Irregular higher-spin generating equations

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Higher-spin problem

$$S_{HS} = \int \sum_s \phi_s \Box_F \phi_s + \sum_{s_1,s_2,s_3} a_{s_1 s_2 s_3} D..D\phi_{s_1} D..D\phi_{s_2} D..D\phi_{s_3} + O(\phi^4)$$

Quadratic action

Fronsdal, Fang and Fronsdal:

$$\delta\phi_{m_1..m_s} = D_{(m_1}\epsilon_{m_2..m_s)}\,, \qquad g^{mn}g^{kl}\phi_{mnkl...} = 0$$

Cubic action

(Bengtsson)[2], Brink; Berends, Burgers, van Dam; Metsaev; Fradkin, Vasiliev; many more

$$s_1 + s_2 + s_3 - 2\min(s_i) \leq N(D) \leq s_1 + s_2 + s_3$$

No go: Minkowski$\rightarrow$ AdS$\rightarrow$ higher-spin symmetry

Higher-spin holography

Klebanov, Polyakov; (Sezgin, Sundell; Leigh, Petkou) [Flato, Fronsdal, 1978]

$$S_{HS}\Big|_{z\to 0} = O(N)\text{-model}$$

3pt test (Giombi, Yin): OK
Maldacena, Zhiboedov:

$$\langle J_{s_1} J_{s_2} J_{s_3}\rangle = \sin^2\phi\langle J_{s_1} J_{s_2} J_{s_3}\rangle_b + \cos^2\phi\langle J_{s_1} J_{s_2} J_{s_3}\rangle_f + \frac{1}{2}\sin 2\phi\langle J_{s_1} J_{s_2} J_{s_3}\rangle_o$$

In the bulk (Tatarenko, Vasiliev)
Holographic reconstruction (Sleight, Taronna):

$$\langle J_{s_1} J_{s_2} J_{s_3}\rangle \to S^{(3)}_{HS}$$

Locality problem

Quartic reconstruction

Bekaert, Erdmenger, Ponomarev, Sleight:

$$\langle J_{s_1} J_{s_2} J_{s_3} J_{s_4} \rangle \to S^{(4)}_{HS} \quad \text{(nonlocal)}$$

Sleight, Taronna: A type of the nonlocality is

$$\frac{1}{\Box} \quad (?)$$

Gelfond: Direct calculation from Vasiliev's equations

(moderate nonlocality)

Locality: state of the art

- HS theory in four dimensions

$$HS_4 = \text{hol} \oplus \text{ahol} \oplus \text{mixed}$$

  Hol: local up to quite higher orders (V.D., Gelfond, Korybut, Vasiliev)
- Hol: generating equations; all-order locality (V.D.)
- Mixed: Moderate nonlocality at quartic order (Gelfond)
- Bosonic HS theory in any dimension: generating equations; all-order locality at the unconstrained level (V.D., Korybut, Povarnin)

Vasiliev's unfolded approach

Stick to the frame-like formalism instead of the metric-like one:

$$\phi_{a_1..a_s} \quad \to \quad \begin{cases} \omega(Y|x) = dx^\mu \omega_\mu(Y|x) & \text{– potentials} \\ C(Y|x) & \text{– curvatures} \end{cases} ,$$

$Y_A = (y_\alpha, \bar{y}_{\dot\alpha})$ are the auxiliary generating variables

Nonlinear HS equations of motion (L_∞-maps)

$$d\omega = -\omega * \omega + \mathcal{V}(\omega,\omega,C) + ... + \mathcal{V}(\omega,\omega,C..C)\,,$$
$$dC = -\omega * C + C * \pi(\omega) + \Upsilon(\omega,C,C) + ... + \Upsilon(\omega,C..C)\,.$$

Consistency

$$d^2 = 0 \quad \to \quad \mathcal{V}(\omega,\omega,C..C)\,, \quad \Upsilon(\omega,C..C) \quad \text{(Vertices)}$$
$$\omega \to \omega + f_\omega(\omega, C..C)\,, \quad C \to C + f_C(C..C) \quad \text{(field redefinition)}$$

- $*$ – associative HS algebra (spanned by Y's)

$$f(Y) * g(Y) = f(Y) e^{i \overleftarrow{\partial_Y} \cdot \overrightarrow{\partial_Y}} g(Y)$$

- Lower order vertices

$$\mathcal{V}(\Omega, \Omega, C) \quad \to \quad \text{Free equations} \qquad (\Omega\text{-vacuum})$$
$$\omega * \omega\,, \quad \omega * C - C * \pi(\omega)\,, \quad \mathcal{V}(\Omega, \omega, C)\,, \quad \Upsilon(\Omega, C, C) \quad \to \quad \text{Cubic int.}$$

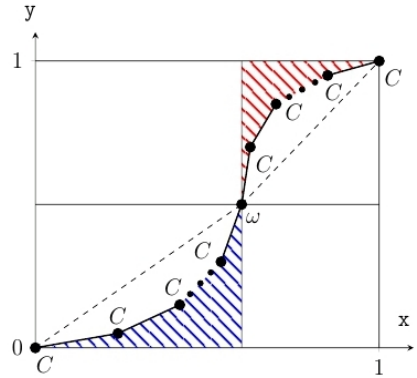
- Example:

$$\Upsilon(\omega, C^n) = (-i\eta)^{n-1} \sum_{k=0}^{n} (-)^{k+1} \int_{\mathcal{D}_n^{[k]}} d\xi d\eta \int \frac{d^2u d^2v}{(2\pi)^2}\, e^{iuv} \times$$
$$\times \left(\prod_{i=1}^{k} \star\, C(\xi_i y + \eta_i v, \bar{y}) \right) \star (y^\alpha \partial_\alpha)^{n-1} \omega(S_n^{[k]} y + u, \bar{y}) \star \left(\prod_{j=k+1}^{n} \star\, C(\xi_j y + \eta_j v, \bar{y}) \right)$$

Geometric interpretation Van Dongen, et al; VD, Povarnin

ω in the k-th position:

$$\mathcal{D}_n^{[k]} = \begin{cases} \eta_i\xi_{i+1} - \eta_{i+1}\xi_i \leqslant 0\,, & i \in [1\,,\, k-1]\,, \\ \eta_i\xi_{i+1} - \eta_{i+1}\xi_i \geqslant 0\,, & i \in [k+1\,,\, n-1]\,. \end{cases} \qquad \frac{1}{2}S_n^k = \text{Area}–\text{Area}$$



Vasiliev equations

$$d_x\omega = \sum_{n=0}^{\infty} \sum_{i+j=n} \eta^i \bar{\eta}^j \mathcal{V}_{i,j}(\omega, \omega, C^n)\,, \tag{1}$$

$$d_x C = \sum_{n=0}^{\infty} \sum_{i+j=n} \eta^i \bar{\eta}^j \Upsilon_{i,j}(\omega, C^{n+1}) \tag{2}$$

$$\pi f(y, \bar{y}) = f(-y, \bar{y})\,, \quad \mathcal{V}_{0,0} = -\omega * \omega\,, \quad \Upsilon_{0,0} = -\omega * C + C * \pi(\omega)$$

Vasiliev's trick:

$$\omega \to W(Z; Y|x) := \omega(Y|x) + W_1(Z; Y|x) + W_2(Z; Y|x) + \dots\,,$$

$$d_x W + W * W = 0$$

star product is extended to (Z, Y) – space

Vasiliev's system: regular case

star product:

$$(f * g)(Z;Y) = \int_{U,V} f\,(Z+U, Y+V)\, g\,(Z-V, Y+V)\, e^{iu_A v^A}\,,$$

Equations:

$$d_x W + W * W = 0\,,$$
$$d_Z W + \{W, \Lambda\}_* + d_x \Lambda = 0; \qquad \Lambda = dZ^A \Lambda_A[C](Z,Y)$$

Consistency: $d_Z^2 = 0$

$$d_Z \Lambda + \Lambda * \Lambda = i\eta\, dz^\alpha dz_\alpha\, B * \kappa + h.c.\,, \quad \kappa = e^{izy}$$
$$d_x B + W * B - B * \pi(W) = 0\,,$$
$$d_Z B + \Lambda * B - B * \pi(\Lambda) = 0\,.$$

Irregular system: $\Lambda * \Lambda = \infty$

Higher-spin locality

Effective star product: VD, Gelfond, Korybut, Vasiliev, '19

$$(f * g)(Z;Y) = \int f(Z+U', Y+V)\, g(Z-V, Y+V+V')\, e^{iU_A V^A + iU'_A V'^A},$$

$$Y_A * = Y_A + i\frac{\partial}{\partial Y^A} - i\frac{\partial}{\partial Z^A}, \qquad * Y_A = Y_A - i\frac{\partial}{\partial Y^A} - i\frac{\partial}{\partial Z^A},$$

$$Z_A * = Z_A + i\frac{\partial}{\partial Y^A} - \cancel{i\frac{\partial}{\partial Z^A}}, \qquad * Z_A = Z_A + i\frac{\partial}{\partial Y^A} + \cancel{i\frac{\partial}{\partial Z^A}}$$

Regularity problem

$$\Lambda[C] * \Lambda[C] = \infty, \qquad \forall C(Y|x), \quad \Lambda_1 = dz^\alpha z_\alpha \int_0^1 d\tau\, \tau C(-\tau z, \bar{y}) e^{i\tau zy}$$

Irregular system

Generating equations:

$$d_x W + W * W = 0\,,$$
$$d_Z W + \{W, \Lambda\}_* + d_x \Lambda = 0; \qquad \Lambda = dZ^A \Lambda_A[C](Z, Y)$$
$$d_Z^2 = ?$$

Need no constraint for $W(Z)$

$$d_Z \{W, \Lambda\}_* - d_x d_Z \Lambda = 0$$

Proper Z-evolution of Λ (Vasiliev, 92)/Projecting out the Z-dependence (VD, 2022)

Functional class

Functional class: $\mathbf{C}^{r_1,r_2}(Z;Y) = \mathbf{C}^{r_1}(z;Y) \otimes \bar{\mathbf{C}}^{r_2}(\bar{z};Y) \qquad (dz)^{r_1} \wedge (d\bar{z})^{r_2}$

General requirements:

$$W \in \mathbf{C}^{0,0} \quad \to \quad \omega(Y|x) \in \mathbf{C}^{0,0}$$

$$\mathbf{C}^r + \mathbf{C}^r \in \mathbf{C}^r\,, \quad r = 0,1,2$$

$$d_z \mathbf{C}^r = \mathbf{C}^{r+1}$$

$$\mathbf{C}^{r_1} * \mathbf{C}^{r_2} = \mathbf{C}^{r_1+r_2}\,, \quad r_1 + r_2 < 2$$

$$H^0(d_z) * \mathbf{C}^2 \in \mathbf{C}^2$$

Source prescription

$$\phi(z,y) = e^{izB+iyA}\phi(z',y')\Big|_{z'=y'=0}\,, \qquad B_\alpha = i\partial^{z'}_\alpha\,, \quad A_\alpha = i\partial^{y'}_\alpha\,.$$

$$\mathbf{C}^r[e^{iyA+izB}] = \int_0^1 d\tau \left(\frac{1-\tau}{\tau}\right)^{1-r} e^{i\tau z(y+B)+i(1-\tau)yA+i\tau AB}$$

Projector identities

Supertrace projector

$$\operatorname{str} f(z,y) := \int_{u,v} f(u,v) e^{-iuv}\,, \qquad \operatorname{str}(f * g) = \operatorname{str}(g * \pi(f))$$

$$\mathbf{h}_a f(z,y) = \operatorname{str} f(z+a,y)$$

Identities

$$d_z(\mathbf{C}^0 * \mathbf{C}^1[\phi]) = \mathbf{h}'_y[\mathbf{C}^0(-z',y) * \mathbf{h}_y \phi(-z, y+y')] * \gamma$$

$$d_z(\mathbf{C}^1[\phi] * \mathbf{C}^0) = \mathbf{h}'_y[\mathbf{h}_y \phi(-z, y+y') * \mathbf{C}^0(-z',-y)] * \gamma$$

$$\gamma = \frac{1}{2} dz_\alpha \wedge dz^\alpha \exp izy\,, \qquad \phi(z,y) = dz^\alpha z_\alpha \psi(z,y)$$

Building up irregular system

Initial data and equations

$$W \in \mathbf{C}^{0,0}\,, \qquad \Lambda[\phi] = dz^\alpha \Lambda_\alpha \in \mathbf{C}^{1,0}\,, \qquad \bar{\Lambda}[\bar{\phi}] = d\bar{z}^{\dot{\alpha}} \bar{\Lambda}_{\dot{\alpha}} \in \mathbf{C}^{0,1}$$

$$\begin{aligned} d_x W + W * W &= 0 \\ d_z W + \{W, \Lambda\}_* + d_x \Lambda &= 0 \\ d_{\bar{z}} W + \{W, \bar{\Lambda}\}_* + d_x \bar{\Lambda} &= 0 \end{aligned}$$

Consistency: $d_x^2 = \{d_x, d_z\} = \{d_x, d_{\bar{z}}\} = 0 \quad \rightarrow \quad$ OK

$$d_z \bar{\Lambda} + d_{\bar{z}} \Lambda + \{\Lambda, \bar{\Lambda}\}_* = 0$$

Consistency: $\{d_z, d_{\bar{z}}\} = 0 \quad \rightarrow \quad$ OK

Consistency consequences: $d_z^2 = d_{\bar{z}}^2 = 0$

$$
\begin{aligned}
& d_z \Lambda = i\eta\, \bar{\mathcal{B}}(\bar{z};\, Y) * \gamma \\
& d_z \mathcal{B} + \bar{\mathbf{h}}'_{\bar{y}} [\Lambda(z, -\bar{z}';\, Y) * \mathcal{B}_{\bar{y}'} - \mathcal{B}_{\bar{y}'} * \bar{\pi} \Lambda(z, -\bar{z}';\, Y)] = 0 \\
& d_x \mathcal{B} + \bar{\mathbf{h}}'_{\bar{y}} \left[W(z, -\bar{z}'; Y) * \mathcal{B}_{\bar{y}'} - \mathcal{B}_{\bar{y}'} * \bar{\pi} W(z, -\bar{z}';\, Y) \right] = 0
\end{aligned}
$$

Definitions

$$
\begin{aligned}
i\bar{\eta}\, \mathcal{B}(z;\, Y) &:= \bar{\mathbf{h}}_{\bar{y}} \bar{\phi}(z, -\bar{z};\, y, \bar{y})\,, & i\eta\, \bar{\mathcal{B}}(\bar{z};\, Y) &:= \mathbf{h}_y \phi(-z, \bar{z};\, y, \bar{y}) \\
i\bar{\eta}\, \mathcal{B}_{\bar{y}'}(z;\, Y) &:= \bar{\mathbf{h}}_{\bar{y}} \bar{\phi}(z, -\bar{z};\, y, \bar{y} + \bar{y}')\,, & i\eta\, \bar{\mathcal{B}}_{y'}(\bar{z};\, Y) &:= \mathbf{h}_y \phi(-z, \bar{z};\, y + y', \bar{y})
\end{aligned}
$$

b-constraint

$$
\mathbf{H}'_Y \left[\mathcal{B}_{\bar{y}'}(-z';\, Y) * \bar{\pi} \bar{\mathcal{B}}_{y'}(-\bar{z}';\, Y) - \bar{\mathcal{B}}_{y'}(-\bar{z}';\, Y) * \pi \mathcal{B}_{\bar{y}'}(-z';\, Y) \right] = 0
$$

Holomorphic sector: $\bar{\Lambda} = 0$

Generating equations

$$\bar{\eta} = 0 \quad \rightarrow \quad \mathcal{B} = 0$$

b-constraint is trivially satisfied

No further constraints for $\Lambda[\phi(z, y)]$. The simplest option: $\phi(z, y) := C(-z)$

$$d_x W + W * W = 0$$
$$d_z W + \{W, \Lambda\}_* + d_x \Lambda = 0$$
$$d_x C + (W(z'; Y) * C - C * \pi_y W(z'; Y))\Big|_{z'=-y} = 0$$

VD, 22'

Mixed sector: $\eta\bar{\eta} \neq 0$

Perturbation theory

$$\mathcal{B} = C + \mathcal{B}_2[C,C] + \dots\,, \qquad \bar{\mathcal{B}} = C + \bar{\mathcal{B}}_2[C,C] + \dots$$
$$\Lambda = \mathbf{\Lambda}[C] + \Lambda_2[C,C] + \dots\,, \qquad \bar{\Lambda} = \bar{\mathbf{\Lambda}}[C] + \bar{\Lambda}_2[C,C] + \dots$$

$$\mathbf{\Lambda} = i\eta\, dz^\alpha z_\alpha \int_0^1 d\tau\tau C(-\tau z, \bar{y}) e^{i\tau zy}\,, \qquad \bar{\mathbf{\Lambda}} = i\bar{\eta}\, d\bar{z}^{\dot\alpha} \bar{z}_{\dot\alpha} \int_0^1 d\tau\tau C(y, -\tau\bar{z}) e^{i\tau\bar{z}\bar{y}}\,.$$

$$\Lambda = \sum_{i\geq 1, j\geq 0} \eta^i \bar{\eta}^j \Lambda_{i,j}\,, \qquad \bar{\Lambda} = \sum_{i\geq 0, j\geq 1} \eta^i \bar{\eta}^j \bar{\Lambda}_{i,j}\,, \quad \mathbf{\Lambda} := \eta\Lambda_{1,0}\,, \quad \bar{\mathbf{\Lambda}} = \bar{\eta}\bar{\Lambda}_{0,1}$$

$$\Lambda_{i,0} = \bar{\Lambda}_{0,i} = 0 \qquad \text{for} \qquad i > 1$$

b-constriant $\rightarrow$ chiral perturbation theory

$$\mathbf{H}'_Y \left[\mathcal{B}_{\bar{y}'}(-z';Y) * \bar{\pi}\bar{\mathcal{B}}_{y'}(-\bar{z}';Y) - \bar{\mathcal{B}}_{y'}(-\bar{z}';Y) * \pi\mathcal{B}_{\bar{y}'}(-z';Y)\right] = 0$$

$$d_{\bar{z}}\Lambda + d_z\bar{\Lambda} + \{\Lambda,\bar{\Lambda}\}_* = 0 \qquad \text{Solvable?}$$

Order C^2 $\rightarrow$ $\eta^n\bar{\eta}\,\Upsilon_{n,1}(\omega, C^{n+1})$

$$C * \bar{\pi}C - C * \pi C = 0 \quad \rightarrow \quad C(y,-\bar{y}) = C(-y,\bar{y}) \quad \rightarrow \quad \text{bosonic}$$

$$\Lambda = \mathbf{\Lambda} + \eta\bar{\eta}\Lambda_{1,1} + \ldots \qquad \bar{\Lambda} = \bar{\mathbf{\Lambda}} + \eta\bar{\eta}\bar{\Lambda}_{1,1} + \ldots$$

Order C^3 $\rightarrow$ $\eta^n\bar{\eta}^2\,\Upsilon_{n,1}(\omega, C^{n+2})$

$$\left(\mathcal{B}_2(z;Y) * \bar{\pi}C - C * \pi\mathcal{B}_2(z;Y)\right)\Big|_{z=-y} = 0 \quad \rightarrow \quad \text{satisfied}$$

$$\Lambda = \mathbf{\Lambda} + \eta\bar{\eta}\Lambda_{1,1} + \eta\bar{\eta}^2\Lambda_{1,2} + \eta^2\bar{\eta}\Lambda_{2,1}\ldots \qquad \bar{\Lambda} = \bar{\mathbf{\Lambda}} + \eta\bar{\eta}\Lambda_{1,1} + \eta\bar{\eta}^2\bar{\Lambda}_{1,2} + \eta^2\bar{\eta}\bar{\Lambda}_{2,1}\ldots$$

Conclusion

- Irregular type generating equations are proposed for addressing HS interaction problem
- The system should be viewed as complementary to the classical Vasiliev equations. Both are designed to extract HS vertices, but unlike Vasiliev equations, the consistency of our system is not obvious
- When consistency is achieved, the system becomes very powerful. For example, all-order self-dual vertices are found in this way
- For mixed interaction, consistency is checked for $N^{k=0,1,2}LO$

The End