Measurement of momentum spread in space charge-dominated ion beams

Competition for the JINR Awards for Young Scientists and Specialists
(Scientific and technical applied works)

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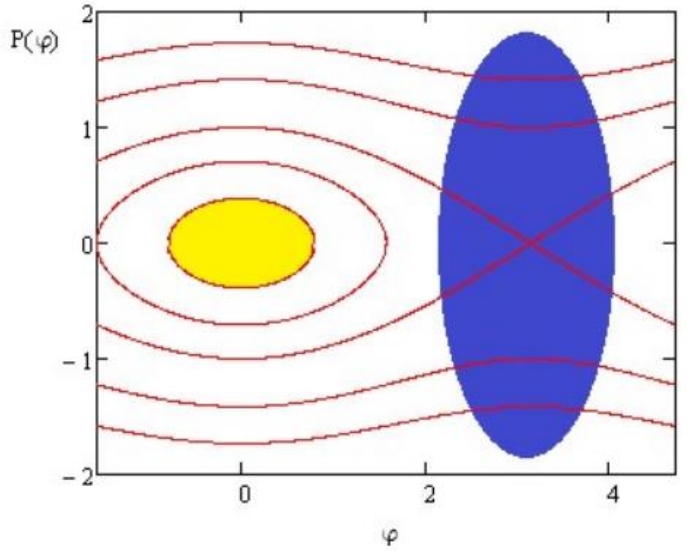
Why do we need to know momentum spread?

- **Intensity** of beam is **crucial** for colliders at each stage;
- NICA Booster synchrotron [1]:
  - accumulates ions (up to 10x);
  - accelerates beam (3.2 MeV/n → 578 MeV/n);
  - extracts beam to Nuclotron;



Scheme of repeated single-lap injection [2]

- Beam fills in available phase space;
- **No accumulation** using only magnetic optics – we need **cooling force**;
- Cooling **reduces volume** of beam **in phase space**;

Measured!

Longitudinal space distribution

Momentum spread distribution

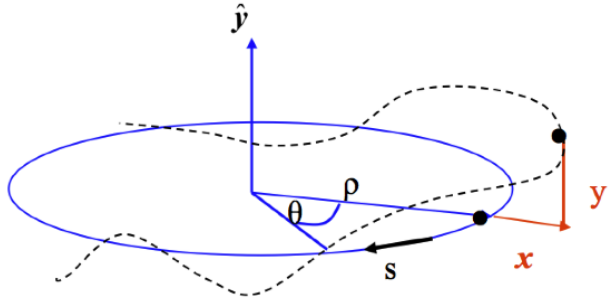
???

- Injection → Cooling → Phase space reduced → Repeat;
- Phase space reconstruction – estimate space required for new injection

[1] NICA Booster: a new-generation superconducting synchrotron. DOI: 10.3367/UFNe.2021.12.039138
[2] Acceleration in Booster and Nuclotron, V. Lebedev, September, 2023

Fokker-Planck equation

- Co-moving coordinate frame: longitudinal coordinate s OR phase ϕ w.r.t. ideal synchronous particle [3]



- Fokker-Planck equation: evolution of particle distribution function $\psi = \psi(\phi, \delta, t)$; [4]

$$\frac{\partial \psi}{\partial t} + \frac{\partial \phi}{\partial t}\frac{\partial \psi}{\partial \phi} + \frac{\partial \delta}{\partial t}\frac{\partial \psi}{\partial \delta} = -\frac{\partial}{\partial \delta}F_{EC}(\delta)\psi + \frac{1}{2}\frac{\partial}{\partial \delta}\left(D(\delta,\phi)\frac{\partial \psi}{\partial \delta}\right)$$

$$\phi = \frac{2\pi h s}{C_a}$$

- $\delta = \frac{\Delta p}{p}$ is relative momentum spread of particle;
- $F_{EC}(\delta)$ is cooling force; $D(\delta, \phi)$ is diffusion power.
- @ equilibrium RHS=0, stationary solution $\psi(\phi, \delta) = \rho(\phi)\lambda(\delta)$

C_a is accelerator circumference

Haissinski equation:

$$\rho(\phi) = \rho_0 \cdot \exp\left(-\frac{U(\phi)}{kT_{\parallel}}\right);$$

- $\rho(\phi)$ is measured longitudinal beam profile
- $\lambda(\delta)$ is unknown, BUT:
- Core of $\lambda(\delta)$ is Gaussian.
- Use σ_δ as RMS relative momentum spread

$kT_{\parallel}$ is longitudinal beam temperature

$U(\phi)$ is Hamiltonian potential part

[3] Beam Dynamics and Beam Losses - https://cds.cern.ch/record/2206741/plots

[4] Haïssinski J. Exact longitudinal equilibrium distribution of stored electrons in the presence of self-fields // Il Nuovo Cimento B. – 1973. – Vol. 18. – № 1. – P. 72-82

Longitudinal beam Hamiltonian with space charge

$$H(\delta,\phi) = \frac{1}{2}h\omega_0\eta\delta^2 + \frac{ZeV_{RF}\omega_0}{2\pi\beta^2 E}\left[\cos\phi_s - \cos\phi + (\phi_s - \phi)\sin\phi_s\right] + U_{SC}(\phi)$$

Kinetic term

Potential term, RF bunching

Potential term, space charge

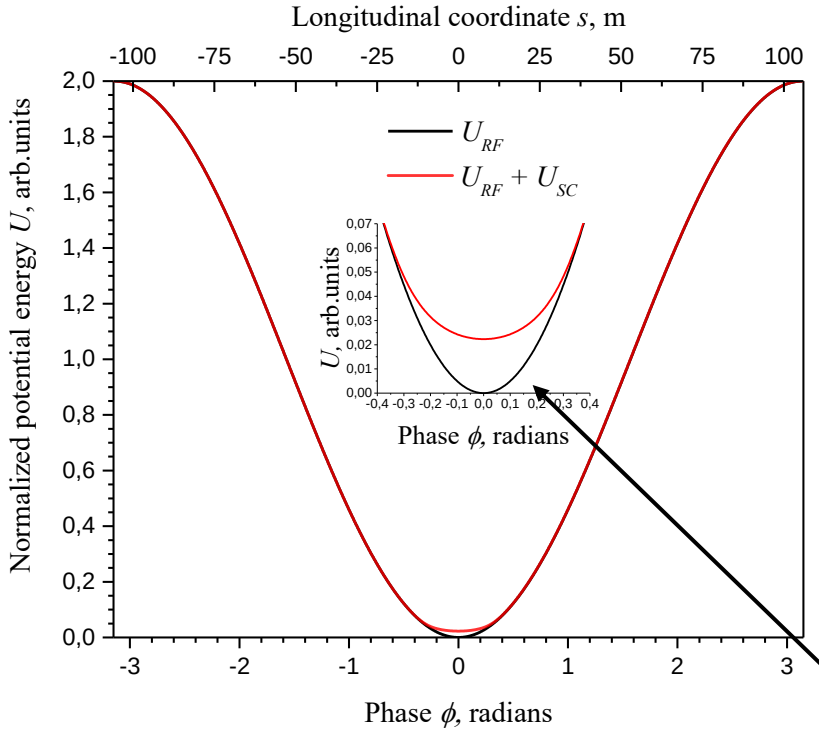
Accumulation: $\phi_s = 0$

Now we construct normalized Hamiltonian:

$$H(\delta,\phi) = \frac{1}{2}\zeta^2 + (1-\cos\phi) + \kappa\cdot\rho(\phi)$$

$\kappa = \frac{Ze\cdot h^2\omega_0 N_0}{V_{RF}}\cdot\frac{Z_\parallel}{n}$ determines space charge

$\zeta = \frac{h\omega_0\eta\delta}{\omega_s} = \frac{h\eta\delta}{\nu_s}$ is conjugate momentum



$Z = 26$, e is elementary charge, $\omega_0 = 2\pi f_0$ is circulation frequency, N_0 is total number of particles, V_{RF} is RF voltage amplitude, $h = 1$ is RF harmonic, $\eta = 0.94$ is slip factor, ω_s is synchrotron oscillations frequency, ν_s is synchrotron tune, $\frac{Z_\parallel}{n}$ is the imaginary part of longitudinal space charge impedance (See next slide)

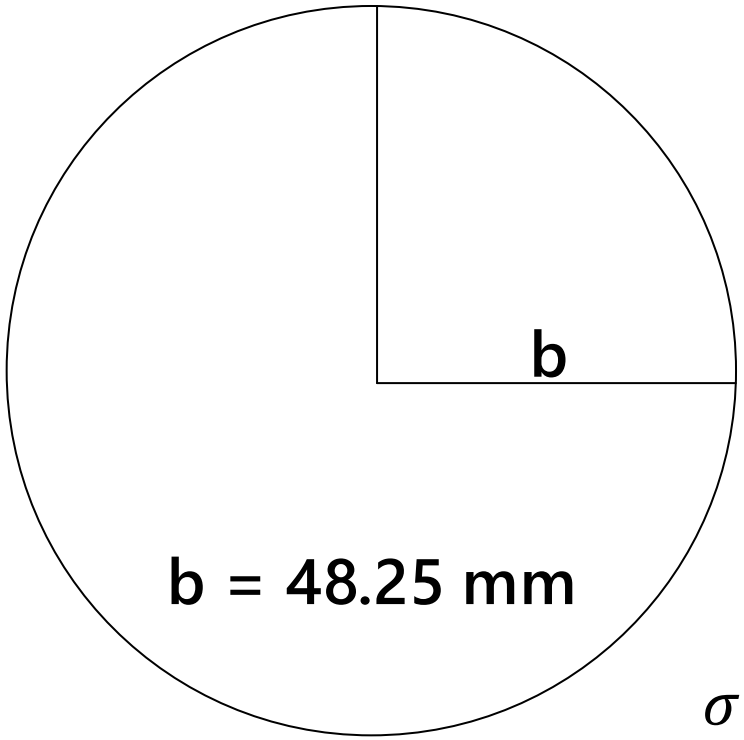
Haissinski equation:

$$\rho(\phi) = \rho_0\cdot\exp\left(-\frac{(1-\cos\phi) + \kappa\cdot\rho(\phi)}{\zeta^2(\sigma_\delta)}\right);$$

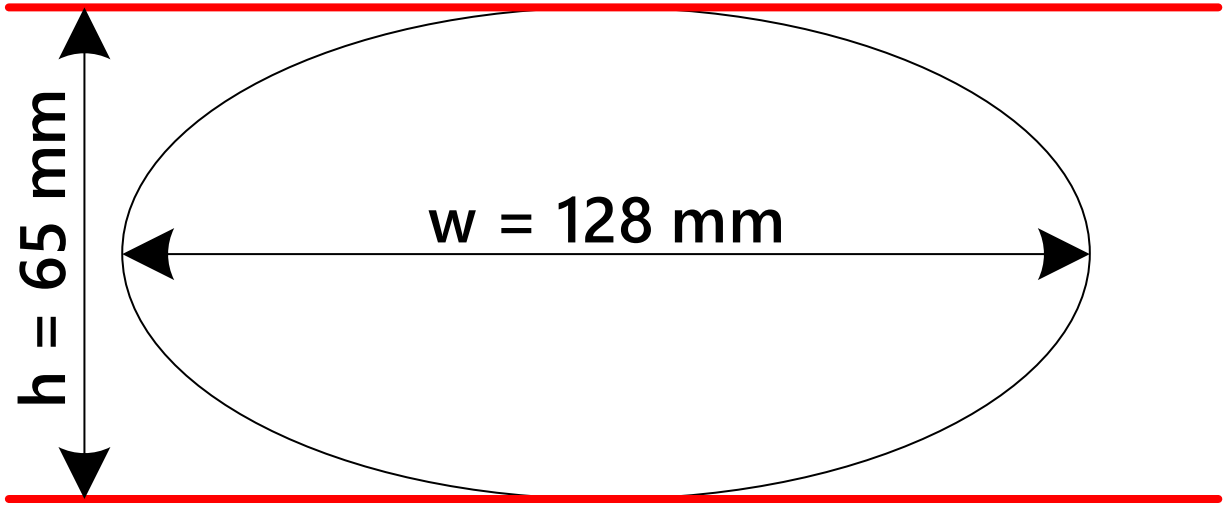
Express beam temperature with $\zeta^2(\sigma_\delta)$ for whole ensemble

Space charge results in potential well flattening

Geometric factor g_0



$$b = \frac{w + h}{4}$$



$\sigma = 4.9$ mm is ⊥ RMS beam radius

$\frac{\mathcal{Z}_\parallel}{n}(g_0) = \frac{\mathcal{Z}_0 g_0}{\beta\gamma^2}$ is the Im part of longitudinal impedance due to SC;

$$\kappa = \kappa\left(\frac{\mathcal{Z}_\parallel}{n}\right) = \kappa(g_0)$$

Beam and chamber geometry factor g_0 (averaged over ⊥ beam distribution):

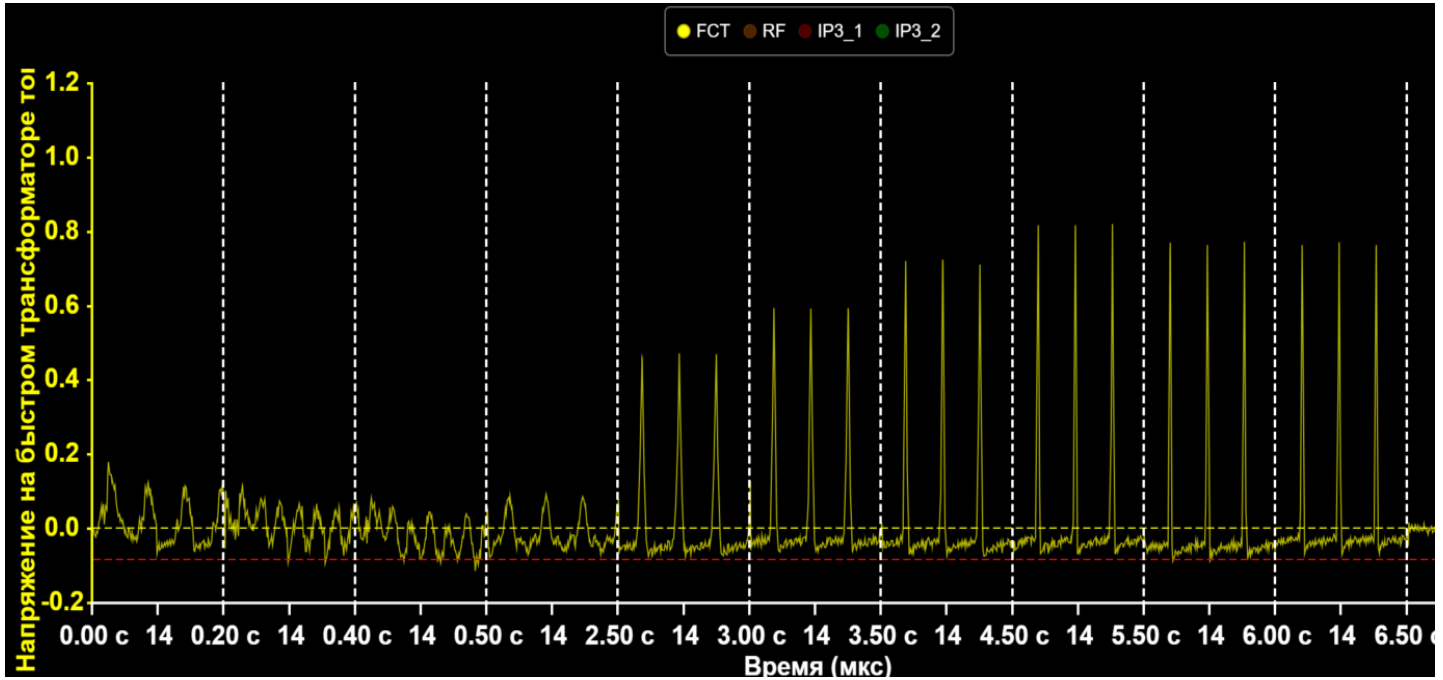
Round chamber, round beam	Elliptic chamber (∥ plates), round beam
$g_0 = \frac{1}{4} + \ln\frac{b}{\sigma} = 2.54$	$g_0 = 0.783 + \ln\left(\frac{h}{1.5\sigma}\right) = 2.27$

*$\mathcal{Z}_0 = 377\ \Omega$ (SI) is the impedance of free space

Longitudinal beam distribution diagnostics



Beam fast current transformer (FCT) manufactured by Bergoz [5]



FCT signal, e-cooling is on

Distribution over time → over phase

$$f(t) \rightarrow f(\phi) = \omega_0 f(t)$$

Measured longitudinal beam profiles

Approximate FCT data with Haissinski equation: vary g_0 and σ_δ to get best fit. Goal: find σ_δ of beam ...and calculate $\kappa(g_0)$

Haissinski equation:

$$\rho(\phi) = \rho_0 \cdot \exp\left(-\frac{(1-\cos\phi) + \kappa(g_0)\cdot\rho(\phi)}{\zeta^2(\sigma_\delta)}\right);$$

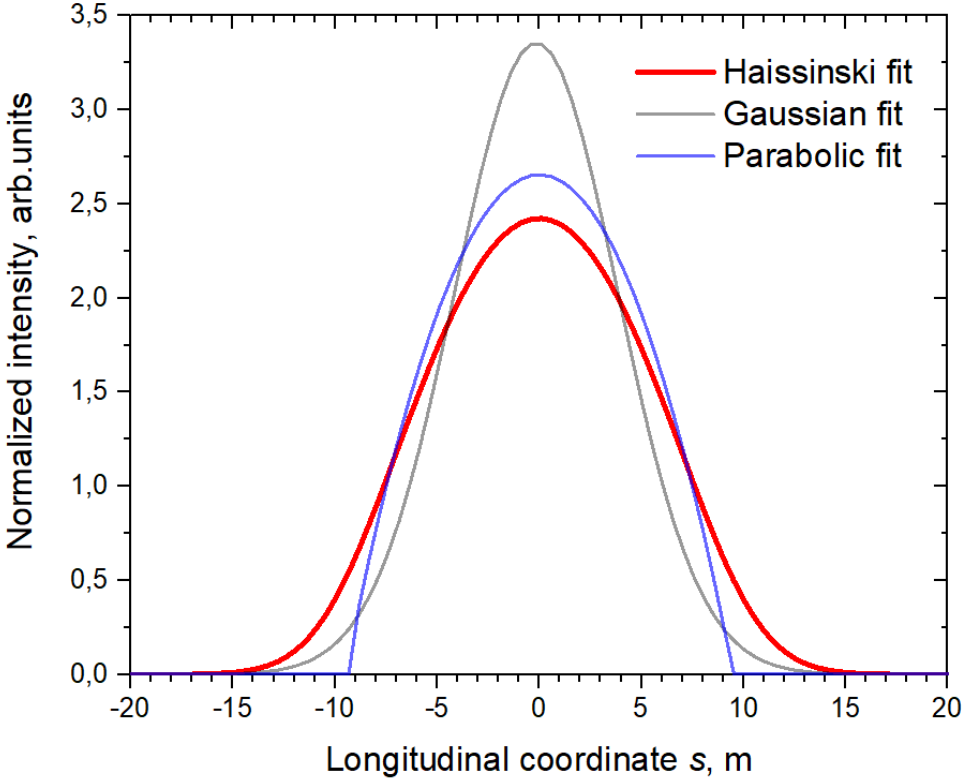
[5] Bergoz FCT: Fast Non-Destructive pulsed or CW beams observation https://www.bergoz.com/products/fct/

Accelerator parameters

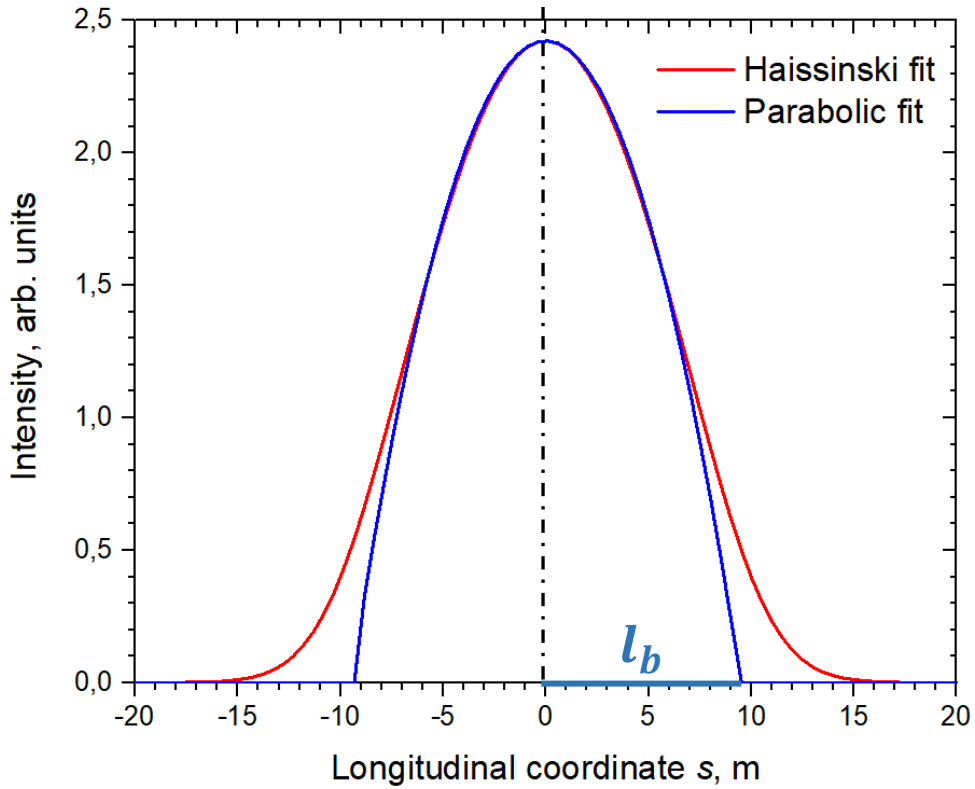
NICA accelerator complex Booster synchrotron
Parameters in 29.06.2025

- Circumference $C_a = 210.96$ m;
- Circulating ions $^{124}Xe^{26+}$ (Target ion charge number $Z = 26$);
- Injection energy 3.2 MeV/nucleon → $\beta = 0.0826; \gamma = 1.003$;
- Beam circulation frequency at injection energy $\frac{\omega_0}{2\pi} = 117550$ Hz;
- RF amplitude $V_{RF} = 40$ V, harmonic $h = 1$;
- Slip factor $|\eta| = 0.94$;
- Measured synchrotron oscillations frequency $\frac{\omega_s}{2\pi} = 51$ Hz is the same as calculated for given V_{RF};
- Number of particles in bunch is calculated via integration of FCT signal with gain coefficient;
- Electron cooling was turned on with electron beam current $I_e = 15$ mA;
- Electron gun cathode voltage $U_c = 1773$ V;
- Longitudinal cooling time is $\tau_{cool} \approx 70$ ms.

Equilibrium bunch distributions



Haissinski fit:
$\sigma_\delta = 5.6 \cdot 10^{-5}$;
$g_0 = 2.23$
Gaussian fit:
$\sigma_\delta = 5.6 \cdot 10^{-5}$;
$g_0 = 0$;
Parabolic fit:
$\sigma_\delta = 0$;
$g_0 = 2.23$;



$$l_b = C_a \cdot \left(\frac{3ZeN_0 g_0}{8\pi^2 \varepsilon_0 C_a h V_{RF}} \right)^{\frac{1}{3}}$$

Distorted potential well → bunch shape changes.

Zero space charge: Gaussian, $\sigma_\delta = 2\pi \frac{\sigma_l}{C_a} \sqrt{\frac{ZeV_{RF}h}{2\pi\beta^2 E_s \cdot \eta}}$ [6] (σ_l is RMS bunch length, m) – grey line;

Cooling ON: Haissinski distribution tends to parabolic with Gaussian tails, $\sigma_\delta \downarrow$, impedance $Z_\parallel/n \uparrow$ - red line;

Limiting case: $\sigma_\delta = 0,\ g_0 \neq 0$ - parabolic - blue line.

[6] M.Bryzgunov et al. First Experiments on Electron Cooling of Ion Beam in the NICA Booster. doi: 10.1134/S1547477124700122

Haissinski equation construction algorithm

Iterative method of successive approximations

ρ_0 from normalization: $\int_{-\pi}^{\pi} \rho(\phi) d\phi = 1$.

$$\rho(\phi) = \rho_0 \cdot \exp\left(-\frac{(1-\cos\phi) + \kappa(g_0)\cdot\rho(\phi)}{\zeta^2(\sigma_\delta)}\right);$$

Use method of iterations. Initial guess is Gaussian-like function with σ_δ estimate from bunch length:

$$\rho^0(\phi) = \rho_0 \cdot \exp\left(-\frac{(1-\cos\phi)}{\zeta^2(\sigma_\delta)}\right)$$

Each iteration adds to function the following correction term:

$$f^a(\phi) = \exp\left(-\frac{(1-\cos\phi)+\kappa(g_0)\cdot\rho(\phi)}{\zeta^2(\sigma_\delta)}\right),$$

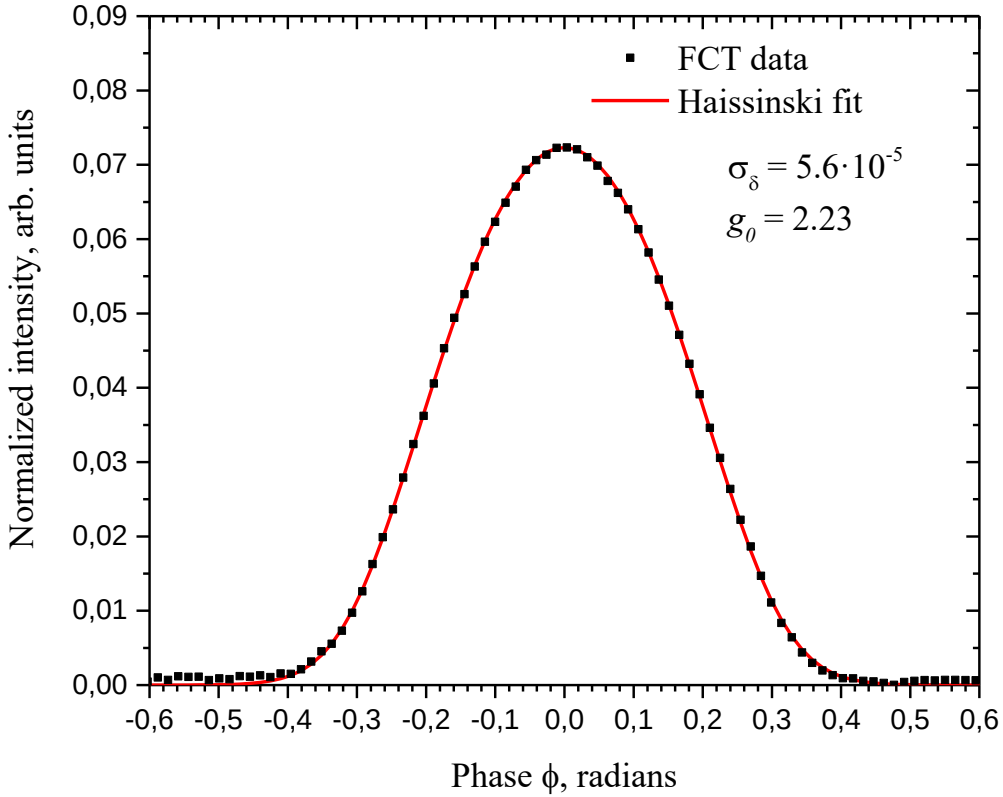
so after i-th iteration the value of j-th distribution point is:

$$\rho_j^i(\phi) = \rho_j^{i-1}(\phi)\cdot(1-\alpha) + \alpha \cdot f_j^a(\phi), 0 < \alpha < 1$$

Converges after 50-100 iterations

Then we run over $[\sigma_\delta; g_0]$ grid to find distribution that fits data with least squares algorithm

Haissinski fit results

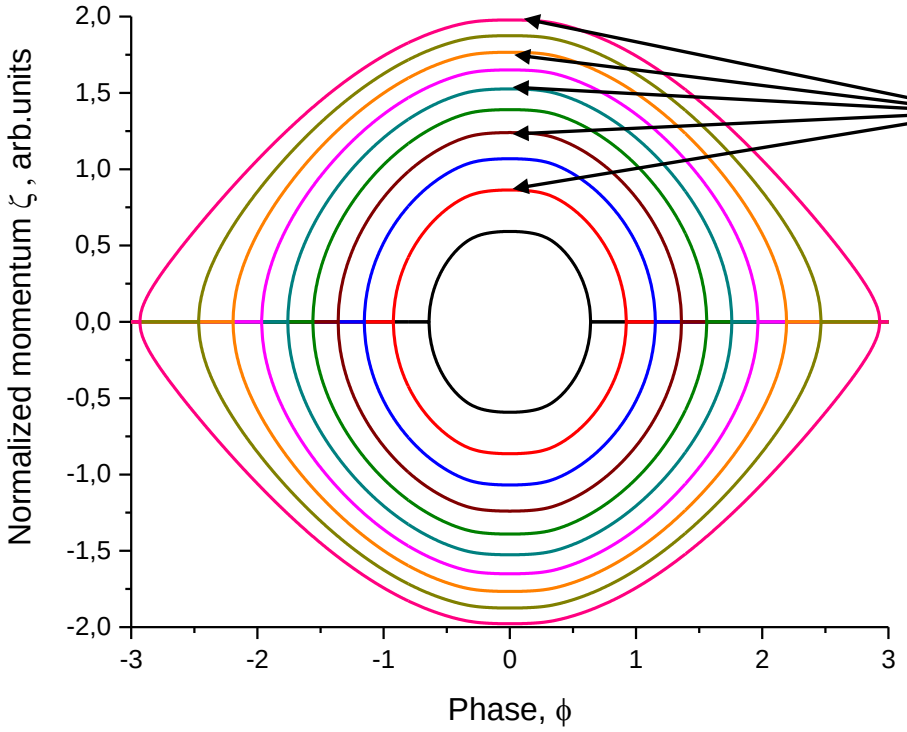


Longitudinal beam profile averaged over 20 turns; cooling ON during 2 s after injection.

- $\sigma_\delta = 5.6 \cdot 10^{-5}$;
- From fit $g_0 = 2.23$, and calculated from ‖ plates approximation $g_0 = 2.27$, discrepancy about 1.8%
- g_0 is calculated close enough to one from fit → we can fit with only one variable σ_δ!
- Now we have parameterized $\rho(\phi)$!

Momentum distribution reconstruction

$$H(\delta,\phi)=\frac{1}{2}\zeta^2(\delta)+(1-\cos\phi)+\kappa\cdot\rho(\phi)$$



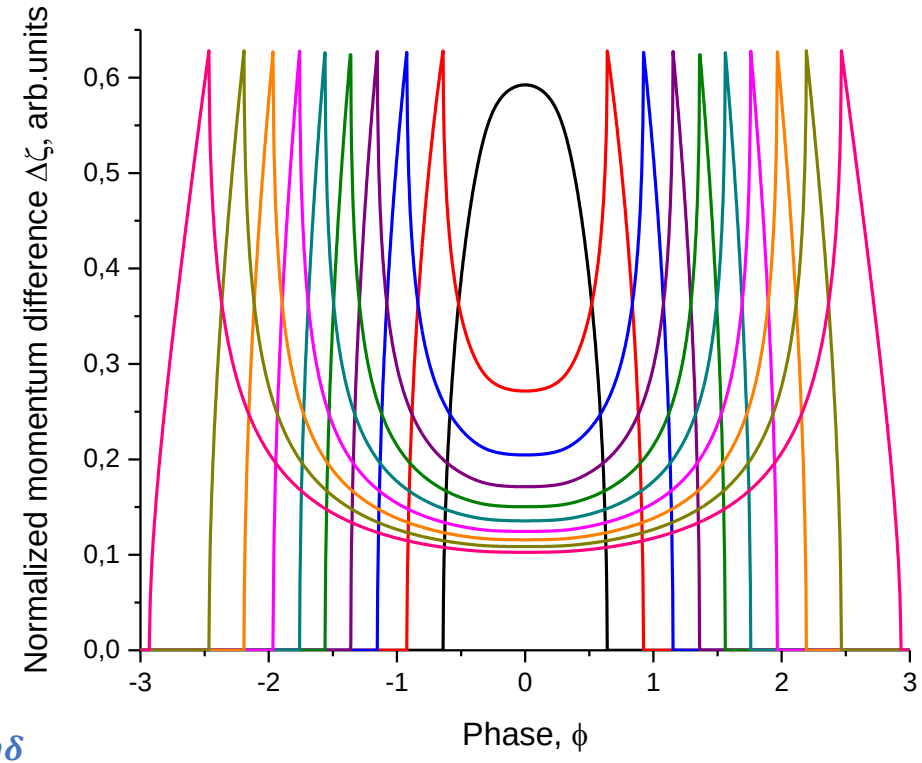
known ζ for each ring

known δ^{maxi} for each ring

$$\zeta=\frac{h\omega_0\eta\delta}{\omega_s}=\frac{h\eta\delta}{\nu_s}$$

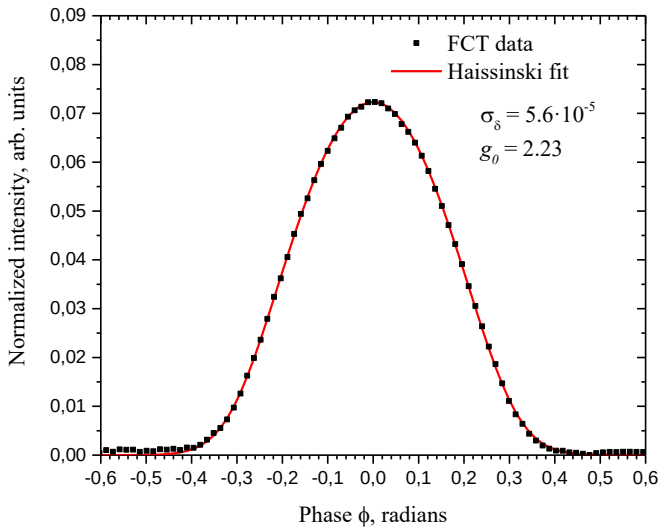
$\approx 2.3\cdot 10^3\delta$ for case under study

$F_i(\phi)$

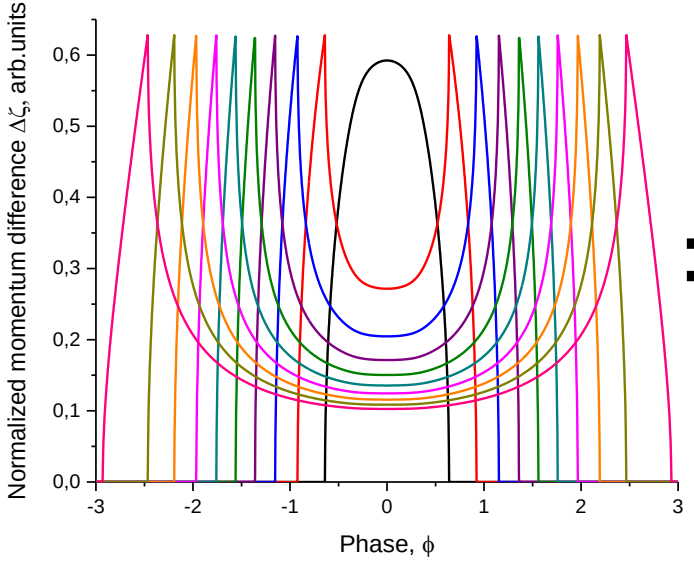


- Construct separatrix for Hamiltonian above;
- Divide into rings of equal area = equal action;
- Construct functions of difference between adjacent rings $\Delta\zeta_i(\phi)$;
- $\rho(\phi)$ is projection of phase space density on ϕ → decompose $\rho(\phi)$ onto $\Delta\zeta_i(\phi)$;

Momentum spread distribution reconstruction



&



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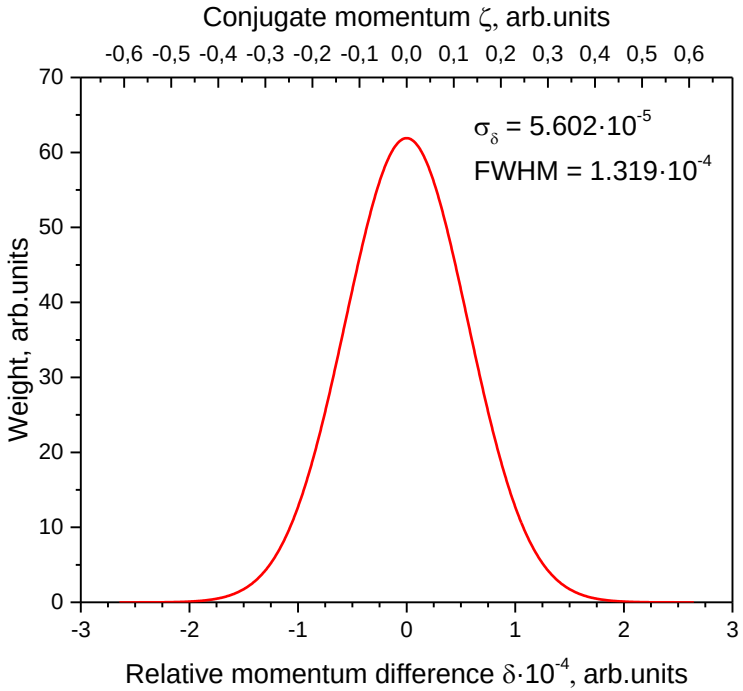

$$\rho(\phi) = \sum_i C_i F_i \qquad F_i \qquad \lambda_i\left(\delta_i^{maxi}\right) = C_i$$

- $\lambda(\delta)$ is unknown, BUT assume core of $\lambda(\delta)$ for equilibrium matched beam is Gaussian;
- We use significantly more rings and F_i ($i \approx 1000$), because No. of points in $\rho(\phi)$ from Haissinski fit can be increased;
- Phase space rings and F_i were used to get weights C_i restoring phase space density distribution;
- Momentum spread distribution is a projection of phase space density distribution onto $\zeta(\delta)$;
- Gaussian fit for reconstructed $\lambda(\delta)$ gives RMS momentum spread $\sigma_\delta = 5.602 \cdot 10^{-5}$

Summary

- Method fits well for **deeply cooled ion beams**
- Only if beam is **well matched** with separatrix and is in **equilibrium state**;
- Algorithm requires knowledge of chamber geometry and ⊥ beam size;
- If these are known, only σ_δ is variable for Haissinski fit, otherwise σ_δ and g_0;
- With fit procedure, one can reconstruct momentum spread distribution;
- Fit results for bunched beam of $2.1 \cdot 10^7$ $^{124}Xe^{26+}$ ions after 2 s of cooling (RMS transverse size $\sigma_\perp = 4.9$ mm): $\sigma_\delta = 5.6 \cdot 10^{-5}$; $g_0 = 2.23$.

Future plans:

- Analysis of beam losses during accumulation;
- Analysis of betatron tune shift due to space charge during accumulation;
- Analysis of effective impedance in dependence on ⊥ beam size;
- Haissinski fit with **barrier-like RF** voltage.

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