On the inconsistency of the Wess-Zumino model and ultraviolet behaviour in QED

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Outline

1. Introduction
2. Finite wave function renormalization
3. Infinite wave function renormalization

4 Ultraviolet behaviour in QED.

5.Conclusions

1. Introduction

All renormalizable models with single coupling constant:

1. Asymptotically free (QCD, ..)

2. Asymptotically non free (QED, WZ…)

3. Free (N = 4 SYM)

Old hypothesis – asymptotically non free models don’t exist as consistent local field theories

Hint – Landau pole singularity in leading log approximation

1.Introduction

History:

In 1983 N.V.K and H.Nikolai presented an

evidence that the WZ model is internally inconsistent as a local field theory

Phys.Lett. B121,2591983)

See however:

S.Aramaki and K.Morita, 1984

C.T.Sachrajda and G.Thompson, 1983

Proof for finite wave renormalization:

N.V.K., 1985 – finite wave function renormalization

The aim of this talk is the investigation of the supersymmetric Wess-Zumino model plus

I shall discuss QED

$$L = L_0 + W + W^* ,$$

$$L_0 = \int \sum^{N} \phi_k^*(x,\theta,\bar\theta)\phi_k(x,\theta,\bar\theta)d^2\theta d^2\bar\theta ,$$

$$W = \int \sum_{i,j,k} [\frac{g_{ijk}}{3}\phi_i(x,\theta)\phi_j(x,\theta)\phi_k(x,\theta) + \frac{m_{ij}}{2}\phi_i(x,\theta)\phi_j(x,\theta)]d^2\theta ,$$

Here $\phi_k(x,\theta,\bar\theta) = \phi_k(x^\mu + i\theta\sigma^\mu\bar\theta,\theta)$ and $\phi_k(x,\theta) = \phi_k(x) + \sqrt{2}\psi_k(x)\theta + \theta\theta F_k(x)$ is chiral scalar superfield. In terms of component fields $(\phi_k(x), \psi_k(x), F_k(x))$ the Lagrangians (2), (3) have the form

$$L_0 = \int \sum_{k=1}^{N} [\partial^\mu\phi_k\partial_\mu\phi_k^* - i\partial_\mu\bar\psi_k\bar\sigma^\mu\psi_k + F_k^*F_k] , \qquad (4)$$

$$W = \int \sum_{i,j,k} [g_{ijk}(F_i\phi_j\phi_k - \phi_i\psi_j\psi_k) + \sum_{i,j} m_{ij}(F_i\phi_j - \frac{1}{2}\psi_i\psi_j) . \qquad (5)$$

1.Introduction

The Lagrangian (1) is invariant under the supersymmetry transformations

$$\delta\phi_k = \sqrt{2}\zeta\psi_k\,,$$

$$\delta\psi_k = i\sqrt{2}\sigma^\mu\bar{\zeta}\partial_\mu\phi_k + \sqrt{2}\zeta F_k\,,$$

$$\delta F_k = i\sqrt{2}\bar{\zeta}\bar{\sigma}^\mu\partial_\mu\psi_k\,.$$

The WZ model describes interaction of massive scalar and majorana fields.

The WZ model with the superpotential (3) is renormalizable model [7] - [11]. Moreover all ultraviolet divergencies can be eliminated by the introduction of the wave function renormalization counterterm [7] - [11]

$$\Delta L = \int \sum_{k=1}^{N} \Delta Z_k \phi_k^*(x,\theta,\bar{\theta})\phi_k(x,\theta,\bar{\theta}) d^2\theta d^2\bar{\theta}\,.$$

1.Introduction

WZ model regularization

$$L_{0,reg} = L_0 + \int \sum_{k=1}^{N} \Phi_k^*(x,\theta,\bar{\theta})\Phi_k(x,\theta,\bar{\theta}) d^2\theta d^2\bar{\theta} + \left(\int \sum_k M_k \frac{\Phi_k(x,\theta)\Phi_k(x,\theta)}{2} d^2\theta + h.c. \right) .$$

The regularization of the interaction superpotential

$$W_{int} = \int \sum_{l,j,k} [g_{ljk}\phi_l(x,\theta)\phi_j(x,\theta)\phi_k(x,\theta) d^2\theta$$

and its hermitean conjugate W_{int}^* is the following:

$$W_{int} \to \int \sum_{l,j,k} [g_{ljk}(\phi_l(x,\theta) + i\Phi_l(x,\theta))(\phi_j(x,\theta) + i\Phi_j(x,\theta))(\phi_k(x,\theta) + i\Phi_k(x,\theta)) d^2\theta ,$$

$$W_{int}^* \to \int \sum_{l,j,k} [g_{ljk}^*(\phi_l^*(x,\bar{\theta}) + i\Phi_l^*(x,\bar{\theta}))(\phi_j^*(x,\bar{\theta}) + i\Phi_j^*(x,\bar{\theta}))(\phi_k^*(x,\bar{\theta}) + i\Phi_k^*(x,\bar{\theta})) .$$

$$g_{ijk} \to g_{ijk} \exp\left(-\frac{\vec{p}_i^2}{\Lambda^2} - \frac{\vec{p}_j^2}{\Lambda^2} - \frac{\vec{p}_k^2}{\Lambda^2} \right) .$$

1.Introduction

Canonical quantization

$$L = \partial_\mu \phi_k(x) \partial^\mu \phi_k^*(x) - V(\phi_k(x), \phi_k^*(x))$$

$$[\phi_k(t,\vec{x}),\ \phi_l(t,\vec{y})] = [\phi_k^*(t,\vec{x}),\ \phi_l(t,\vec{y})] = 0\,,$$

$$[\frac{\partial_k \phi_k(t,\vec{x})}{dt},\ \phi_l(t,\vec{y})] = 0\,,$$

$$[\frac{\partial_k \phi_k(t,\vec{x})}{dt},\ \phi_l^*(t,\vec{y})] = 0 \ at\ k \neq l\,,$$

$$i[\frac{\partial_k \phi_k(t,\vec{x})}{dt},\ \phi_k^*(t,\vec{y})] = \delta(\vec{x}-\vec{y})\,.$$

$$\phi_{kr\Lambda}(x) = Z^{-1/2}(\Lambda,...)\phi_k(x)$$

$$i[\frac{\partial_k \phi_{kr}(t,\vec{x})}{dt},\ \phi_{kr}^*(t,\vec{y})] = \frac{1}{Z_k(\infty,...)}\delta(\vec{x}-\vec{y})\,.$$

1.Introduction

KI representation:

$$< 0|[\phi_{kr}(x), \phi^*_{lr}(y)]|0 >= \frac{1}{i}\int_0^\infty D(x-y,t)\rho_{kl}(t)dt\,,$$

$$D(x-y,m^2) = \frac{i}{(2\pi)^3}\int \exp\left[-ik(x-y)\right]\epsilon(k^0)\delta(k^2-m^2)d^4k\,,$$

$$\rho_{kl}(t) = c_k^2\delta_{kl}\delta(t-m_k^2) + \Delta\rho_{kl}(t)$$

$$\int_0^\infty \rho_{kl}(t)dt = 0 \;at\; k \neq l\,,$$

$$\int_0^\infty \rho_{kk}(t)dm^2 = \frac{1}{Z_k(\infty,...)}\,.$$

For the often used normalization condition $c_k^2 = 1$ well known inequality

$$0 \leq Z(\infty,...) = [1 + \int_0^\infty \Delta\rho_{kk}(t)dt]^{-1} \leq 1\,,$$

1.Introduction

Schwinger's equations

$$\int \prod_k d[\phi_k] \prod_i \frac{\delta}{\delta \bar{\phi}_i(x_i)} [R(\phi_k) \exp(iS(\Phi_k)] = 0 .$$

Here ϕ_k denotes the multiplet (ϕ_k, F_k, ψ_k), the action $S(\Phi_k) = \int d^4x L + \int J_k \phi_k d^4x$ and $R[\phi_k]$ an arbitrary function; $\bar{\phi}_i$ may be taken to be any of the fields ϕ_k F_k and ψ_k. The crusial point is that for renormalized Lagrangian (16, 17) and for finite regularization Λ we can derive Schwinger's equations for any Λ. It should be stressed that we assume finite limit for $Z(\Lambda, ...)$ at $\Lambda \to \infty$. This assumption is not valid in the perturbation theory where at one loop level $Z = 1 - a g_r^2 \ln(\frac{\Lambda}{\mu})$, $a > 0$.

2 .Finite wave function renormalization $Z(\infty, ...) \neq 0$

As an example consider the WZ model with three chiral superfields $\phi_k(x,\theta) = \phi_k(x) + \sqrt{2}\psi_k(x)\theta + F_k(x)\theta\theta$ $(k = 1, 2, 3)$ and with the superpotential

$$W = W_2 + W_3\,, \tag{35}$$

$$W_2 = \frac{m}{2}(\phi_1^2(x,\theta) + \phi_2^2(x,\theta) + \phi_3^2(x,\theta))\,, \tag{36}$$

$$W_3 = g\phi_1(x,\theta)\phi_2(x,\theta)\phi_3(x,\theta)\,. \tag{37}$$

The kinetic term has standard form

$$L_0 = \int d^2\theta d^2\bar{\theta} \sum_{k=1}^{3} \phi_k^*(x,\theta,\bar{\theta})\phi_k(x,\theta,\bar{\theta})\,, \tag{38}$$

The Lagrangian with the superpotential (35, 36, 37) is invariant under the discrete transformations $\phi_k \to \phi_l$, $\phi_l \to \phi_k$. As a consequence the counterterm

$$\Delta L = \int d^2\theta d^2\bar{\theta} \sum_{k=1}^{3} (Z(\Lambda, g_r, \mu, m_r) - 1)\phi_k^*(x,\theta,\bar{\theta})\phi_k(x,\theta,\bar{\theta}) \tag{39}$$

2 .Finite wave function renormalization

$Z(\infty,...) \neq 0$

$$[\mu\frac{\partial}{\partial\mu} + \beta(g_r)\frac{\partial}{\partial g_r} + m_r\gamma_m(g_r)\frac{\partial}{\partial m_r} + n\gamma(g_r)]G_n = 0\,,$$

$$\beta(g_r) = 3g_r\gamma(g_r) \text{ and } \gamma_m(g_r) = 2\gamma(g_r).$$

$$Z(\infty,...)\delta(x)+iZ^2(\infty,...) < 0|T(F^*_{r1}(x), F_{r1}(0)|0 >= ig_r^2 < 0|T(\phi_{r2}(x)\phi_{r3}(x), \phi^*_{r3}(0)\phi^*_{r2}(0) > \quad (54)$$

$$Z(\infty,...) + Z^2(\infty,...)D_{F_{r1},F^*_{r1}}(p^2) = g_r^2 D_{\phi_{r2}\phi_{r3},\phi^*_{r3}\phi^*_{r2}}(p^2)\,.$$

$$Z^2(\infty,...)t\rho_{\phi_{r1},\phi^*_{r1}}(t) = g_r^2\rho_{\phi_{r2}\phi_{r3},\phi^*_{r3}\phi^*_{r2}}(t)\,.$$

3 .Finite wave function renormalization

$Z(\infty, ...) \neq 0$

Due to assumed finiteness of $Z^{-1}(\infty, ...) = \int_0^\infty \rho_{\phi_{r1},\phi^*_{r1}}(t)dt$ we find that $lim_{t\to\infty}(t\rho_{\phi_{r1},\phi^*_{r1}}(t)) = 0$. The spectral density $g_r^{2/3}\rho_{\phi_{r1},\phi^*_{r1}}(t)$ has zero anomalous dimension and due to the commutation relations (25) $Z(\infty, ...) = (\frac{g_r}{g_{r,fp}})^{2/3}$. The spectral density $Z(\infty, ...)\rho_{\phi_{r1}\phi^*_{r1}}(t)$ can be represented in the form

$$Z(\infty, ...)\rho_{\phi_{r1}\phi^*_{r1}}(t) = \delta(t) + \frac{1}{t}\Delta\rho(\frac{t}{\Lambda^2}), \tag{57}$$

where Λ satisfies the renormalization group equation $(\mu\frac{\partial}{\partial\mu} + \beta(g_r)\frac{\partial}{\partial g_r})\Lambda = 0$ and the $\Lambda = 0$ corresponds to the case of fixed point $g_{r,fp}$ with $\beta(g_{r,fp}) = 0$. At fixed point

$Z(\infty, ...)\rho_{\phi_{r1}\phi^*_{r1}}(t) = \delta(t)$ and the $lim_{t\to\infty}\Delta\rho(\frac{t}{\Lambda^2}) = 0$. The right hand side of the equation (56) can be represented in the form

$$g_r^2\rho_{\phi_{r2}\phi^*_{r3}\phi^*_{r3}\phi^*_{r2}}(t) = Z(\infty, ...)\Phi(t/\Lambda^2). \tag{58}$$

2.Finite wave function renormalization

As it has been mentioned before the case $\Lambda = 0$ or the limit $t \to \infty$ corresponds to the fixed point. According to Pohlmeyer theorem [23] if the propagator of the local scalar field is proportional to the propagator of massless free scalar field then the scalar field is free massless field. As a consequence of Pohlmeyer theorem the right hand side of the equation (56) at $g_r = g_{r,fp}$ is equal to $g_r^2 Z^{-2}(\infty, ...)\frac{1}{16\pi^2}$. It means that

$$lim_{t\to\infty}(g_r^2 \rho_{\phi_{r2}\phi_{r3},\phi^*_{r3}\phi^*_{r2}}(t)) = g_r^2 Z^{-2}(\infty, ...)\frac{1}{16\pi^2}\,. \qquad (59)$$

So we find the contradiction: the left hand side of the equation (56) vanishes in the limit $t \to \infty$ while the right hand side of the equation (56) is different from zero in this limit.

Note that the proof of the absence of fixed point in the WZ model is valid also for the WZ models with different superpotentials. For instance, in full analogy with the previous case one can prove that for the WZ model with single chiral superfield $\phi(x, \theta)$ and with the superpotential $W_3 = \frac{m}{2}\phi(x, \theta) + g\phi^3(x, \theta)$ fixed point behaviour is not possible [14].

3.Infinite wave function renormalization

$$-Z(\Lambda,...) < 0|T(\phi^*_{r1}(x,\Lambda), F^*_{r1}(y,\Lambda)|0 >= m_r < 0|T(\phi^*_{r1}(x,\Lambda), \phi_{r1}(y,\Lambda)|0 > +g_r < 0|T(\phi^*_{r1}(x,\Lambda), (\phi_{r2}(y,\Lambda) \quad (41)$$

We assume that the limit $\Lambda \to \infty$ exists for the Green's functions of the renormalized fields $\phi_{rk}(x,\Lambda)$ namely, we assume that

$$(\phi_{rk}(x,\Lambda), \psi_{rk}(x,\Lambda), F_{rk}(x,\Lambda) \to (\phi_{rk}(x), \psi_{rk}(x), F_{rk}(x)) \; at \; \Lambda \to \infty \qquad (42)$$

As a consequence of the assumption (42) the equation (41) takes the form

$$m_r < 0|T(\phi^*_{r1}(x), \phi_{r1}(y)|0 > +g_r < 0|T(\phi^*_{r1}(x), (\phi_{r2}(y)\phi_{r3}(y)))|0 >= 0\,. \qquad (43)$$

The analogous relation for the spectral densities of the KL representations for the propagators reads

$$m_r \rho_{\phi^*_{r1},\phi_{r1}}(t) + g_r \rho_{\phi^*_{r1},\phi_{r2}\phi_{r3}}(t) = 0\,. \qquad (44)$$

As a consequence of the commutation relations (21 - 23) we find that

$$[\frac{\partial \phi^*_{r1}(t,\vec{x})}{\partial t}, (\phi_{2r}(t,\vec{y})\phi_{3r}(t,\vec{y}))_{reg}] = 0\,,$$

3. Infinite wave function renormalization $Z(\infty, ...) = 0$

For the commutation relation (45) the equation (29) takes the form

$$\int_0^\infty \rho_{\phi^*_{r1},\phi_{r2}\phi_{r3}}(t)dt = 0\,. \tag{46}$$

Due to the assumed positivity of the metric for the states ($< n|n > \;\; > 0$) the spectral density $\rho_{\phi^*_{1r},\phi_{1r}}(t) \geq 0$ is non negative. As a consequence of the relation (44) the spectral density

$$m_r g_r \rho_{\phi^*_{r1},\phi_2\phi_3}(t) \leq 0 \tag{47}$$

is non positive. Moreover the relations (47) and (46) lead to zero spectral density $m_r g_r \rho_{\phi^*_{r1},\phi_2\phi_3}(t) = 0$ and $\rho_{\phi^*_{r1},\phi_{r1}}(t) = 0$. So the scalar propagator vanishes that contradicts to the expectations from the perturbation theory that the spectrum contains the scalar massive state.

The limit $Z(\infty,...)=0$ is described by the ultralocal Lagrangian

$$L_{ultra}=\int d^2\theta[(g_r\phi_{r1}(x,\theta)\phi_{r2}(x,\theta)\phi_{r3}(x,\theta)+\frac{m_r}{2}\phi^2_{r1}(x,\theta)+...]+h.c. \qquad (51)$$

without derivatives which has been considered in [20]. For the Lagrangian (51) all scalar Green's functions are equal to zero. It is easy to prove this fact using the continual integral formalism for the Lagrangian (51). Really for the Lagrangian L_{utra} after the integration over the axiliary fields $F(x)$, $F^*(x)$ we find the delta functions

$$\int dF_{rk}(x)dF^*_{rk}(x)d...exp(iL_{ultra})\sim\int d...\delta(m_r\phi_{r1}(x)+g_r\phi_{r2}(x)\phi_{r3}(x))...e^{iL_{ultra}}. \qquad (52)$$

that allows to calculate the continual integral for L_{ultra} exactly [20]. The spectrum for the Lagrangian L_{ultra} is an empty, i.e. it does not contain any physical states.

It is possible also to consider the standard case with single scalar chiral superfield $\phi(x,\theta)$ and with the superpotential $W=\frac{m}{2}\phi^2(x,\theta)+\frac{g}{3}\phi^3(x,\theta)$. For such potential the equations (43, 44) are also valid with the replacement $g_r\phi_{r2}(y)\phi_{r3}(y)\to g_r\phi^2_r(y)$. Note that in the derivation of sum rule $\int_0^\infty\rho_{\phi^*_r,\phi_r\phi_r}(t)dt=0$ we have to use in addition the commutation relation (25). As in a previous case we find that for $Z(\infty,...)=0$ the WZ model is internally inconsistent.

4.Ultraviolet behaviour in QED

We consider QED with N identical massless fermions

$$L_{int} = \frac{e}{\sqrt{N}} \sum_{k=1}^{N} \bar{\psi}_k \gamma^\mu \psi_k A^\mu \, .$$

$$\bar{\alpha}(x, \alpha, N) = \frac{\alpha}{1 + \alpha \Pi(x, \alpha, N)}$$

$$\Pi(x, \alpha, N) = \int_0^\infty [\frac{\rho(t, \alpha, N)}{t + x)} - \frac{\rho(t, \alpha, N)}{t + 1)}]$$

4.Ultraviolet behavior in QED

is valid. The renormalization group equation for the invariant charge $\bar{\alpha}(x, \alpha, N)$ has the form

$$x\frac{d}{dx}\bar{\alpha}(x, \alpha, N) = \psi(\bar{\alpha}, N)\,, \tag{4}$$

where

$$\psi(\bar{\alpha}, N) = \beta_2\alpha^2 + \sum_{k=1}^{\infty}(1/N)^k\psi_k(\alpha)\,, \tag{5}$$

$\beta_2 = \frac{1}{3\pi}$ and

$$\psi_k(\alpha) = \sum_{l=k+2}^{\infty} c_{l,k}\alpha^l\,.$$

$$\frac{d\bar{\alpha}}{dt} = \beta_2\bar{\alpha}^2\,,$$

4.Ultraviolet behavior in QED

Large N expansion

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Invariant charge at complex momenta

$$\bar{\alpha}(\frac{p^2}{\mu^2},\alpha)=\frac{\alpha}{1-\alpha\beta_2\ln(\frac{p^2}{\mu^2})}=\frac{1}{-\beta_2\ln(\frac{p^2}{\Lambda^2})},$$

$$\bar{\alpha}(\frac{\exp(i\phi)|p^2|}{\Lambda^2})=\frac{\ln(\frac{|p^2|}{\Lambda^2})-i\phi)}{-\beta_2(\ln^2(\frac{|p^2|}{\Lambda^2})+\phi^2)}.$$

$$\bar{\alpha}_{Re,\phi}(\frac{|p^2|}{\lambda^2})\equiv Re\ \bar{\alpha}(\frac{\exp(i\phi)|p^2|}{\Lambda^2})=\frac{\ln(\frac{|p^2|}{\Lambda^2})}{-\beta_2(\ln^2(\frac{|p^2|}{\Lambda^2})+\phi^2)}.$$

The most interesting case $\phi=\pi$ corresponds to time-like region $p^2<0$. At time-like region the real part of the invariant charge has maximum(minimum) at $\ln(\frac{|p^2|}{\Lambda^2})=-\pi(\pi)$ equal to $\frac{1}{2\pi\beta_2}(-\frac{1}{2\pi\beta_2})$. We can define the invariant charge in time-like region as real part of the invariant charge , i.e. $\bar{\alpha}_{Re,\phi}=\frac{\bar{\alpha}+\bar{\alpha^*}}{2}$. At two-loop approximation (10) for $\phi=\pi$ and $\ln(\frac{|p^2|}{\Lambda^2})=-\pi$

$$\bar{\alpha}_{Re,\phi=\pi}(\frac{|p^2|}{\Lambda^2}=\exp(-\pi))=\frac{1}{2\pi\beta_2}-\frac{\beta_3}{8\pi\beta_2^3}. \qquad (12)$$

Invariant charge at complex momenta

In QED with the Lagrangian (1) $\beta_2 = \frac{1}{3\pi}$, $\beta_3 = \frac{1}{4N\pi^2}$ and two-loop correction at (12) is less than 10 percent for $N \geq 6$. It means that we can trust one-loop approximation for $\bar{\alpha}_{Re,\phi}(\frac{|p^2|}{\lambda^2}, N)$ at $N \gg 1$.

It is interesting to note that at imaginary $p^2 = ix$, $Im\ x = 0$ the dispersion relation for $\bar{\alpha}_{Re,\phi=\frac{\pi}{2}}(\frac{|p^2|}{\Lambda^2})$ can be represented in the form

$$\bar{\alpha}_{Re,\phi=\frac{\pi}{2}}(\frac{x}{\Lambda^2}) = x^2 \int_0^\infty \frac{\rho(t)dt}{x^2+t^2}\,, \tag{13}$$

where $\rho(t) \geq 0$. As a consequence of the representation (13) $\bar{\alpha}_{Re,\phi=\frac{\pi}{2}}(x)$ is increasing function at variable x, i.e. $\frac{d\bar{\alpha}_{Re,\phi=\frac{\pi}{2}}(\frac{x}{\Lambda^2})}{dx} \geq 0$. It means that one-loop approximation (11) is not applicable at $\frac{|p^2|}{\Lambda^2} \geq exp(-\frac{\pi}{2})$.

So we have found that the invariant charge at complex p^2 including negative p^2 does not have Landau pole singularity and we can define new invariant charge as a real part of the originally defined invariant charge at $p^2 = \exp(i\phi)|p^2|$. For such definition of the invariant charge we don't have Landau pole singularity. In one-loop approximation new invariant charge has extremum equal to $\mp\frac{1}{2\beta_2\phi}$ at $\ln(\frac{|p^2|}{\Lambda^2}) = \pm\phi$. Probably the most natural way is the definition of new invariant charge as real part of old invariant charge at negative p^2.

In the next slides I shall consider 1/N expansion in QED for photon propagator or invariant charge

The main conclusion:

Ultraviolet behavior of the photon propagator is determined by leading Log approximztion

1/N expansion

It is useful to define the so called Adler D-function as

$$D(x,\alpha,N) = -x\frac{d}{dx}\Pi(x,\alpha,N) = \frac{\psi(\bar{\alpha},N)}{\bar{\alpha}^2}\,. \tag{16}$$

The D-function (16) has zero anomalous dimension, i.e. it obeys the equation $[x\frac{d}{dx} - \psi(\alpha,N)]D(x,\alpha,N) = 0$. One can find that in QED with the Lagrangian (1)

$$D(x=1,\alpha,N) = \frac{\psi(\alpha,N)}{\alpha^2} = \beta_2 + \frac{\alpha\beta_3}{N} + \sum_{k=2}^{\infty} c_k(N)\alpha^k\,, \tag{17}$$

where $\beta_2 = \frac{1}{3\pi}$ and $\beta_3 = \frac{1}{4\pi^2}$ [1]. For the D-function the $1/N$ expansion is represented in the form

$$D(x,\alpha,N) = \sum_{k=0}^{\infty}(\frac{1}{N})^k D_k(\alpha,x)\,, \tag{18}$$

where $D_k(x,\alpha) = \sum_{n=k}^{\infty}\alpha^n\bar{C}_{n,k}(x)$ and $D_0(x,\alpha) = \beta_2$.

1/N expansion

For the invariant charge $\bar{\alpha}(x, \alpha, N)$ the $1/N$ perturbation theory coincides with the standard perturbation theory except the use of the effective photon propagator

$$\frac{\bar{\alpha}(\frac{p^2}{\mu^2}, \alpha)}{p^2} = -\frac{1}{\beta_2 p^2 \ln(\frac{p^2}{\Lambda^2})} \tag{18}$$



The $1/N$ corrections to the invariant charge are described by the diagrams depicted in Figure 2.



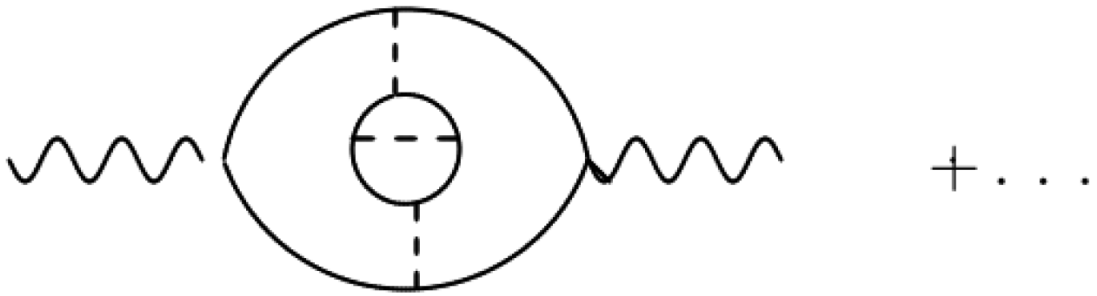

Figure 3: The bubble type $1/N$ subdiagram giving infinite contribution to the invariant charge



Figure 4: Ultraviolet finite $1/N^2$ corrections to the photon propagator

Modified ultraviolet finite 1/N perturbation theory

$$D^{imp}_{eff}(p^2,\Lambda^2)=\frac{1}{p^2(-\beta_2\ln(\frac{p^2}{\Lambda^2})+\frac{1}{N}\Pi_2(\frac{p^2}{\Lambda^2}))}\,.$$

The first terrm in (21) is the leading contribution of the diagrams of Fig.1 and the Π_2 is the $1/N$ contribution of the diagrams depicted at Fig.2. One can find that the improved effective propagator (21) leads to ultraviolet finite perturbation theory. The ultraviolet asymptotics of the improved effective propagator is



$$D^{imp}_{eff}(p^2,\Lambda^2)=\frac{1}{p^2(-\beta_2\ln(\frac{p^2}{\Lambda^2})+\frac{\beta_3}{N\beta_2}\ln(-\ln(\frac{p^2}{e\Lambda^2}))+o(1))}\,,$$

Our results demonstrate that the ultraviolet behavior in QED is determined by leading log approximation. It is correct at least for the 1/N expansion. In QED with the imaginary charge(nonphysical model) we have asymptotic freedom and the leading log approximation with Landau pole singularity is valid. Probably the same situation takes place in other non asymptotically free models.

5.Conclusions

For WZ model our proof is based on the following assumptions:

1. We assumed that the WZ model is the model with positively defined metric of states - there are no ghost states in the spectrum. As a consequence of this assumption we used the KL inequality $0 \leq Z(\infty, g_r, \mu, m_r) \leq 1$ for the wave function renormalization.

2. In the perturbation theory the Green's functions become ultraviolet finite by the introduction of the $\Delta Z_k \int d^2\theta d^2\bar{\theta} \Phi_k^*(x, \theta, \bar{\theta}) \Phi_k(x, \theta, \bar{\theta})$ counterterms. We assumed that this structure of the counterterms is valid irrespective of the perturbation theory.

3.We assumed that commutation relations among generalized coordinates and momenta are valid in the renormalized field theory.

5.Conclusions

Our proof of the inconsistency of the finite wave function renormalization $Z(\infty,...) \neq 0$ was based on the use of Polhlmeyer theorem while the proof for the case of infinite wave function renormalization $Z(\infty,...) = 0$ was based on the use of the commutation relations (21 - 24). So our proof excludes $Z(\infty,...) \neq \pm\infty$. The case $Z(\infty,...) = \pm\infty$ corresponds to the spectrum with negative norm states (so called Landau pole singularities). For $Z(\infty, m, \mu, g) = \infty$ the Schwinger's equations (34) for renormalized fields are not well defined and our proof based on the use of Schwinger's equations does not work. So our results is another hint that the spectrum of the WZ model contains the states with negative norm, so called Landau poles. The calculations in leading log approximation confirm this hypothesis.

5.Conclusions

We considered QED and using 1/N expansion

and demonstrated that the ultraviolet behavior of photon propagator coincides with leading log approximation

As a consequence : negative norm states in

the spectrum

THE END

Another way to understand the inconsistency of the case $Z(\infty,...)=0$ is the following [6]. Take the differential operator in (34) to be $\frac{\delta}{\delta F_{r1}(x)}\frac{\delta}{\delta F^*_{r1}(x)}$. In the limit $\Lambda\to\infty$ of the regularization removing one can find [6] that

$$<0|T(m_r\phi_{r1}(x)+g_r\phi_{r2}(x)\phi_{r3}(x),m_r\phi^*_{r1}(y)+g_r\phi^*_{r2}(y)\phi^*_{r3}(y)|0>=0\,. \qquad (48)$$

Using the equation (43) one can find that

$$<0|T(m_r\phi_{r1}(x),m_r\phi^*_{r1}(y))|0>=<0|T(g_r\phi_{r2}(x)\phi_{r3}(x),g_r\phi^*_{r2}(y)\phi^*_{r3}(y)|0>\,. \qquad (49)$$

Due to the absence of radiative corrections to the superpotential (3) $m_{ij}\phi_i(x,\theta)\phi_j(x,\theta)=m_{rij}\phi_{ri}x,\theta)\phi_{rj}(x,\theta)$. As a consequence the composite renormalized operator $(\phi_i(x,\theta)\phi_j(x,\theta))_r=\phi_{ri}(x,\theta)\phi_{rj}(x,\theta)$ and the same equality $(\phi_i(x)\phi_j(x))_r=\phi_{ri}(x)\phi_{rj}(x)$ takes place for scalar fields $\phi_{ri}(x)$ and $\phi_{rj}(x)$. Using the KL representation for (50) one can find that the spectral densities satisfy the equation

$$m_r^2\rho_{\phi_{r1},\phi^*_{r1}}(t)=g_r^2\rho_{(\phi_1\phi_2)_r,(\phi^*_1\phi^*_2)_r)} \qquad (50)$$

and all terms in (50) are well defined within perturbation theory at sufficiently small renormalization coupling constant g_r. For small g_r we have $m_r^2 \rho_{\phi_{r1},\phi_{r1}^*}(t) \sim m_r^2 \delta(t - m_r^2)$ while the right hand side of the equation (50) is equal to $\frac{g_r^2}{16\pi^2}(1 - \frac{4m_r^2}{t})^{1/2} + O(g_r^4)$. So we find the contradiction. The same contradiction takes place for the equation (44).