Unfolded hypermultiplet in harmonic superspace

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PMMP'26, JINR Dubna 18.02.26

main statements

- unfolded formulation of hypermultiplet in harmonic superspace is constructed
- harmonic contribution arises from "vielbeinization" of R-symmetry gauge fields in the unfolded system
- background universality: Minkowskian, superspace and harmonic-superspace realizations arise from a single unfolded system

outline

- motivation: higher-spin unfolding & harmonic superspace
- harmonic superspace
- unfolding
- unfolded hypermultiplet
- background universality
- towards off-shell

motivation: higher-spin unfolding & harmonic superspace

- to formulate **higher-spin theory** in a manifestly diffeomorphism- and gauge-invariant way [Vasiliev'89-94], a special first-order formalism was developed called **unfolded dynamics approach** [Vasiliev hep-th/0504090];
- to efficiently realize manifest **off-shell** $\mathcal{N} = 2$ **supersymmetry**, it is handy to formulate field theories in **harmonic superspace** [Galperin, Ivanov, Kalitzin, Ogievetsky, Sokatchev' 84];
- unfolding: infinitely many auxiliary fields arising through expansion in additional (spinor) variables – basis of differential descendants of dynamical fields;
- harmonic superspace: infinitely many auxiliary fields arising through expansion in additional harmonic variables – geometrization of R-symmetry transformations;
- **an old question** – how are they related to each other?
- unfolded susy: [Ponomarev, Vasiliev 1012.2903; NM, Vasiliev 1301.2230; Buchbinder, Snegirev, Zinoviev 1606.02475; Khabarov, Zinoviev 2001.07903; NM 2201.01674]
- higher-spin fields in harmonic superspace: [Buchbinder, Ivanov, Zaigraev 2109.07639, 2202.08196, 2211.09501, 2212.14114, 2404.19016, 2402.05704; Zaigraev 2408.00668]

$\mathcal{N} = 2$ supersymmetry

- $\mathcal{N} = 2$ Poincaré superalgebra

$$[M_{\alpha\alpha}, M_{\beta\beta}] = \epsilon_{\alpha\beta} M_{\alpha\beta}, \quad [\bar{M}_{\dot\alpha\dot\alpha}, \bar{M}_{\dot\beta\dot\beta}] = \epsilon_{\alpha\beta} \bar{M}_{\dot\alpha\dot\beta}, \tag{1}$$

$$[M_{\alpha\alpha}, P_{\beta\dot\beta}] = \epsilon_{\alpha\beta} P_{\alpha\dot\beta}, \quad [\bar{M}_{\dot\alpha\dot\alpha}, P_{\beta\dot\beta}] = \epsilon_{\dot\alpha\dot\beta} P_{\beta\dot\alpha}, \tag{2}$$

$$[M_{\alpha\alpha}, Q^i_\beta] = \epsilon_{\alpha\beta} Q^i_\alpha, \quad [\bar{M}_{\dot\alpha\dot\alpha}, \bar{Q}_{\dot\beta j}] = \epsilon_{\dot\alpha\dot\beta} \bar{Q}_{\dot\alpha j}, \tag{3}$$

$$\{Q^i_\alpha, \bar{Q}_{\dot\alpha j}\} = \delta^i{}_j P_{\alpha\dot\alpha}, \tag{4}$$

$$[T_A, Q^i_\alpha] = \frac{1}{2}(\sigma_A)^i{}_j Q^j_\alpha, \quad [T_A, \bar{Q}_{\dot\alpha j}] = -\frac{1}{2}(\sigma_A)^i{}_j \bar{Q}_{\dot\alpha i}, \tag{5}$$

  T_A correspond to ***su*(2) *R*-symmetry** of $\mathcal{N} = 2$ susy.

- susy transformations are naturally realized in **superspace** $\mathbb{R}^{4|4\mathcal{N}}$ with coordinates $\{x^a, \theta^\alpha_i, \bar\theta^{\dot\alpha, i}\}$ as translations in θ's.
- natural objects to realize representations are **superfields** – functions in $\mathbb{R}^{4|4\mathcal{N}}$.
- general superfields lead to reducible reps, so one has to impose **constraints**.
- this works very well for $\mathcal{N} = 1$ (lower-spin) theories, but is problematic for $\mathcal{N} > 1$: constraints are not always known or cannot be explicitly resolved.

harmonic superspace

- for harmonic superspace, it is handy to switch to Cartan–Weyl basis

$$T^0 = 2T_3, \quad T^{\pm\pm} = T_1 \pm iT_2 \tag{6}$$

and introduce

$$Q^+_\alpha = Q^1_\alpha,\ Q^-_\alpha = Q^2_\alpha,\ \bar{Q}^+_{\dot\alpha} = -\bar{Q}_{\dot\alpha,2},\ \bar{Q}^-_{\dot\alpha} = \bar{Q}_{\dot\alpha,1}, \tag{7}$$

so that

$$\{Q^+_\alpha, \bar{Q}^-_{\dot\alpha}\} = -\{Q^-_\alpha, \bar{Q}^+_{\dot\alpha}\} = P_{\alpha\dot\alpha}. \tag{8}$$

$$[T^0, Q^\pm_{\hat\alpha}] = \pm Q^\pm_{\hat\alpha}, \quad [T^{\pm\pm}, Q^\mp_{\hat\alpha}] = Q^\pm_{\hat\alpha}. \tag{9}$$

$$[T^{++}, T^{--}] = T^0, \quad [T^0, T^{\pm\pm}] = \pm 2T^{\pm\pm}. \tag{10}$$

- **harmonic superspace** $\mathbb{R}^{4|8} \times S^2$ arises from tensoring $\mathcal{N} = 2$ superspace with a two-sphere $S^2 = SU(2)/U(1)$ which emerges through factorization of R-symmetry group over T^0.
- coordinates on harmonic superspace are $\{x^{\alpha\dot\alpha}, \theta^{\pm\alpha}, \bar\theta^{\pm\dot\alpha}, u^\pm_i\}$, $u^{+i}u^-_i = 1$.

harmonic superspace: hypermultiplet

- **hypermultiplet** – $\mathcal{N}=2$ irrep containing $su(2)$-doublet of complex scalars and two Weyl spinors

$$f^{i=1,2}(x), \quad \lambda_\alpha(x), \quad \bar{\varkappa}_{\dot\alpha}(x), \tag{11}$$

$$\Box f^i = 0, \quad \not\partial\lambda = 0, \quad \not\partial\bar{\varkappa} = 0. \tag{12}$$

- realization in $\mathcal{N}=2$ **superspace** $\mathbb{R}^{4|8}$: doublet of complex superfields $q^i(x,\theta,\bar\theta)$

$$D_\alpha{}^{(i} q^{j)} = \bar D_{\dot\alpha}{}^{(i} q^{j)} = 0. \tag{13}$$

- realization in **harmonic superspace**: complex superfield $q^+(x,\theta^\pm,\bar\theta^\pm,u^{\pm i})$ with

$$D^0 q^+ = q^+, \quad D^+_\alpha q^+ = \bar D^+_{\dot\alpha} q^+ = 0, \quad D^{++} q^+ = 0 \Rightarrow (D^{--})^2 q^+ = 0. \tag{14}$$

- problem of **going off-shell**: it is impossible to have susy off-shell formulation of hypermultiplet with a finite number of auxiliary fields.
- in harmonic superspace, an **off-shell extension** is achieved simply by **relaxing**

$$D^{++} q^+ = 0 \to D^{++} q^+ \neq 0. \tag{15}$$

infinite number of auxiliary fields are now encoded in the $u^{\pm i}$-expansion of q^+.

unfolding: general concept

- **unfolded equations** are first-order exterior-form equations

$$\mathrm{d}W^A(z) + G^A(W) = 0, \tag{16}$$

- **unfolded fields** are exterior forms; dynamical field theories require ∞ of unfolded fields encoding all d.o.f. (auxiliary generating variables are handy);
- **diffeomorphism** invariance; $\mathrm{d} = \mathrm{d}z^i \frac{\partial}{\partial z^i}$ acts on a base manifold $\mathcal{M}^D$ with coordinates $z^{i=\overline{1,D}}$.
- **consistency** condition

$$\mathrm{d}^2 \equiv 0 \quad \Rightarrow \quad G^B \frac{\delta G^A}{\delta W^B} \equiv 0. \tag{17}$$

- **unfolded gauge symmetries**

$$\delta W^A = \mathrm{d}\varepsilon^A(z) + \frac{\delta G^A}{\delta W^B}\varepsilon^B. \tag{18}$$

- unfolded fields form modules for all unfolded symmetries of the theory.
- gauge symmetries are associated with 1-forms, 0-forms correspond to physical d.o.f. (in the linear order, they transform only under vacuum symmetries).

unfolding: global symmetries and representations

- unfolded vacuum: consider 1-form $\Omega = \Omega^A(z)\, T_A$ from Lie algebra $\{T_A\}$ subject to

$$\mathrm{d}\Omega + \frac{1}{2}[\Omega \wedge \Omega] = 0. \tag{19}$$

- this gives rise to **local** symmetries with parameters $\varepsilon = \varepsilon^A(z)\, T_A$

$$\delta\Omega = \mathrm{d}\varepsilon + [\Omega, \varepsilon]. \tag{20}$$

- choosing a particular Ω_0 breaks them down to leftover **global** symmetries $\varepsilon_0(z)$

$$\mathrm{d}\varepsilon_0 + [\Omega_0, \varepsilon_0] = 0, \quad \delta\Omega_0 = 0. \tag{21}$$

- linearized system for 0-forms Φ is

$$\mathrm{d}\Phi + \Omega_0^A\, \hat{T}_A^{\mathcal{R}}\Phi = 0, \tag{22}$$

so that Φ is a **module** of representation $\mathcal{R}$ of algebra of **vacuum global symmetries**

$$\delta\Phi = \varepsilon_0^A\, \hat{T}_A^{\mathcal{R}}\Phi. \tag{23}$$

unfolding: covariant derivatives and e.o.m.

- for 0-form $\Phi(z)$ to depend non-trivially on all $z^{i=\overline{1,D}}$, all $\mathrm{d}z^i$ must enter Ω_0^A, otherwise $\partial_{i_0}\Phi = 0$ for absent $\mathrm{d}z^{i_0}$ follows directly from

$$\mathrm{d}\Phi = -\Omega_0^A \hat{T}_A^{\mathcal{R}}\Phi. \tag{24}$$

- so, among vacuum 1-forms Ω_0^A on $\mathcal{M}^D$, there must be D **(super)vielbeins** $\{E_0^{i=\overline{1,D}}\} \subset \{\Omega_0^A\}$; then **covariant derivatives** in E_0^i-basis arise via expanding

$$\mathrm{d} = E_0^i D_i. \tag{25}$$

- then linearized unfolded equations relate the fiber representation $\mathcal{R}$ to background geometry, in particular,

$$D_i\Phi = -\hat{T}_i^{\mathcal{R}}\Phi. \tag{26}$$

- this implies that symmetry transformations of Φ associated to (super)vielbeins are realized geometrically on $\mathcal{M}^D$, while the rest are realized algebraically.
- transition from a fiber space to a base can impose **differential constraints**; this depends both on the **representation** $\mathcal{R}$ and on the **dimension** D of the base.
- consistent unfolded system can be set on any base $\mathcal{M}^D$. This generates different theories in different spaces, but with the same symmetries and physical d.o.f., and explains higher-spin AdS/CFT [Vasiliev 1203.5554] between 4d dynamical fields in AdS and 3d conformal conserved currents.

unfolded hypermultiplet: vacuum connection

- harmonic superspace vacuum 1-form connection is

$$\Omega = e^{\alpha\dot\alpha} P_{\alpha\dot\alpha} + \omega_L^{\alpha\beta} M_{\alpha\beta} + \bar\omega_L^{\dot\alpha\dot\beta} \bar M_{\dot\alpha\dot\beta} + \psi^{\hat\alpha,\pm} Q^{\mp}_{\hat\alpha} + \omega^0 T^0 + \omega^{\pm\pm} T^{\mp\mp}, \tag{27}$$

$$\mathrm{d}\Omega + \frac{1}{2}[\Omega \wedge \Omega] = 0, \tag{28}$$

- Ω can be set on different backgrounds and (28) can be solved in different ways.
- **Minkowski space** – $\mathbb{R}^{1,3}$ with Cartesian coordinates $\{x^{\alpha\dot\alpha}\}$,

$$e^{\alpha\dot\alpha} = \mathrm{d}x^{\alpha\dot\alpha}, \quad \omega_L = 0, \quad \psi = 0, \quad \omega = 0. \tag{29}$$

- $\mathcal{N} = 2$ **superspace** – $\mathbb{R}^{4|8}$ with Cartesian coordinates $\{x^{\alpha\dot\alpha}, \theta^{\alpha,i}, \bar\theta^{\dot\alpha,i}\}$,

$$e^{\alpha\dot\alpha} = \mathrm{d}x^{\alpha\dot\alpha} + \frac{1}{2}\theta_i^\alpha \mathrm{d}\bar\theta^{\dot\alpha,i} + \frac{1}{2}\bar\theta^{\dot\alpha,i}\mathrm{d}\theta_i,\ \omega_L = 0,\ \psi_i^\alpha = \mathrm{d}\theta_i^\alpha,\ \bar\psi^{\dot\alpha,i} = \mathrm{d}\bar\theta^{\dot\alpha,i},\ \omega = 0 \tag{30}$$

- **harmonic superspace** – $\mathbb{R}^{4|8} \times S^2$ with Cartesian coordinates $\{x^{\alpha\dot\alpha}, \theta^{\alpha,i}, \bar\theta^{\dot\alpha,i}, u_i^\pm\}$,

$$e^{\alpha\dot\alpha} = \mathrm{d}x^{\alpha\dot\alpha} \pm \frac{1}{2}(\bar\theta^{\mp\dot\alpha}\mathrm{d}\theta^{\pm\alpha} + \theta^{\pm\alpha}\mathrm{d}\bar\theta^{\mp\dot\alpha}) \pm \theta^{\pm\alpha}\bar\theta^{\pm\dot\alpha}\omega^{\mp\mp} + (\theta^{+\alpha}\bar\theta^{-\dot\alpha} + \theta^{-\alpha}\bar\theta^{+\dot\alpha})\omega^0,$$

$$\omega_L = 0,\ \psi^{\pm\hat\alpha} = \mathrm{d}\theta^{\pm\hat\alpha} + \theta^{\mp\hat\alpha}\omega^{\pm\pm} \mp \theta^{\pm\hat\alpha}\omega^0,\ \omega^0 = u_i^- \mathrm{d}u^{+i},\ \omega^{\pm\pm} = u_i^\pm \mathrm{d}u^{\pm i}. \tag{31}$$

- from the base point of view, symmetries associated to (super)vielbeins are realized geometrically, the rest – algebraically.

unfolded hypermultiplet: component fields

- unfolded **scalar** $q = q(z|y\bar{y})$ (here $\mathrm{d}_L = \mathrm{d} + \omega_L^{\alpha\beta} y_\alpha \partial_\beta + \bar{\omega}_L^{\dot\alpha\dot\beta} \bar{y}_{\dot\alpha} \bar{\partial}_{\dot\beta}$)

$$\mathrm{d}_L q = \frac{1}{\hat\nu + 1} e^{\alpha\dot\alpha} \partial_\alpha \bar{\partial}_{\dot\alpha} q, \tag{32}$$

in Cartesian coordinates in $\mathbb{R}^{1,3}$

$$q(x|y,\bar{y}) = \exp\{y^\alpha \bar{y}^{\dot\alpha} \frac{\partial}{\partial x^{\alpha\dot\alpha}}\} q(x) = q(x + y\bar{y}), \quad \Box q = 0. \tag{33}$$

- unfolded **left Weyl** spinor $\lambda = \lambda_\alpha(z|y\bar{y}) y^\alpha$

$$\mathrm{d}_L \lambda(z|y,\bar{y}) = \frac{1}{\hat\nu + 3/2} e^{\alpha\dot\alpha} \partial_\alpha \bar{\partial}_{\dot\alpha} \lambda, \tag{34}$$

in Cartesian coordinates in $\mathbb{R}^{1,3}$

$$\lambda(x|y,\bar{y}) = y^\alpha \lambda_\alpha(x + y\bar{y}), \quad \partial^{\alpha\dot\alpha} \lambda_\alpha(x) = 0. \tag{35}$$

- unfolded **right Weyl** spinor $\bar{\varkappa} = \bar{\varkappa}_{\dot\alpha}(z|y\bar{y}) \bar{y}^{\dot\alpha}$

$$\mathrm{d}_L \bar{\varkappa} = \frac{1}{\hat\nu + 3/2} e^{\alpha\dot\alpha} \partial_\alpha \bar{\partial}_{\dot\alpha} \bar{\varkappa}, \tag{36}$$

in Cartesian coordinates in $\mathbb{R}^{1,3}$

$$\bar{\varkappa}(x|y,\bar{y}) = \bar{y}^{\dot\alpha} \bar{\varkappa}_{\dot\alpha}(x + y\bar{y}), \quad \partial^{\alpha\dot\alpha} \bar{\varkappa}_{\dot\alpha}(x) = 0. \tag{37}$$

unfolded hypermultiplet: harmonic superfield

- we start with a harmonic scalar superfield $q^+(z)$ in $\mathbb{R}^{4|8} \times S^2$ and impose constraints

$$D_0 q^+ = q^+, \quad D^+_\alpha q^+ = 0, \quad \bar{D}^+_{\dot\alpha} q^+ = 0, \quad D^{++} q^+ = 0. \tag{38}$$

- for non-zero covariant derivatives, we introduce auxiliary fields

$$D^-_\alpha q^+ = \lambda_\alpha, \quad \bar{D}^-_{\dot\alpha} q^+ = \bar{\varkappa}_{\dot\alpha}, \quad D^{--} q^+ = q^-. \tag{39}$$

- then we promote all the fields to unfolded ones

$$q^+(z|y,\bar{y}), \quad q^-(z|y,\bar{y}), \quad \lambda(z|y,\bar{y}), \quad \bar{\varkappa}(z|y,\bar{y}). \tag{40}$$

- next, we write an ansatz with unknown $\#_{\hat\nu}$ dependent on $\hat\nu = y^\alpha \partial_\alpha + \bar{y}^{\dot\alpha} \bar{\partial}_{\dot\alpha}$

$$\mathrm{d}_L q^\pm = \frac{1}{\hat\nu + 1} e^{\alpha\dot\alpha} \partial_\alpha \bar{\partial}_{\dot\alpha} q^\pm + \#_{\hat\nu} \psi^{\pm\alpha} \partial_\alpha \lambda + \#_{\hat\nu} \bar{\psi}^{\pm\dot\alpha} \bar{\partial}_{\dot\alpha} \bar{\varkappa} + \omega^{\pm\pm} q^\mp \pm \omega^0 q^\pm, \tag{41}$$

$$\mathrm{d}_L \lambda = \frac{1}{\hat\nu + 3/2} e^{\alpha\dot\alpha} \partial_\alpha \bar{\partial}_{\dot\alpha} \lambda + \#_{\hat\nu} \bar{\psi}^{-\dot\alpha} \bar{\partial}_{\dot\alpha} q^+ + \#_{\hat\nu} \bar{\psi}^{+\dot\alpha} \bar{\partial}_{\dot\alpha} q^-, \tag{42}$$

$$\mathrm{d}_L \bar{\varkappa} = \frac{1}{\hat\nu + 3/2} e^{\alpha\dot\alpha} \partial_\alpha \bar{\partial}_{\dot\alpha} \bar{\varkappa} + \#_{\hat\nu} \psi^{-\alpha} \partial_\alpha q^+ + \#_{\hat\nu} \psi^{+\alpha} \partial_\alpha q^-, \tag{43}$$

- finally, we solve for the consistency condition $\mathrm{d}^2 \equiv 0$, fixing $\#_{\hat\nu}$.

unfolded hypermultiplet: system of equations

- resulting consistent unfolded system

$$\mathrm{d}_L q^+ = \frac{1}{\hat{\nu}+1} e^{\alpha\dot{\alpha}} \partial_\alpha \bar{\partial}_{\dot{\alpha}} q^+ + \frac{1}{\hat{\nu}+1} \psi^{+\alpha} \partial_\alpha \lambda + \frac{1}{\hat{\nu}+1} \bar{\psi}^{+\dot{\alpha}} \bar{\partial}_{\dot{\alpha}} \bar{\varkappa} + \omega^{++} q^- + \omega^0 q^+, \tag{44}$$

$$\mathrm{d}_L q^- = \frac{1}{\hat{\nu}+1} e^{\alpha\dot{\alpha}} \partial_\alpha \bar{\partial}_{\dot{\alpha}} q^- - \frac{1}{\hat{\nu}+1} \psi^{-\alpha} \partial_\alpha \lambda - \frac{1}{\hat{\nu}+1} \bar{\psi}^{-\dot{\alpha}} \bar{\partial}_{\dot{\alpha}} \bar{\varkappa} + \omega^{--} q^+ - \omega^0 q^-, \tag{45}$$

$$\mathrm{d}_L \lambda = \frac{1}{\hat{\nu}+3/2} e^{\alpha\dot{\alpha}} \partial_\alpha \bar{\partial}_{\dot{\alpha}} \lambda + \bar{\psi}^{-\dot{\alpha}} \bar{\partial}_{\dot{\alpha}} q^+ + \bar{\psi}^{+\dot{\alpha}} \bar{\partial}_{\dot{\alpha}} q^-, \tag{46}$$

$$\mathrm{d}_L \bar{\varkappa} = \frac{1}{\hat{\nu}+3/2} e^{\alpha\dot{\alpha}} \partial_\alpha \bar{\partial}_{\dot{\alpha}} \bar{\varkappa} - \psi^{-\alpha} \partial_\alpha q^+ - \psi^{+\alpha} \partial_\alpha q^-. \tag{47}$$

- full vacuum connection 1-form Ω is subject to

$$\mathrm{d}\Omega + \frac{1}{2}[\Omega \wedge \Omega] = 0. \tag{48}$$

- at this level, background manifold $\mathcal{M}^D$ and coordinates on it remain unspecified: for any choice, the system is self-consistent.

unfolded hypermultiplet: symmetries

- flat 1-form connection of $\mathcal{N}=2$ Poincaré superalgebra

$$\Omega = e^{\alpha\dot\alpha} P_{\alpha\dot\alpha} + \omega_L^{\alpha\beta} M_{\alpha\beta} + \bar\omega_L^{\dot\alpha\dot\beta} \bar M_{\dot\alpha\dot\beta} + \psi^{\pm\hat\alpha} Q^{\mp}_{\hat\alpha} + \omega^0 T^0 + \omega^{\pm\pm} T^{\mp\mp}$$

  generates unfolded gauge symmetries with local parameters

$$\varepsilon^{\alpha\dot\alpha}(z),\ \varepsilon^{\alpha\alpha}(z),\ \bar\varepsilon^{\dot\alpha\dot\alpha}(z),\ \varepsilon^{\pm\alpha}(z),\ \bar\varepsilon^{\pm\dot\alpha}(z),\ \varepsilon^{0}(z),\ \varepsilon^{\pm\pm}(z).$$

- independently of background manifold $\mathcal{M}^D$, a representation of the superalgebra is realized on unfolded 0-form master-fields as

$$\begin{aligned}
\delta q^{\pm} &= \varepsilon^{\alpha\beta} y_\alpha \partial_\beta q^{\pm} + \bar\varepsilon^{\dot\alpha\dot\beta} \bar y_{\dot\alpha} \bar\partial_{\dot\beta} q^{\pm} - \frac{1}{\hat\nu+1} \varepsilon^{\alpha\dot\alpha} \partial_\alpha \bar\partial_{\dot\alpha} q^{\pm} \mp \\
&\mp \frac{1}{\hat\nu+1} \varepsilon^{\pm\alpha} \partial_\alpha \lambda \mp \frac{1}{\hat\nu+1} \bar\varepsilon^{\pm\dot\alpha} \bar\partial_{\dot\alpha} \bar\varkappa - \varepsilon^{\pm\pm} q^{\mp} \mp \varepsilon^0 q^{\pm}, \qquad (49)\\
\delta\lambda &= \varepsilon^{\alpha\beta} y_\alpha \partial_\beta \lambda + \bar\varepsilon^{\dot\alpha\dot\beta} \bar y_{\dot\alpha} \bar\partial_{\dot\beta} \lambda - \frac{1}{\hat\nu+3/2} \varepsilon^{\alpha\dot\alpha} \partial_\alpha \bar\partial_{\dot\alpha} \lambda - \bar\varepsilon^{\pm\dot\alpha} \bar\partial_{\dot\alpha} q^{\mp}, \qquad (50)\\
\delta\bar\varkappa &= \varepsilon^{\alpha\beta} y_\alpha \partial_\beta \bar\varkappa + \bar\varepsilon^{\dot\alpha\dot\beta} \bar y_{\dot\alpha} \bar\partial_{\dot\beta} \bar\varkappa - \frac{1}{\hat\nu+3/2} \varepsilon^{\alpha\dot\alpha} \partial_\alpha \bar\partial_{\dot\alpha} \bar\varkappa + \varepsilon^{\pm\alpha} \partial_\alpha q^{\mp}. \qquad (51)
\end{aligned}$$

background universality

- this is a manifestation of **background universality of unfolding**: at the linear level, all theories possessing the same algebras of global symmetries and the same spectra of physical d.o.f. forming their representations lead to equivalent unfolded systems.
- thus, it is **global symmetries** and **spectra of 0-forms** that constitute an "invariant" content of the theory (at the free level) and determine its universality class, while backgrounds, constraints, e.o.m. can vary within the class.
- in our case, global symmetries are Poincaré, supersymmetry and R-symmetry
- choosing different backgrounds for the unfolded system leads to different realizations of these symmetries and different statuses of 0-forms q^+, q^-, λ and $\bar{\varkappa}$.
- consider three cases: harmonic superspace, $\mathcal{N} = 2$ superspace and Minkowski space.

background universality: harmonic superspace

- **harmonic superspace** with coordinates $\{x^{\alpha\dot\alpha}, \theta^{\pm\alpha}, \bar\theta^{\pm\dot\alpha}, u^\pm_i\}$, $u^{+i}u^-_i = 1$

$$e^{\alpha\dot\alpha} = \mathrm{d}x^{\alpha\dot\alpha} \pm \frac{1}{2}(\bar\theta^{\mp\dot\alpha}\mathrm{d}\theta^{\pm\alpha} + \theta^{\pm\alpha}\mathrm{d}\bar\theta^{\mp\dot\alpha}) \pm \theta^{\pm\alpha}\bar\theta^{\pm\dot\alpha}\omega^{\mp\mp} + (\theta^{+\alpha}\bar\theta^{-\dot\alpha} + \theta^{-\alpha}\bar\theta^{+\dot\alpha})\omega^0,$$

$$\omega_L = 0,\ \psi^{\pm\hat\alpha} = \mathrm{d}\theta^{\pm\hat\alpha} + \theta^{\mp\hat\alpha}\omega^{\pm\pm} \mp \theta^{\pm\hat\alpha}\omega^0,\ \omega^0 = u^-_i\,\mathrm{d}u^{+i},\ \omega^{\pm\pm} = u^\pm_i\mathrm{d}u^{\pm i}, \quad (52)$$

$$\mathrm{d} = e^{\alpha\dot\alpha}D_{\alpha\dot\alpha} + \psi^{\alpha,\pm}D^\mp_\alpha + \bar\psi^{\dot\alpha,\pm}\bar D^\mp_{\dot\alpha} + \omega^0 D^0 + \omega^{\pm\pm}D^{\mp\mp}, \quad (53)$$

  all symmetries are geometric (except for a spin part of Lorentz);

- the only primary field is q^+ subject to

$$D_0 q^+ = q^+, \quad D^+_\alpha q^+ = 0, \quad \bar D^+_{\dot\alpha} q^+ = 0, \quad D^{++}q^+ = 0, \quad (54)$$

  while

$$q^- = D^{--}q^+, \quad \lambda_\alpha = D^-_\alpha q^+, \quad \bar\varkappa_{\dot\alpha} = \bar D^-_{\dot\alpha} q^+. \quad (55)$$

- alternatively, one can choose q^- as the only independent field, then $q^+ = -D^{++}q^-$.

background universality: $\mathcal{N} = 2$ superspace

- $\mathcal{N} = 2$ **superspace** with coordinates $\{x^{\alpha\dot{\alpha}}, \theta_i^\alpha, \bar{\theta}^{\dot{\alpha},i}\}$.

$$\psi^{+\alpha} = \psi_1^\alpha, \quad \psi^{-\alpha} = \psi_2^\alpha, \quad \bar{\psi}^{+\dot{\alpha}} = -\bar{\psi}^{\dot{\alpha},2}, \quad \bar{\psi}^{-\dot{\alpha}} = \bar{\psi}^{\dot{\alpha},1}, \quad q^1 = q^+, \quad q^2 = q^-. \tag{56}$$

$$e^{\alpha\dot{\alpha}} = \mathrm{d}x^{\alpha\dot{\alpha}} + \frac{1}{2}\theta_i^\alpha \mathrm{d}\bar{\theta}^{\dot{\alpha},i} + \frac{1}{2}\bar{\theta}^{\dot{\alpha},i}\mathrm{d}\theta_i, \quad \omega_L = 0, \quad \psi_i^\alpha = \mathrm{d}\theta_i^\alpha, \quad \bar{\psi}^{\dot{\alpha},i} = \mathrm{d}\bar{\theta}^{\dot{\alpha},i}, \quad \omega = 0, \tag{57}$$

$$\mathrm{d} = e^{\alpha\dot{\alpha}} D_{\alpha\dot{\alpha}} + \psi^{\alpha,i} D_{\alpha,i} + \bar{\psi}^{\dot{\alpha},i} \bar{D}_{\dot{\alpha},i}, \tag{58}$$

  Poincaré and susy are geometric, while R-symmetry is algebraic;

- primary fields are now q^+ and q^-, while

$$\lambda_\alpha = D_\alpha^- q^+, \quad \bar{\varkappa}_{\dot{\alpha}} = \bar{D}_{\dot{\alpha}}^- q^+ \tag{59}$$

  and

$$D_\alpha^{(i} q^{j)} = \bar{D}_{\dot{\alpha}}^{(i} q^{j)}. \tag{60}$$

background universality: Minkowski space

- **Minkowski space** with coordinates $\{x^{\alpha\dot{\alpha}}\}$,

$$e^{\alpha\dot{\alpha}} = \mathrm{d}x^{\alpha\dot{\alpha}}, \quad \omega_L = 0, \quad \psi^{\pm\alpha} = \bar{\psi}^{\pm\dot{\alpha}} = \omega^0 = \omega^{\pm\pm} = 0, \quad D_{\alpha\dot{\alpha}} = \frac{\partial}{\partial x^{\alpha\dot{\alpha}}}. \quad (61)$$

  Poincaré is geometric, while susy and R-symmetry are algebraic.

- all fields q^+, q^-, λ and $\bar{\varkappa}$ are primary with

$$\Box q^{\pm} = 0, \quad \partial^{\alpha\dot{\alpha}}\lambda_\alpha(x) = 0, \quad \partial^{\alpha\dot{\alpha}}\bar{\varkappa}_{\dot{\alpha}}(x) = 0. \quad (62)$$

towards off-shell

- the true power of harmonic superspace is revealed when one tries to keep $\mathcal{N} = 2$ susy manifest off-shell;
- it is impossible with a finite number of auxiliary fields in usual $\mathcal{N} = 2$ superspace;
- in harmonic superspace, **off-shell extension** is achieved by **relaxing**

$$D^{++} q^{+} = 0, \tag{63}$$

  which leads to ∞ number of auxiliary fields via the u-expansion of $q^{+}(x, \theta, \bar{\theta}, u)$;
- abandoning (63) generates new infinite sequences of descendants like e.g.

$$q^{++\cdots+} = (D^{++})^{n} q^{+}, \quad q^{--\cdots-} = (D^{--})^{n} q^{+}. \tag{64}$$

towards off-shell: R-symmetry in the fiber

- introduce a **new auxiliary variable** v encoding $u(1)$-charge and a single master-field

$$q(z|y^\alpha, \bar{y}^{\dot\alpha}, v) = q^+ e^{-iv} + q^- e^{iv} \equiv \sum_{n=\pm 1} q^{(n)} e^{-inv}. \tag{65}$$

- then the unfolded hypermultiplet equations turn into

$$\mathrm{d}_L q(z|y,\bar{y},v) = \frac{1}{\hat\nu+1} e^{\alpha\dot\alpha}\partial_\alpha\bar\partial_{\dot\alpha} q + \frac{1}{\hat\nu+1}(e^{iv}\psi^{+\alpha} - e^{-iv}\psi^{-\alpha})\partial_\alpha\lambda +$$
$$+ \frac{1}{\hat\nu+1}(e^{iv}\bar\psi^{+\dot\alpha} - e^{-iv}\bar\psi^{-\dot\alpha})\bar\partial_{\dot\alpha}\bar\varkappa + i(\omega^0\frac{\partial}{\partial v} \pm \omega^{\mp\mp} e^{\mp 2iv}\frac{\partial}{\partial v})q, \tag{66}$$

$$\mathrm{d}_L \lambda(z|y,\bar{y},v) = \frac{1}{\hat\nu+3/2} e^{\alpha\dot\alpha}\partial_\alpha\bar\partial_{\dot\alpha}\lambda + (e^{-iv}\bar\psi^{+\dot\alpha} + e^{+iv}\bar\psi^{-\dot\alpha})\bar\partial_{\dot\alpha} q, \tag{67}$$

$$\mathrm{d}_L \bar\varkappa(z|y,\bar{y},v) = \frac{1}{\hat\nu+3/2} e^{\alpha\dot\alpha}\partial_\alpha\bar\partial_{\dot\alpha}\bar\varkappa - (e^{-iv}\psi^{+\alpha} + e^{+iv}\psi^{-\alpha})\partial_\alpha q. \tag{68}$$

- v-operators form a representation of $su(2)$ via the identification

$$T^0 = i\frac{\partial}{\partial v}, \quad T^{\pm\pm} = \pm\frac{i}{2} e^{\mp 2iv}\frac{\partial}{\partial v}. \tag{69}$$

- **generating variable** v for $u^\pm_i$ is what $\{y^\alpha, \bar{y}^{\dot\alpha}\}$ is for $x^{\alpha\dot\alpha}$.
- relaxing $D^{++}q^+ = 0$ amounts to invoking **all odd** n in the sum (65) and **all even charges** for fermions.

conclusions

- **unfolded** formulation of hypermultiplet in **harmonic superspace** has been constructed
- **relation** between unfolded and harmonic approaches has been established: harmonic contribution arises from "**vielbeinization**" of R-symmetry gauge fields
- **background universality** of unfolding:
  a single unfolded system generates formulations in different (super)spaces, while an invariant content includes symmetries and a set of physical d.o.f.
- **to do**: massive case, off-shell extension, super-Yang–Mills theory