Chiral higher spin theories and generalized self-dual double copy

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Presentation structure

1. Motivation
2. Classical double copy
3. Classical self-dual double copy
4. Chiral higher spin theory and its solution
5. The analysis at the level of Weyl tensors
6. Conclusion

Motivation

- Chiral higher-spin theories, which represent generalizations of self-dual Yang-Mills (SDYM) and self-dual gravity (SDGR) to higher spins, provide a consistent framework that circumvents no-go theorems
- Solving their (CHS) EoM allows one to generalize the double copying procedure to higher spins

Classical double copy

- Kerr-Schild anzats in GR: $g_{\mu\nu} = \eta_{\mu\nu} + \varphi k_\mu k_\nu = \eta_{\mu\nu} + h_{\mu\nu}$ where k is null $k^2 = 0$ and geodesic $k \cdot \partial k^\mu = 0$ w.r.t. the Minkowski metric η.

Double copy [Monteiro et al.'14]:

$$\begin{array}{ccc} s=0 & s=1 & s=2 \\ \varphi = \varphi & A_\mu = k_\mu \varphi & h_{\mu\nu} = k_\mu k_\nu \varphi \end{array}$$

- at the level of Weyl tensors[Luna et.al'18]:

$$\begin{array}{ccc} s=0 & s=1 & s=2 \\ \varphi = [\mu\lambda]^{-1/2} & F_{\dot\alpha\dot\alpha} = \frac{\mu_{\dot\alpha}\lambda_{\dot\alpha}}{[\mu\lambda]^{-3/2}} & C_{\dot\alpha\dot\alpha\dot\alpha\dot\alpha} = \frac{\mu_{\dot\alpha}\lambda_{\dot\alpha}\mu_{\dot\alpha}\lambda_{\dot\alpha}}{[\mu\lambda]^{-5/2}} \end{array}$$

Classical self-dual double copy

In the lightcone gauge:

$$S_{\text{SDGR}} = \int d^4x \Phi^{-2} \Box \Phi^{+2} + \Phi^{-2} D^{\dot{\alpha}} D^{\dot{\beta}} \Phi^{+2} D_{\dot{\alpha}} D_{\dot{\beta}} \Phi^{+2}$$

$$S_{\text{SDMax}} = \int d^4x \Phi^{-1} \Box \Phi^{+1}, \quad S_{Scalar} = \int d^4x \Phi^{0} \Box \Phi^{0}.$$

the SDGR equation:

$$\Box \Phi^{+2} + D^{\dot{\alpha}} D^{\dot{\beta}} \Phi^{+2} D_{\dot{\alpha}} D_{\dot{\beta}} \Phi^{+2} = 0, \quad D_{\dot{1},\dot{2}} = \partial_x \pm i \partial_v,$$

and the Kerr-Schild condition then implies:

$$D^{\dot{\alpha}} D^{\dot{\beta}} \Phi^{+2} D_{\dot{\alpha}} D_{\dot{\beta}} \Phi^{+2} = 0.$$

The self-dual double copy means[Berman et al.' 18; Tod' 82] that:

$$\Phi^0 = \Phi^{+1} = \Phi^{+2}$$

The corresponding Weyl tensors are:

$$C_{\dot{\alpha}\dot{\alpha}\dot{\alpha}\dot{\alpha}} = \mu_{\dot{\alpha}} \mu_{\dot{\alpha}} \lambda_{\dot{\alpha}} \lambda_{\dot{\alpha}}, \quad F_{\dot{\alpha}\dot{\alpha}} = \mu_{\dot{\alpha}} \mu_{\dot{\alpha}}$$

Chiral (self-dual) higher spin theory [Metsaev'91; Ponomarev'17]

Action

$$S = \int d^4x \sum_{\lambda\in\mathbb{Z}} \Phi^{-\lambda}\Box\Phi^{\lambda} + g \sum_{\lambda_1,\lambda_2,\lambda_3} C^{\lambda_1\lambda_2\lambda_3} \int d^4x \Phi^{\lambda_1}\Phi^{\lambda_2}\left(\overleftarrow{\partial_x}\overrightarrow{\partial_v} - \overleftarrow{\partial_v}\overrightarrow{\partial_x}\right)^{\lambda_1+\lambda_2+\lambda_3}\Phi^{\lambda_3},$$

where: $C^{\lambda_1\lambda_2\lambda_3} = 2\frac{I^{\lambda_1+\lambda_2+\lambda_3-1}}{(\lambda_1+\lambda_2+\lambda_3-1)!}$.

It has a contracted version with only two-derivative terms $C^{\lambda_1\lambda_2\lambda_3} = 2I\delta(\lambda_1+\lambda_2+\lambda_3-2)$ called *Poisson Chiral HS*

The EoM of Poisson Chiral HS

$$\Box\Phi^{\lambda_1} + \sum_{\substack{\lambda_{2,3}\in\mathbb{Z},\\ \lambda_2+\lambda_3=2-\lambda_1}} D^{\dot\alpha}D^{\dot\beta}\Phi^{\lambda_2}D_{\dot\alpha}D_{\dot\beta}\Phi^{\lambda_3} = 0 \qquad (*)$$

It is quite obvious that

$$\Phi^{\lambda} = C^{\lambda}\Phi^{+2},$$

solves the equation above if Φ^2 is an SDGR solution in the KS-form. This is our first result.

The analysis at the level of Weyl tensors

For lower spins (type D case):

$$C_{\dot\alpha\dot\alpha\dot\alpha\dot\alpha} = \mu_{\dot\alpha}\mu_{\dot\alpha}\tau_{\dot\alpha}\tau_{\dot\alpha}, \quad F_{\dot\alpha\dot\alpha} = \mu_{\dot\alpha}\mu_{\dot\alpha}$$

Higher spin multicopy

We assume that there is a multicopy if:

$$C^{\dot\alpha(2s)} = \mu_1^{\dot\alpha} \dots \mu_{2s}^{\dot\alpha}$$

is expressed algebraically via $C^{\dot\alpha\dot\alpha\dot\alpha\dot\alpha}$

It turns out that for SD theories

$$C^{\dot\alpha(2s)} = \underbrace{D^{\dot\alpha} \dots D^{\dot\alpha}}_{2s} \Phi^s, \qquad (**)$$

as it is in free theory. Taking into account $\Phi^{+s} = C^s\Phi^2$, the question of the presence of a multicopy comes down to whether the higher derivatives in (**) are expressed through the first 4 algebraically. We studied this issue via some version of the unfolding procedure.

The analysis at the level of Weyl tensors

- Kerr-Schild condition $D^{\dot\alpha}D^{\dot\beta}\Phi D_{\dot\alpha}D_{\dot\beta}\Phi = 0 \iff Det(D_{\dot\alpha}D_{\dot\beta}\Phi) = 0 \iff \exists \mu_{\dot\alpha} : D_{\dot\alpha}D_{\dot\beta}\Phi = \mu_{\dot\alpha}\mu_{\dot\beta}$.
- The consistency condition for μ then reads:

$$D^{\dot\alpha}\left(\mu_{\dot\alpha}\mu_{\dot\beta}\right) = 0 \iff \exists \lambda_{\dot\alpha} : D_{\dot\alpha}\mu_{\dot\beta} = \lambda_{(\dot\alpha}\mu_{\dot\beta)} + \frac{1}{6}\epsilon_{\dot\alpha\dot\beta}[\lambda\mu]$$

- Analogously we may continue to differentiate the newly introduced functions observing that at each step only two new algebraically independent functions actually arise:

$$\exists A, \Lambda : D_{\dot\alpha}\lambda_\beta = \frac{2}{3}\lambda_{\dot\alpha}\lambda_{\dot\beta} - 2\frac{\Lambda}{[\lambda\mu]}\lambda_{(\dot\alpha}\mu_{\dot\beta)} + \left(\frac{\Lambda}{[\lambda\mu]}\right)^2 A\mu_{\dot\alpha}\mu_{\dot\beta} - \frac{1}{2}\epsilon_{\dot\alpha\dot\beta}\Lambda, \quad \text{and so forth}$$

- For special cases, such as Petrov type D, additional conditions appear.

The analysis at the level of Weyl tensors

Case I. An arbitrary solution of the KS equations

$$C^{\dot\alpha(4)} = 2\mu^{\dot\alpha}\mu^{\dot\alpha}\left(D^{\dot\alpha}\lambda^{\dot\alpha} + 2\lambda^{\dot\alpha}\lambda^{\dot\alpha}\right),$$

$$C^{\dot\alpha(2s)} = 2\mu^{\dot\alpha}\mu^{\dot\alpha}\mathcal{F}\left(\mu^{\dot\alpha}, \lambda^{\dot\alpha}, F_1^{(2s)}, F_2^{(2s)}\right), \quad \textit{for} \quad s > 2,$$

i.e. in each order there are two new functions $F_1^{(2s)}, F_2^{(2s)}$ that are algebraically independent of $C^{\dot\alpha(4)}$.

Case II. KS + Petrov type N

$$C^{\dot\alpha(4)} \propto \mu^{\dot\alpha}\mu^{\dot\alpha}\mu^{\dot\alpha}\mu^{\dot\alpha}$$

$$C^{\dot\alpha(2s)} = F_3^{(2s)}\mu^{\dot\alpha(2s)}.$$

contains an arbitrary function that is not expressed algebraically through $C^{\dot\alpha(4)}$, but the tensor structure of the expression is clear.

The analysis at the level of Weyl tensors

Case III. KS + Type D

$$C^{\dot\alpha(4)} = 2\mu^{\dot\alpha}\mu^{\dot\alpha}\tau^{\dot\alpha}\tau^{\dot\alpha},$$

$$C^{\dot\alpha(2s)} = 2\mu^{\dot\alpha}\mu^{\dot\alpha}\tau^{\dot\alpha}\tau^{\dot\alpha}\left(\sum_{k=0}^{s-2}(-1)^k\frac{1}{3}C^k_{s-2}\frac{(2s-5)!!}{(2s-2k-5)!!}(4s-2k-5)!!v_1^{\dot\alpha(2k)}v_2^{\dot\alpha(2s-2k-4)}\right),$$

where $\tau^{\dot\alpha} = \sqrt{\frac{8}{3}}\left(\lambda^{\dot\alpha} - \frac{3}{8}X\mu^{\dot\alpha}\right)$, $v_1^{\dot\alpha} = \frac{2}{3}\lambda^{\dot\alpha} + \frac{1}{2}X\mu^{\dot\alpha}$, $v_2^{\dot\alpha} = \frac{2}{3}\lambda^{\dot\alpha} - X\mu^{\dot\alpha}$, $(-1)!! = 1$, $X = \frac{\Lambda}{[\lambda\mu]}$.

Case IV. KS + Type D + $\left(D\lambda = \frac{2}{3}\lambda\lambda\right)$

$$C^{\dot\alpha(2s)} = \frac{2}{9}\left(\frac{2}{3}\right)^{2s-4}(2s)!\mu^{\dot\alpha(2)}\lambda^{\dot\alpha(2s-2)}.$$

Example: Eguchi-Hanson metric. The same principal null spinors as in the gravity appear to be principal null spinors for higher spins, i.e. in case IV we have a straightforward multycopying procedure.

Conclusions

- We studied the possibility to generalize SD classical double copy to higher spins
- At the level of prepotentials a straightforward generalization exists. This allowed us to find a family of self-dual higher-spin solutions that generalizes an arbitrary solution of SDGR in KS-form.
- The presence of a pattern at the level of Weyl tensors depends on the Petrov types of the original metric.