Heavy quark polarisation in QCD medium

Sourav Dey

Based on:

S. Dey and A. Jaiswal, arXiv:2502.20352

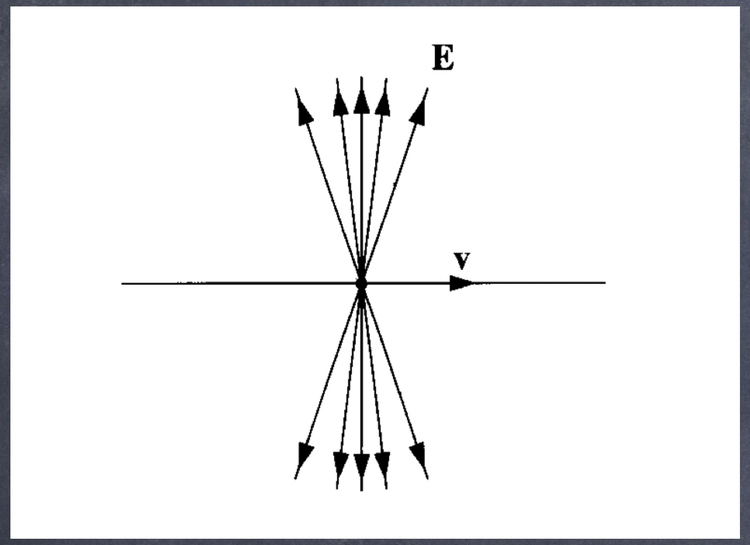
Workshop on physics performance studies at NICA (NICA-2025)

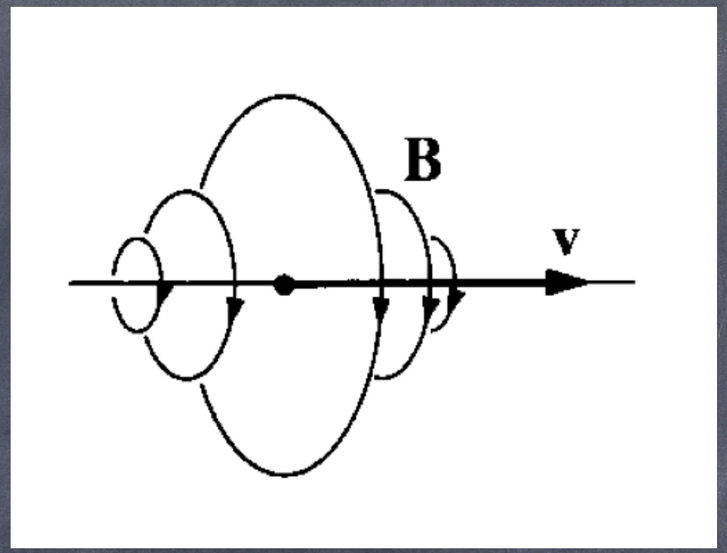
17 Dec 2025



Moving charges create a magnetic field:

- A moving charged particle generates magnetic field due to its motion.





$$\mathbf{B}(t,\mathbf{r}) = \mathbf{v}\times\mathbf{E}(t,\mathbf{r})$$

$$e\mathbf{E}(t,\mathbf{r}) = \alpha_{em}\frac{(1-\mathbf{v}^2)\mathbf{R}}{\mathbf{R}^3\left[1-\frac{(\mathbf{R}\times\mathbf{v})^2}{\mathbf{R}^2}\right]^{\frac{3}{2}}} \quad \text{where,} \quad \mathbf{R} = \mathbf{r} - \mathbf{V}t$$

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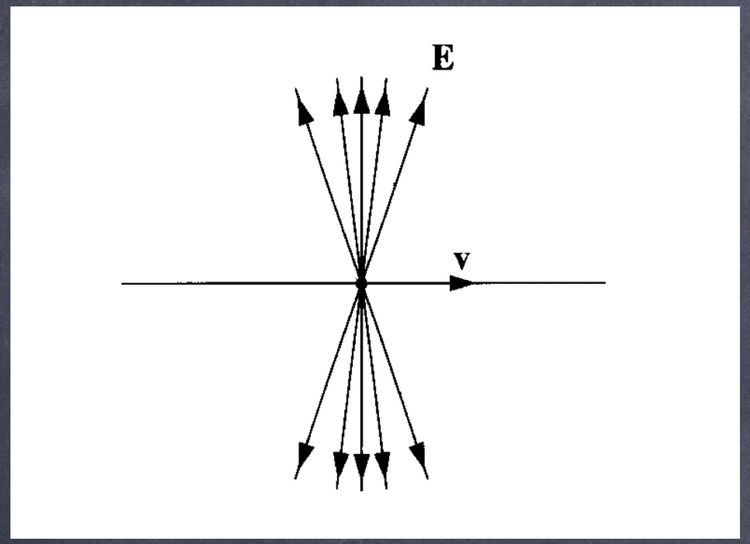
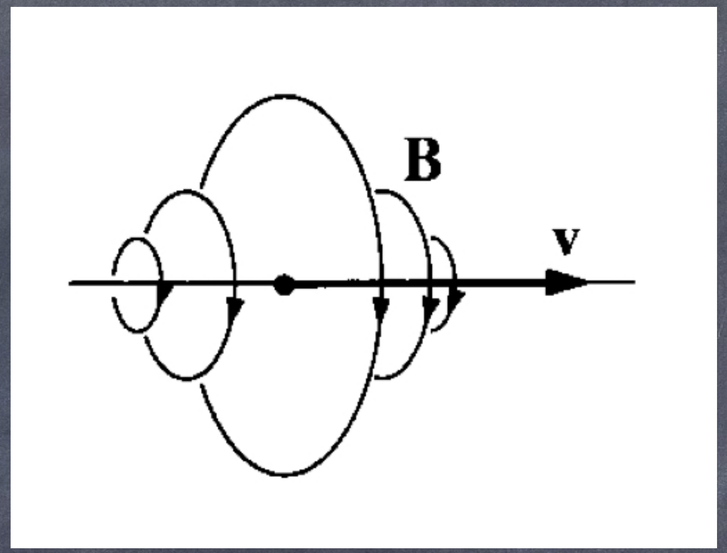
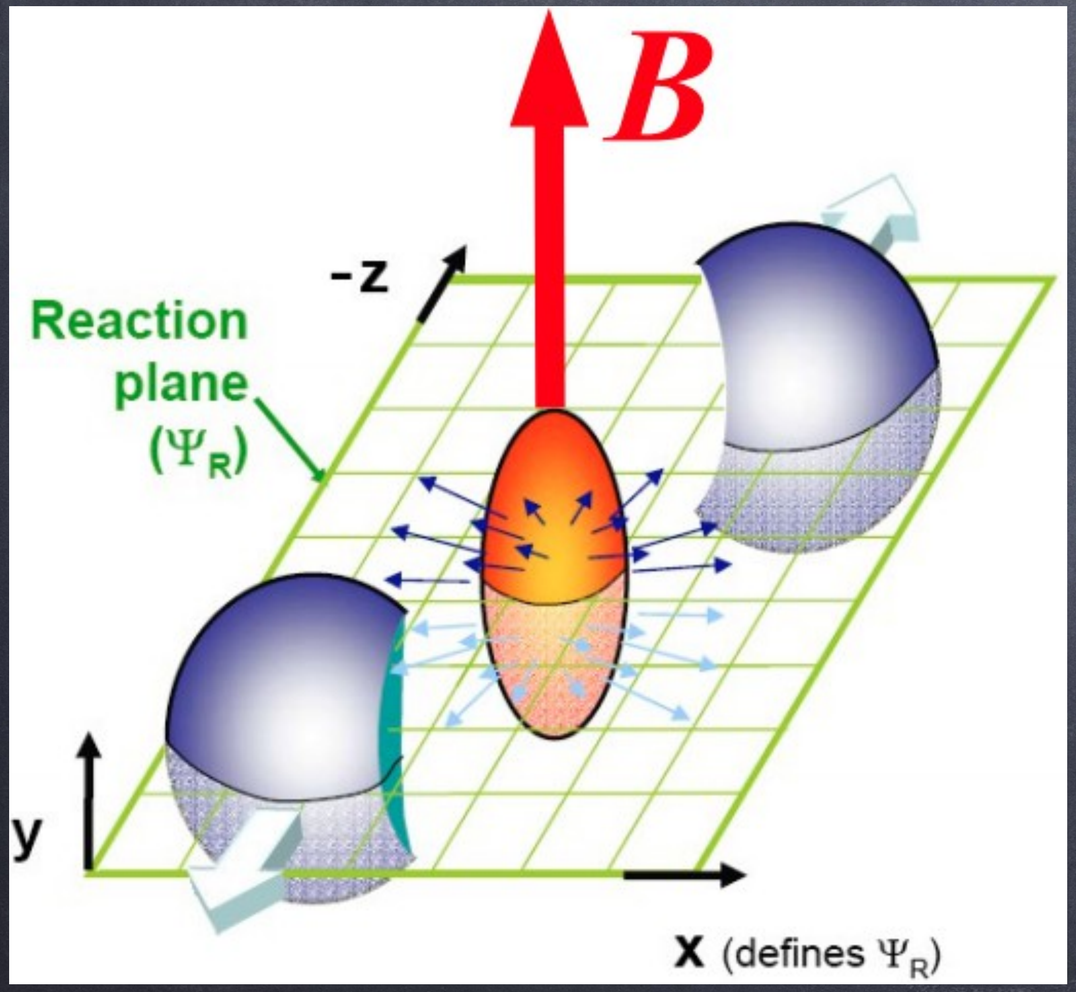




$$\mathbf{B}(t,\mathbf{r}) = \mathbf{v} \times \mathbf{E}(t,\mathbf{r})$$

$$e\mathbf{E}(t,\mathbf{r}) = \alpha_{em} \sum_i \frac{Z_i(1-\mathbf{v}_i^2)\mathbf{R}_i}{\mathbf{R}_i^3\left[1-\frac{(\mathbf{R}_i \times \mathbf{v}_i)^2}{\mathbf{R}_i^2}\right]^{\frac{3}{2}}} \quad \text{where,} \quad \mathbf{R} = \mathbf{r} - \mathbf{V}t$$

Magnetic field in heavy ion collision



- **B** generated in HICs by spectator nucleons, which is perpendicular to the reaction plane.

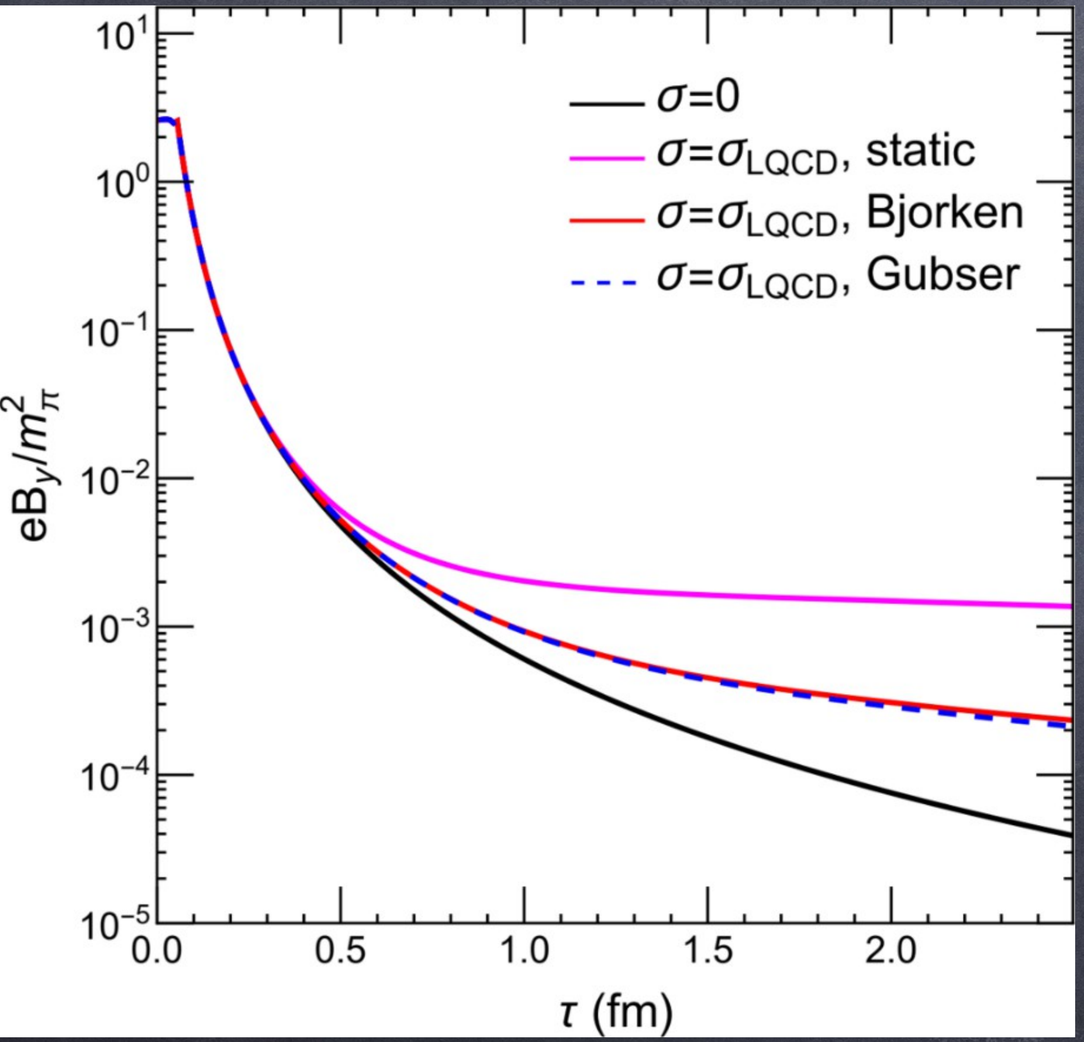
$$e\mathbf{E}(t,\mathbf{r}) = \alpha_{em}\sum_i \frac{Z_i(1-\mathbf{v}_i^2)\mathbf{R}_i}{\mathbf{R}_i^3\left[1-\frac{(\mathbf{R}_i\times\mathbf{v}_i)^2}{\mathbf{R}_i^2}\right]^{\frac{3}{2}}}$$

$$\mathbf{B}(t,\mathbf{r}) = \mathbf{v}\times\mathbf{E}(t,\mathbf{r})$$

$$\mathbf{B} \sim 10^{14} T$$

[Adapted from D. Kharzeev @ CPOD 2013.]

Magnetic field in heavy ion collision



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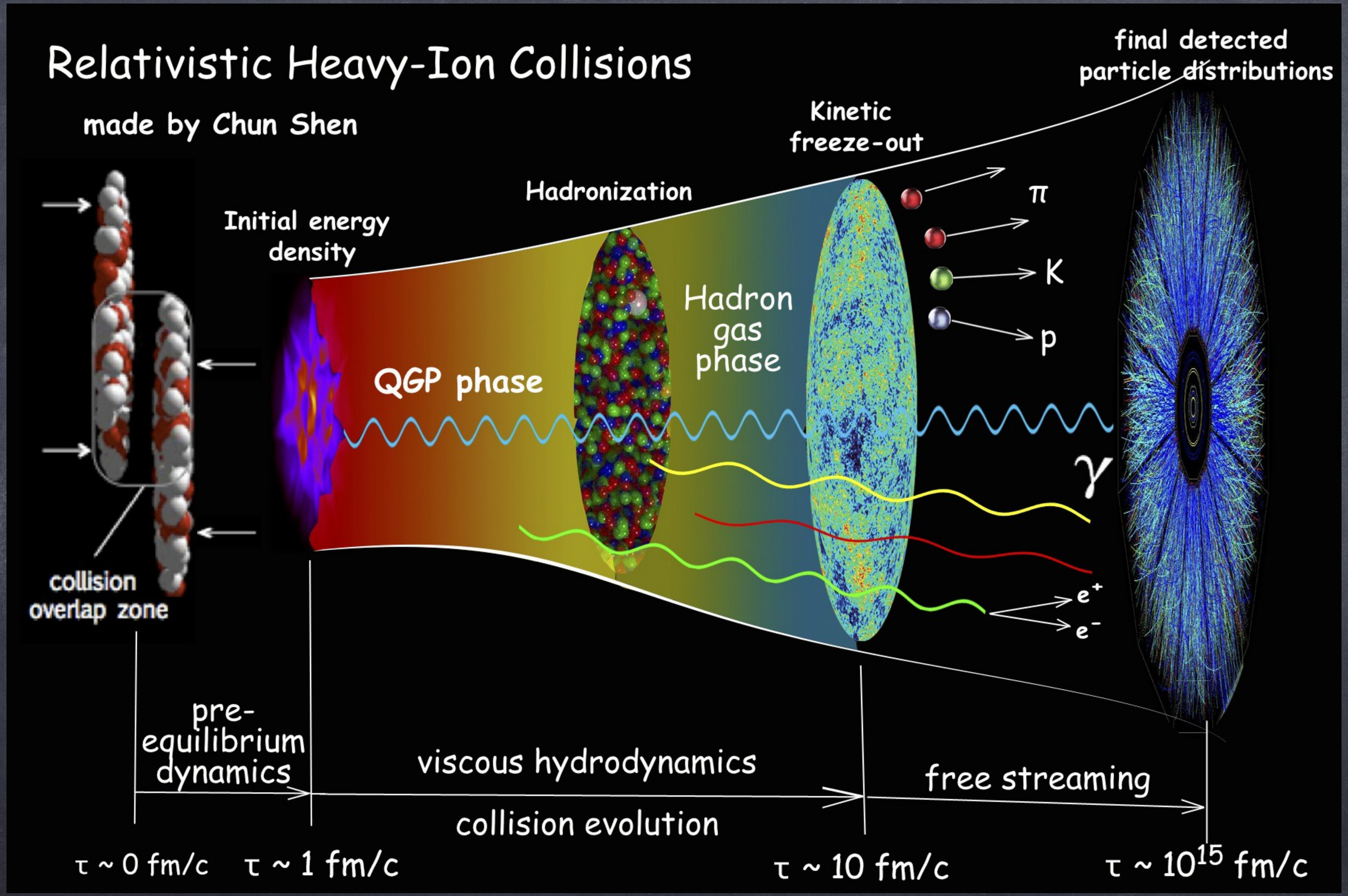
$$e\mathbf{B} \sim m_\pi^2$$

[A. Huang, D. She, S. Shi, M. Huang and J. Liao, Phys. Rev. C 107, 034901 (2023).]


Relativistic Heavy-Ion Collisions
made by Chun Shen
final detected particle distributions
Kinetic freeze-out
Hadronization
Initial energy density
π
K
p
Hadron gas phase
QGP phase
γ
e+
e-
collision overlap zone
pre-equilibrium dynamics
viscous hydrodynamics
free streaming
collision evolution
τ ~ 0 fm/c
τ ~ 1 fm/c
τ ~ 10 fm/c
τ ~ 10^15 fm/c

Heavy quarks in HICs

- Heavy quarks (charm and bottom) has long been recognised as an excellent probe of transport properties of QCD medium. [S. K. Das, J.-e. Alam, and P. Mohanty, PRC 82 (2010) 014908; D. Banerjee, S. Datta, R. Gavai, and P. Majumdar, PRD 85 (2012) 014510; ...]

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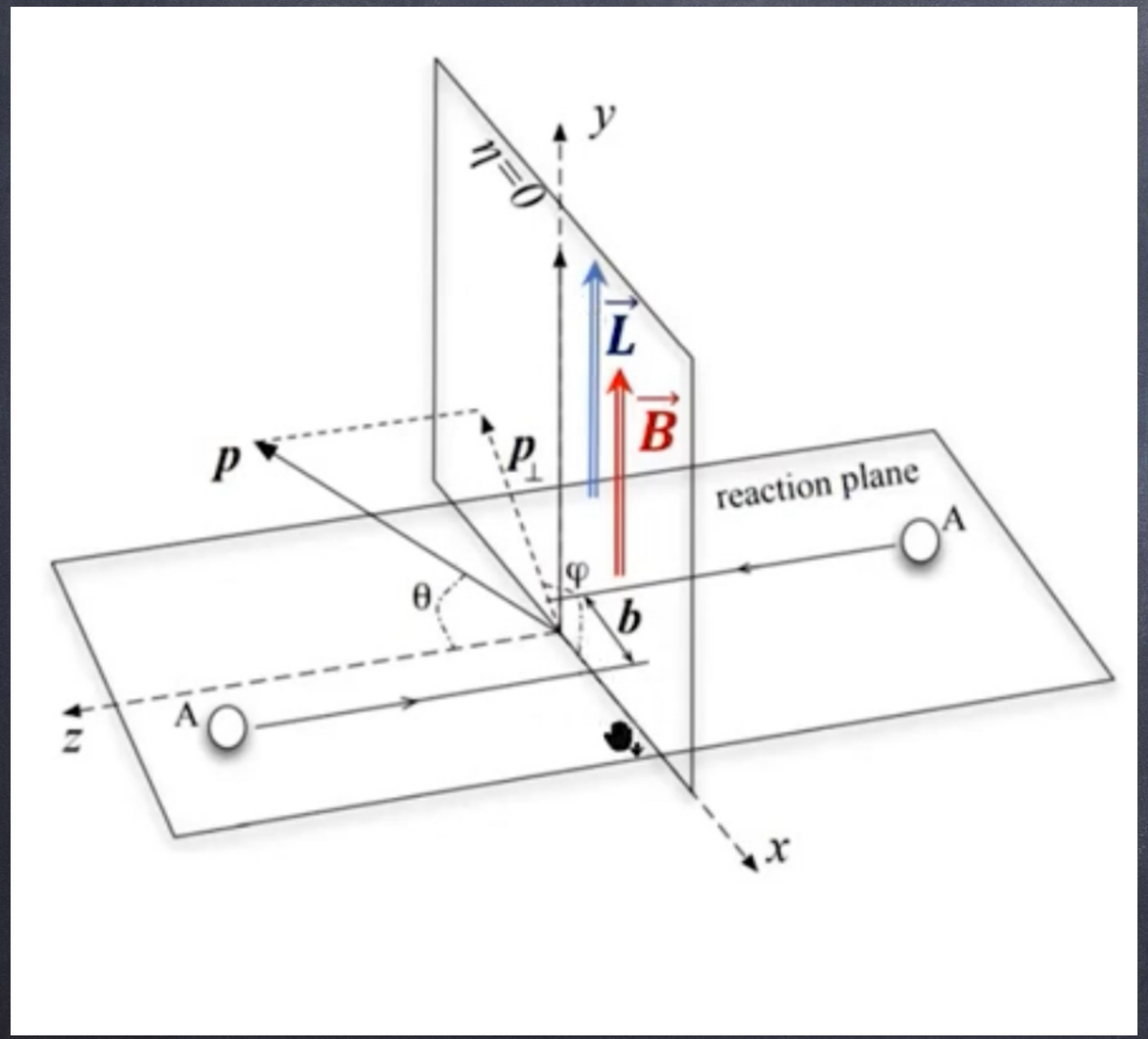
- Strong transient magnetic fields produced which are significant only during the early stages of the collision.
- Heavy quarks: ideal for observable signals of initial magnetic field.
- Transverse momentum dependence of open heavy-flavor hadron polarisation: distinctive signal for initial strong magnetic field.

Spin polarization of hadrons in HICs

- Spin polarisation is a relatively new topic in heavy ion collisions.



- Attributed to angular momentum deposition in off-central Coll.

$$L \simeq A\sqrt{s_{NN}}\frac{b}{2}$$

$L \sim 5 \times 10^4$ at $\sqrt{s_{NN}} = 200\,GeV$ (RHIC Au-Au Coll)

$L \sim 1.4 \times 10^7$ at $\sqrt{s_{NN}} = 5.5\,TeV$ (LHC Pb-Pb Coll)

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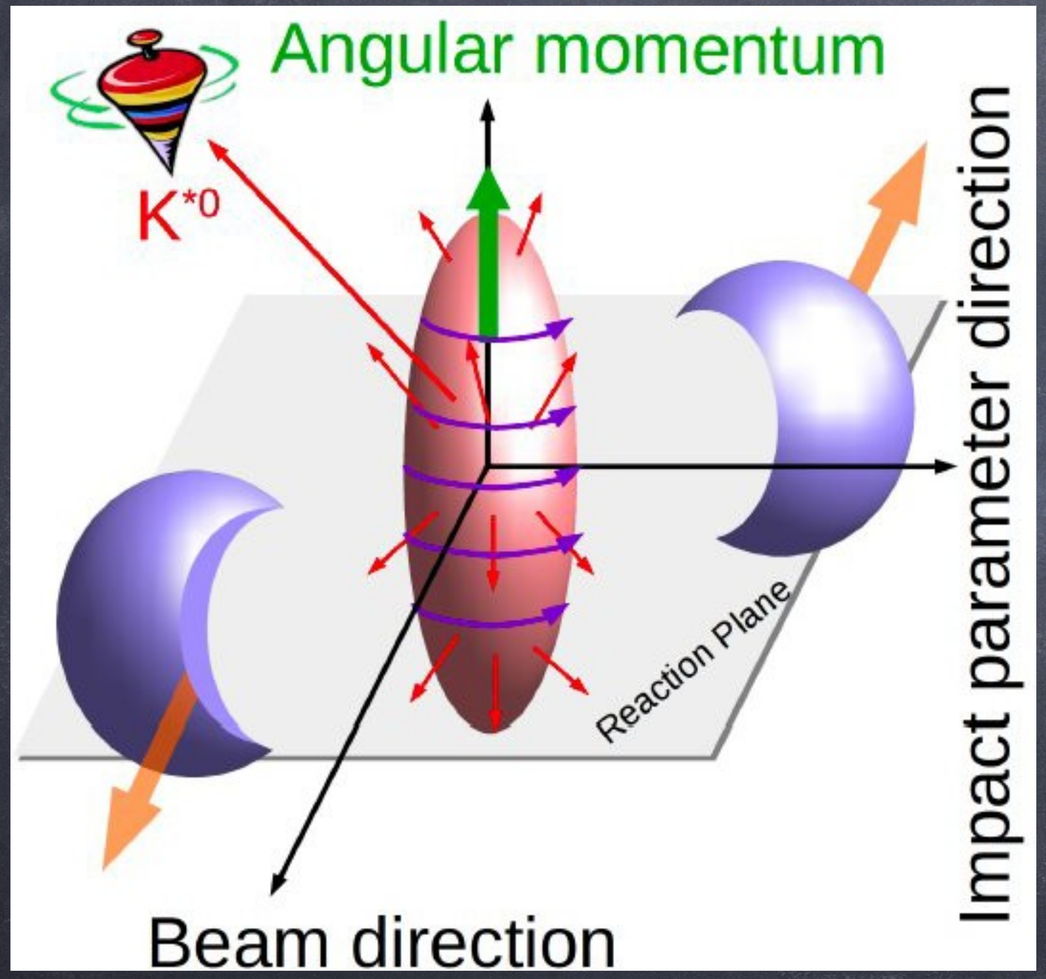

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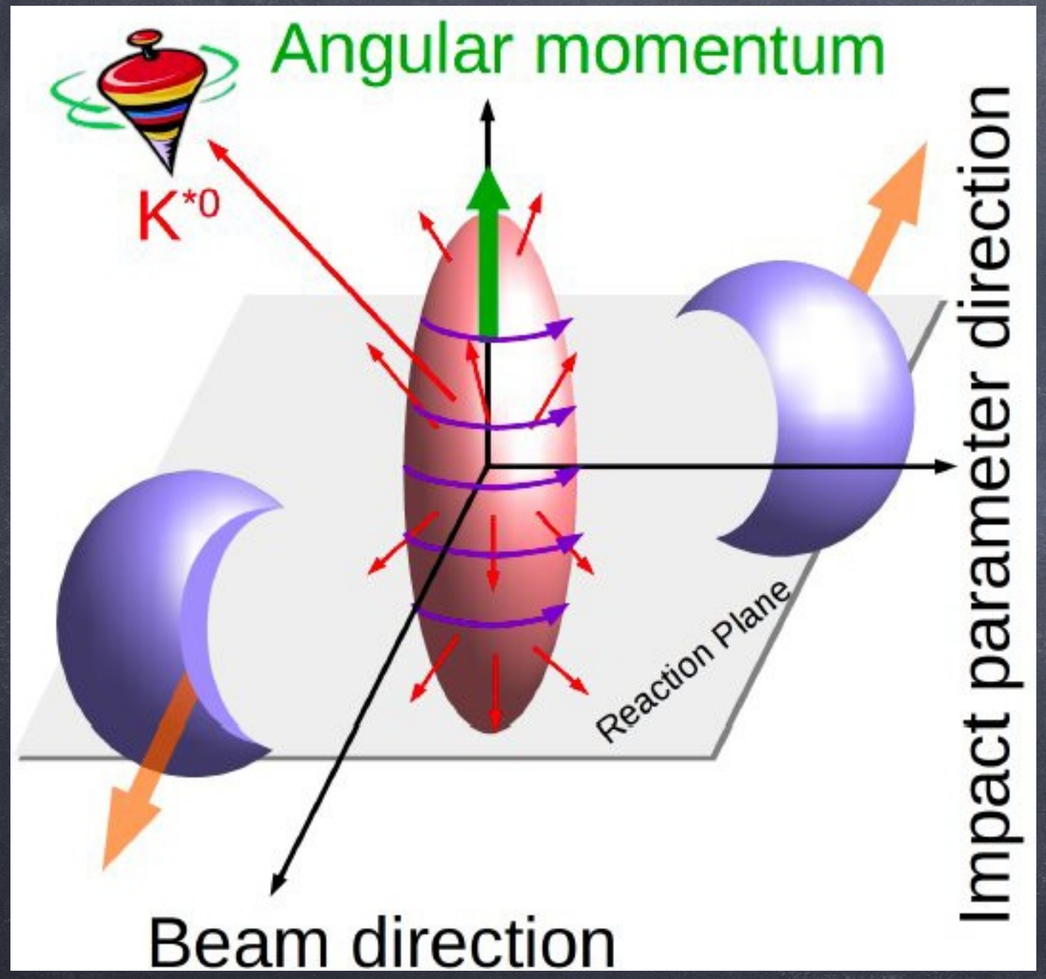

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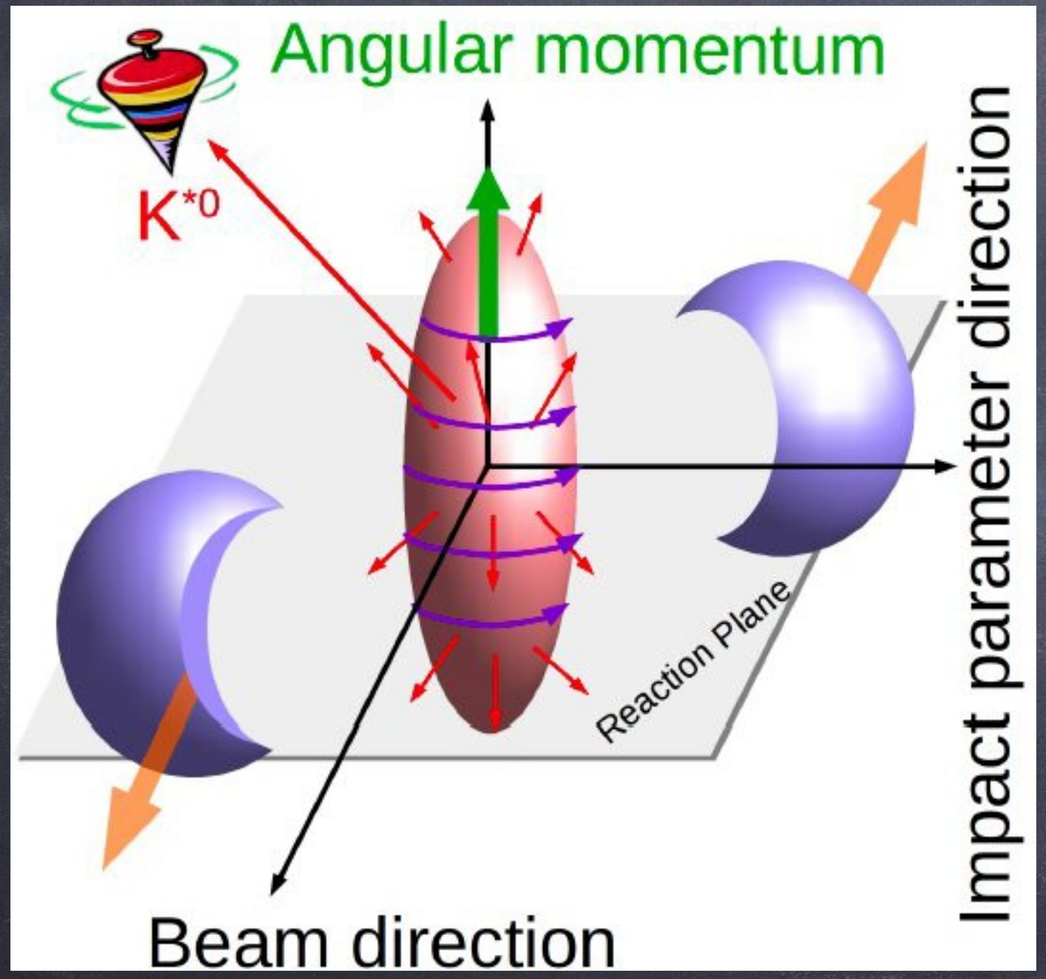

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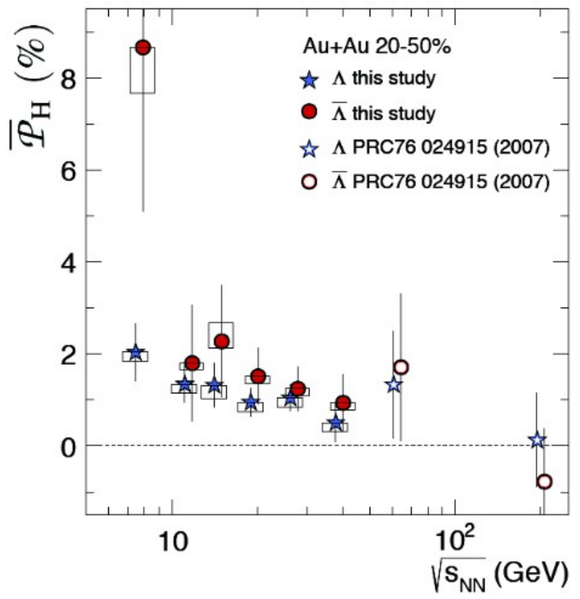
STAR Collaboration, *Global Lambda hyperon polarization in nuclear collisions,* Nature 548 62-65, 2017





First evidence of a quantum effect in (relativistic) hydrodynamics

- QGP most vortical fluid

$$\omega \sim 10^{22} s^{-1}$$

Adapted from F. Becattini 'Subatomic Vortices'

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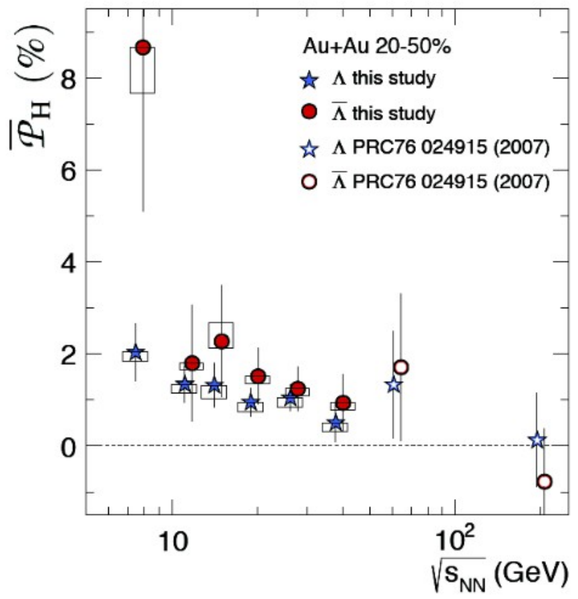



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$$\vec{\mathscr{P}} \approx \frac{\beta\vec{\omega}}{2} + \beta\mu_q \vec{B}$$

[Z.T.Liang, X.N.Wang '05; Becattini, Karpenko, Lisa, Upsal, Voloshin '17,; Y.Yang, R.Fang, Q. Wang, X. Wang'18]

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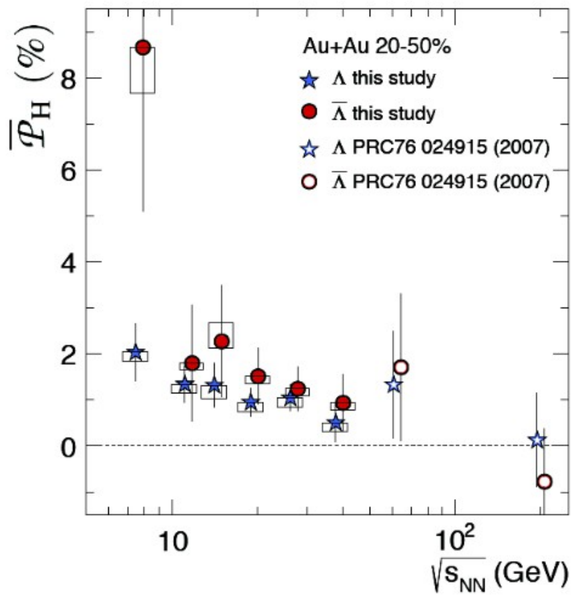



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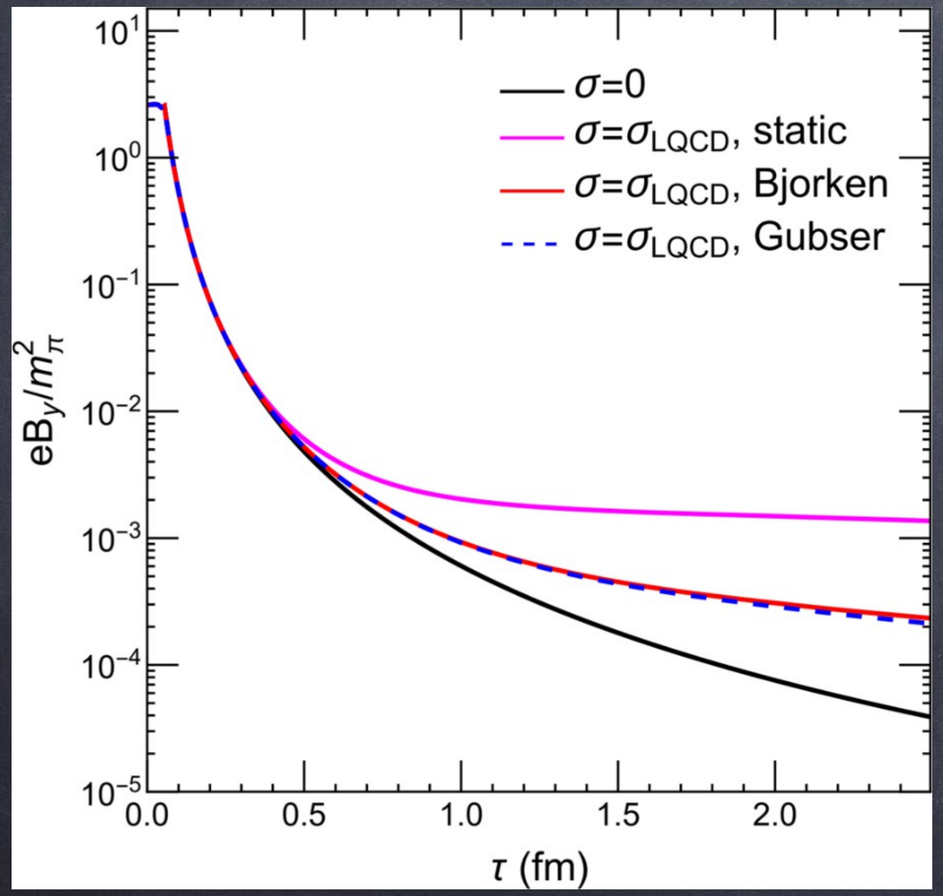
Polarisation measurement for open heavy flavor underway.

[Z.T.Liang, X.N.Wang '05; Becattini, Karpenko, Lisa, Upsal, Voloshin '17,; Y.Yang, R.Fang, Q. Wang, X. Wang'18]

Heavy quarks: charged, spin 1/2 particles

- Charm and Bottom quarks are produced in the first phases of coll

$$\tau_c < \frac{\hbar}{m_c} \sim 0.1\, fm/c\,, \qquad \tau_b < \frac{\hbar}{m_b} \sim 0.03\, fm/c\,.$$



$$\tau_B \sim 0.4\, fm/c\,,$$

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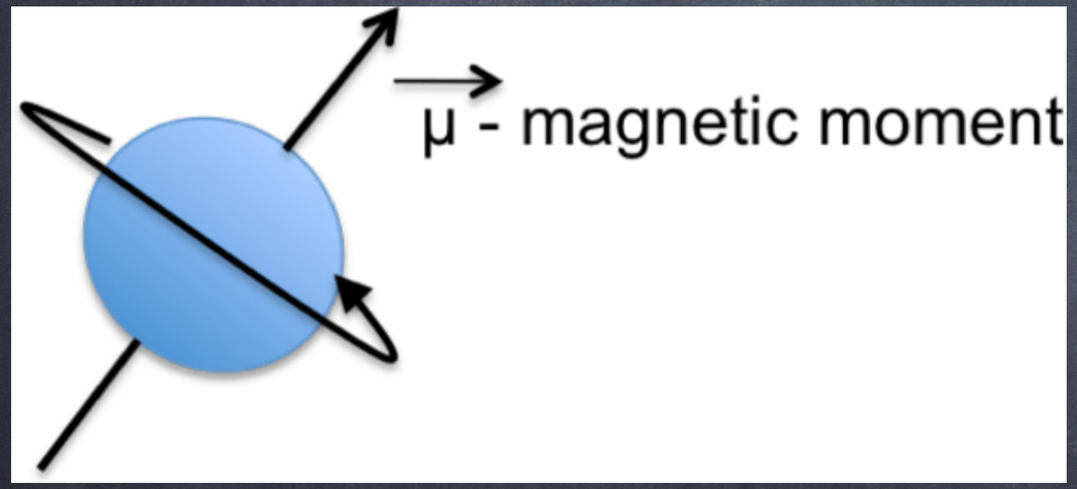
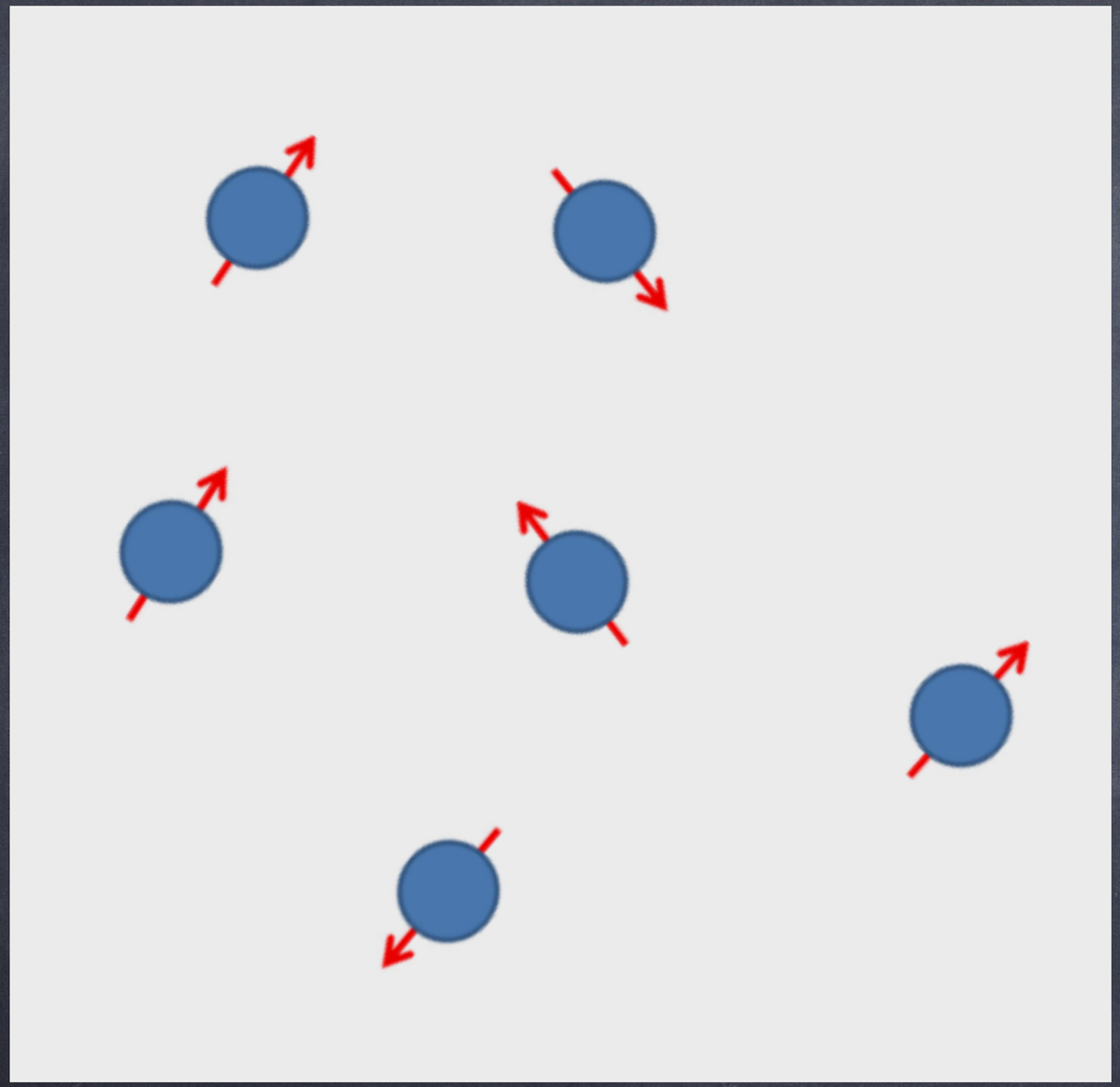


- Interaction with magnetic field:

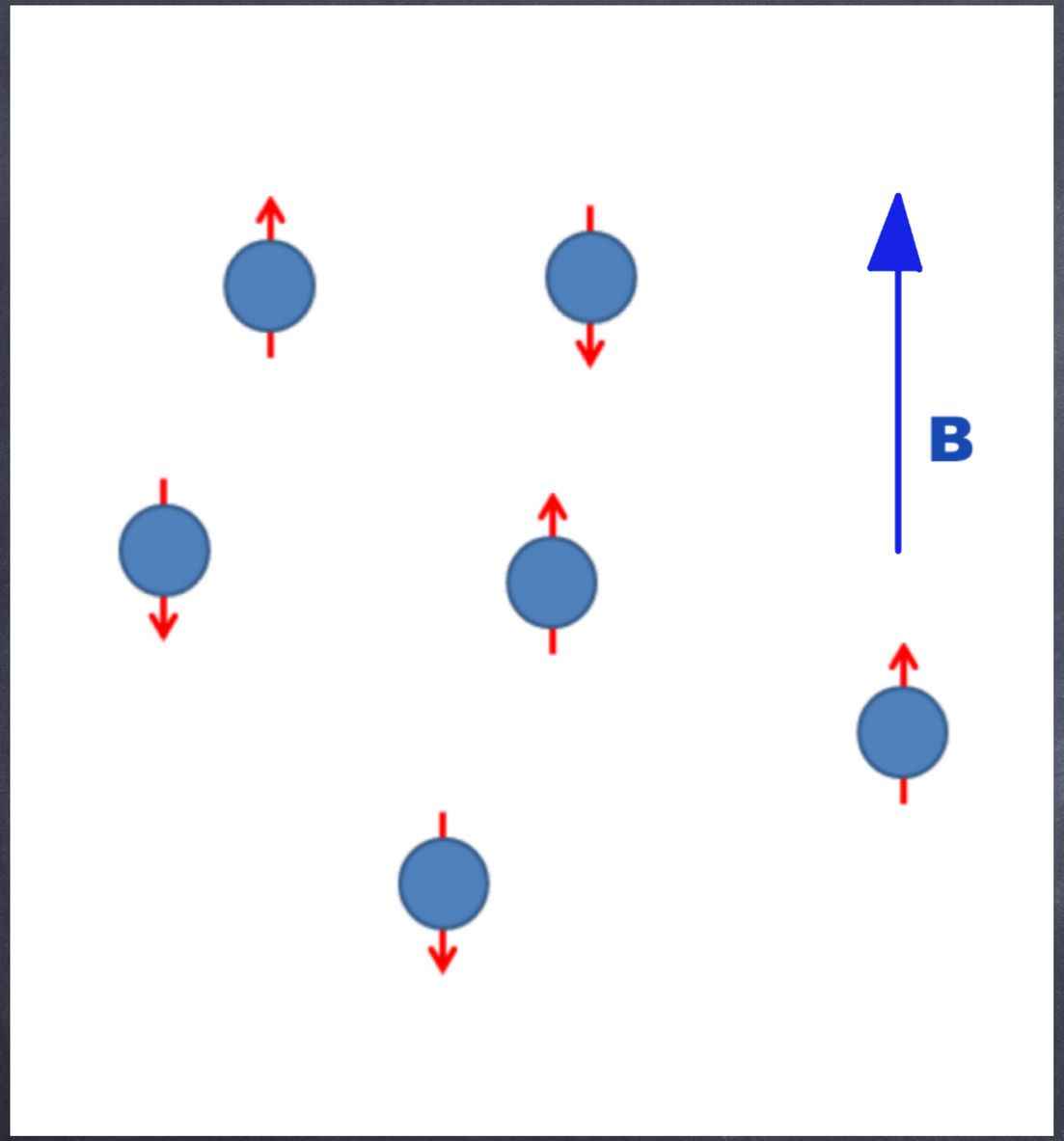
$$H = -\vec{\mu} \cdot \vec{B}$$

Magnetic moment and spin:

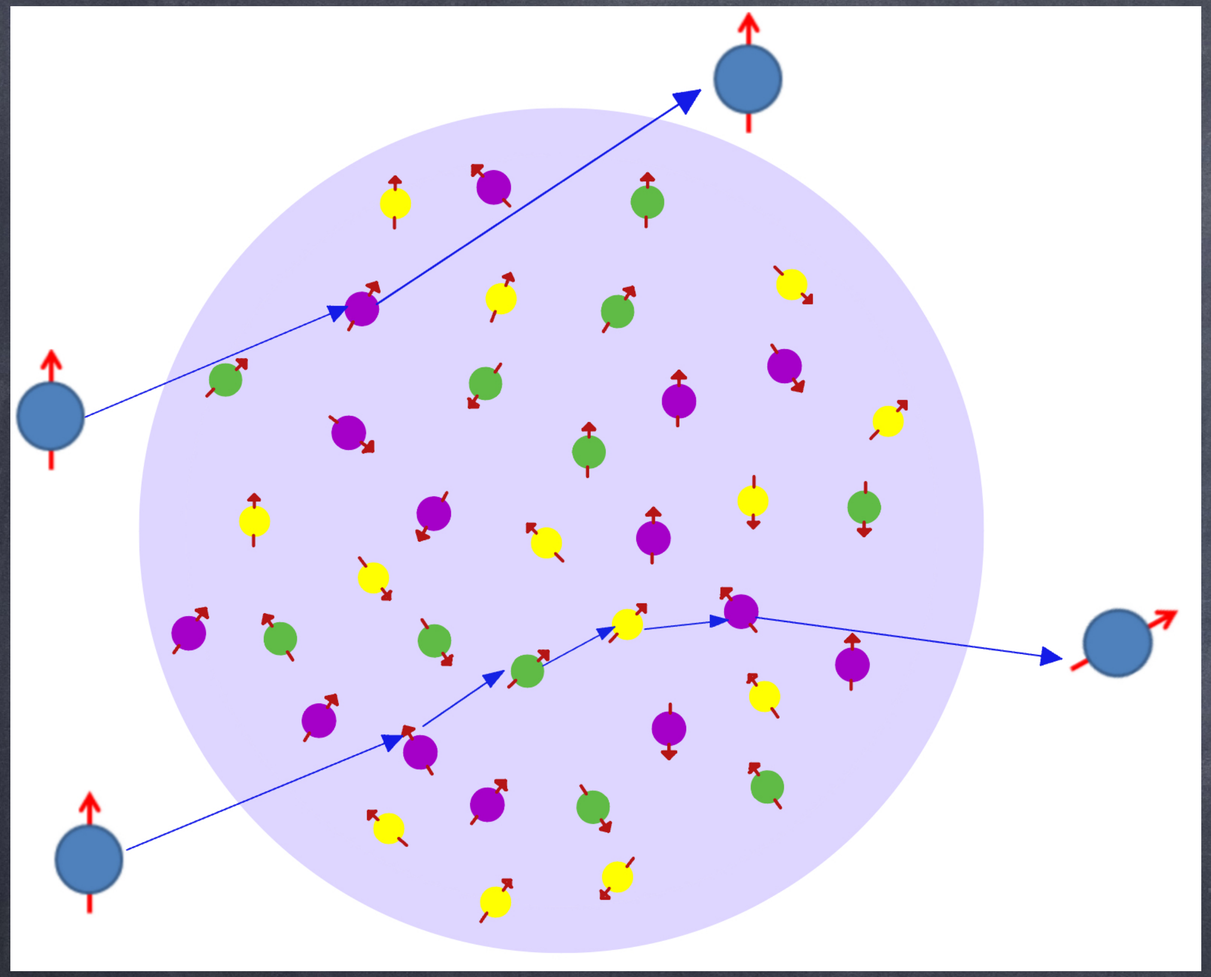
$$\vec{\mu} = \gamma \vec{S}$$



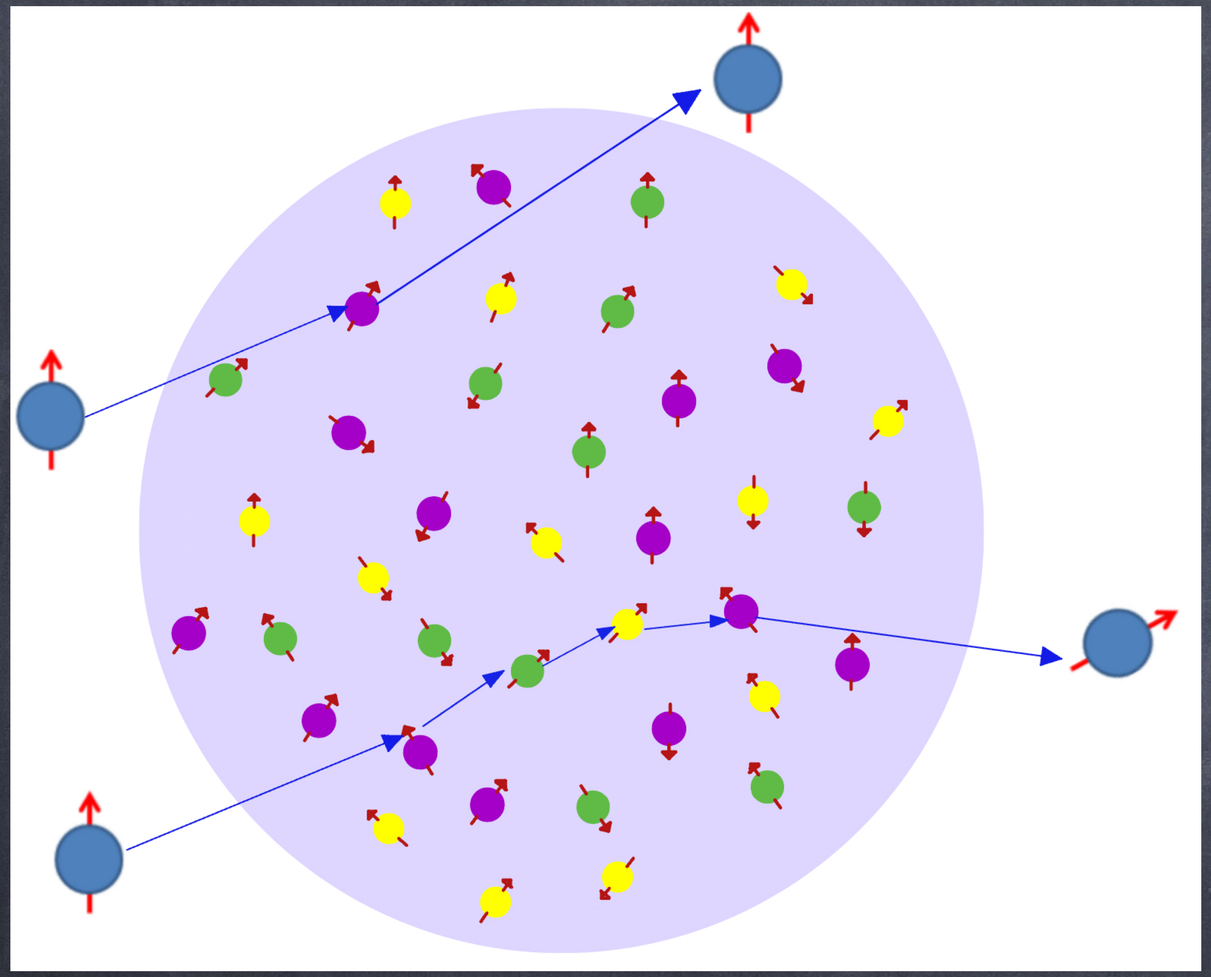
- Randomly aligned heavy quarks



- Aligned spin of heavy quarks in presence of magnetic field.



- Interaction with the medium.



- Interaction with the medium.

- De-polarisation larger for larger time spent in the medium.

Heavy Quark Rotational Brownian Motion

Rotational Brownian Motion

- Random rotational motion (orientation and angular velocity) of a microscopic particle due to background fluctuations caused by collisions with surrounding medium particles.
- Rotational Brownian motion problem: first applied by Debye.
- For classical spins, the Langevin equation corresponds to the stochastic Landau-Lifshitz-Gilbert equation.

$$\frac{d\mathbf{s}}{dt} = \mathbf{s} \times \left[\tilde{\mathbf{B}} + \boldsymbol{\xi}(t)\right] - \lambda\, \mathbf{s} \times \left(\mathbf{s} \times \tilde{\mathbf{B}}\right)$$

Here, $\tilde{\mathbf{B}} = \gamma \mathbf{B}$

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Damping coefficient

Here,

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Random torque on the particle by the medium

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Random torque on the particle by the medium

$$\begin{cases} \langle \xi_i(\tau) \rangle = 0 \\ \langle \xi_i(\tau_1)\xi_j(\tau_2) \rangle = 2D_s \delta_{ij} \delta(\tau_1 - \tau_2) \end{cases}$$

Langevin to Fokker-Planck equation

- Generalised multivariate Langevin equation

$$\frac{dy_i}{dt} = A_i(y,t) + C_{ik}(y,t)\,\xi_k(t),$$

- Using the Kramers-Moyal expansion, one arrives at the Fokker-Planck equation

$$\frac{\partial \mathscr{P}}{\partial t} = -\frac{\partial}{\partial y_i}\left[A_i(y,t) + D\,C_{jk}(y,t)\frac{\partial C_{ik}(y,t)}{\partial y_j}\right]\mathscr{P} + D\,\frac{\partial^2}{\partial y_i \partial y_j}\left[C_{ik}(y,t)C_{jk}(y,t)\mathscr{P}\right]$$

Where, $\mathscr{P} = \mathscr{P}(y,t;y_0,t_0)$ is the transition probability of $(y_0,t_0) \to (y,t)$

Fokker-Planck equation for spin

- Fokker-Planck equation corresponding to the stochastic Landau-Lifshitz-Gilbert equation.

By setting $y_i = s_i$

$$\frac{\partial \mathscr{P}}{\partial \tau} = \frac{\partial}{\partial s_i}\left[\epsilon_{ijk}\, s_j \tilde{B}_k + \lambda(s^2\delta_{ik} - s_i\, s_k)\tilde{B}_k - 2D_s s_i\right]\mathscr{P} + D_s\,\frac{\partial^2}{\partial s_i\,\partial s_j}\left[s^2\delta_{ij} - s_i s_j\right]\mathscr{P}$$

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- Assuming that the field $\mathbf{B}$ is independent of particle spin $\mathbf{s}$

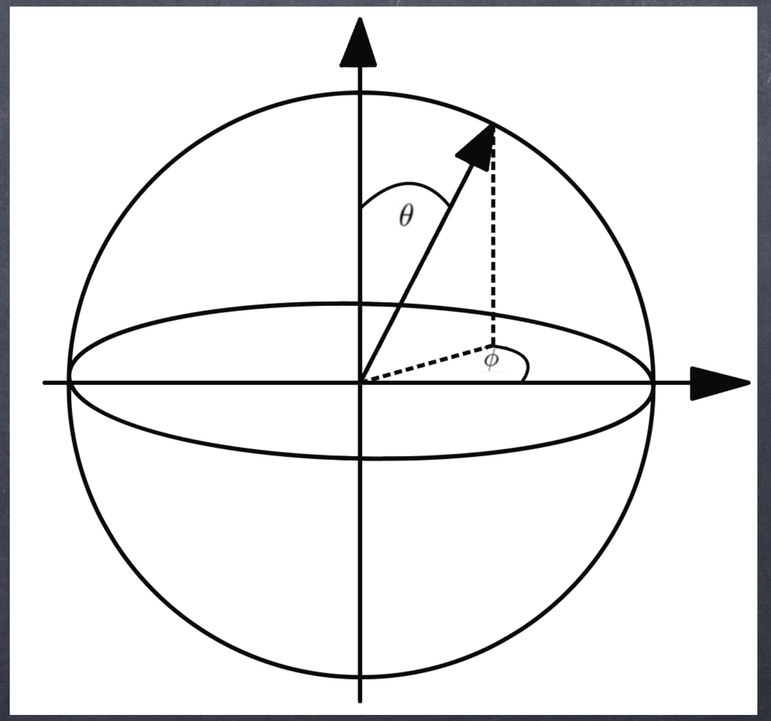
$$\frac{\partial \mathscr{P}}{\partial \tau} = D_s\,\frac{\partial}{\partial \mathbf{s}}\cdot\left[\frac{\lambda}{D_s}\mathbf{s}\times\left(\mathbf{s}\times\left(\tilde{\mathbf{B}} - \frac{\partial}{\partial \mathbf{s}}\right)\right)\right]\mathscr{P}$$

Fokker-Planck equation for spin

- To find: Probability of a spin-polarised particle having an instantaneous orientation in the direction (θ, ϕ).

- Consider a sphere in spin-space of fixed radius $|\mathbf{s}| = s$, i.e., $\mathbf{s} = (s, \theta, \phi)$:

Each point on the surface represents a different spin orientation.



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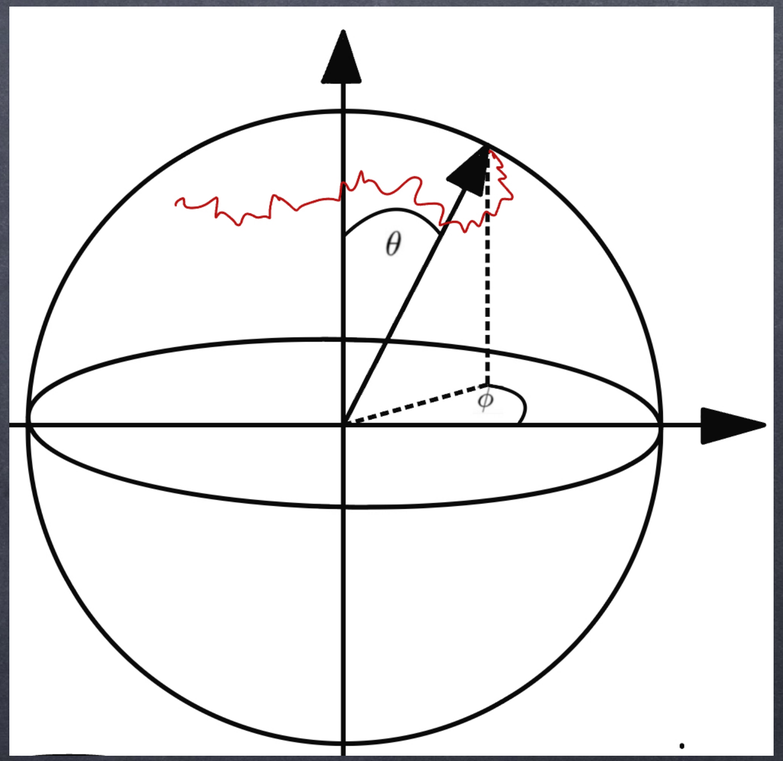

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- Consider a sphere in spin-space of fixed radius $|\mathbf{s}| = s$, i.e., $\mathbf{s} = (s, \theta, \phi)$:

1) Choose $\mathbf{z}$-axis to be along $\mathbf{B}$ 2) Axially symmetric Hamiltonian $\mathscr{H} = -\mu \cdot \mathbf{B}$

$$\tau_s \frac{\partial \mathscr{P}}{\partial t} = \frac{1}{\sin\theta} \frac{\partial}{\partial \theta} \left[\sin\theta \left(\frac{\lambda}{D_s} \frac{\partial \mathscr{H}}{\partial \theta} \mathscr{P} + \frac{\partial \mathscr{P}}{\partial \theta} \right) \right]$$

Where, $\tau_s = \dfrac{1}{D_s}$ Spin relaxation time

Fokker-Planck equation for spin

- Furthermore, for spatially homogeneous and time varying magnetic field

$$\tau_s \frac{\partial \mathscr{P}}{\partial \tau} = \frac{1}{\sin\theta} \frac{\partial}{\partial \theta} \left[\sin\theta \left(\frac{\partial}{\partial \theta} + \frac{\lambda}{D_s} \mu B(\tau) \sin\theta \right) \right] \mathscr{P}$$

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- This equation can be written in shorthand notation as

$$\tau_s \partial_\tau \mathscr{P}(\theta,\tau) = \mathscr{L}_\theta(\tau)\mathscr{P}(\theta,\tau) \quad \Longleftrightarrow \quad \tau_s \partial_\tau |\mathscr{P},\tau\rangle = \hat{\mathscr{L}}(\tau)|\mathscr{P},\tau\rangle$$

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$$|\mathscr{P},\tau\rangle = \exp\left(\frac{1}{\tau_s}\int_0^\tau d\tau' \hat{\mathscr{L}}(\tau')\right)|\mathscr{P},0\rangle$$

Fokker-Planck equation for spin

- The generic solution has the structure:

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- Considering the time dependence of the magnetic field to be of the form

$$\mathbf{B}(\tau) = \mathbf{B}_0\phi(\tau)$$

Fokker-Planck equation for spin

- The generic solution has the structure:

$$|\mathscr{P},t\rangle = \exp\left(\frac{1}{2\tau_s}\int_0^t dt'\hat{\mathscr{L}}(t')\right)|\mathscr{P},0\rangle$$

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$$\hat{\mathscr{L}}(\tau) = \hat{\mathscr{L}}^0 + \alpha\hat{\mathscr{L}}'(\tau)\begin{cases} \hat{\mathscr{L}}^0 = \dfrac{1}{\sin\theta}\dfrac{\partial}{\partial\theta}\left(\sin\theta\dfrac{\partial}{\partial\theta}\right) \\ \hat{\mathscr{L}}'(\tau) = \dfrac{\phi(\tau)}{\sin\theta}\dfrac{\partial}{\partial\theta}\sin^2\theta \end{cases}$$

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Fluctuation-dissipation theorem

$D_s = \lambda T$

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- For $eB_0 \approx e\gamma_v B_{lab}$, $T = 300\,MeV$ and average momentum $\sim 50GeV$

$$\alpha = \frac{\mu_Q B_0}{T} \ll 1 \begin{cases} 0.034 \text{ for charm} \\ -0.007 \text{ for bottom} \end{cases}$$

Where, $eB_{lab} \lesssim 0.1 m_\pi^2$

Fokker-Planck equation for spin

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$$\mathscr{P}(\theta,0) = \delta(cos\theta - cos\theta_0)$$

$$\alpha = \frac{\mu_Q B_0}{T} \ll 1 \begin{cases} 0.034 \text{ for charm} \\ -0.007 \text{ for bottom} \end{cases}$$

and

$$\mathbf{B}(\tau) = \mathbf{B}_0 \exp\left(-\frac{\tau}{\tau_B}\right)$$

Fokker-Planck equation for spin

- Vector polarisation (baryons) is

$$\langle \cos\theta \rangle = \cos\theta_0 \, e^{-2\tau/\tau_s} - \frac{2\alpha\tau_B}{3} e^{-2\tau/\tau_s} \left[\frac{1 - \exp(-\frac{(\tau_s - 2\tau_B)\tau}{\tau_s\tau_B})}{\tau_s - 2\tau_B} - \frac{1 - \exp(-\frac{(4\tau_B + \tau_s)\tau}{\tau_s\tau_B})}{4\tau_B + \tau_s} \right]$$

- Tensor polarisation (vector mesons) is

$$\langle \cos^2\theta \rangle = \frac{1}{3} + \frac{2}{3} e^{-6\tau/\tau_s} + \frac{2\alpha\tau_B}{5} e^{-6\tau/\tau_s} \left[\frac{1}{(\tau_s - 4\tau_B)} \left(1 - \exp\left[- \frac{(\tau_s - 4\tau_B)\tau}{\tau_s\tau_B} \right] \right) \right.$$

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Predictions

- Initially $\theta_0 = (0 \text{ or } \pi)$ depends on charge of the quark.
  - Open heavy baryon and anti-baryon have opposite polarisation.
  - Open heavy vector meson and its anti-particle, will have same sign.
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- Assume static uniform fireball of constant temperature.

$$\tau = \frac{Lm_Q}{|\mathbf{p}|}$$

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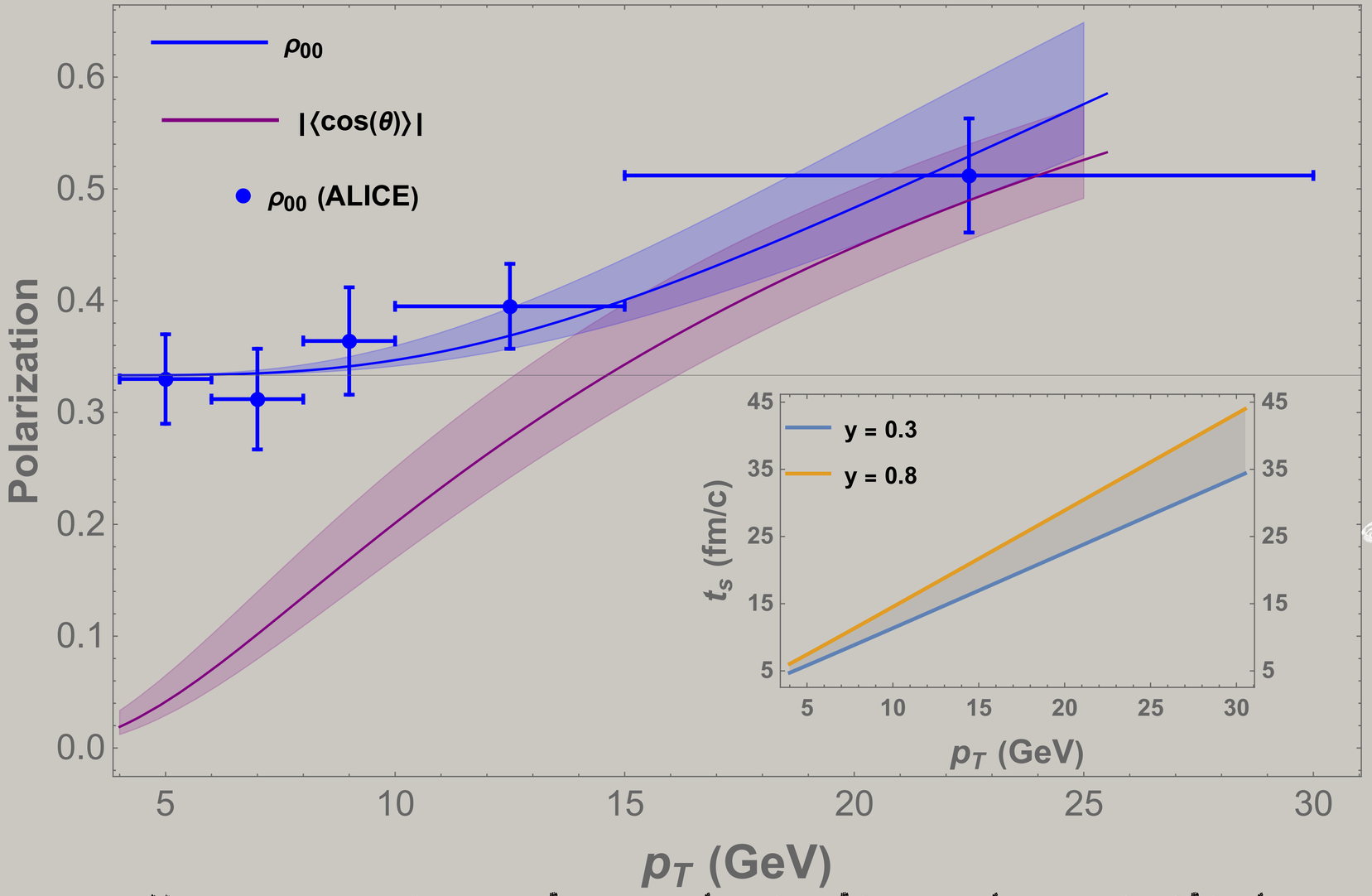
With the longitudinal rapidity y

$$L \sim 10\, fm$$

Result



- Vector polarisation (baryons)

$$\langle \cos\theta \rangle$$

- Tensor polarisation (vector mesons)

$$\rho_{00} = \frac{1}{3} + \frac{5}{2}\left[\langle \cos^2\theta \rangle - \frac{1}{3}\right]$$

Heavy quark vector polarisation along initial magnetic field direction

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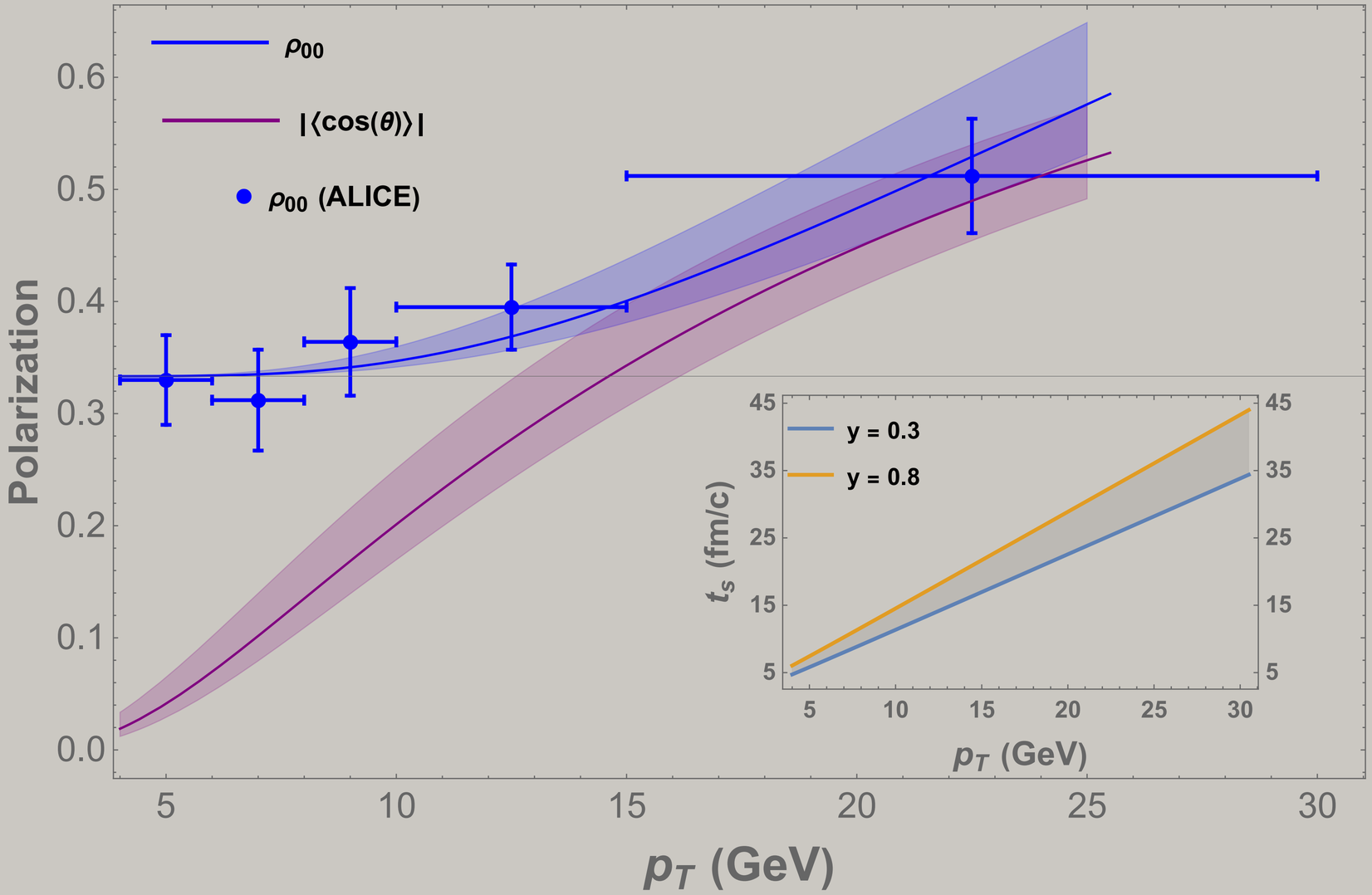

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$$0.3 < |y| < 0.8$$

$\tau_s = 1.31 fm$ For, $y = 0.5$

Heavy quark vector polarisation along initial magnetic field direction

Summary

- Very strong magnetic field produced in early stages of relativistic HIcs.
- Heavy quarks are primarily generated in the initial hard scattering of partons.
- Couples with the strong magnetic field due to possession of magnetic moment, and which makes spin polarised quarks along the strong field direction.
- Interaction with QGP medium $\longrightarrow$ de-polarisation.
- De-polarisation larger for larger time spent in the medium.
- Heavy quark polarisation increasing with p_T is a signature of initial strong magnetic field.
- Prediction for difference between polarisation of open heavy vector meson and baryons.

Ongoing and future works

- Fireball assumed to be static with constant average temperature. More realistic space-time evolution of the fireball and external magnetic field necessary.
- Predictions at forward rapidities.
- Calculation of spin relaxation time τ_S for heavy quarks.
- Derivation of an Einstein-Stokes-like relation between the spin diffusion coefficient and the dissipative parameters in spin hydrodynamics.
- Derivation of rotational Fokker-Planck equation from Kinetic theory with non-local collision terms.

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Thank you