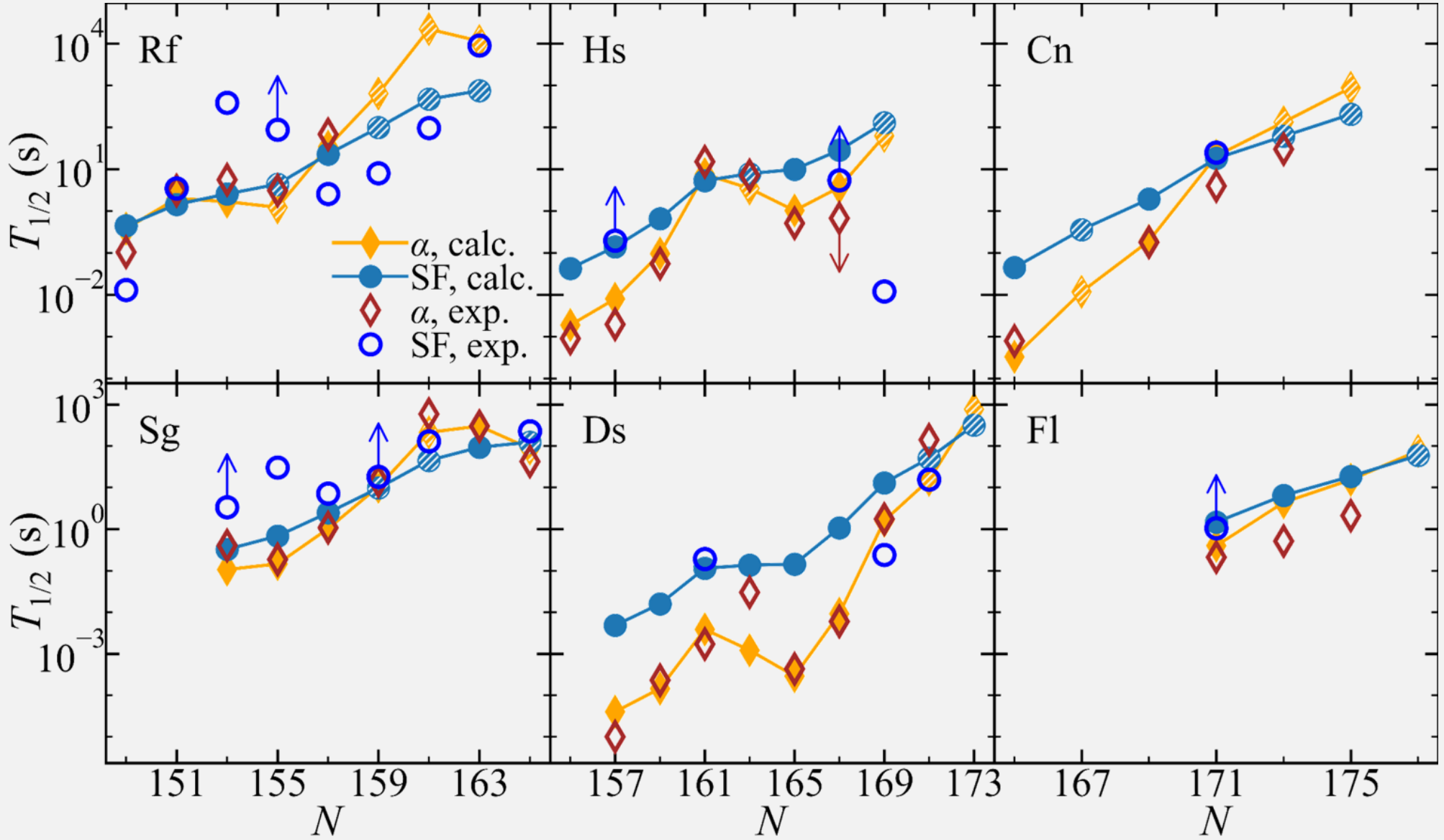




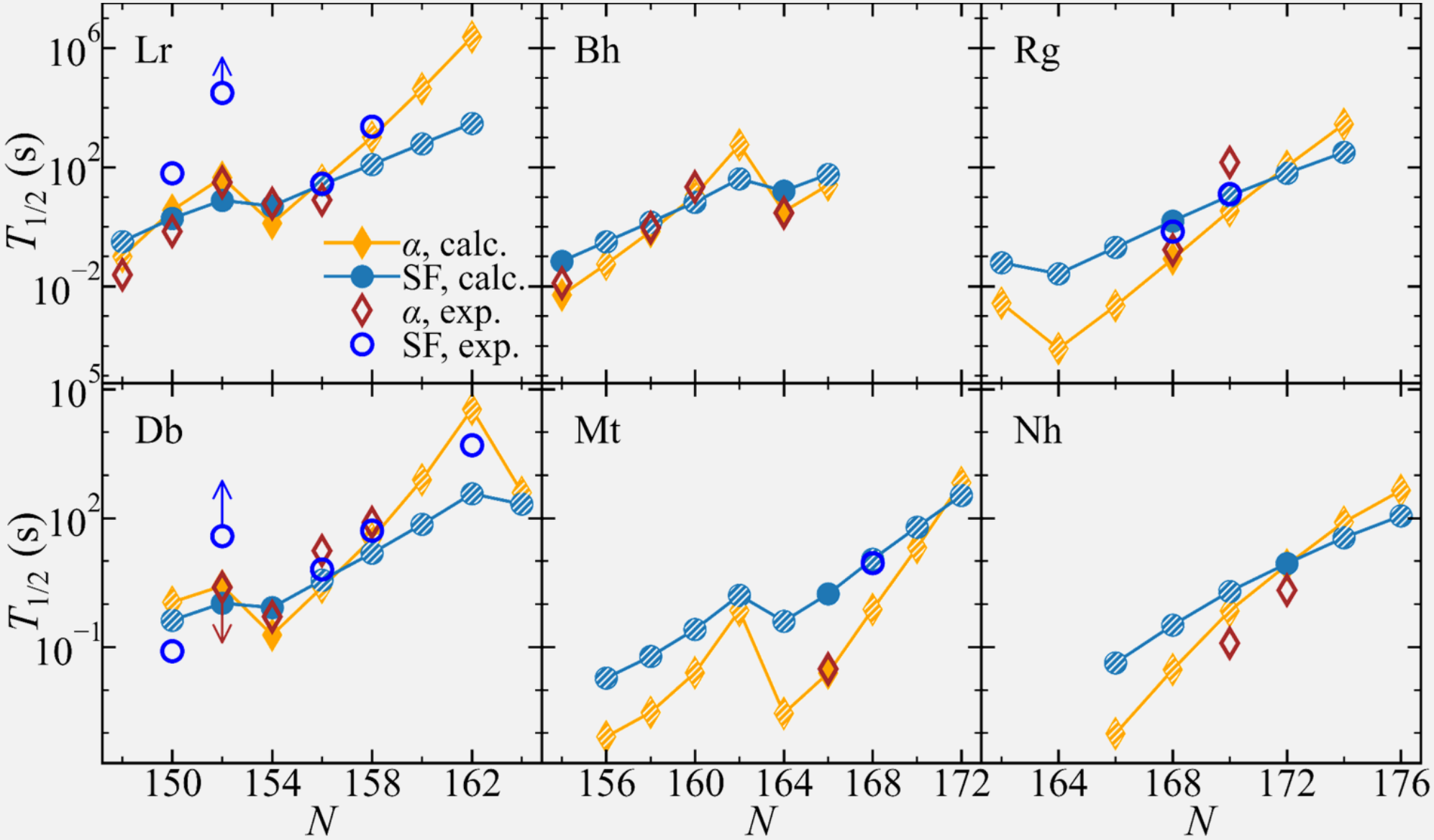
Influence of the choice of mass tables on the predictions of spontaneous fission (SF) half-lives calculated using new formula. Lines indicate calculations based on different mass models, crosses represent calculations based on experimental values and systematics. Open circles denote experimental data.



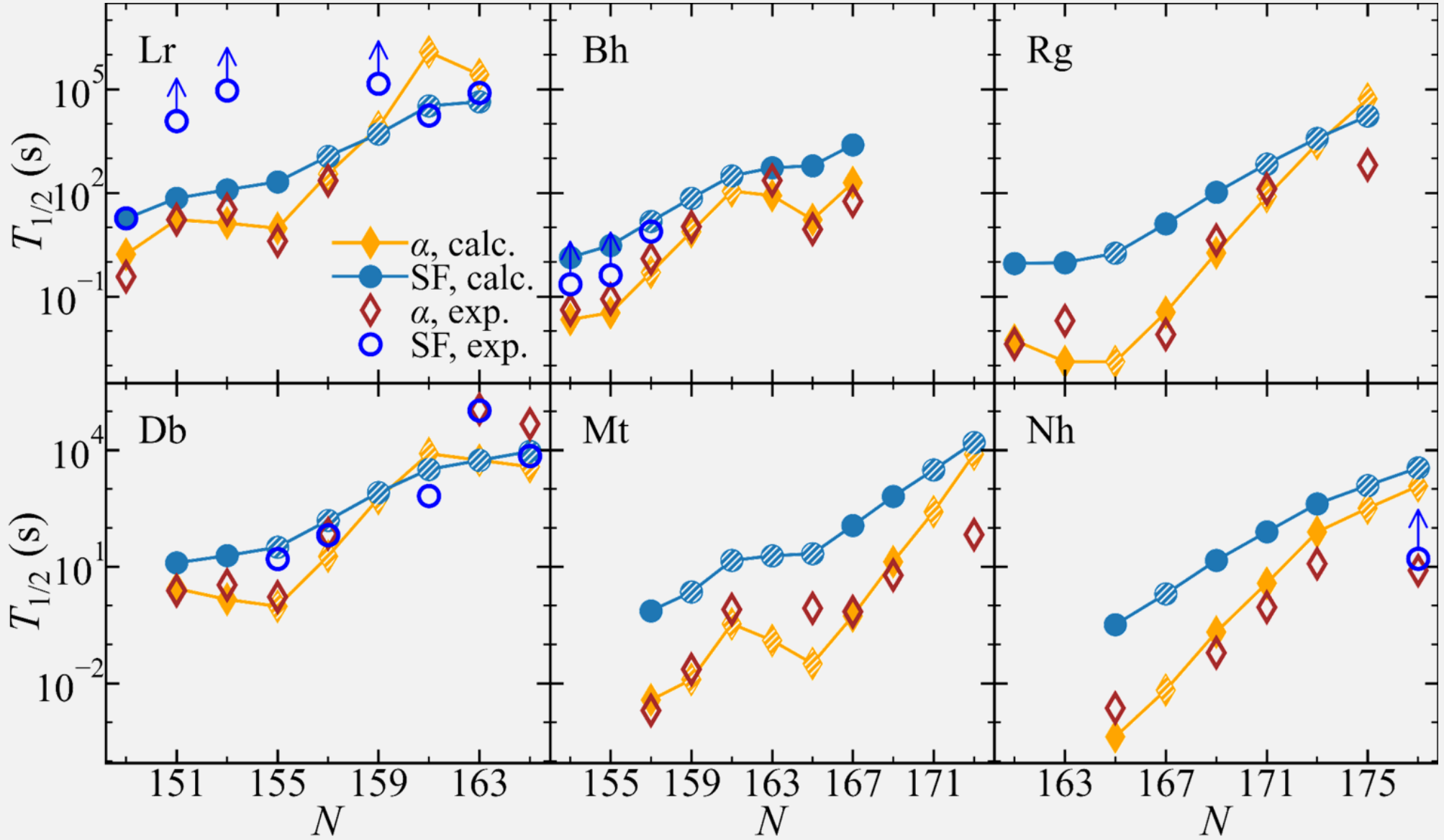
Calculated α-decay (solid diamonds) and spontaneous fission (solid circles) half-lives of **even-even** SHN. Solid (hatched) symbols represent calculations using experimental (systematic) Q_α values [*Wang, 2021*]. Experimental α-decay (open diamonds) and spontaneous fission (open circles) half-lives are taken from [*Kondev, 2021*]. The down (up) arrow indicates only the upper (lower) limit of the half-life of the corresponding decay mode.



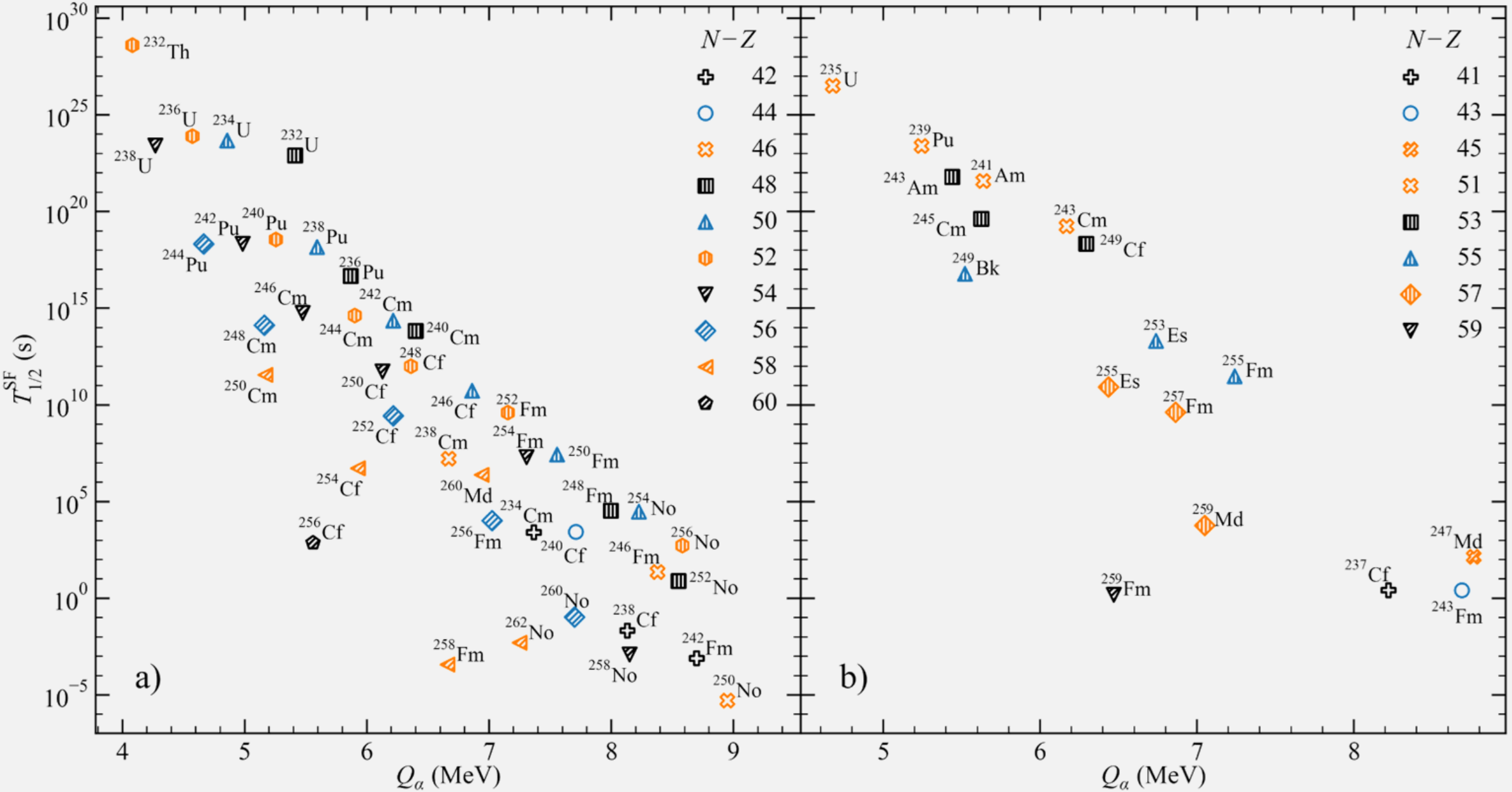
Calculated α-decay (solid diamonds) and spontaneous fission (solid circles) half-lives of **even-odd** SHN. Solid (hatched) symbols represent calculations using experimental (systematic) Q_α values [*Wang, 2021*]. Experimental α-decay (open diamonds) and spontaneous fission (open circles) half-lives are taken from [*Kondev, 2021*]. The down (up) arrow indicates only the upper (lower) limit of the half-life of the corresponding decay mode.



Calculated α-decay (solid diamonds) and spontaneous fission (solid circles) half-lives of **odd-even** SHN. Solid (hatched) symbols represent calculations using experimental (systematic) Q_α values [*Wang, 2021*]. Experimental α-decay (open diamonds) and spontaneous fission (open circles) half-lives are taken from [*Kondev, 2021*]. The down (up) arrow indicates only the upper (lower) limit of the half-life of the corresponding decay mode.



Calculated α-decay (solid diamonds) and spontaneous fission (solid circles) half-lives of **odd-odd** SHN. Solid (hatched) symbols represent calculations using experimental (systematic) Q_α values [*Wang, 2021*]. Experimental α-decay (open diamonds) and spontaneous fission (open circles) half-lives are taken from [*Kondev, 2021*]. The down (up) arrow indicates only the upper (lower) limit of the half-life of the corresponding decay mode.



Spontaneous fission half-life versus alpha separation energy for nuclei with $90 \leq Z \leq 102$, shown for even (left) and odd (right) neutron excess $N - Z$. Experimental data from [Wang, 2021; Kondev, 2021].

Classification of the competition between ground-state α decay and SF according to the ordering of half-lives relative to the time of maximum competition $T_c = 0.232$ s and the formula with the corresponding ranges of neutron excesses $N - Z$.

Ordering of half-lives	$\frac{T^{\alpha}_{1/2}}{T_c}$	$\frac{T^{\mathrm{SF}}_{1/2}}{T_c}$	$f \equiv \log_{\left(T^{\alpha}_{1/2}/T_c\right)}\left(T^{\mathrm{SF}}_{1/2}/T_c\right)$	$N - Z$ range	Dominant decay mode
$T_c < T^{\alpha}_{1/2} < T^{\mathrm{SF}}_{1/2}$	>1	>1	$f > 1$	$N - Z < (N - Z)_{\mathrm{cr1}}$	α
$T^{\alpha}_{1/2} < T_c < T^{\mathrm{SF}}_{1/2}$	<1	>1	$f < 0$	$N - Z > (N - Z)_{\mathrm{cr2}}$	α
$T^{\alpha}_{1/2} < T^{\mathrm{SF}}_{1/2} < T_c$	<1	<1	$0 < f < 1$	$(N - Z)_{\mathrm{cr1}} < N - Z < (N - Z)_{\mathrm{cr2}}$	α
$T_c < T^{\mathrm{SF}}_{1/2} < T^{\alpha}_{1/2}$	>1	>1	$0 < f < 1$	$(N - Z)_{\mathrm{cr1}} < N - Z < (N - Z)_{\mathrm{cr2}}$	SF
$T^{\mathrm{SF}}_{1/2} < T_c < T^{\alpha}_{1/2}$	>1	<1	$f < 0$	$N - Z > (N - Z)_{\mathrm{cr2}}$	SF
$T^{\mathrm{SF}}_{1/2} < T^{\alpha}_{1/2} < T_c$	<1	<1	$f > 1$	$N - Z < (N - Z)_{\mathrm{cr1}}$	SF

$$f \equiv \log_{T^{\alpha}_{1/2}/T_c}\left(T^{\mathrm{SF}}_{1/2}/T_c\right) = c_0 + c_1\left[N - Z + (-1)^{N-Z} - 1\right]$$

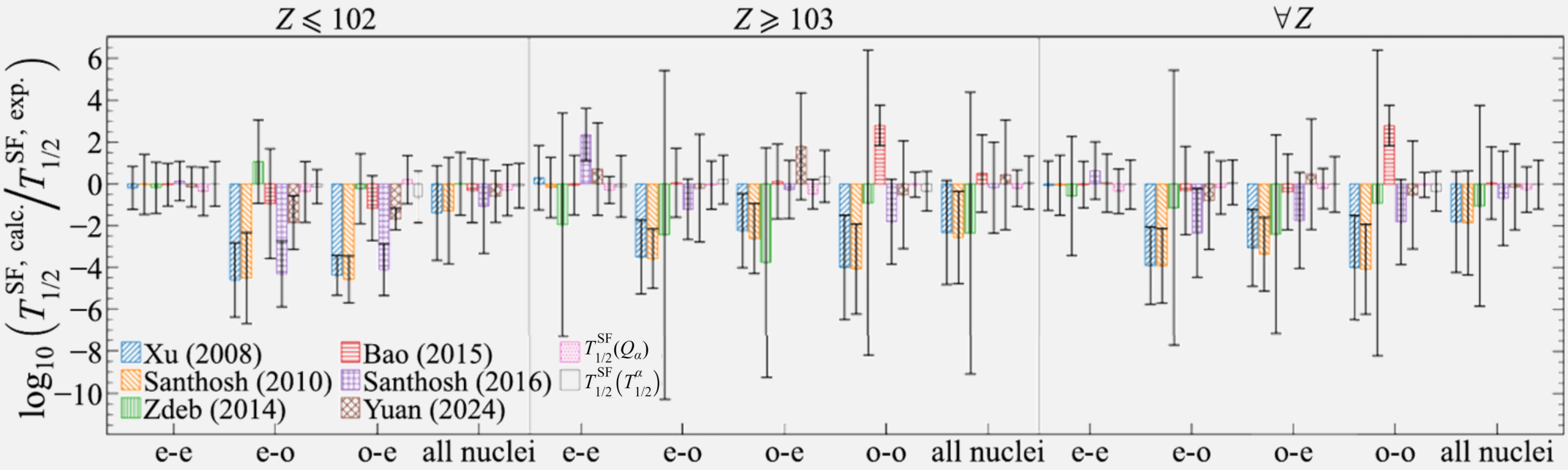
$$c_0 = 7.912357,$$
$$c_1 = -0.1212846,$$
$$T_c = 0.232 \text{ s}.$$

$f = 1$

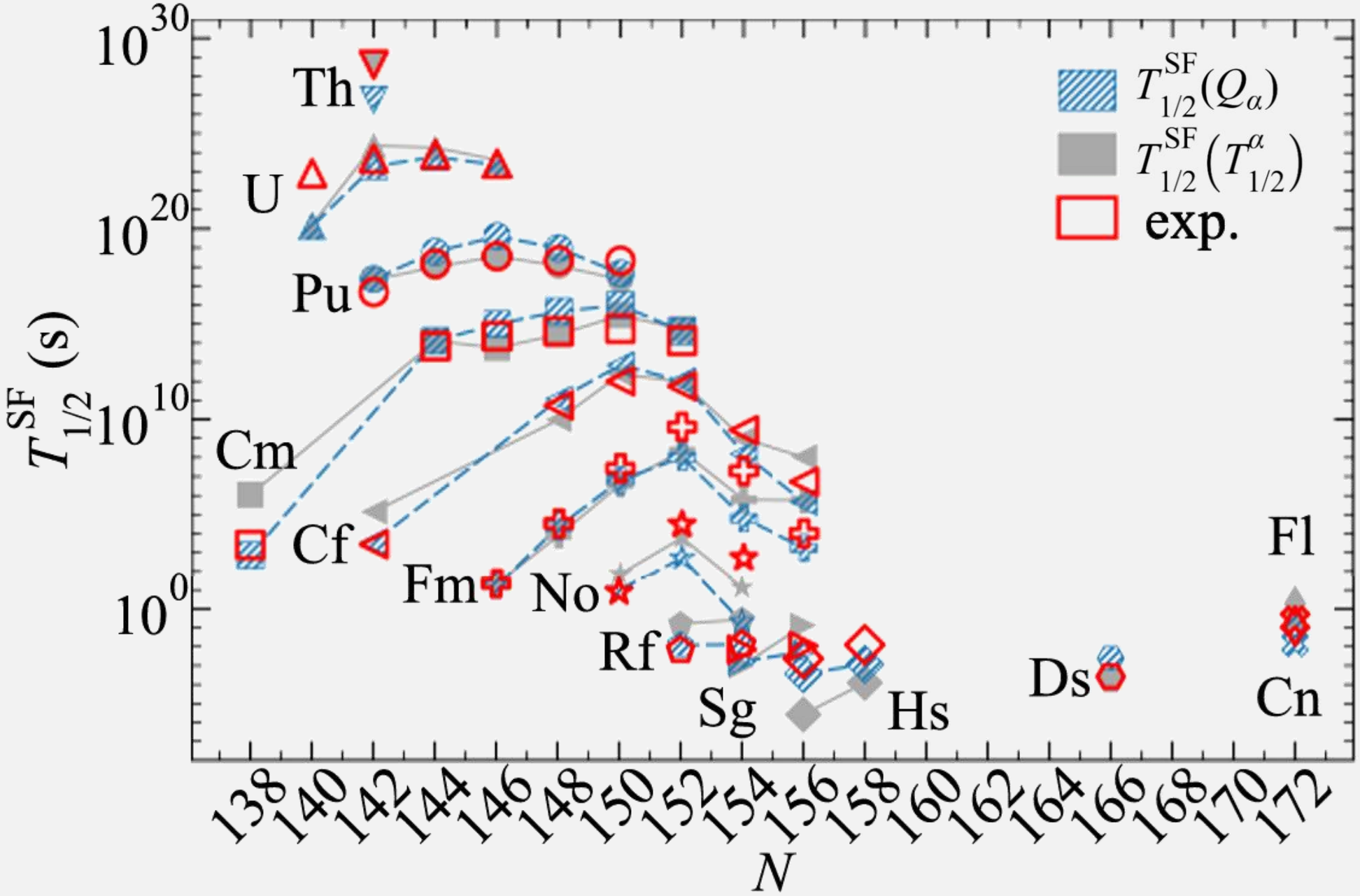
$$(N - Z)_{\mathrm{cr1}} = \begin{cases} \frac{1-c_0}{c_1} = 56.99 & \text{for e-e, o-o nuclei,} \\ \frac{1-c_0}{c_1} + 2 = 58.99 & \text{for e-o, o-e nuclei,} \end{cases}$$

$f = 0$

$$(N - Z)_{\mathrm{cr2}} = \begin{cases} -\frac{c_0}{c_1} = 65.24 & \text{for e-e, o-o nuclei,} \\ -\frac{c_0}{c_1} + 2 = 67.24 & \text{for e-o, o-e nuclei.} \end{cases}$$



Mean values of the decimal logarithms of the ratio of calculated-to-experimental SF half-lives. The calculations were performed using the empirical formulas and the two proposed formulas. Experimental data are taken from [Kondev, 2021]. The comparison is made for three nuclear groups: with proton numbers $Z \leq 102$, $Z \geq 103$, and for the combined dataset ($\forall Z$). Within each group, nuclei are categorized by the parity of protons and neutrons: e-e (even-even), e-o (even-odd), o-e (odd-even), o-o (odd-odd), and all nuclei. The error bars represent the standard deviations.



SF half-lives for 40 even-even nuclei, calculated with formulas $T_{1/2}^{SF}(Q_\alpha)$ (dashed blue lines connecting hatched symbols) and $T_{1/2}^{SF}(T_{1/2}^{\alpha})$ (solid gray lines connecting closed symbols), compared to experimental values (open red symbols).