AdS Harmonic Superspace

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Summary and outlook

Supersymmetry and superfields

- The concept of supersymmetry is in the heart of the modern mathematical and quantum physics. It allowed to construct a lot of new theories with remarkable features: supergravities, superstrings, superbranes, $\mathcal{N} = 4$ super Yang-Mills (the first example of the UV-finite quantum field theory), etc. It also exhibited unexpected relations between them, e.g., "gravity/gauge" duality.

- Supersymmetric theories are adequately formulated off shell in terms of superfields defined on various superspaces.

- The natural generalization of Minkowski space x^m to supersymmetry is $\mathcal{N}$ **extended Minkowski superspace**

$$\mathcal{M}^{(4|4\mathcal{N})} = \left(x^m\,,\ \theta_i^\alpha\,,\ \bar{\theta}^{\dot{\alpha}\,i}\right),\ i = 1, \ldots, \mathcal{N}$$

  where $\theta_i^\alpha\,, \bar{\theta}^{\dot{\alpha}\,i}$ are Grassmann coordinates, $\{\theta, \theta\}, \{\theta, \bar{\theta}\} = 0$.

- The basic $4D, \mathcal{N} = 1$ superspace is chiral superspace (x_L^m, θ^α), with half the original $\mathcal{N} = 1$ Grassmann coordinates. Its non-trivial $\mathcal{N} = 2$ analog is **Harmonic Superspace**.

Harmonic flat superspace

- In $4D$, the only self-consistent off-shell superfield formalism for $\mathcal{N}=2$ (and $\mathcal{N}=3$) theories is the harmonic superspace approach (Galperin, Ivanov, Kalitzin, Ogievetsky, Sokatchev, 1984, 1985).

- Harmonic $\mathcal{N}=2$ superspace:

$$Z=(x^m,\ \theta^\alpha_i,\ \bar\theta^{\dot\alpha\, j}, u^{\pm i}), \quad u^{\pm i}\in SU(2)/U(1),\ u^{+i}u^-_i=1$$

- Analytic harmonic $\mathcal{N}=2$ superspace:

$$\zeta_A=(x^m_A,\theta^{+\alpha},\bar\theta^{+\dot\alpha},u^{\pm i}),\ \theta^{+\alpha,\dot\alpha}:=\theta^{\alpha,\dot\alpha i}u^+_i,\ x^m_A:=x^m-2i\theta^{(i}\sigma^m\bar\theta^{j)}u^+_iu^+_j$$

- All basic $\mathcal{N}=2$ superfields are analytic:

$$\underline{\text{SYM}}: \quad V^{++}(\zeta_A),\ \underline{\text{matter hypermultiplets}}:\ q^+(\zeta_A),\ \bar q^+(\zeta_A)$$

$$\underline{\text{supergravity}}: \quad H^{++m}(\zeta_A),\ H^{++\hat\alpha+}(\zeta_A),\ H^{++5}(\zeta_A),\ \hat\alpha=(\alpha,\dot\alpha)$$

$\mathcal{N} = 2$ spin 1 multiplet

- An instructive example is Abelian $\mathcal{N} = 2$ gauge theory,

$$V^{++}(\zeta_A)\,, \quad \delta V^{++} = D^{++}\Lambda(\zeta_A)\,,\ D^{++} = \partial^{++} - 4i\theta^{+\alpha}\bar{\theta}^{+\dot{\alpha}}\partial_{\alpha\dot{\alpha}}$$

- Wess-Zumino gauge (8 + 8 off-shell degrees of freedom):

$$V^{++}(\zeta_A) = (\theta^+)^2\phi + (\bar{\theta}^+)^2\bar{\phi} + 2i\theta^{+\alpha}\bar{\theta}^{+\dot{\alpha}}A_{\alpha\dot{\alpha}}$$
$$+ (\bar{\theta}^+)^2\theta^{+\alpha}\psi^i_\alpha u^-_i + (\theta^+)^2\bar{\theta}^+_{\dot{\alpha}}\bar{\psi}^{\dot{\alpha}i}u^-_i + (\theta^+)^2(\bar{\theta}^+)^2 D^{(ik)}u^-_i\,u^-_k$$

- Invariant action:

$$S \sim \int d^{12}Z\ V^{++}V^{--}\,,\ D^{++}V^{--} - D^{--}V^{++} = 0\,,\ \delta V^{--} = D^{--}\Lambda\,,$$
$$[D^{++}, D^{--}] = D^0, \quad D^0\,V^{\pm\pm} = \pm 2\,V^{\pm\pm}$$

Supersymmetry and higher spins

- Free massless bosonic and fermionic higher spin field theories: Fronsdal, 1978; Fang, Fronsdal, 1978.
- The complete off-shell $\mathcal{N} = 1$ superfield Lagrangian formulation of $\mathcal{N} = 1, 4D$ free higher spins: Kuzenko et al, 1993, 1994.
- An off-shell superfield Lagrangian formulation for higher-spin **extended** supersymmetric theories, with all supersymmetries manifest, was unknown for long time even for free theories.
- This gap was filled in I. Buchbinder, E. Ivanov, N. Zaigraev, JHEP 12 (2021) 016. An off-shell manifestly $\mathcal{N} = 2$ supersymmetric unconstrained formulation of $4D, \mathcal{N} = 2$ super Fronsdal theory for integer spins was constructed in the harmonic superspace approach.
- Manifestly $\mathcal{N} = 2$ supersymmetric off-shell cubic couplings of $4D, \mathcal{N} = 2$ to the matter hypermultiplets were further constructed in I. Buchbinder, E. Ivanov, N. Zaigraev, 2022, 2023.
- Later on, we generalized HSS non-conformal construction to the case of $\mathcal{N} = 2$ superconformal multiplets and their hypermultiplet coupling (JHEP 08 (2024) 120).

Towards AdS background

- In view of the celebrated AdS/CFT correspondence, it is of high importance to explicitly construct $\mathcal{N}=2$ higher spins in the AdS background, with the superconformal symmetry $SU(2,2|2)$ being broken down to the AdS supersymmetry $OSp(2|4;R)$. Actually, even the $\mathcal{N}=2$ hypermultiplet self-couplings and couplings to gauge superfields on AdS_4 background were not manifestly constructed, as opposed to $\mathcal{N}=1$ AdS.

- One way is to start from the $AdS_4, \mathcal{N}=2$ superspace defined as the coset $OSp(2|4;R)/[SO(2)\times SL(2,C)]$, thus generalizing the $AdS_4, \mathcal{N}=1$ superfield approach by Ivanov & Sorin, 1979, 1980. This way was chosen by Kuzenko, Tartaglino-Mazzucchelli, 2008, based on the so called projective superspace techniques. No clear connection with the harmonic analyticity was established on this way.

- Our approach (Ivanov & Zaigraev, PLB 2025) proceeds from the analyticity-preserving realization of the superconformal symmetry in $\mathcal{N}=2$ HSS and identifies $\mathcal{N}=2$ AdS supersymmetry $OSp(2|4;R)$ as its subalgebra, $OSp(2|4;R)\subset SU(2,2|2)$. So the super AdS supersymmetry is already implied by the superconformal symmetry. Once again, the harmonic Grassmann analyticity plays the decisive role in our construction.

- Schematically, the (anti)commutation relations of the $\mathcal{N}=2,4D$ superconformal algebra $su(2,2|2)$ look as follows

$$[K,P]\propto L+D,\ [D,P]\propto P\,,\ [D,K]\propto K\,,$$
$$\{Q,Q\}\propto P,\{S,S\}\propto K,[Q,K]\propto S\,,[S,P]\propto Q\,,[Q,P]=[S,K]=0\,,$$
$$\{Q,S\}\propto L+D+I+R\,,[D,S]\propto S\,,[D,Q]\propto Q\,,[I,Q]\propto Q\,,[I,S]\propto S,$$

  where P,K,D,L,R,I are the bosonic generators of translations, conformal boosts, dilatations, Lorentz transformations, internal R- and $SU(2)_c$ symmetries, while Q and S are generators of the Poincaré and special conformal $\mathcal{N}=2,4D$ supersymmetries.

- The standard $\mathcal{N}=2$ superspace can be defined as the supercoset

$$(x^m,\theta^\alpha_i,\bar\theta^{\dot\alpha i})=\frac{SU(2,2|2)}{\{L,K,D,S,R,I\}}$$

- How to define the relevant HSS and to find (if exists) the appropriate analytic subspace? The answer was found in A. Galperin, E. Ivanov, V. Ogievetsky, E. Sokatchev, 1985. The idea was to extend $SU(2,2|2)$ by an extra automorphism $SU(2)_A$ which uniformly rotates all isodoublet indices of the $SU(2,2|2)$ generators and to introduce the harmonic variables $u^{\pm i}$ just on this $SU(2)_A\propto\{T^{(ik)}\}$. Some combination of $u^{\pm i}$ and group parameters of $SU(2)_c$ are just the new harmonics $w^{+i},w^{-i}=u^{-i}$ needed to define the correct analytic HSS.

- So we start with the semidirect product,

$$\widehat{SU}(2,2|2) = SU(2,2|2) \rtimes SU(2)_A,$$

and identify the appropriate analytic subspace with the supercoset

$$(x^m_{an}, \theta^{+\alpha}, \bar{\theta}^{+\dot{\alpha}}, w^+, w^-) = \frac{\widehat{SU}(2,2|2)}{\{L, K, D, S, R, I^{++}, I^0, (I^{--} - T^{--})\}}$$

- The complete set of the analyticity-preserving superconformal transformations of the coordinates can be found by the standard coset routine, from the left shifts on this supercoset. The $\mathcal{N} = 2, 4D$ SCA is a closure of the rigid $\mathcal{N} = 2$ supersymmetry and special conformal symmetry

$$\delta_\epsilon \theta^{+\hat{\alpha}} = \epsilon^{\hat{\alpha} i} w^+_i \,,\ \delta_\epsilon x^{\alpha\dot{\alpha}} = -4i \left(\epsilon^{\alpha i} \bar{\theta}^{+\dot{\alpha}} + \theta^{+\alpha} \bar{\epsilon}^{\dot{\alpha} i} \right) w^-_i \,, \hat{\alpha} = (\alpha, \dot{\alpha}) \,,$$

$$\delta_k \theta^{+\alpha} = x^{\alpha\dot{\beta}} k_{\beta\dot{\beta}} \theta^{+\beta} \,,\ \delta_k x^{\alpha\dot{\alpha}} = x^{\rho\dot{\alpha}} k_{\rho\dot{\rho}} x^{\dot{\rho}\alpha} \,,\ \delta_k w^{+i} = (4i\theta^{+\alpha} \bar{\theta}^{+\dot{\alpha}} k_{\alpha\dot{\alpha}}) w^{-i}$$

- So, in order to check the superconformal invariance of some object it is enough to check invariance under these selected transformations.

- The next step towards the AdS_4 HSS is to single out $\mathcal{N}=2,4D$ AdS supergroup $OSp(4|2,R)$ from $\widehat{SU}(2,2|2)$
- The embedding of the $\mathcal{N}=2$ AdS superalgebra into $SU(2,2|2)$ is realized through the identification (Gates, Grisaru, Rocek, Siegel, 1983; Bandos, Ivanov, Lukierski, Sorokin, 2002)

$$\Psi^i_\alpha = Q^i_\alpha + c^{ik} S_{k\alpha}, \quad \bar\Psi^i_{\dot\alpha} = \overline{\Psi^i_\alpha} = \bar Q_{\dot\alpha i} + c_{ik}\bar S^k_{\dot\alpha},$$
$$c^{ik} = c^{ki} \qquad \overline{c^{ik}} = c_{ik} = \varepsilon_{il}\varepsilon_{kj}c^{lj}$$

- The $SU(2,2|2)$ commutation relations imply for super AdS generators

$$\{\Psi^i_\alpha, \Psi^k_\beta\} = -4c^{ik}L_{(\alpha\beta)} + 4i\varepsilon_{\alpha\beta}\varepsilon^{ik} I, \quad I := c_{lm}T^{lm}, \quad [I,\Psi^i_\alpha] = c^{ik}\Psi_{k\alpha},$$
$$\{\Psi^i_\alpha, \bar\Psi_{\dot\beta k}\} = 4\delta^i_k R_{\alpha\dot\beta}, \quad R_{\alpha\dot\beta} = P_{\alpha\dot\beta} + \frac{1}{2}c^2 K_{\alpha\dot\beta}, \quad c^2 := c^{ik}c_{ik} \sim \frac{1}{R^2_{AdS}},$$
$$[R_{\alpha\dot\alpha}, R_{\gamma\dot\gamma}] = \frac{1}{2}c^2\left(\varepsilon_{\alpha\gamma}\bar L_{\dot\alpha\dot\gamma} + \varepsilon_{\dot\alpha\dot\gamma}L_{\alpha\gamma}\right), \quad [R_{\alpha\dot\beta}, \Psi^i_\beta] \sim \varepsilon_{\alpha\beta}\,\bar\Psi^i_{\dot\beta} \text{ (and c.c.)}$$

- These relations are recognized as those of $\mathcal{N}=2,4D$ AdS supergroup $OSp(2|4;R)$. Choosing the special $SU(2)_A$ frame $c^{ik}=\delta^{ik}m$, we recover the standard form of the $osp(2|4;R)$ relations, with $SO(2)$ as the internal symmetry group.

Realization on analytic coordinates

- The **super AdS** transformation:

$$\delta_\epsilon x^{\alpha\dot\alpha} = -4i\left[\epsilon^{\alpha i}\bar\theta^{+\dot\alpha} + \theta^{+\alpha}\bar\epsilon^{\dot\alpha i} - c^{ik}\left(x^{\alpha\dot\beta}\bar\epsilon_{\dot\beta k}\bar\theta^{+\dot\alpha} + x^{\beta\dot\alpha}\epsilon_{\beta k}\theta^{+\alpha}\right)w_i^-\right],$$
$$\delta_\epsilon \theta^{+\alpha} = \left(\epsilon^{\alpha i} - x^{\alpha\dot\alpha}c^{ik}\bar\epsilon_{\dot\alpha k}\right)w_i^+ - 2i(\theta^+)^2 c^{ki}\epsilon_k^\alpha w_i^-\,,$$
$$\delta_\epsilon \bar\theta^{+\dot\alpha} = \left(\bar\epsilon^{\dot\alpha i} + x^{\alpha\dot\alpha}c^{ik}\epsilon_{\alpha k}\right)w_i^+ + 2i(\bar\theta^+)^2 c^{ik}\bar\epsilon_k^{\dot\alpha} w_i^-\,,$$
$$\delta_\epsilon w^{+i} = -4i\left[c^{kl}w_k^+(\epsilon_{\alpha l}\theta^{+\alpha} + \bar\epsilon_{\dot\alpha l}\bar\theta^{+\dot\alpha})\right]w^{-i}\,.$$

These transformations follow from the coordinate superconformal ones upon identifying the conformal and Poincaré supersymmetry infinitesimal parameters as $\eta^i_{\hat\alpha} = c^{ik}\epsilon_{\hat\alpha k}$.

- **Nonlinear AdS** translations:

$$\delta_a x^{\alpha\dot\alpha} = a^{\alpha\dot\alpha} + \frac{1}{2}c^2 a_{\beta\dot\beta}x^{\alpha\dot\beta}x^{\beta\dot\alpha} = a^{\alpha\dot\alpha}\left(1 - \frac{1}{4}c^2x^2\right) + \frac{1}{2}c^2(ax)x^{\alpha\dot\alpha}\,,$$
$$\delta_a\theta^{+\alpha} = \frac{1}{2}c^2 a_{\beta\dot\beta}x^{\alpha\dot\beta}\theta^{+\beta} = \frac{1}{4}c^2(ax)\theta^{+\alpha} + \frac{1}{2}c^2 x^{(\alpha\dot\beta}a_{\beta)\dot\beta}\theta^{+\beta}\,,$$
$$\delta_a\bar\theta^{+\dot\alpha} = \frac{1}{2}c^2 a_{\beta\dot\beta}x^{\beta\dot\alpha}\bar\theta^{+\dot\beta} = \frac{1}{4}c^2(ax)\bar\theta^{+\dot\alpha} + \frac{1}{2}c^2 x^{\beta(\dot\alpha}a_{\beta\dot\beta)}\bar\theta^{+\dot\beta}\,,$$
$$\delta_a w^{+i} = 2i\left(c^2 a_{\alpha\dot\alpha}\theta^{+\alpha}\bar\theta^{+\dot\alpha}\right)w^{-i}\,,\quad c^2 = c^{ik}c_{ik} := 2m^2\,.$$

Hypermultiplet couplings

- The construction of interactions in the theory of higher spins is a very important (albeit difficult) task. There is an extensive literature related to the construction of cubic higher spin interactions (e.g., Bengtsson et al, 1983; Fradkin, Metsaev, 1991; Metsaev, 1993; Manvelyan, Mkrtchyan, Ruehl, 2010, 2011, and many others).
- In Buchbinder, Ivanov, Zaigraev, JHEP 05 (2022) 104 we have constructed the off-shell manifestly $\mathcal{N}=2$ supersymmetric cubic couplings $(\frac{1}{2},\frac{1}{2},\mathbf{s})$ of an arbitrary higher integer superspin $\mathbf{s}$ gauge $\mathcal{N}=2$ multiplet to the hypermultiplet matter in $4D,\mathcal{N}=2$ harmonic superspace.
- In our approach $\mathcal{N}=2$ supersymmetry of cubic vertices is always manifest and off-shell, in contrast, e.g., to the non-manifest light-cone formulations (Metsaev, 1905.11357, 1909.05241).
- The harmonic analyticity-preserving locally superconformal couplings of the hypermultiplets to $\mathcal{N}=2,4D$ higher-spin gauge superfields was given in (Buchbinder, Ivanov, Zaigraev, JHEP 08 (2024) 120) Rigid $\mathcal{N}=2,4D$ superconformal symmetry played a crucial role in fixing the structure of the theory.
- It was of big interest to generalize all this to the AdS case and, first of all, to the hypermultiplet as the basic $\mathcal{N}=2,4D$ matter multiplet.

AdS Hypermultiplet

- The starting point is the $\mathcal{N}=2$ hypermultiplet off-shell free action:

$$S=\int d\zeta^{(-4)}\ \mathcal{L}^{+4}_{free}=-\int d\zeta^{(-4)}\ \frac{1}{2}q^{+a}\mathcal{D}^{++}q^{+}_{a}\,,\ a=1,2$$

- It is superconformally invariant up to total derivative, if

$$\delta_{sc}q^{+a}=-\hat{\Lambda}_{sc}q^{+a}-\frac{1}{2}\Omega q^{+a}\,,\quad \Omega:=(-1)^{P(M)}\partial_{M}\lambda^{M}\,,$$
$$\hat{\Lambda}_{sc}:=\lambda^{\alpha\dot{\alpha}}_{sc}\partial_{\alpha\dot{\alpha}}+\lambda^{\hat{\alpha}+}_{sc}\partial_{\hat{\alpha}+}+\lambda^{++}_{sc}\partial^{--}\,,$$

  where $\lambda^{M},\ M=(\alpha\dot{\beta},\hat{\alpha},++)$, are just the infinitesimal coordinate transformations.

- Since the free q^{+} action written above is invariant under the $\mathcal{N}=2$ superconformal transformations it is obviously invariant under the $\mathcal{N}=2$ super AdS ones.

- To see this, we write the AdS SUSY transformation (its holomorphic part) of the analytic integration measure:

$$\delta_\epsilon d\zeta^{(-4)} = 8i \left(c^{kl} u_k^- \epsilon_{\alpha l} \theta^{+\alpha} \right) d\zeta^{(-4)}$$

- To compensate this variation of the integration measure in the action of q^{+a}, the latter should include the appropriate weight factor in its transformation

$$\delta_\epsilon q^{+a} = -4i \left(c^{kl} u_k^- \epsilon_{\alpha l} \theta^{+\alpha} \right) q^{+a}.$$

- The harmonic derivative $\mathcal{D}^{++}$ transforms under AdS SUSY as

$$\delta_\epsilon D^{++} = 4i\, c^{kl} w_k^+ \epsilon_{\alpha l} \theta^{+\alpha}\, D^0\,, \quad D^0 q^{+a} = q^{+a},\ q^{+a} q_a^+ = 0$$

- So the free q^+ action is indeed invariant under the $\mathcal{N} = 2$ super AdS symmetry as a consequence of its $\mathcal{N} = 2$ superconformal invariance. The question is how to construct the actions invariant ONLY under $OSp(2|4; R)$ symmetry. To accomplish this, one needs to properly break $SU(2,2|2)$.

Two ways of deriving $OSp(2|4;R)$ invariants

- We start with the $\mathcal{N}=2$ AdS invariant mass term. The basic idea is to extend the coordinate action of generators of AdS supersymmetry by extra "matrix" pieces of an external $SO(2)$ realized on q^{+a}. In the flat limit such modified $\mathcal{N}=2$ AdS contracts into the central-charge extended $4D, \mathcal{N}=2$ supersymmetry.
- Defining formally this extra $SO(2)$ generator as ∂_5 and introducing

$$q^+(\zeta, x^5) = e^{-im_q x^5} q^+(\zeta), \quad \tilde{q}^+(\zeta, x^5) = e^{im_q x^5} \tilde{q}^+(\zeta), \quad [m_q] = l^{-1},$$

we have found the corresponding realization of $OSp(2|4;R)$ by the following Killing super-vector

$$\hat{\Lambda}_{AdS} = \lambda^{\alpha\dot{\alpha}}_{AdS}\partial_{\alpha\dot{\alpha}} + \lambda^{\hat{\alpha}+}_{AdS}\partial^-_{\hat{\alpha}} + \lambda^{++}_{AdS}\partial^{--} + \lambda^5_{AdS}\partial_5, \quad \lambda^5_{AdS} = \lambda^5_\epsilon + \lambda^5_{so(2)},$$

$$\lambda^5_\epsilon = 2ie(x)\Big[(\theta^+\epsilon^-) - x^{\alpha\dot{\alpha}}\bar{\theta}^+_{\dot{\alpha}}(\epsilon^+_\alpha c^{--} - \epsilon^-_\alpha y)\Big] \times$$

$$\times\Big\{1 - 2i\left[(\theta^+)^2 - (\bar{\theta}^+)^2\right] e(x) c^{--}\Big\} + (c.c.),$$

$$\lambda^5_{so(2)} = \frac{\gamma}{2} + i\left[(\theta^+)^2 - (\bar{\theta}^+)^2\right]\gamma c^{--} e(x) - 6(\theta^+)^4\gamma\left(c^{--}\right)^2 e(x)^2,$$

where $y = c^{ij} w^+_i u^-_j$, $e(x) = \frac{1}{1+m^2x^2/2}$.

- In order to construct invariant q^+ action, one needs to lengthen $\mathcal{D}^{++}$ as

$$\mathcal{D}^{++} \;\Rightarrow\; \mathcal{D}^{++} + H^{++5}_{AdS}\partial_5\,,$$
$$H^{++5}_{AdS} = i\left[(\theta^+)^2 - (\bar\theta^+)^2\right] e(x) - 6(\theta^+)^4 e(x)^2\, c^{--}\,,\ \delta_{AdS} H^{++5}_{AdS} = \mathcal{D}^{++}\lambda^5_{AdS}$$

- After elimination of infinite tail of the auxiliary fields and integrating over Grassmann and harmonic variables we obtain the bosonic part of the massive AdS hypermultiplet as

$$S_{scalar} = -\frac{1}{2}\int d\zeta^{(-4)}\, q^{+a}\left(\mathcal{D}^{++} + H^{++5}_{AdS}\partial_5\right) q^+_a\ |_{bos}$$
$$= \int d^4x\left(\partial_n f^i \partial^n \bar f_i - m_q^2 e(x)^2 f^i \bar f_i - i m_q e(x)^2 f^i \bar f^j c_{(ij)}\right).$$

  where we observe the explicit $SU(2)$ breaking part in the mass term, with the residual $SO(2)$ invariance.

- After rescaling $f^i \to e(x)\hat f^i$ we obtain AdS covariant scalar field

$$\delta_a \hat f^i(x) = 0 \qquad (1)$$

  and gain the standard kinetic action for the massive scalars given on AdS metric $g_{mn} = e(x)^2\eta_{mn}$ (with the scalar curvature $R = 4\Lambda = -48 c^{ik}c_{ik}$):

$$S_{scalar} = \int d^4x\sqrt{-g}\left(g^{mn}\partial_m \hat f^i \partial_n \bar{\hat f}_i + \frac{R}{6}\hat f^i \bar{\hat f}_i - m_q^2 \hat f^i \bar{\hat f}_i - i m_q \hat f^i \bar{\hat f}^j c_{(ij)}\right)$$

- After passing to the special frame $c_{ij} = m\delta_{ij}$, diagonalizing mass matrix and rewriting $\hat{f}^i = (f_+, f_-)$, we obtain the splitting of mass for the scalar fields proportional to the parameter m,

$$m_{\pm}^2 = m_q^2 \pm m m_q$$

- For the fermionic fields we need to make a similar redefinition, $\psi_\alpha(x) \to e(x)^{\frac{3}{2}} \hat{\psi}_\alpha(x)$ and, using the AdS covariant derivative $\nabla_{\alpha\dot{\alpha}} = \left(1 + \frac{m^2}{2} x^2\right) \partial_{\alpha\dot{\alpha}} = e(x)^{-1} \partial_{\alpha\dot{\alpha}}$, we obtain the action for Dirac spinor on AdS space:

$$S_{fer} = \int d^4x \sqrt{-g} \Big\{ i\bar{\hat{\psi}}^{\dot{\alpha}} \left(\nabla_{\dot{\alpha}}^{\alpha} - \frac{3}{2} m^2 x_{\dot{\alpha}}^{\alpha} \right) \hat{\psi}_\alpha + i\bar{\hat{\kappa}}^{\dot{\alpha}} \left(\nabla_{\dot{\alpha}}^{\alpha} - \frac{3}{2} m^2 x_{\dot{\alpha}}^{\alpha} \right) \hat{\kappa}_\alpha + \frac{m_q}{2} \left(\hat{\psi}^\alpha \hat{\kappa}_\alpha + \bar{\hat{\psi}}_{\dot{\alpha}} \bar{\hat{\kappa}}^{\dot{\alpha}} \right) \Big\}. \tag{2}$$

So, $m_f = m_q$, $\quad m_+^2 + m_-^2 = 2m_f^2$, like in $\mathcal{N} = 1$ AdS case (Ivanov & Sorin, 1980).

Superfield Weyl transformation

- In order to get a more general class of $\mathcal{N}=2$ AdS invariant systems we need to redefine the superfield q^{+a} in such a way that it transforms as a scalar superfield with zero weight under the AdS subgroup of $\mathcal{N}=2$ superconformal group. This is the superfield Weyl-type transformation preserving $\mathcal{N}=2$ harmonic analyticity.
- We start from the free q^+ action, $S_{free} = -\frac{1}{2}\int d\zeta^{(-4)} q^{+a} \mathcal{D}^{++} q^+_a$. It is superconformally invariant and hence invariant under super $\mathcal{N}=2$ super AdS_4 group. Then we make Weyl-type rescaling of q^+,

$$q^{+a} = G^{\frac{1}{2}}\,\hat{q}^{+a}\,,\ G = \frac{\left(1+\frac{y^2}{m^2}\right)}{\left(1+\frac{m^2x^2}{2}\right)^2}\Big(1+\theta\ terms\Big),\ y := c^{+-} = c^{ik}w^+_i u^-_k\ ,$$

so that $\hat{q}^{+a}$ is a scalar under the $\mathcal{N}=2$ super AdS_4 group. The $\hat{q}^+$ action takes the form manifestly invariant under this group

$$S_{free} = -\frac{1}{2}\int d\zeta^{(-4)}\, G\, \hat{q}^{+a}\mathcal{D}^{++}\hat{q}^+_a\ ,\quad \delta_{AdS}\hat{q}^{+a} = 0\,,$$

- The new integration measure $d\zeta^{(-4)}G$ is invariant under $OSp(2|4;R)$. So one can add to the Lagrangian any proper function of $\hat{q}^{+a}$ without breaking of $OSp(2|4;R)$.

- In particular, one can add an arbitrary $\mathcal{L}^{+4}(\hat{q}^{+a}, u^-)$ and so gain a wide class of the hyper-Kähler type sigma model actions on the AdS_4 background. To be more precise, one can consider the following action

$$S^{AdS}_{HK} = \int d\zeta^{(-4)} G \left[\hat{q}^+_a \mathcal{D}^{++} \hat{q}^{+a} + L^{(+4)}(\hat{q}^+, \hat{w}^+, u^-)\right],$$

  where

$$\hat{w}^{+i} := w^{+i} - u^{-i} c^{++} \frac{y}{y^2 + m^2}, \qquad \delta_{AdS} \hat{w}^{+i} = 0,$$

  is a new harmonic variable inert under $OSp(2|4)$.

- In the properly defined flat limit this action goes over just to the general action of hyper-Kähler $\mathcal{N} = 2$ sigma models. It generically does not exhibit any external isometry (besides the intrinsic $SO(2)$), quite similar to its flat $\mathcal{N} = 2$ counterparts. It would be interesting to find out the relevant deformation of the hyper-Kähler geometry and the role of the breaking parameter c^{ik} in it.

Summary and outlook

The theory of $\mathcal{N} = 2$ supersymmetric higher spins $s \geq 3$ recently opened a new promising direction of applications of the harmonic superspace approach which earlier proved to be indispensable for description of more conventional $\mathcal{N} = 2$ theories with maximal spins $s \leq 2$. Once again, the basic property underlying these new higher-spin theories **is the harmonic Grassmann analyticity** (all basic gauge potentials are unconstrained analytic superfields involving an infinite number of degrees of freedom off shell before fixing WZ-type gauges). As we saw on the example of hypermultiplet, HSS admits a nice generalization to the case of AdS background where **the harmonic Grassmann analyticity principle** also plays the decisive role. We expect that the same principle will nicely work as well for various $\mathcal{N} = 2$ AdS supergravities and allow us to get further insights into the geometric and quantum structure of these notable theories. One of the most interesting problems here is to construct AdS analogs of analytic prepotenials of diverse $\mathcal{N} = 2$ supergravities on the flat background and to find the higher-spin counterparts of these AdS analytic prepotentials.

Some further related issues and applications will be subject of the afternoon talk by Nikita Zaigraev.

THANK YOU FOR ATTENTION!