T-even hadronic structure functions in the Drell-Yan process with γ/Z^0 interference

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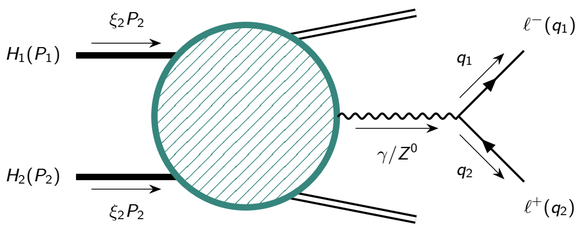
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ABSTRACT

We present a study of T-even hadronic structure functions in the Drell-Yan process. Our calculations are performed in perturbative QCD using collinear factorization at order α_s, incorporating γ/Z^0 interference in the dilepton decay. We analyze the dependence on the transverse momentum of the dilepton pair and perform a statistical comparison of angular coefficients with the geometric method as well as experimental data from the ATLAS and CMS Collaborations at the LHC.

1 INTRODUCTION



The experimental and theoretical study of the Drell-Yan process is an essential tool for investigating hadron structure, as it provides direct access to parton distribution functions.

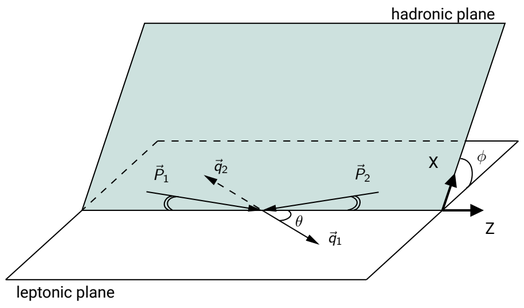
Schematically, the Drell-Yan process can be written as follows:

$$H_1(P_1) + H_2(P_2) \to \gamma/Z^0 + X \to \ell^-(q_1) + \ell^+(q_2) + X\,,$$

where P_1, P_2 – hadron momenta;
$q_1,\ q_2$ – lepton momenta;
$q = q_1 + q_2 - = (Q_0, Q_T, 0, Q_z)$ – boson momentum.

2 BASICS OF THE DRELL-YAN REACTIONS

To describe the Drell-Yan process we use Collins-Soper dilepton center-of-mass frame. In this frame, the hadronic and leptonic degrees of freedom factorize, leading to a cross section of the following form:



$$\frac{\mathrm{d}\sigma}{\mathrm{d}Q^2\,\mathrm{d}Q_T\,\mathrm{d}y\,\mathrm{d}\Omega} = \frac{\alpha^2}{4(2\pi)^4 Q^4 s^2} L_{\mu\nu}(q_1, q) W^{\mu\nu}(P_1, P_2, q)\,.$$

As a result, the lepton angular distribution can be parameterized by eight angular coefficients A_{0-8}:

$$\begin{aligned}\frac{\mathrm{d}N}{\mathrm{d}\Omega} &= \frac{\mathrm{d}\sigma}{\mathrm{d}Q^2\,\mathrm{d}Q_T\,\mathrm{d}y\,\mathrm{d}\Omega}\left(\frac{\mathrm{d}\sigma}{\mathrm{d}Q^2\,\mathrm{d}Q_T\,\mathrm{d}y}\right)^{-1} \\ &= \frac{3}{16\pi}\big[1 + \cos^2\theta + \frac{1}{2}A_0\big(1 - 3\cos^2\theta\big) + A_1 \sin 2\theta \cos\phi \\ &+ \frac{1}{2}A_2 \sin^2\theta \cos 2\phi + A_3 \sin\theta \cos\phi + A_4 \cos\theta \\ &+ A_5 \sin^2\theta \sin 2\phi + A_6 \sin 2\theta \sin\phi + A_7 \sin\theta \sin\phi\big]\,.\end{aligned}$$

However, at α_s order there are only five non-zero angular coefficients associated with T-odd hadronic structure functions $W_{T/L/\Delta/\Delta\Delta/T_P/\nabla_P}$:

$$A_0 = \frac{2W_L}{2W_T + W_L}\,,\quad A_1 = \frac{2W_\Delta}{2W_T + W_L}\,,\quad A_2 = \frac{4W_{\Delta\Delta}}{2W_T + W_L}\,,$$
$$A_3 = \frac{2W_{\nabla_P}}{2W_T + W_L}\,,\quad A_4 = \frac{2W_{T_P}}{2W_T + W_L}\,.$$

Hadronic structure functions W_{I} are related to the partonic-level structure functions $\tilde{\omega}_{\mathrm{I}}$ by the QCD collinear factorization formula:

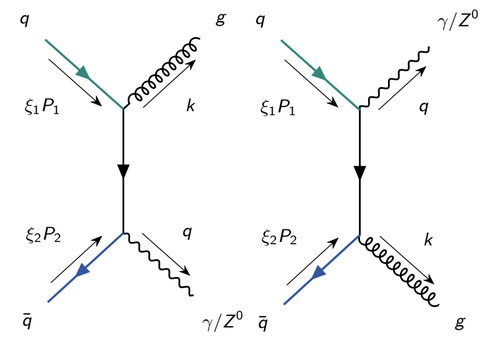
$$W_{\mathrm{I}} = \frac{1}{x_1 x_2}\sum_{\mathrm{a,b}}\int_{x_1}^1 \mathrm{d}z_1 \int_{x_2}^1 \mathrm{d}z_2\, f_{\mathrm{a}/1}\left(\frac{x_1}{z_1}\right)\tilde{\omega}_{\mathrm{I}}^{\mathrm{ab}}(z_1, z_2)\delta\big((\hat{s} + \hat{t} + \hat{u} - Q^2)/\hat{s}\big) f_{\mathrm{b}/2}\left(\frac{x_2}{z_2}\right),$$

where $f_{a/H}(\xi)$ with $\xi = x/z$ is the PDF describing the collinear ξ distribution of partons of type a in a hadron H and $x_{1,2} = e^{\pm y}\sqrt{(Q^2 + Q_T^2)/s}$ are momentum fractions of the light-cone components of the finale vector boson.

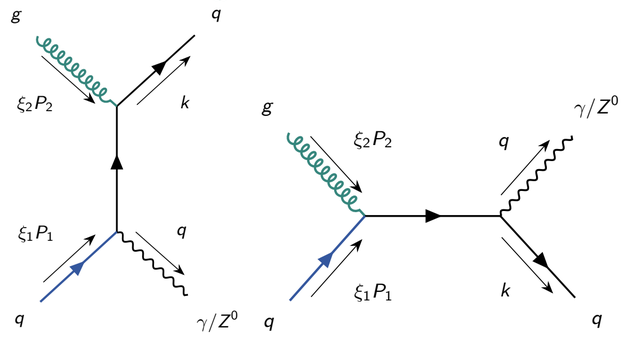
3 ANALYTICAL RESULTS

Collinear factorization allows us to consider the Drell–Yang process as a sum over all possible partonic subprocesses, in which a dilepton pair can arise. At α_s order, there are two such subprocesses:

Quark-antiquark annihilation



Quark-gluon scattering



Interference of γ and Z^0

For consideration of the γ/Z^0 interference, we require the following specific electroweak couplings

$$g_{\mathrm{EW};1}^{Z\gamma} = 1 + 2\,g_{Zq}^V g_{Zl}^V \,\mathrm{Re}[D_Z(Q^2)] + \Big((g_{Zq}^V)^2 + (g_{Zq}^A)^2\Big)\Big((g_{Z\ell}^V)^2 + (g_{Z\ell}^A)^2\Big)\,|D_Z(Q^2)|^2\,,$$

$$g_{\mathrm{EW};2}^{Z\gamma} = 2\,g_{Zq}^A\left[2\,g_{Zq}^V\left(g_{Z\ell}^A\, g_{Z\ell}^V\right)|D_Z(Q^2)|^2 + g_{Z\ell}^A\,\mathrm{Re}[D_Z(Q^2)]\right].$$

Here, g_{Zf}^V and g_{Zf}^A are the standard vector and axial-vector couplings of the Z^0 boson to fermions. And $D_Z(Q^2)$ is the Breit-Wigner propagator.

$$C_{q\bar{q}} = \frac{C_F}{N_c} = \frac{N_c^2 - 1}{2N_c^2} = \frac{4}{9}\,,$$
$$g_{q\bar{q};i} = (8\,\pi^2 e_q^2\,\alpha_s)\,C_{q\bar{q}}\,g_{\mathrm{EW};i}^{Z\gamma} = G_i C_{q\bar{q}}\,.$$

$$C_{qg} = \frac{T_F}{N_c} = \frac{1}{2N_c} = \frac{1}{6}\,,$$
$$g_{qg;i} = (8\,\pi^2 e_q^2\,\alpha_s)\,C_{qg}\,g_{\mathrm{EW};i}^{Z\gamma} = G_i C_{qg}\,.$$

Partonic-level structure functions

$$\tilde{\omega}_T^{q\bar{q}} = g_{q\bar{q};1}\frac{1}{\rho^2}\left(1 + \frac{\rho^2}{2}\right)\frac{z_1^2 + z_2^2}{z_1 z_2}\,,$$

$$\tilde{\omega}_L^{q\bar{q}} = 2\,\tilde{\omega}_{\Delta\Delta}^{q\bar{q}} = g_{q\bar{q};1}\frac{z_1^2 + z_2^2}{z_1 z_2}\,,$$

$$\tilde{\omega}_\Delta^{q\bar{q}} = g_{q\bar{q};1}\frac{1}{\rho}\frac{z_1^2 - z_2^2}{z_1 z_2}\,,$$

$$\tilde{\omega}_{T_P}^{q\bar{q}} = g_{q\bar{q};2}\frac{\sqrt{1+\rho^2}}{\rho^2}\frac{z_1^2 + z_2^2}{z_1 z_2}\,,$$

$$\tilde{\omega}_{\nabla_P}^{q\bar{q}} = g_{q\bar{q};2}\frac{\sqrt{1+\rho^2}}{\rho}\frac{z_1^2 - z_2^2}{z_1 z_2}\,.$$

$$\tilde{\omega}_T^{qg} = g_{qg;1}\frac{1}{\rho^2}\frac{1 - z_2}{z_1 z_2}\Big(z_2^2 + (1 - z_1 z_2)^2 + \rho^2\Big(1 - \frac{z_1^2}{2} - z_1 z_2(z_1 + z_2)\Big)\Big)\,,$$

$$\tilde{\omega}_L^{qg} = 2\,\tilde{\omega}_{\Delta\Delta}^{qg} = g_{qg;1}\frac{1 - z_2}{z_1 z_2}\Big(z_2^2 + (z_1 + z_2)^2\Big)\,,$$

$$\tilde{\omega}_\Delta^{qg} = g_{qg;1}\frac{1}{\rho}\frac{1 - z_2}{z_1 z_2}\Big(z_1^2 - 2z_2^2\Big)\,,$$

$$\tilde{\omega}_{T_P}^{qg} = g_{qg;2}\frac{\sqrt{1+\rho^2}}{\rho^2}\frac{1 - z_2}{z_1 z_2}\Big(z_2^2 + (1 - z_2)^2 - (1 - z_1)^2\Big)\,,$$

$$\tilde{\omega}_{\nabla_P}^{qg} = g_{qg;2}\frac{\sqrt{1+\rho^2}}{\rho}\frac{1 - z_2}{z_1 z_2}\Big(1 - 2z_2^2 - (1 - z_1)^2 + 2z_2(1 - z_1)\Big)\,.$$

Small Q_T^2/Q^2-expansion

The method developed in Ref. [2] allows a small $\rho^2 = Q_T^2/Q^2$-expansion of structure functions to arbitrary order. For example, at leading order:

$$\delta\left((1 - z_1)(1 - z_2) - \frac{\rho^2}{1 + \rho^2}z_1 z_2\right) = \frac{\delta(1 - z_1)}{(1 - z_1)_+} + \frac{\delta(1 - z_2)}{(1 - z_2)_+} - \delta(1 - z_1)\delta(1 - z_2)\log\rho^2)\,,$$

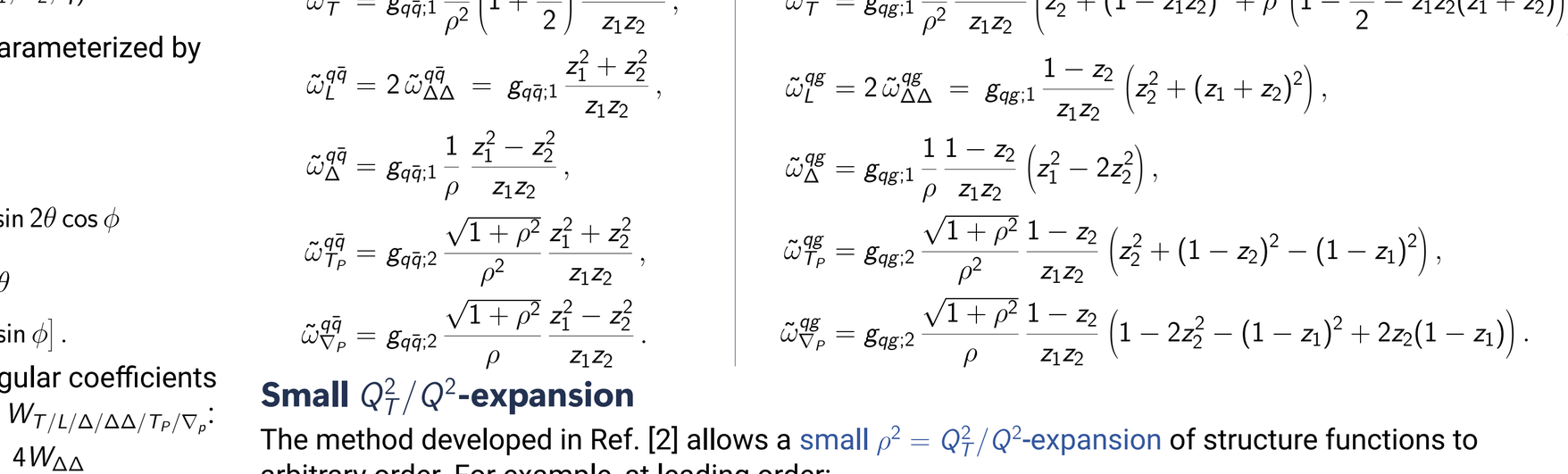
$$\begin{aligned}W_T^{\mathrm{LP};q\bar{q}}(x_1^0, x_2^0, L_\rho) &= \frac{1}{\rho^2}\,W_L^{\mathrm{LP};q\bar{q}}(x_1^0, x_2^0, L_\rho) = \frac{g_{q\bar{q};1}}{g_{q\bar{q};2}}\,W_{T_P}^{\mathrm{LP};q\bar{q}}(x_1^0, x_2^0, L_\rho) \\ &= \frac{g_{q\bar{q};1}}{\rho^2 x_1^0 x_2^0}\frac{1}{C_F}\Big[-C_F(2L_\rho + 3)\,q_1(x_1^0)\,\bar{q}_2(x_2^0) + q_1(x_1^0)\Big(P_{qq}\otimes\bar{q}_2\Big)(x_2^0) + \Big(P_{qq}\otimes q_1\Big)(x_1^0)\,\bar{q}_2(x_2^0)\Big]\,,\end{aligned}$$

$$W_T^{\mathrm{LP};qg}(x_1^0, x_2^0, L_\rho) = \frac{g_{qg;1}}{g_{qg;2}}\,W_{T_P}^{\mathrm{LP};qg}(x_1^0, x_2^0, L_\rho) = \frac{2g_{qg;1}}{\rho^2 x_1^0 x_2^0}\,q_1(x_1^0)\Big(P_{qg}^+\otimes g_2\Big)(x_2^0)\,,$$

$$W_L^{\mathrm{LP};qg}(x_1^0, x_2^0, L_\rho) = \frac{2g_{qg;1}}{x_1^0 x_2^0}\,q_1(x_1^0)\Big(P_{qg}^-\otimes g_2\Big)(x_2^0)\,,$$

4 NUMERICAL RESULTS

We statistically compare the predictions for the Drell-Yan angular coefficients with available data from the LHC experiments (CMS, ATLAS and LHCb Collaborations) as well as with the geometrical method [3], using statistical χ^2-test to asses agreement. To perform numerical calculations, we utilize the LHAPDF library, specifically utilizing the CT18NLO parametrization for PDFs including scale Q evolution. We perform a numerical simulation by randomly sampling from a normal distribution within the same Q range as in the corresponding experiment.



5 CONCLUSIONS

- We presented analytical results for the Drell-Yan T-even hadronic structure functions in the framework of the pQCD based on the collinear factorization scheme at α_s order;
- We developed systematic Q_T^2/Q^2-expansion for structure functions up to desired order;
- Based on our analytical pQCD derivations, we numerically calculated angular coefficients A_0–A_4,
- We statistically compared our theoretical predictions with experimental data from the ATLAS, CMS, and LHCb Collaborations, as well as with results obtained via the geometrical method.

6 REFERENCES

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[3] W.-C. Chang, R.E. McClellan, J.-C. Peng and O. Teryaev, Phys. Rev. D. 96, 054020 (2017).