Asymptotic freedom in Fermi theory

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Introduction

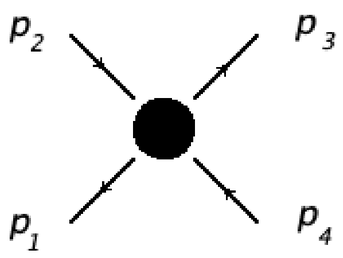
- The Lagrangian

$$\mathcal{L} = i\bar{\Psi}\hat{\partial}\Psi - \frac{G}{\sqrt{2}}(\bar{\Psi}\hat{\mathcal{O}}_i\Psi)(\bar{\Psi}\hat{\mathcal{O}}_i\Psi),$$

- The operator

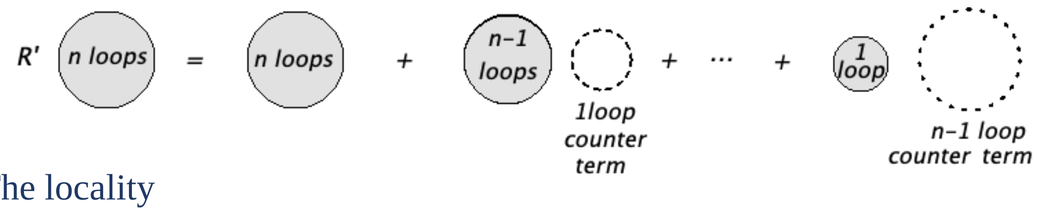
$$\hat{\mathcal{O}} = \gamma^\mu(1-\gamma^5)/2$$

- The object



Methods

- The Bogolyubov-Parasyuk R-operation



- The locality

$$A_n^{(n)} = (-1)^{n+1}\frac{A_1^{(n)}}{n}$$

- Recurrence relations

$$nS_n(s,t,u) = -8s\int_0^1 dx \sum_{k=0}^{n-1}\sum_{p=0}^{k} \frac{[s(-s-u)]^p[x(1-x)]^{p+1}}{p!(p+1)!(p+2)^{-1}} \times \times \frac{d^pA_k(s,-s-u',u')}{du'^p}\frac{d^pA_{n-1-k}(s,-s-u',u')}{du'^p}\Big|_{u'\to -sx}$$

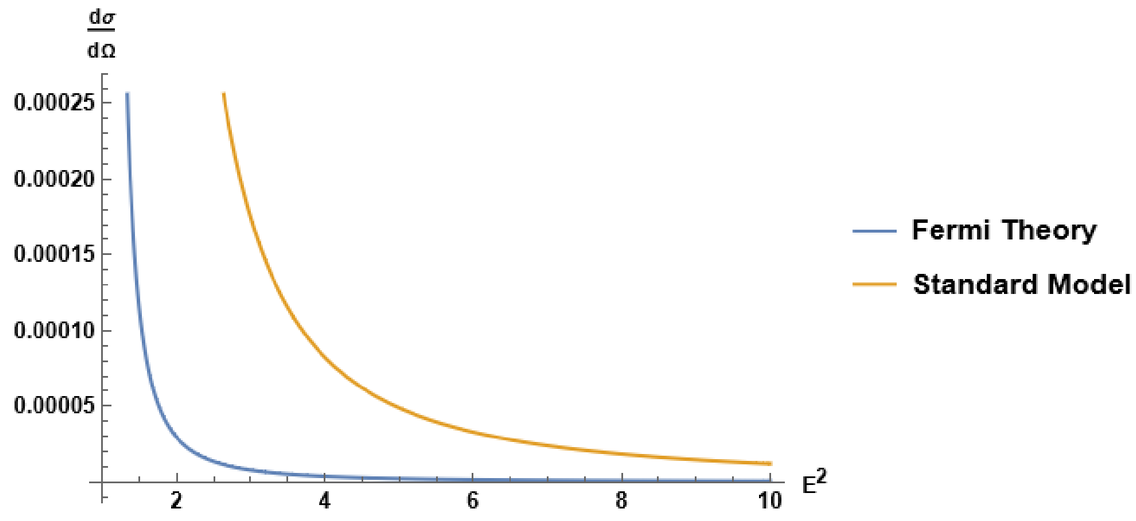
$$nT_n(s,t,u) = -8t\int_0^1 dx \sum_{k=0}^{n-1}\sum_{p=0}^{k} \frac{[t(-t-u)]^p[x(1-x)]^{p+1}}{p!(p+1)!(p+2)^{-1}} \times \times \frac{d^pA_k(-t-u',t,u')}{du'^p}\frac{d^pA_{n-1-k}(-t-u',t,u')}{du'^p}\Big|_{u'\to -tx}$$

$$nU_n(s,t,u) = 8u\int_0^1 dx \sum_{k=0}^{n-1}\sum_{p=0}^{k} \frac{[u(-s-u)]^p[x(1-x)]^{p+1}}{p!(p+1)!(p+3)^{-1}} \times \times \frac{d^pA_k(s',-s'-u,u)}{ds'^p}\frac{d^pA_{n-1-k}(s',-s'-u,u)}{ds'^p}\Big|_{s'\to -ux}$$
$$+8u\int_0^1 dx \sum_{k=0}^{n-1}\sum_{p=0}^{k} \frac{[u(-t-u)]^p[x(1-x)]^{p+1}}{p!(p+1)!(p+3)^{-1}} \times \times \frac{d^pA_k(-t'-u,t',u)}{dt'^p}\frac{d^pA_{n-1-k}(-t'-u,t',u)}{dt'^p}\Big|_{t'\to -ux}$$

- Generalised RG equations

$$-\frac{dA(s,t,u)}{dz} =$$
$$= -8s\int_0^1 dx \sum_{p=0}^{\infty} \frac{[s(-s-u)]^p[x(1-x)]^{p+1}}{p!(p+1)!(p+2)^{-1}}\left(\frac{d^pA(s,-s-u',u')}{du'^p}\right)^2$$
$$-8t\int_0^1 dx \sum_{p=0}^{\infty} \frac{[t(-t-u)]^p[x(1-x)]^{p+1}}{p!(p+1)!(p+2)^{-1}}\left(\frac{d^pA(-t-u',t,u')}{du'^p}\right)^2$$
$$+8u\int_0^1 dx \sum_{p=0}^{\infty} \frac{[u(-s-u)]^p[x(1-x)]^{p+1}}{p!(p+1)!(p+3)^{-1}}\left(\frac{d^pA(s',-s'-u,u)}{ds'^p}\right)^2$$
$$+8u\int_0^1 dx \sum_{p=0}^{\infty} \frac{[u(-t-u)]^p[x(1-x)]^{p+1}}{p!(p+1)!(p+3)^{-1}}\left(\frac{d^pA(-t'-u,t',u)}{dt'^p}\right)^2$$

Results



Conclusions

- The Bogolyubov-Parasyuk R-operation and the locality condition allow to construct recurrence relations, which reproduces the leading UV singularities in arbitrary order of perturbation theory. Then, it is possible to move on generalized RG equations, that represent the high energy behavior of the theory.

- The numerical analysis of the obtained RG equations shows that leading logarithm approximation of the Fermi demonstrates asymptotically freedom behavior in the high energy regime, just as in the gauge theory.

- As a consequence we see that the unitarity condition, which violates at the tree level, restores by the summation of the leading contribution.

$$\frac{d\sigma}{d\Omega} \approx \frac{G^2 s}{16\pi^2}\frac{k^2}{G^2 s^2\,\log^2(s/\mu^2)} = \frac{C^2}{s\log^2(s/\mu^2)} < \frac{1}{4\pi^2 s}$$

References

- Loop corrections to the four-fermion interaction, *Eur.Phys.J.C* 85 (2025) 9, 1000

- High-energy behaviour of Fermi theory, *Eur.Phys.J.C* (2026)

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