$\mathcal{N} = 2$ superconformal higher-spin theories in the harmonic superspace

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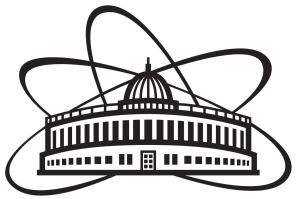

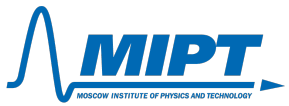

- Harmonic superspace [Galperin, Ivanov, Kalitsyn, Ogievetsky, Sokatchev 1984] is the universal method to deal with off-shell $\mathcal{N}=2$ supersymmetry theories:

$$\mathbb{HR}^{4+2|8}=\mathbb{R}^{4|8}\times S^2=\{x^m,\theta^{\alpha i},\bar{\theta}^{\dot{\alpha} i},u_i^{\pm}\}.$$

- $\mathcal{N}=2$ higher-spin theories have the natural geometric formulation in terms of unconstrained analytic prepotentials [Buchbinder, Ivanov, N.Z. 2021-2024]:

$$\left\{h^{++\alpha(s-1)\dot{\alpha}(s-1)},h^{++\alpha(s-1)\dot{\alpha}(s-2)+},h^{++\alpha(s-2)\dot{\alpha}(s-1)+},h^{++\alpha(s-2)\dot{\alpha}(s-2)}\right\}.$$

- These prepotentials are the higher-spin generalization of the linearized (over flat $\mathcal{N}=2$ harmonic superspace) $\mathcal{N}=2$ "minimal" Einstein supergravity [Fradkin, Vasiliev 1979], [de Wit, van Holten 1979] (obtained using $\mathcal{N}=2$ vector and non-linear $\mathcal{N}=2$ tensor compensators).
- These higher-spin prepotentials naturally couple to the hypermultiplet:

$$\mathcal{D}^{++}\quad\rightarrow\quad\mathcal{D}^{++}+\kappa_s h^{++\alpha(s-2)\dot{\alpha}(s-2)M}\partial_M\partial^{s-2}_{\alpha(s-2)\dot{\alpha}(s-2)}(J)^{P(s)}.$$

$\mathcal{N} = 2$ superconformal higher-spin multiplets

Hypermultiplet in harmonic superspace

- Hypermultiplet is the unique $\mathcal{N}=2$ matter multiplet.
- On-shell hypermultiplet contain doublet of complex scalars $f^i(x)$ and a pair of singlet spinors $\psi_\alpha(x), \kappa_\alpha(x)$.
- A finite set of auxiliary fields does not exist for hypermultiplet ("no-go theorems" in extended SUSY).
- **Hypermultiplet** in HSS is described by an unconstrained analytic superfield $q^+(\zeta)$. It contains a doublet of complex scalars f^i and a pair of singlet spinors $\psi_\alpha, \kappa_\alpha$, as well as an infinite set of auxiliary fields which comes from the harmonic S^2 expansions:

$$q^+(\zeta) = f^i u_i^+ + \theta^{+\alpha}\psi_\alpha + \bar\theta^+_{\dot\alpha}\bar\kappa^{\dot\alpha} + \text{auxiliary fields}.$$

- The free hypermultiplet action has the form:

$$S_{free} = -\int d\zeta^{(-4)}\tilde q^+ \mathcal{D}^{++} q^+ = -\frac{1}{2}\int d\zeta^{(-4)}\, q^{+a}\mathcal{D}^{++} q^+_a .$$

  Here $q^{+a} = (\tilde q^+, q^+)$, $q^+_a = \epsilon_{ab}q^{+b} = (q^+, -\tilde q^+)$.

- Equation of motion $\mathcal{D}^{++}q^+ = 0$ allows to exclude all auxiliary fields and it lead to the free Klein-Gordon and Weyl equations: $\Box f^i = 0$, $\partial^{\alpha\dot\alpha}\bar\psi_{\dot\alpha} = 0$, $\partial^{\alpha\dot\alpha}\bar\kappa_{\dot\alpha} = 0$.

$\mathcal{N}=2$ superconformal symmetry in harmonic superspace

- $\mathcal{N}=2$ superconformal symmetry is realized on hypermultiplet as:

$$\delta q^{+a} = -\hat{\Lambda} q^{+a} - \frac{1}{2}\Omega q^{+a},$$

where

$$\hat{\Lambda} := \lambda^M \partial_M = \lambda^{\alpha\dot{\alpha}}\partial_{\alpha\dot{\alpha}} + \lambda^{+\hat{\alpha}}\partial^-_{\hat{\alpha}} + \lambda^{++}\partial^{--},$$
$$\Omega := (-1)^{P(M)}\partial_M \lambda^M = \partial_{\alpha\dot{\alpha}}\lambda^{\alpha\dot{\alpha}} - \partial^-_{\hat{\alpha}}\lambda^{+\hat{\alpha}} + \partial^{--}\lambda^{++}.$$

- This is the most general one-derivative transformations of hypermultiplet. If parameters satisfy equation

$$[\mathcal{D}^{++}, \hat{\Lambda}] = \lambda^{++}\mathcal{D}^0,$$

the free hypermultiplet action is invariant.

- The most general solution of this equation are $\mathcal{N}=2$ superconformal transformations:

$$\begin{cases}
\lambda^{\alpha\dot{\alpha}}_{sc} = & a^{\alpha\dot{\alpha}} - 4i\left(\epsilon^{\alpha i}\bar{\theta}^{+\dot{\alpha}} + \theta^{+\alpha}\bar{\epsilon}^{\dot{\alpha} i}\right) u^-_i + x^{\dot{\alpha}\rho} k_{\rho\dot{\rho}} x^{\dot{\rho}\alpha} + b x^{\alpha\dot{\alpha}} \\
& -4i\theta^{+\alpha}\bar{\theta}^{+\dot{\alpha}}\lambda^{(ij)}u^-_i u^-_j - 4i\left(x^{\alpha\dot{\rho}}\eta^i_{\dot{\rho}}\bar{\theta}^{+\dot{\alpha}} + \theta^{+\alpha}\eta^i_{\rho}x^{\rho\dot{\alpha}}\right)u^-_i, \\
\lambda^{+\alpha}_{sc} = & \epsilon^{\alpha i}u^+_i + \frac{1}{2}\theta^{+\alpha}(b+i\gamma) + x^{\alpha\dot{\beta}}k_{\beta\dot{\beta}}\theta^{+\beta} + x^{\alpha\dot{\alpha}}\eta^i_{\dot{\alpha}}u^+_i \\
& +\theta^{+\alpha}\left(\lambda^{(ij)}u^+_i u^-_j + 4i\theta^{+\rho}\eta^i_\rho u^-_i\right), \\
\bar{\lambda}^{+\dot{\alpha}}_{sc} = & \epsilon^{\dot{\alpha} i}u^+_i + \frac{1}{2}\bar{\theta}^{+\dot{\alpha}}(b-i\gamma) + x^{\dot{\alpha}\beta}k_{\beta\dot{\beta}}\bar{\theta}^{+\dot{\beta}} + x^{\alpha\dot{\alpha}}\eta^i_{\alpha}u^+_i \\
& +\bar{\theta}^{+\dot{\alpha}}\left(\lambda^{(ij)}u^+_i u^-_j - 4i\bar{\theta}^{+\dot{\rho}}\eta^i_{\dot{\rho}}u^-_i\right), \\
\lambda^{++}_{sc} = & \lambda^{ij}u^+_i u^+_j + 4i\theta^{+\alpha}\bar{\theta}^{+\dot{\alpha}}k_{\alpha\dot{\alpha}} + 4i\left(\theta^{+\alpha}\eta^i_\alpha + \eta^i_{\dot{\alpha}}\bar{\theta}^{+\dot{\alpha}}\right)u^+_i.
\end{cases}$$

$\mathcal{N}=2$ Weyl multiplet

- $\mathcal{N}=2$ conformal supergravity can be obtained by gauging $\mathcal{N}=2$ superconformal group [Galperin, Ivanov, Ogievetsky, Sokatchev 1987].
- Supergravity prepotentials are analytic superfields and appear as vielbeins in covariant analytic harmonic derivative $\mathcal{D}^{++}$:

$$\mathcal{D}^{++} \quad \rightarrow \quad \mathfrak{D}^{++} = \mathcal{D}^{++} + \kappa_2 \hat{\mathcal{H}}^{++}_{s=2},$$

$$\hat{\mathcal{H}}^{++}_{s=2} = h^{++\beta\dot{\beta}}\partial_{\beta\dot{\beta}} + h^{++\beta+}\partial^-_{\beta} + h^{++\dot{\beta}+}\partial^-_{\dot{\beta}} + h^{(+4)}\partial^{--},$$

$$\delta^{full}_{\lambda}\hat{\mathcal{H}}^{++}_{(s=2)} = [\mathcal{D}^{++} + \kappa_2\hat{\mathcal{H}}^{++}_{(s=2)}, \hat{\Lambda}_{(s=2)}].$$

- Component content of $\mathcal{N}=2$ Weyl multiplet:

$$\begin{cases} h^{++\alpha\dot{\alpha}} = -4i\theta^+_\rho\bar{\theta}^+_{\dot{\rho}}\Phi^{(\alpha\rho)(\dot{\alpha}\dot{\rho})} - (\bar{\theta}^+)^2\theta^+_\rho\psi^{(\alpha\rho)\dot{\alpha}i}u^-_i + (\theta^+)^2\bar{\theta}^+_{\dot{\rho}}\bar{\psi}^{\alpha(\dot{\alpha}\dot{\rho})i}u^-_i \\ \qquad\qquad + (\theta^+)^4 V^{\alpha\dot{\alpha}(ij)}u^-_i u^-_j, \\ h^{++\alpha+} = (\theta^+)^2\bar{\theta}^+_{\dot{\alpha}}P^{\alpha\dot{\alpha}} + (\bar{\theta}^+)^2\theta^+_\beta T^{(\alpha\beta)} + (\theta^+)^4\chi^{\alpha i}u^-_i, \\ h^{++\dot{\mu}+} = \widetilde{h^{++\mu+}}, \\ h^{(+4)} = (\theta^+)^4 D. \end{cases}$$

- $\mathbf{32_B + 32_F}$ off-shell degrees of freedom.

Superconformal cubic vertex

- The general form of the cubic vertex of hypermultiplet with higher spin superfields has form:

$$S^{(s)}_{sc-cubic,s} = -\frac{\kappa_s}{2}\int d\zeta^{(-4)}\, q^{+a} h^{++M_1\dots M_{s-1}} \partial_{M_{s-1}}\dots\partial_{M_1} (J)^{P(s)}\, q^+_a$$
$$+ \text{lower derivative terms}\,,$$

  Here we denote:

$$\partial_M := \left(\partial_{\alpha\dot\alpha}, \partial^-_\alpha, \partial^-_{\dot\alpha}, \partial^{--}\right), \quad J\tilde q^+ = i\tilde q^+, \quad Jq^+ = -iq^+, \qquad P(s) := \frac{1-(-1)^s}{2}.$$

- Superconformal invariance of the cubic interaction requires inclusion of all possible combinations of derivatives and presence of terms with $s-1$, $s-3$, ... derivatives:

$$S_{free} + S_{sc-cubic,s} = -\frac{1}{2}\int d\zeta^{(-4)}\, q^{+a}\left(\mathcal{D}^{++} + \hat{\mathbb{H}}^{++}_{(s)}(J)^{P(s)}\right) q^+_a,$$

  where we introduced the analytic differential operator of degree $s-1$ with the odd number of superspace derivatives:

$$\hat{\mathbb{H}}^{++}_{(s)} := h^{++M_1\dots M_{s-1}}\partial_{M_{s-1}}\dots\partial_{M_1} + h^{++M_1\dots M_{s-3}}\partial_{M_{s-3}}\dots\partial_{M_1} + \dots.$$

- $\mathcal{N}=2$ superconformal group is linearly realized on $h^{++M_1\dots}$ superfields:

$$\delta_{sc}\hat{\mathbb{H}}^{++}_{(s)} = [\hat{\mathbb{H}}^{++}_{(s)}, \hat\Lambda] + \frac{1}{2}[\hat{\mathbb{H}}^{++}_{(s)}, \Omega].$$

Gauge freedom

- Total hypermultiplet action with the higher-spin cubic coupling is:

$$S_{free} + S_{sc-cubic,s} = -\frac{1}{2}\int d\zeta^{(-4)}\, q^{+a}\left(\mathcal{D}^{++} + \hat{\mathbb{H}}^{++}_{(s)}(J)^{P(s)}\right) q^{+}_{a}.$$

- Constructed action has enormous gauge freedom ($k = s, s-2, \dots$):

$$\delta^{(s,k)}_{\lambda} q^{+}_{a} = -\frac{\kappa_s}{2}\left\{\hat{\Lambda}^{M_1\dots M_{k-2}} + \frac{1}{2}\Omega^{M_1\dots M_{k-2}}, \partial_{M_{k-2}}\dots\partial_{M_1}\right\}_{AGB}(J)^{P(s)} q^{+}_{a},$$

$$\delta^{(s,k)}_{\lambda}\hat{\mathbb{H}}^{++}_{(s)} = \frac{1}{2}\left[\mathcal{D}^{++}, \left\{\hat{\Lambda}^{M_1\dots M_{k-2}}, \partial_{M_{k-2}}\dots\partial_{M_1}\right\}_{AGB}\right],$$

$$\hat{\Lambda}^{M_1\dots M_{k-2}} := \lambda^{M_1\dots M_{k-2}N}\partial_N, \qquad \Omega^{M_1\dots M_{k-2}} := (-1)^{P(N)}\partial_N\lambda^{NM_1\dots M_{k-2}}.$$

- The different values of k correspond to the gauge freedom for different spin contributions appearing in the superspace operator $\hat{\mathbb{H}}^{++}_{(s)}$.
- It is also possible to extend cubic coupling to arbitrary $\mathcal{N} = 2$ conformal supergravity background:

$$\mathcal{D}^{++} \quad \to \quad \mathfrak{D}^{++}.$$

Superconformal higher-spin multiplet: Wess-Zumino gauge

- Using this gauge freedom, one can impose the Wess-Zumino-type gauge:

$$
\begin{aligned}
h^{++\alpha(s-1)\dot\alpha(s-1)}_{WZ} &= -4i\theta^+_\rho\bar\theta^+_{\dot\rho}\Phi^{(\rho\alpha(s-1))(\dot\rho\dot\alpha(s-1))} - (\bar\theta^+)^2\theta^+_\rho\psi^{(\rho\alpha(s-1))\dot\alpha(s-1)i}u^-_i \\
&\quad - (\theta^+)^2\bar\theta^+_{\dot\rho}\bar\psi^{\alpha(s-1)(\dot\alpha(s-1)\dot\rho)i}u^-_i + (\theta^+)^4 V^{\alpha(s-1)\dot\alpha(s-1)ij}u^-_i u^-_j\,, \\
h^{++\alpha(s-1)\dot\alpha(s-2)+}_{WZ} &= (\theta^+)^2\bar\theta^+_{\dot\nu}P^{\alpha(s-1)(\dot\alpha(s-2)\dot\nu)} + (\bar\theta^+)^2\theta^+_\nu T^{(\alpha(s-1)\nu)\dot\alpha(s-2)} \\
&\quad + (\theta^+)^4\chi^{\alpha(s-1)\dot\alpha(s-2)i}u^-_i\,, \\
h^{++\alpha(s-2)\dot\alpha(s-1)+}_{WZ} &= h^{++\alpha\widetilde{(s-1)\dot\alpha}(s-2)+}_{WZ}\,, \\
h^{(+4)\alpha(s-2)\dot\alpha(s-2)}_{WZ} &= (\theta^+)^4 D^{\alpha(s-2)\dot\alpha(s-2)}\,.
\end{aligned}
$$

- All other potentials are purely gauge degrees of freedom. In the gauge that we have fixed, they are set to zero.
- In principle, it is possible to initially fix the gauge in which these fields are zero, but in this case the superconformal group will be implemented non-linearly. This situation is typical for the conformal higher-spin theories, see, e.g., [Kuzenko, Ponds, Raptakis 2022].

Gauge freedom of components

• Bosonic fields $\Phi^{\alpha(s)\dot\alpha(s)}$, $V^{\alpha(s-1)\dot\alpha(s-1)ij}$, $P^{(\alpha(s-1)\dot\alpha(s-1))}$, $D^{\alpha(s-2)\dot\alpha(s-2)}$ are conformal Fradkin-Tseytlin fields [Fradkin, Tseytlin 1985] with gauge freedom of the form:

$$\delta\Phi^{\alpha(s)\dot\alpha(s)} \sim \partial^{(\alpha(\dot\alpha}a^{\alpha(s-1))\dot\alpha(s-1))}, \quad \delta V^{\alpha(s-1)\dot\alpha(s-1)ij} \sim \partial^{(\alpha(\dot\alpha}v^{\alpha(s-2))\dot\alpha(s-2))(ij)}, \ldots.$$

• Complex field $T^{\alpha(s)\dot\alpha(s-2)}$ have the gauge freedom:

$$\delta T^{\alpha(s)\dot\alpha(s-2)} \sim \partial^{(\alpha(\dot\alpha}t^{\alpha(s-1))\dot\alpha(s-3))}.$$

This is more general type of the conformal field. In the simplest non-trivial $s=3$ case, these fields are equivalent to the traceless hook gauge field $T^{a[bc]}$ with the algebraic symmetries corresponding to the simplest hook Young diagram $\begin{array}{|c|c|}\hline & \\ \hline \\ \cline{1-1}\end{array}$.

• Fields $\psi^{\alpha(s)\dot\alpha(s-1)i}$ and $\chi^{\alpha(s-1)\dot\alpha(s-2)i}$ are conformal fermionic gauge fields:

$$\delta\psi^{\alpha(s)\dot\alpha(s-1)i} \sim \partial^{(\alpha(\dot\alpha}\xi^{\alpha(s-1))\dot\alpha(s-2))i}, \quad \delta\chi^{\alpha(s-1)\dot\alpha(s-2)i} \sim \partial^{\alpha\dot\alpha}\zeta^{\alpha(s-2))\dot\alpha(s-3))i}.$$

• Finally, the $\mathcal{N}=2$ superconformal spin **s** multiplet is encompassed by $\mathbf{8(2s-1)_B + 8(2s-1)_F}$ off-shell degrees of freedom.

• For $s \geq 3$ all field are gauge fields, so there are no auxiliary fields!

• The simplest $s=1,2$ multiplets (in contrast to $s \geq 3$) contain auxiliary fields and comprise $\mathcal{N}=2$ Maxwell and $\mathcal{N}=2$ conformal supergravity (Weyl) gauge multiplets.

Non-abelian deformation of gauge freedom

$$\hat{\mathbb{H}}^{++} := \sum_{s=1}^{\infty} \kappa_s \hat{\mathbb{H}}^{++}_{(s)} (J)^{P(s)}.$$

- Action of infinite tower of integer $\mathcal{N} = 2$ superconformal higher spins on arbitrary $\mathcal{N} = 2$ conformal supergravity background is:

$$S_{full} = -\frac{1}{2} \int d\zeta^{(-4)}\, q^{+a} \left(\mathfrak{D}^{++} + \hat{\mathbb{H}}^{++} \right) q^{+}_{a}.$$

- Then by proper transformation of $\hat{\mathbb{H}}^{++}$, one can obtain the gauge invariance in any order in couplings constant.
- We denote sum of hypermultiplet higher-spin gauge transformations for all spins as

$$\delta_\lambda q^{+a} := -\hat{\mathcal{U}} q^{+a} = -\sum_{s=1}^{\infty} \kappa_s \hat{\mathcal{U}}_s q^{+a}.$$

- For the tower of gauge fields we obtain the non-abelian transformation law:

$$\delta_\lambda \hat{\mathbb{H}}^{++} = -\left[\mathfrak{D}^{++} + \hat{\mathbb{H}}^{++}, \hat{\mathcal{U}} \right].$$

This transformations act linearly on the hypermultiplet and mix superfields with different spins.

$\mathcal{N} = 2$ superconformal higher-spin action

- Harmonic derivative in the basis of covariant derivatives:

$$\mathfrak{D}^{++} = \mathcal{D}^{++} + G^{++\alpha\dot{\alpha}}\partial_{\alpha\dot{\alpha}} + G^{++\hat{\alpha}-}\mathcal{D}^{+}_{\hat{\alpha}} + G^{++\hat{\alpha}+}\mathcal{D}^{-}_{\hat{\alpha}} + G^{(+4)}\mathcal{D}^{--}.$$

  Here G^{++}-vielbeins are related to the analytic h^{++}-vielbeins by relations:

$$\begin{aligned}
G^{++\alpha\dot{\alpha}} &= h^{++\alpha\dot{\alpha}} + 4ih^{++\alpha+}\bar{\theta}^{-\dot{\alpha}} + 4i\theta^{-\alpha}h^{++\dot{\alpha}+} - 4ih^{(+4)}\theta^{-\alpha}\bar{\theta}^{-\dot{\alpha}},\\
G^{++\hat{\alpha}-} &= h^{++\hat{\alpha}-},\\
G^{++\hat{\alpha}+} &= -h^{++\hat{\alpha}+} + h^{(+4)}\theta^{-\hat{\alpha}},\\
G^{(+4)} &= h^{(+4)}.
\end{aligned}$$

- The main feature of G^{++}-superfields is their manifest invariance under rigid $\mathcal{N} = 2$ supersymmetry.

- The linearized gauge transformations of G^{++}-superfields read

$$\delta_\lambda G^{++\alpha\dot\alpha} = \mathcal{D}^{++}\Lambda^{\alpha\dot\alpha},$$
$$\delta_\lambda G^{++\hat\alpha-} = \mathcal{D}^{++}\Lambda^{-\hat\alpha} + \Lambda^{+\hat\alpha},$$
$$\delta_\lambda G^{++\hat\alpha+} = \mathcal{D}^{++}\Lambda^{+\hat\alpha},$$
$$\delta_\lambda G^{(+4)} = \mathcal{D}^{++}\Lambda^{++},$$

  where the gauge parameters are defined through the expansion of the first-rank superdifferential operator $\hat\Lambda$ over covariant derivatives:

$$\Lambda^{\alpha\dot\alpha} = \lambda^{\alpha\dot\alpha} + 4i\lambda^{\alpha+}\bar\theta^{-\dot\alpha} + 4i\theta^{-\alpha}\lambda^{+\dot\alpha} - 4i\lambda^{++}\theta^{-\alpha}\bar\theta^{-\dot\alpha},$$
$$\Lambda^{-\hat\alpha} = \lambda^{-\hat\alpha},$$
$$\Lambda^{+\hat\alpha} = -\lambda^{+\hat\alpha} + \lambda^{++}\theta^{-\hat\alpha},$$
$$\Lambda^{++} = \lambda^{++}.$$

- By defining the appropriate zero-curvature equations, one can construct the superfields with negative harmonic charges:

$$\mathcal{D}^{++}G^{--\alpha\dot\alpha} = \mathcal{D}^{--}G^{++\alpha\dot\alpha},$$
$$\mathcal{D}^{++}G^{--\hat\alpha+} = \mathcal{D}^{--}G^{++\hat\alpha+},$$
$$\mathcal{D}^{++}G^{--\hat\alpha-} = G^{--\hat\alpha+}.$$

- Let us consider the complex dimensionless superfield:

$$\mathcal{H}^{++}_{(\alpha\beta)} = \partial^{\dot\beta}_{(\alpha} G^{++}_{\beta)\dot\beta} + \mathcal{D}^{-}_{(\alpha} G^{+++}_{\beta)},$$

- which satisfies the "half-analyticity" condition:

$$\bar{\mathcal{D}}^{+}_{\dot\rho} \mathcal{H}^{++}_{(\alpha\beta)} = 0.$$

  This condition determines the relative coefficient .

- The superfield $\mathcal{H}^{++}_{(\alpha\beta)}$ has a simple gauge transformation law:

$$\delta_\lambda \mathcal{H}^{++}_{(\alpha\beta)} = \mathcal{D}^{++}\Lambda_{(\alpha\beta)} + \underbrace{\mathcal{D}^{+}_{(\alpha}\Lambda^{+}_{\beta)}}_{0}, \qquad \Lambda_{(\alpha\beta)} = \partial^{\dot\beta}_{(\alpha}\Lambda_{\beta)\dot\beta} + \mathcal{D}^{-}_{(\alpha}\Lambda^{+}_{\beta)},$$

  where the composite gauge parameter $\Lambda_{(\alpha\beta)}$ satisfies the half-analyticity condition $\bar{\mathcal{D}}^{+}_{\dot\rho}\Lambda_{(\alpha\beta)} = 0$.

- Using the zero-curvature equation

$$\mathcal{D}^{++}\mathcal{H}^{--}_{(\alpha\beta)} = \mathcal{D}^{--}\mathcal{H}^{++}_{(\alpha\beta)}\,,$$

  we define a superfield with the negative charge,

$$\mathcal{H}^{--}_{(\alpha\beta)} = \partial^{\dot\beta}_{(\alpha} G^{--}_{\beta)\dot\beta} + \mathcal{D}^{-}_{(\alpha} G^{--+}_{\beta)} - \mathcal{D}^{+}_{(\alpha} G^{---}_{\beta)},$$

  subject to the gauge transformation law $\delta_\lambda \mathcal{H}^{--}_{(\alpha\beta)} = \mathcal{D}^{--}\Lambda_{(\alpha\beta)}$.

- Using these superfields one can construct the action:

$$S_{Weyl} = \int d^4x d^8\theta du\, \mathcal{H}^{++(\alpha\beta)}\mathcal{H}^{--}_{(\alpha\beta)} + \text{c.c.}\,.$$

- Gauge invariance of this action can be easily checked:

$$\begin{aligned}\delta_\lambda S_{Weyl} &= \int d^4x d^8\theta du\, \left\{\mathcal{D}^{++}\Lambda^{(\alpha\beta)}\mathcal{H}^{--}_{(\alpha\beta)} + \mathcal{H}^{++(\alpha\beta)}\mathcal{D}^{--}\Lambda_{(\alpha\beta)}\right\} \\ &= 2\int d^4x d^8\theta du\, \mathcal{H}^{++(\alpha\beta)}\mathcal{D}^{--}\Lambda_{(\alpha\beta)} \\ &= \frac{1}{8}\int d^4x d^4\theta du\, \left(\bar{\mathcal{D}}^-\right)^2\left\{\mathcal{H}^{++(\alpha\beta)}\underbrace{\left(\bar{\mathcal{D}}^+\right)^2\left[\mathcal{D}^{--}\Lambda_{(\alpha\beta)}\right]}_{0}\right\} = 0\end{aligned}$$

- Thus the invariance of the action is ensured by the half-analyticity condition. This is analogous to what happens in $\mathcal{N}=2$ Maxwell theory, where the invariance is secured due to the analyticity condition.

Chiral representation

- Constructed action admits a chiral representation. Using $\mathcal{H}^{--}_{(\alpha\beta)}$ one can construct the gauge-invariant supertensor
$$\mathcal{W}_{(\alpha\beta)} = (\bar{\mathcal{D}}^+)^2 \mathcal{H}^{--}_{(\alpha\beta)}.$$
Its gauge invariance is a consequence of the half-analyticity condition.
- In components, this supertensor contains the linearized Weyl tensor
$$\mathcal{W}_{(\alpha\beta)} \sim \theta^{+\gamma}\theta^{-\delta} C_{(\alpha\beta\gamma\delta)} + \dots, \qquad C_{(\alpha\beta\gamma\delta)} = \partial^{\dot\alpha}_{(\alpha}\partial^{\dot\beta}_{\beta}\Phi_{\gamma\delta)(\dot\alpha\dot\beta)}.$$
Therefore we can identify $\mathcal{W}_{(\alpha\beta)}$ with the holomorphic part of linearized $\mathcal{N}=2$ super Weyl tensor.
- As another consequence of the half-analyticity, $\mathcal{W}_{(\alpha\beta)}$ is $\mathcal{N}=2$ chiral superfield. Moreover, $\mathcal{N}=2$ super Weyl tensor satisfies Bianchi identity:
$$(\mathcal{D}^{+\alpha}\mathcal{D}^{-\beta})\mathcal{W}_{(\alpha\beta)} = (\bar{\mathcal{D}}^{+\dot\alpha}\bar{\mathcal{D}}^{-\dot\beta})\overline{\mathcal{W}}_{(\dot\alpha\dot\beta)}.$$
- Action in chiral representation
$$S_{Weyl} = \int d^4x d^8\theta du\, \mathcal{H}^{++(\alpha\beta)}\mathcal{H}^{--}_{(\alpha\beta)} + \text{c.c.} = \frac{1}{16}\int d^4x_L d^4\theta\, \mathcal{W}^{(\alpha\beta)}\mathcal{W}_{(\alpha\beta)} + \text{c.c.}\,.$$
In components, this action contains the square of the linearized Weyl tensor, so it indeed describes the linearized $\mathcal{N}=2$ conformal supergravity.

Spin **3** example

- Superconformal spin **3** supermultiplet is described by analytic differential operator:

$$\begin{aligned}\hat{\mathcal{H}}^{++}_{(s=3)} = {} & h^{++\alpha\dot\alpha\beta\dot\beta}\partial_{\beta\dot\beta}\partial_{\alpha\dot\alpha} + h^{++\hat\alpha\beta\dot\beta+}\partial_{\beta\dot\beta}\partial^-_{\hat\alpha} + h^{++\beta\dot\beta++}\partial_{\beta\dot\beta}\partial^{--} \\ & + h^{++\alpha+\beta+}\partial^-_\beta\partial^-_\alpha + h^{++\dot\alpha+\dot\beta+}\partial^-_{\dot\beta}\partial^-_{\dot\alpha} + h^{++\alpha+\dot\beta+}\partial^-_{\dot\beta}\partial^-_\alpha \\ & + h^{++\hat\alpha+++}\partial^{--}\partial^-_{\hat\alpha} + h^{(+6)}\partial^{--}\partial^{--} + h^{++},\end{aligned}$$

  where $h^{++\cdots}$ are unconstrained analytical superfields.

- Gauge transformations of analytical prepotentials are given by:

$$\begin{aligned}\delta_\lambda h^{++\alpha\dot\alpha\beta\dot\beta} = {} & \mathcal{D}^{++}\lambda^{\alpha\beta\dot\alpha\dot\beta} + 2i\left(\lambda^{\alpha\dot\alpha\beta+}\bar\theta^{+\dot\beta} + \lambda^{\beta\dot\beta\alpha+}\bar\theta^{+\dot\alpha}\right) \\ & - 2i\left(\bar\lambda^{\alpha\dot\alpha\dot\beta+}\theta^{+\beta} + \bar\lambda^{\beta\dot\beta\dot\alpha+}\theta^{+\alpha}\right), \\ \delta_\lambda h^{++\alpha\dot\alpha\beta+} = {} & \mathcal{D}^{++}\lambda^{\alpha\dot\alpha\beta+} - 8i\lambda^{[\alpha+\beta]+}\bar\theta^{+\dot\alpha} - 8i\lambda^{\beta+\dot\alpha+}\theta^{+\alpha} - \lambda^{\alpha\dot\alpha++}\theta^{+\beta}, \\ \delta_\lambda h^{++\alpha\dot\alpha++} = {} & \mathcal{D}^{++}\lambda^{++\alpha\dot\alpha} + 4i\lambda^{++\alpha+}\bar\theta^{+\dot\alpha} - 4i\lambda^{++\dot\alpha+}\theta^{+\alpha}. \\ & \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\end{aligned}$$

- Expansion in the basis of covariant derivatives is

$$\hat{\mathcal{H}}^{++}_{(s=3)} = G^{++\alpha\dot\alpha\beta\dot\beta}\partial_{\beta\dot\beta}\partial_{\alpha\dot\alpha} + G^{++\hat\alpha+\beta\dot\beta}\partial_{\beta\dot\beta}\mathcal{D}^-_{\hat\alpha} + G^{++\beta\dot\beta++}\partial_{\beta\dot\beta}\mathcal{D}^{--}$$
$$+ G^{++\alpha+\beta+}\mathcal{D}^-_\beta\mathcal{D}^-_\alpha + G^{++\dot\alpha+\dot\beta+}\bar{\mathcal{D}}^-_{\dot\beta}\bar{\mathcal{D}}^-_{\dot\alpha} + G^{++\alpha+\dot\beta+}\bar{\mathcal{D}}^-_{\dot\beta}\mathcal{D}^-_\alpha$$
$$+ G^{++\hat\alpha+++}\mathcal{D}^{--}\mathcal{D}^-_{\hat\alpha} + G^{(+6)}\mathcal{D}^{--}\mathcal{D}^{--} + G^{++}.$$

- Covariant superfields in terms of analytic prepotentials:

$$G^{++(\alpha\beta)(\dot\alpha\dot\beta)} = h^{++(\alpha\beta)(\dot\alpha\dot\beta)} + 4ih^{++(\alpha\beta)(\dot\alpha+}\bar\theta^{-\dot\beta)} + 4i\theta^{-(\alpha}h^{++\beta)(\dot\alpha\dot\beta)+}$$
$$- 4ih^{++(\alpha(\dot\alpha++}\theta^{-\beta)}\bar\theta^{-\dot\beta)} - 16h^{++(\alpha+(\dot\alpha+}\theta^{-\beta)}\bar\theta^{-\dot\beta)},$$

$$G^{++(\alpha\beta)\dot\alpha+} = -h^{++(\alpha\beta)\dot\alpha+} + h^{++(\alpha\dot\alpha++}\theta^{-\beta)} + \dots$$

- Gauge transformations of symmetrized superfields:

$$\delta_\lambda G^{++(\alpha\beta)(\dot\alpha\dot\beta)} = \mathcal{D}^{++}\Lambda^{(\alpha\beta)(\dot\alpha\dot\beta)},$$
$$\delta_\lambda G^{++(\alpha\beta)\dot\alpha+} = \mathcal{D}^{++}\Lambda^{(\alpha\beta)\dot\alpha+},$$
$$\dots$$

- As the generalization of spin **2** half-analytic superfield $\mathcal{H}^{++}_{(\alpha_1\alpha_2)}$, we introduce

$$\mathcal{H}^{++}_{(\alpha_1\alpha_2\alpha_3\alpha_4)} := \partial^{\dot\beta_1}_{(\alpha_1}\partial^{\dot\beta_2}_{\alpha_2}G^{++}_{\alpha_3\alpha_4)(\dot\beta_1\dot\beta_2)} + \partial^{\dot\beta}_{(\alpha_1}\mathcal{D}^-_{\alpha_2}G^{+++}_{\alpha_3\alpha_4)\dot\beta},$$

which satisfy half-analyticity condition

$$\bar{\mathcal{D}}^+_{\dot\rho}\mathcal{H}^{++}_{\alpha(4)} = 0.$$

- Under gauge transformations half analytic superfield have quite simple transformation law

$$\delta_\lambda\mathcal{H}^{++}_{\alpha(4)} = \mathcal{D}^{++}\Lambda_{\alpha(4)},$$

where gauge superparameter is given by

$$\Lambda_{(\alpha_1\alpha_2\alpha_3\alpha_4)} = \partial^{\dot\beta_1}_{(\alpha_1}\partial^{\dot\beta_2}_{\alpha_2}\Lambda_{\alpha_3\alpha_4)(\dot\beta_1\dot\beta_2)} + \partial^{\dot\beta}_{(\alpha_1}\mathcal{D}^-_{\alpha_2}\Lambda^{++}_{\alpha_3\alpha_4)\dot\beta}.$$

- Then we introduce gauge-invariant spin **3** super Weyl tensor

$$\mathcal{W}_{\alpha(4)} = (\bar{\mathcal{D}}^+)^2\mathcal{H}^{--}_{\alpha(4)} \sim \theta^{+\alpha}\theta^{-\alpha}C_{\alpha(6)}, \qquad C_{\alpha(6)} = (\partial^3)^{\dot\alpha(3)}_{\alpha(3)}\Phi_{\alpha(3)\dot\alpha(3)}.$$

- $\mathcal{W}_{\alpha(4)}$ satisfy harmonic independence and chirality conditions.
- Gauge invariant action

$$S_{Weyl-3} = \int d^4xd^8\theta du\ \mathcal{H}^{++\alpha(4)}\mathcal{H}^{--}_{\alpha(4)} + c.c. = \frac{1}{16}\int d^4x_Ld^4\theta\ \mathcal{W}^{\alpha(4)}\mathcal{W}_{\alpha(4)} + c.c.$$

$\mathcal{N} = 2$ superconformal gravitino

- $\mathcal{N}=4$ vector multiplet $\left[(1), 4(\frac{1}{2}), 6(0)\right]$ in the $\mathcal{N}=2$ harmonic superspace is described by the action:

$$\boxed{S^{\mathcal{N}=4} = \frac{1}{4}\int d^4x d^8\theta du\; V^{++}V^{--} - \frac{1}{2}\int d\zeta^{(-4)} q^{+a}\mathcal{D}^{++}q^+_a.}$$

- Here $q^{+a} = (\tilde{q}^+, q^+)$ is the hypermutiplet $[2(\frac{1}{2}), 4(0)]$ superfield:

$$q^+ = f^i u^+_i + \theta^{+\alpha}\psi_\alpha + \bar{\theta}^+_{\dot{\alpha}}\bar{\chi}^{\dot{\alpha}} + 4i\theta^{+\alpha}\bar{\theta}^{+\dot{\alpha}}\partial_{\alpha\dot{\alpha}} f^i u^-_i + \dots$$

- The analytic superfield V^{++} with gauge freedom $\delta^{(0)}_\lambda V^{++} = \mathcal{D}^{++}\lambda$ describes $\mathcal{N}=2$ vector multiplet $[(1), 2(\frac{1}{2}), 2(0)]$:

$$\begin{aligned} V^{++}_{WZ} =& i(\theta^+)^2\phi - i(\bar{\theta}^+)^2\bar{\phi} - 4i\theta^{+\alpha}\bar{\theta}^{+\dot{\alpha}}A_{\alpha\dot{\alpha}} \\ &+ (\bar{\theta}^+)^2\theta^{+\alpha}\psi^i_\alpha + (\theta^+)^2\bar{\theta}^{+\dot{\alpha}}\bar{\psi}^i_{\dot{\alpha}}u^-_i + (\theta^+)^4 D^{(ij)}u^-_i u^-_j. \end{aligned}$$

- The V^{--} superfield is defined as the solution of the zero curvature equation

$$\mathcal{D}^{++}V^{--} = \mathcal{D}^{--}V^{++}.$$

- The action is manifestly invariant under $\mathcal{N}=2$ supersymmetry ($\epsilon^\pm_{\hat{\alpha}} := \epsilon^i_{\hat{\alpha}} u^\pm_i$):

$$\delta_\epsilon V^{++} = 0, \qquad \delta_\epsilon q^+_a = 0,$$

which is realized on coordinates as:

$$\delta_\epsilon x_{\alpha\dot{\alpha}} = -4i\left(\epsilon^-_\alpha\bar{\theta}^+_{\dot{\alpha}} + \theta^+_\alpha\bar{\epsilon}^-_{\dot{\alpha}}\right), \quad \delta_\epsilon\theta^+_\alpha = \epsilon^+_\alpha, \quad \delta_\epsilon\bar{\theta}^+_{\dot{\alpha}} = \bar{\epsilon}^+_{\dot{\alpha}}.$$

Hidden supersymmetry

$$\boxed{S^{\mathcal{N}=4} = \frac{1}{4}\int d^4x d^8\theta du\; V^{++}V^{--} - \frac{1}{2}\int d\zeta^{(-4)} q^{+a}\mathcal{D}^{++}q^+_a.}$$

- This action is invariant under global transformations

$$\delta_\varepsilon V^{++} = (\varepsilon^{\alpha a}\theta^+_\alpha + \bar{\varepsilon}^a_{\dot\alpha}\bar\theta^{+\dot\alpha})q^+_a,$$
$$\delta_\varepsilon q^+_a = -\frac{1}{2}(\mathcal{D}^+)^4\left[(\varepsilon^\alpha_a\theta^-_\alpha + \bar\varepsilon_{a\dot\alpha}\bar\theta^{-\dot\alpha})V^{--}\right]$$

  with rigid fermionic parameters $\varepsilon^{\alpha a}$ and $\bar\varepsilon^{\dot\alpha a} = \widetilde{\varepsilon^{\alpha a}}$. These transformations corresponds to extra (hidden) supersymmetries.

- Corresponding analytic $\mathcal{N}=2$ supercurrent:

$$\mathcal{J}^{++a}_\alpha = \frac{1}{2}(\mathcal{D}^+)^4\left(q^{+a}\theta^-_\alpha V^{--}\right) = \frac{1}{32}q^{+a}(\mathcal{D}^+)^2\left(\theta^-_\alpha\mathcal{W}\right) \approx \frac{1}{16}q^{+a}\mathcal{D}^+_\alpha\mathcal{W}$$

  satisfy conservation equation:

$$\mathcal{D}^{++}\mathcal{J}^{++a}_\alpha \approx 0.$$

Hidden R symmetries

$$\boxed{S^{\mathcal{N}=4} = \frac{1}{4}\int d^4x d^8\theta du\; V^{++}V^{--} - \frac{1}{2}\int d\zeta^{(-4)} q^{+a}\mathcal{D}^{++}q^+_a.}$$

- There is another class of global symmetries [Buchbinder, Ivanov, Ivanovskiy 2020]:

$$\delta_r V^{++} = 2\left(r^{-a}(\theta^+)^2 + \bar{r}^{-a}(\bar{\theta}^+)^2\right) q^+_a,$$
$$\delta_r q^+_a = -\frac{1}{2}(\mathcal{D}^+)^4\left[\left\{r^+_a(\theta^-)^2 - 2r^-_a(\theta^+\theta^-) + \bar{r}^+_a(\bar{\theta}^-)^2 - 2\bar{r}^-_a(\bar{\theta}^+\bar{\theta}^-)\right\}V^{--}\right]$$

  with rigid parameters $r^{\pm a} = r^{ai}u^\pm_i$. These transformations corresponds to hidden R-symmetries and extend the $R-$symmetry group from $SU(2)_R \times SU(2)_{PG}$ to $SU(4)$.

- Corresponding analytic $\mathcal{N}=2$ supercurrent

$$\mathcal{J}^{+a} = \frac{1}{2}(\mathcal{D}^+)^4\left(q^{+a}(\theta^-)^2 V^{--}\right) \approx \frac{1}{8}q^{+a}\mathcal{W} + \frac{1}{8}q^{+a}\theta^{-\alpha}\mathcal{D}^+_\alpha\mathcal{W}$$

  satisfy conservation equation:

$$\mathcal{D}^{++}\mathcal{J}^{+a} \approx 2\theta^{+\alpha}\mathcal{J}^{++a}_\alpha, \qquad (\mathcal{D}^{++})^2\mathcal{J}^{+a} \approx 0.$$

Hidden higher-spin supersymmetry

$$\boxed{S^{\mathcal{N}=4} = \frac{1}{4}\int d^4x d^8\theta du\; V^{++}V^{--} - \frac{1}{2}\int d\zeta^{(-4)} q^{+a}\mathcal{D}^{++}q^+_a.}$$

- Higher-spin supersymmetry:

$$\delta_\varepsilon V^{++} = (-1)^s \partial^s_{\alpha(s)\dot\alpha(s)}\left[(\varepsilon^{\alpha(s+1)\dot\alpha(s)a}\theta^+_\alpha - \bar\varepsilon^{\alpha(s)\dot\alpha(s+1)a}\bar\theta^+_{\dot\alpha})\, q^+_a\right],$$

$$\delta_\varepsilon q^+_a = -\frac{1}{2}(\mathcal{D}^+)^4\left[(\varepsilon_a^{\alpha(s+1)\dot\alpha(s)}\theta^-_\alpha - \bar\varepsilon_a^{\alpha(s)\dot\alpha(s+1)}\bar\theta^-_{\dot\alpha})\partial^s_{\alpha(s)\dot\alpha(s)}V^{--}\right]$$

  with rigid fermionic parameters.

- Corresponding analytic $\mathcal{N}=2$ higher-spin supercurrent:

$$\mathcal{J}^{++a}_{\alpha(s+1)\dot\alpha(s)} = \frac{1}{2}(\mathcal{D}^+)^4\left(q^{+a}\theta^-_{(\alpha}\partial^s_{\alpha(s))\dot\alpha(s)}V^{--}\right) \approx \frac{1}{16}q^{+a}\mathcal{D}^+_{(\alpha}\partial^s_{\alpha(s))\dot\alpha(s)}\mathcal{W}$$

  satisfy conservation equation:

$$\mathcal{D}^{++}\mathcal{J}^{++a}_{\alpha(s+1)\dot\alpha(s)} \approx 0.$$

Hidden higher-spin R symmetries

$$\boxed{S^{\mathcal{N}=4} = \frac{1}{4}\int d^4x d^8\theta du\; V^{++}V^{--} - \frac{1}{2}\int d\zeta^{(-4)} q^{+a}\mathcal{D}^{++}q^+_a.}$$

- Higher-spin R-symmetries:

$$\delta_r V^{++} = (-1)^s \partial^s_{\alpha(s)\dot\alpha(s)}\left[2\left(r^{\alpha(s)\dot\alpha(s)-a}(\theta^+)^2 + \bar r^{\alpha(s)\dot\alpha(s)-a}(\bar\theta^+)^2\right) q^+_a\right],$$

$$\delta_r q^+_a = -\frac{1}{2}(\mathcal{D}^+)^4\Big[\Big\{r_a^{\alpha(s)\dot\alpha(s)+}(\theta^-)^2 - 2r_a^{\alpha(s)\dot\alpha(s)-}(\theta^+\theta^-) + \bar r_a^{\alpha(s)\dot\alpha(s)+}(\bar\theta^-)^2 - 2\bar r_a^{\alpha(s)\dot\alpha(s)-}(\bar\theta^+\bar\theta^-)\Big\}\partial^s_{\alpha(s)\dot\alpha(s)}V^{--}\Big]$$

  with rigid parameters $r^{\pm a}_{\alpha(s)\dot\alpha(s)} = r^{ai}_{\alpha(s)\dot\alpha(s)}u^\pm_i$.

- Corresponding analytic $\mathcal{N}=2$ higher-spin supercurrent

$$\mathcal{J}^{+a}_{\alpha(s)\dot\alpha(s)} = \frac{1}{2}(\mathcal{D}^+)^4\left(q^{+a}(\theta^-)^2\partial^s_{\alpha(s)\dot\alpha(s)}V^{--}\right)$$
$$\approx \frac{1}{8}q^{+a}\partial^s_{\alpha(s)\dot\alpha(s)}\mathcal{W} + \frac{1}{8}q^{+a}\theta^{-\alpha}\mathcal{D}^+_\alpha\partial^s_{\alpha(s)\dot\alpha(s)}\mathcal{W}$$

  satisfy conservation equation:

$$\mathcal{D}^{++}\mathcal{J}^{+a}_{\alpha(s)\dot\alpha(s)} \approx 2\theta^{+\alpha}\mathcal{J}^{++a}_{\alpha(s+1)\dot\alpha(s)}, \qquad (\mathcal{D}^{++})^2\mathcal{J}^{+a}_{\alpha(s)\dot\alpha(s)} \approx 0.$$

Analytic $\mathcal{N}=2$ gravitino prepotentials

- We introduce unconstrained analytic prepotentials $h_a^{++\alpha}$ and h_a^{+++} with gauge transformations:

$$\delta_\lambda h_a^{++\alpha} = \mathcal{D}^{++}\lambda_a^\alpha + 2\theta^{+\alpha}\lambda_a^+, \qquad \delta_\lambda h_a^{+++} = \mathcal{D}^{++}\lambda_a^+.$$

- One can construct the cubic interaction

$$S_{int} = -\kappa \int d\zeta^{-4} \left[h^{+++a}\mathcal{J}_a^+ + h^{++\alpha a}\mathcal{J}_{\alpha a}^{++} + (\tilde{}\ \text{conj}) \right].$$

- Requirement of invariance under rigid $\mathcal{N}=2$ supersymmetry lead to supersymmetry transformations of analytic prepotentials:

$$\delta_\epsilon \mathcal{J}_a^+ = 2\epsilon^{-\alpha}\mathcal{J}_{\alpha a}^{++}, \quad \delta_\epsilon \mathcal{J}_{\alpha a}^{++} = 0 \quad \Rightarrow \quad \delta_\epsilon h^{++\alpha a} = -2\epsilon^{-\alpha} h^{+++a}, \quad \delta_\epsilon h^{+++a} = 0.$$

- Full action $S^{\mathcal{N}=4} + S_{int}$ is gauge invariant in the first order on coupling constant κ.
- Due to the gauge invariance (in zero order in κ) of $\mathcal{N}=2$ supercurrents

$$\delta_\lambda^{(0)} \mathcal{J}_\alpha^{++a} = \frac{1}{2}(\mathcal{D}^+)^4 \left(q^{+a}\theta_\alpha^- \mathcal{D}^{--}\lambda \right) = 0, \qquad \delta_\lambda^{(0)} \mathcal{J}_a^+ = \frac{1}{2}(\mathcal{D}^+)^4 \left(q^{+a}(\theta^-)^2 \mathcal{D}^{--}\lambda \right) = 0$$

we obtain cubic $(\frac{3}{2}, 1, \frac{1}{2})$ coupling, consistent in the leading order.

Wess-Zumino type gauge

- Using gauge freedom

$$\delta_\lambda h^{++\alpha} = \mathcal{D}^{++}\lambda^\alpha + 2\theta^{+\alpha}\lambda^+, \qquad \delta_\lambda h^{+++} = \mathcal{D}^{++}\lambda^+.$$

one can impose the Wess-Zumino type gauge:

$$\begin{aligned} h^{++\alpha}_{WZ} &= (\bar\theta^+)^2\psi^\alpha - 4i\theta^{+\beta}\bar\theta^{+\dot\beta}\Psi^\alpha_{\beta\dot\beta} + 8i(\theta^+)^2\bar\theta^{+\dot\beta}C^{\alpha i}_{\dot\beta}u^-_i \\ &\quad + (\bar\theta^+)^2\theta^{+\alpha}F^i u^-_i + (\bar\theta^+)^2\theta^{+\beta}\mathcal{F}^{\alpha)i}_{(\beta}u^-_i + (\theta^+)^4 D^{\alpha(ij)}u^-_i u^-_j, \\ h^{+++}_{WZ} &= (\theta^+)^2\bar\theta^{+\dot\beta}\bar\rho_{\dot\beta} - 4(\bar\theta^+)^2\theta^{+\beta}\kappa_\beta + (\theta^+)^4 C^i u^-_i. \end{aligned}$$

- In this gauge residual gauge freedom spanned by parameters:

$$\begin{aligned} \lambda^\alpha &= \epsilon^\alpha + 2\theta^{+\alpha}c^i u^-_i, \\ \lambda^+ &= c^i u^+_i + 2i\bar\theta^{+\dot\beta}\bar\eta_{\dot\beta} + 4i\theta^{+\rho}\bar\theta^{+\dot\rho}\partial_{\rho\dot\rho}c^i u^-_i. \end{aligned}$$

- Residual gauge transformations acts on fields in the WZ gauge as:

$$\delta_\lambda C^i_{\beta\dot\beta} = \partial_{\beta\dot\beta}c^i, \qquad \delta_\lambda C^i = 2\Box c^i, \qquad \delta_\lambda \Psi^\alpha_{\beta\dot\beta} = \partial_{\beta\dot\beta}\epsilon^\alpha + \delta^\beta_\rho\bar\eta_{\dot\rho}, \qquad \delta_\lambda\kappa_\beta = \partial^{\dot\beta}_\beta\bar\eta_{\dot\beta}.$$

Obtained transformations law means that $C^i_{\beta\dot\beta}$ is the doublet of spin 1 gauge fields, $\Psi^\alpha_{\beta\dot\beta}$ is the conformal gravitino field.

- Other fields with non-trivial gauge transformations can be redefined in terms of $\Psi^{\alpha}_{\beta\dot{\beta}}$ and $C^{i}_{\beta\dot{\beta}}$ fields and corresponds to auxiliary fields.
- For the bosonic field C^{i} the redefinition is given by

$$C^{i} = 4\partial^{\beta\dot{\beta}} C^{i}_{\beta\dot{\beta}} + G^{i}$$

so we extract the auxiliary field G^{i}.

- For conformal gravitino, using parameter $\bar{\eta}_{\dot{\alpha}}$ (which can be interpreted as parameter of local conformal supersymmetry) one can impose symmetric gauge for spin $\frac{3}{2}$ field $\Psi_{(\rho\beta)\dot{\rho}}$. In this gauge:

$$\delta\Psi_{(\alpha\beta)\dot{\alpha}} = \partial_{(\alpha\dot{\alpha}}\epsilon_{\beta)} \qquad \Rightarrow \qquad \kappa_{\beta} = -\frac{1}{6}\left(\partial^{\alpha\dot{\alpha}}\Psi_{(\alpha\beta)\dot{\alpha}}\right) + \chi_{\beta}.$$

- As the result, we obtain off-shell content of the conformal spin $\frac{3}{2}$ supermultiplet:

$$\begin{aligned} &\text{Gauge sector:} && \Psi_{(\alpha\beta)\dot{\alpha}}, C^{i}_{\alpha\dot{\alpha}}; \\ &\text{Non-gauge sector:} && \psi_{\alpha}, \rho_{\alpha}, \chi_{\alpha}, G^{i}, F^{i}, \mathcal{F}^{(\alpha\beta)i}, D^{\alpha(ij)}. \end{aligned}$$

Total in the multiplet there are $\mathbf{32_B + 32_F}$ off-shell degrees of freedom.

- We fix the canonical dimension for the gravitino field $[\Psi_{(\alpha\beta)\dot{\alpha}}] = \frac{3}{2}$, then the vector field has non-canonical dimension $[C^{i}_{\alpha\dot{\alpha}}] = 2$. From here we obtain prepotential dimensions $[h^{++\alpha}] = \frac{1}{2}$, $[h^{+++}] = 1$ and coupling constant dimension $[\kappa] = -1$.

Conformal gravitino action

- Thus, from the dimensional analysis, it can be expected that the free $\mathcal{N}=2$ conformal gravitino action in components contain conformal gravitino action with three derivatives and standard Maxwell action for doublet of vector fields.

- Desired gauge-invariant action for conformal gravitino can be constructed in terms of gauge-invariant strengths:

$$\check{C}_{(\dot\alpha\dot\beta\dot\gamma)} := \partial^{\alpha}_{(\dot\alpha}\partial^{\beta}_{\dot\beta}\Psi_{(\alpha\beta)\dot\gamma)}, \qquad \hat{C}_{(\alpha\beta\gamma)} := \partial^{\dot\rho}_{(\alpha}\Psi_{\beta\gamma)\dot\rho},$$

$$S^{\frac{3}{2}}_{conf} = \int d^4x \left(\hat{C}^{(\alpha\beta\gamma)}\breve{\check{C}}_{(\alpha\beta\gamma)} + \check{C}_{(\dot\alpha\dot\beta\dot\gamma)}\hat{\hat{C}}^{(\dot\alpha\dot\beta\dot\gamma)}\right).$$

Supersymmetry invariant superfields and zero curvature equations

- Analytic prepotential $h^{++\alpha}$ have nontrivial transformation law under rigid supersymmetry. One can introduce supersymmetry invariant superfields:

$$G^{++\alpha} = h^{++\alpha} + 2\theta^{-\alpha} h^{+++}, \qquad G^{+++} = h^{+++}.$$

- Corresponding gauge transformations are

$$\delta_\lambda G^{++\alpha} = \mathcal{D}^{++}\Lambda^\alpha, \qquad \delta_\lambda G^{+++} = \mathcal{D}^{++}\Lambda^+,$$

were gauge parameters are defined as

$$\Lambda^\alpha = \lambda^\alpha + 2\theta^{-\alpha}\lambda^+, \qquad \Lambda^+ = \lambda^+.$$

- As the solutions of zero curvature equations one can construct set of negatively charged potentials:

$$\mathcal{D}^{++} G^{--\alpha} = \mathcal{D}^{--} G^{++\alpha},$$
$$\mathcal{D}^{++} G^{--+} = \mathcal{D}^{--} G^{+++},$$
$$\mathcal{D}^{++} G^{---} = G^{--+}.$$

- Zero curvature equations are invariant under gauge transformations, accompanied by gauge transformation laws of negatively charged potentials:

$$\delta_\lambda G^{--\alpha} = \mathcal{D}^{--}\Lambda^\alpha,$$
$$\delta_\lambda G^{--+} = \mathcal{D}^{--}\Lambda^+ - \Lambda^-,$$
$$\delta_\lambda G^{---} = \mathcal{D}^{--}\Lambda^-,$$

where parameter Λ^- satisfy equation $\mathcal{D}^{++}\Lambda^- = \Lambda^+$.

$\mathcal{N}=2$ gravitino supercurvatures

- Using negatively charged potentials one can construct two gauge-invariant superfield strengths:

$$\hat{\mathcal{W}}_\alpha = (\bar{\mathcal{D}}^+)^2 G_\alpha^{--}, \qquad \check{\bar{\mathcal{W}}}_\alpha = (\bar{\mathcal{D}}^+)^2 \bar{\mathcal{H}}_\alpha^{--}.$$

- Second superstrength have interesting geometric structure, analogous to structure of linearized $\mathcal{N}=2$ super Weyl tensor [Ivanov, N.Z. 2024]. We start with composite object, which satisfy "half-analyticity" condition:

$$\bar{\mathcal{H}}_\alpha^{++} = \partial_{\alpha\dot{\alpha}} \bar{G}^{++\dot{\alpha}} + \frac{i}{2}\mathcal{D}_\alpha^- \bar{G}^{+++}, \qquad \bar{\mathcal{D}}_{\dot{\alpha}}^+ \bar{\mathcal{H}}_\alpha^{++} = 0.$$

Using zero curvature equation, one construct $\bar{\mathcal{H}}_\alpha^{--}$

$$\mathcal{D}^{++}\bar{\mathcal{H}}_\alpha^{--} = \mathcal{D}^{--}\bar{\mathcal{H}}_\alpha^{++} \quad \Rightarrow \quad \bar{\mathcal{H}}_\alpha^{--} = \partial_{\alpha\dot{\alpha}} \bar{G}^{--\dot{\alpha}} + \frac{i}{2}\mathcal{D}_\alpha^- \bar{G}^{--+} - \frac{i}{2}\mathcal{D}_\alpha^+ \bar{G}^{---}.$$

- *Harmonic independence*:

$$\mathcal{D}^{\pm\pm}\hat{\mathcal{W}}_\alpha = 0, \qquad \mathcal{D}^{\pm\pm}\check{\bar{\mathcal{W}}}_\alpha = 0.$$

- *Chirality*:

$$\bar{\mathcal{D}}_{\dot{\alpha}}^{\pm}\hat{\mathcal{W}}_\alpha = 0, \qquad \bar{\mathcal{D}}_{\dot{\alpha}}^{\pm}\check{\bar{\mathcal{W}}}_\alpha = 0.$$

$\mathcal{N} = 2$ superfield action

- Since dimensions of superfield strength $[\hat{\mathcal{W}}_\alpha] = \frac{3}{2}$, $[\check{\mathcal{W}}_\alpha] = \frac{5}{2}$ we obtain component composition in the gauge sector:

$$\hat{\mathcal{W}}_\alpha \sim \theta^{+(\beta}\theta^{-\gamma)}\hat{C}_{(\alpha\beta\gamma)} + (\theta^-)^2\theta^{+\beta}\mathcal{F}^i_{(\alpha\beta)}u^+_i + (\theta^+)^2\theta^{-\beta}\mathcal{F}^i_{(\alpha\beta)}u^-_i + \dots,$$

$$\check{\mathcal{W}}_\alpha \sim \theta^{+\beta}\bar{\mathcal{F}}^i_{(\alpha\beta)}u^-_i - \theta^{-\beta}\bar{\mathcal{F}}^i_{(\alpha\beta)}u^+_i + \theta^{+(\beta}\theta^{-\gamma)}\check{C}_{(\alpha\beta\gamma)} + \dots.$$

- Since $\hat{\mathcal{W}}^\alpha$ and $\check{\mathcal{W}}_\alpha$ are chiral superstrengths, we can construct invariant action in chiral superspace:

$$S = \int d^4x d^4\theta\, \hat{\mathcal{W}}^\alpha\overline{\check{\mathcal{W}}}_\alpha + c.c.$$

  In gauge sector, this action gives (up to numerical coefficients):

$$S = \int d^4x \left(\hat{C}^{(\alpha\beta\gamma)}\check{C}_{(\alpha\beta\gamma)} + \mathcal{F}^{(\alpha\beta)}_i\bar{\mathcal{F}}^i_{(\alpha\beta)}\right) + c.c.$$

- The action can be presented in a different form – as the integral over full harmonic superspace:

$$S = \int d^4x d^8\theta du\, \left[G^{++\alpha}\bar{\mathcal{H}}^{--}_\alpha - \bar{G}^{++}_{\dot\alpha}\mathcal{H}^{--\dot\alpha}\right].$$

Thank you for your attention!