# Casimir effect under fast rotation in heavy-ion collisions

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## Introduction

It is established that a vortical fluid is realized during ion collisions [15]. This is due to polarization, which has also been studied at JINR [16]. Moreover, the vortical fluid affects vacuum effects, the most well-known of which are the Unruh effect and the Casimir effect. Therefore, it is interesting to consider the Casimir effect in heavy-ion collisions. Experimental verification of the Casimir effect began with Sparnaay [2]. Subsequent studies have measured the force between various materials [3-4], and important experiments were performed [5]-[9].

From a theoretical point of view, Casimir's original formulation was extended to various geometries, materials, temperatures, and space-time models [10-12]. In this work, the Casimir effect between parallel plates uniformly moving along a circular trajectory of radius $R$ with angular velocity $\Omega$ in the equatorial region is considered, and the result of computing the renormalized vacuum energy density for a massless scalar field up to $\Omega^2 R^2$, taking into account temperature corrections, is presented.

## Calculation

We considered the following metric:

$$ds^2 = (1 + \Omega^2(R^2 + 2Rx))dt^2 - dx^2 - dy^2 - dz^2$$

And we used the Klein-Fock-Gordon equation with Dirichlet boundary conditions $\Psi(0) = \Psi(L) = 0$, $L$ - the distance between the plates .

$$\frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g}g^{\mu\nu}\partial_\nu)\Psi = 0$$

The Casimir energy was obtained by calculating the vacuum expectation value of the stress–energy tensor:

$$\epsilon_{vac,1} = \langle 0|n^t n^t T_{tt}(\Psi)|0\rangle = \int d^2k_{||} \sum_n n^t n^t T_{tt}(\Psi)$$

To calculate the temperature corrections, we need to calculate the renormalized Helmholtz free energy:

$$\Delta_T \mathscr{F}_0^{ren} = 2Ak_B T \sum_{n=0}^{\infty} \int\int \frac{d^2k_{||}}{(2\pi)^3} \log\left[1 - e^{-\tilde{\beta}(k_{||}^2 + \frac{n^2\pi^2}{L^2})^{1/2}}\right] - Vf_{bb}(T)$$

Where $Vf_{bb}(T)$ is the free energy density of black-body radiation, expressed by

$$f_{bb}(T) = k_B T \int\int\int \frac{d^3k}{(2\pi)^3} \log\left[1 - e^{-\tilde{\beta}|k|}\right]$$

Now, we can calculate the renormalized internal energy:

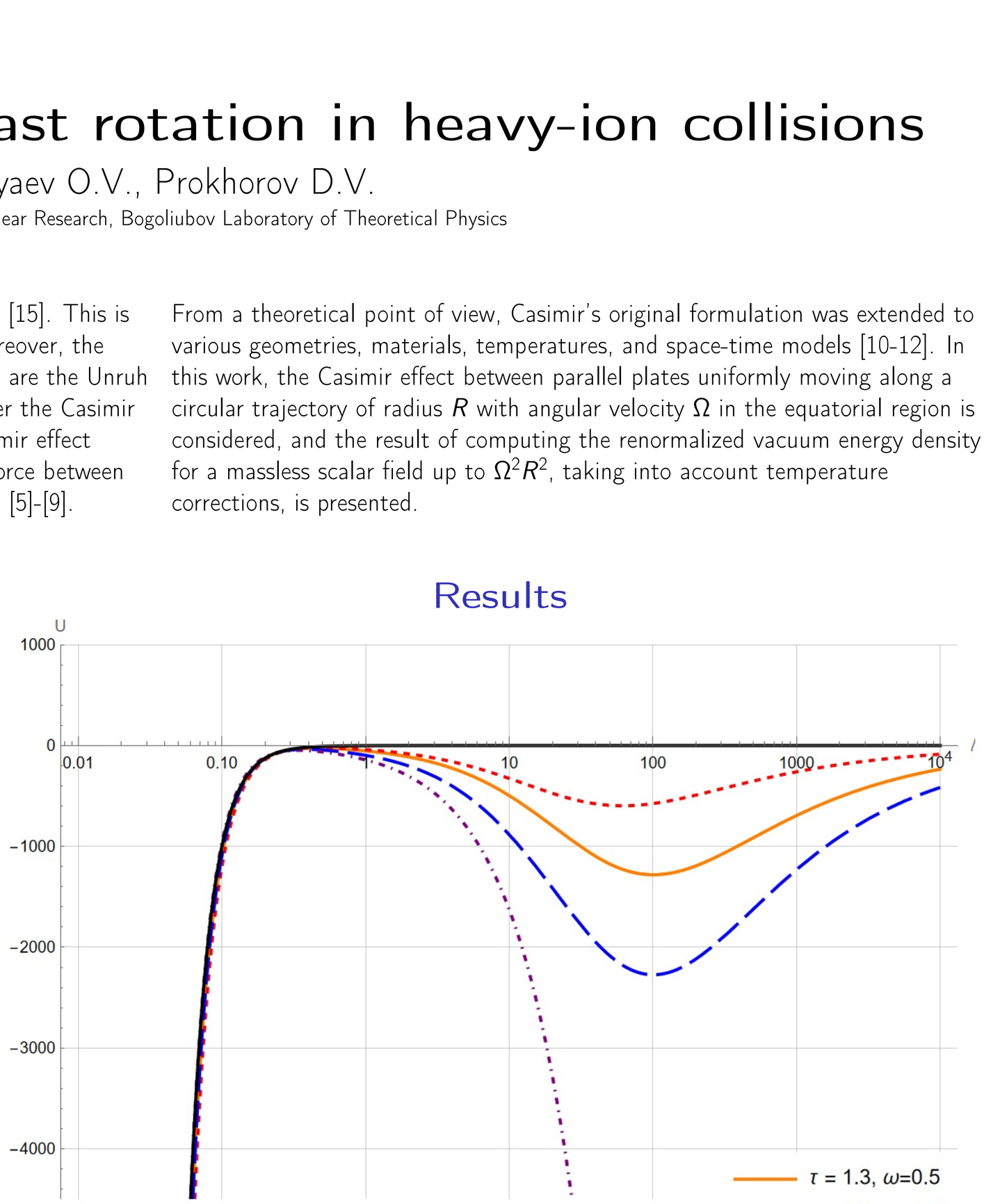
$$U_0^{ren,1} = -T^2 \frac{\partial(\mathscr{F}_0^{(1),ren}/T)}{\partial T}$$

The value of internal energy is expressed in terms of polylogarithm functions: $\mathrm{Li}_s(z) = \sum_{k=1}^{\infty} \frac{z^k}{k^s}$.

The plot of the received internal energy is shown in Figure. The plot was constructed using dimensionless parameters.

On the right side is $\alpha \equiv \sqrt{(1 + \Omega^2 R^2)\left(1 + \frac{LR\Omega^2}{1+\Omega^2R^2}\right)}$

## Results




| Dimensionless parameters: | |
|---|---|
| $l = L/L_0$ | The ratio of the distance between the plates to the characteristic scale $L_0$ |
| $\tau = T/T_0$ | The ratio of temperature to characteristic temperature $T_0 = \hbar c/(k_B L_0)$ |
| $\omega = R\Omega/c$ | The ratio of the linear speed of rotation to the speed of light |

Figure: Plot of internal energy as a function of dimensionless length

$$\bar{\epsilon}_{vac,2}^{ren} = -\sqrt{1 + R^2\Omega^2}\frac{\pi^2}{1440L^4} = \sqrt{1 + R^2\Omega^2}\bar{\epsilon}_{vac}^{ren}$$

$$U_0^{ren,1} \approx -\frac{A\pi^2}{1440L^3} - \frac{Ak_B^4 L\pi^2 T^4}{30\alpha^{3/2}} - \frac{Ak_B^3}{1440\pi^3\alpha}\left(\frac{58\pi^4\alpha^3}{L^3\left(e^{\frac{\pi\alpha}{LT}} - 1\right)} + \frac{4e^{\frac{\pi\alpha}{LT}}\pi^5\alpha^4}{L^4 T\left(e^{\frac{\pi\alpha}{LT}} + 1\right)^2} - \frac{e^{\frac{\pi\alpha}{LT}}\pi^6\alpha^5}{L^5T^2\left(e^{\frac{\pi\alpha}{LT}} - 1\right)^3} - \frac{e^{\frac{2\pi\alpha}{LT}}\pi^6\alpha^5}{L^5T^2\left(e^{\frac{\pi\alpha}{LT}} - 1\right)^3} + \frac{360\pi^3 T\alpha^2 \ln\left(1 - e^{-\frac{\pi\alpha}{LT}}\right)}{L^2} - \frac{1440\pi^2T^2\alpha\mathrm{Li}_2\left(e^{-\frac{\pi\alpha}{LT}}\right)}{L} - 3600\pi T^3\mathrm{Li}_3\left(e^{-\frac{\pi\alpha}{LT}}\right) - \frac{4320LT^4\mathrm{Li}_4\left(e^{-\frac{\pi\alpha}{LT}}\right)}{\alpha} - 1440\pi T^3\zeta(3)\right)$$

## Conclusions

- At low angular velocities $R^2\Omega^2 \ll 1$, the effect is absent.
- A stable equilibrium point appears.
- The existence of the stable equilibrium point is attributed specifically to rotation rather than thermal effects, as it vanishes when $\Omega \to 0$.
- Rotation and temperature have opposing effects: an increase in one effectively corresponds to a decrease in the other.

## References

1. H. Casimir, Proc. K. Ned. Akad. Wet. **51**, 793 (1948).
2. M. J. Sparnaay, Physica **24**, 751 (1958).
3. G. L. Klimchitskaya et al., Rev. Mod. Phys. **81**, 1827 (2009).
4. A. W. Rodriguez et al., Nature Photon. **5**, 211 (2011).
5. C.-C. Chang et al., Phys. Rev. Lett. **107**, 090403 (2011).
6. A. A. Banishev et al., Phys. Rev. B **85**, 045436 (2012).
7. C.-C. Chang et al., Phys. Rev. B **85**, 165443 (2012).
8. A. A. Banishev et al., Phys. Rev. B **85**, 195422 (2012).
9. A. A. Banishev et al., Phys. Rev. Lett. **110**, 137401 (2013).
10. V. M. Mostepanenko and N. N. Trunov, *The Casimir Effect and Its Applications*, Oxford (1997).
9. A. F. Ferrari et al., Mod. Phys. Lett. A **28**, 1350052 (2013).
10. C. R. Muniz et al., Phys. Rev. D **88**, 104035 (2013).
11. V. B. Bezerra et al., arXiv:1401.2084v1 [hep-th].
12. L. D. Landau and E. M. Lifshitz, *Theoretical Physics*, Vol. 2.
13. M. Bordag et al., *Advances in the Casimir Effect*, Oxford (2009).
14. M. Abramowitz and I. A. Stegun, *Handbook of Mathematical Functions*, Dover (1965).
15. The STAR Collaboration. Global Λ hyperon polarization in nuclear collisions. Nature 548, 62–65 (2017). https://doi.org/10.1038/nature23004
16. O. Rogachevsky et al. Phys. Rev. C **82**, 054910, (2010)
17. G. Y. Prokhorov et al. Phys. Rev. D **99**, 071901 (2019)