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МФТИ

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Global quenches and correlator dynamics in de Sitter space

based on arXiv:2511.13563
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Problems of the Modern Mathematical Physics – PMMP'26
Dubna, 2026

Motivation

Quantum quench - a sudden change of a system's parameter.
What is the subsequent relaxation to equilibrium?

Results in literature:

- In holography and condensed matter physics, local quenches probe integrability, chaotic features, and thermalization rates.
- In field theory, global quenches are often studied using the "$m(t)$ quench protocol" (with M $\rightarrow$ m in the sudden limit).
- Quench dynamics has also been studied in the context of early Universe evolution, due to fast phase transitions during inflation.

Our goal:

- Introduce a new class of initial states that yield solutions for subsequent non-equilibrium dynamics.
- Employ the Schwinger-Keldysh functional integral framework

Basement in quench dynamics

In flat spacetime, the post-quench state is often described as a generalized Calabrese-Cardy (gCC) state

$$|\text{in}\rangle = |\text{gCC}\rangle = \exp\left(-\frac{1}{2}\sum_k \kappa(k)\hat{a}^\dagger_{\text{out}}(k)\hat{a}_{\text{out}}(k)\right)|D\rangle\,, \tag{1}$$

where $|D\rangle$ is a Dirichlet boundary state $\hat{\phi}\,|D\rangle = 0$.

For even spacetime dimensions D, the relaxation of the two-point correlation function after a quench

$$\begin{aligned} \langle\phi\phi\rangle_{\text{gCC}} &= \langle\phi\phi\rangle_{\text{GGE}} + \text{tail}(t), \\ \text{tail}(t) &\sim \frac{1}{(mt)^{\frac{D-1}{2}}}\cos(2mt+\delta). \end{aligned} \tag{2}$$

Building blocks

Consider free scalar field theory in curved background

$$S[\phi] = \frac{1}{2}\int_{t_0}^{+\infty} dt \int d^{D-1}\mathbf{x}\sqrt{|g(t,\mathbf{x})|}\left(g^{\mu\nu}\partial_\mu\phi\,\partial_\nu\phi - m^2\phi^2\right). \tag{3}$$

The evolution of the quantum average of some composite operator

$$\begin{aligned}\langle\hat{\mathcal{O}}[\hat{\phi}(\mathbf{x})]\rangle_t \equiv \mathrm{tr}\left\{\hat{\rho}(t)\hat{\mathcal{O}}\right\} = \mathrm{tr}\left\{\hat{\rho}_0\hat{U}^\dagger(t,t_0)\hat{\mathcal{O}}\hat{U}(t,t_0)\right\} \equiv \\ \equiv \mathrm{tr}\left\{\hat{\rho}_0\hat{U}^\dagger(t,t_0)\hat{\mathcal{O}}\hat{U}^\dagger(+\infty,t)\hat{U}(+\infty,t_0)\right\}.\end{aligned} \tag{4}$$

Building blocks

The standard transition from the evolution operator to the path integral

$$\langle \hat{\mathcal{O}}[\hat{\phi}(\mathbf{x})] \rangle_t = \sum_{\xi_1,\xi_2,\chi} \langle \xi_1 | \hat{\rho}_0 | \xi_2 \rangle \langle \xi_2 | \hat{U}^\dagger(t,t_0) \hat{\mathcal{O}} \hat{U}^\dagger(+\infty,t) | \chi \rangle \langle \chi | \hat{U}(+\infty,t_0) | \xi_1 \rangle$$

$$= \int \mathcal{D}\chi(\mathbf{x}) \mathcal{D}\xi_1(\mathbf{x}) \mathcal{D}\xi_2(\mathbf{x}) \langle \xi_1 | \hat{\rho}_0 | \xi_2 \rangle \times \tag{5}$$

$$\times \int_{\phi_+(t_0)=\xi_1}^{\phi_+(+\infty)=\chi} \mathcal{D}\phi_+ \int_{\phi_-(t_0)=\xi_2}^{\phi_-(+\infty)=\chi} \mathcal{D}\phi_- \; e^{iS[\phi_+]-iS[\phi_-]} \mathcal{O}[\phi_-(t,\mathbf{x})].$$

The Schwinger-Keldysh formalism

Introducing the Keldysh rotation $\phi_{cl} = \frac{\phi_+ + \phi_-}{2}, \quad \phi_q = \phi_+ - \phi_-$:

$$\begin{aligned} \mathcal{S}_K[\phi_{cl}, \phi_q] = S[\phi_+] - S[\phi_-] = \int_{t_0}^{+\infty} dt \int d^{D-1}\mathbf{x}\sqrt{|g|}\left(g^{\mu\nu}\partial_\mu\phi_q\,\partial_\nu\phi_{cl} + m^2\phi_q\phi_{cl}\right) = \\ = -\int d^{D-1}\mathbf{x}\,(\xi_1(\mathbf{x}) - \xi_2(\mathbf{x}))\,\pi(t_0,\mathbf{x}) - \int d^D x\sqrt{|g|}\phi_q\left[\frac{1}{\sqrt{|g|}}\partial_\mu\sqrt{|g|}\partial^\mu + m^2\right]\phi_{cl}, \end{aligned} \tag{6}$$

Using the further convenient notations $\beta(\mathbf{x}) = \xi_1(\mathbf{x}) - \xi_2(\mathbf{x})$ and $\Phi(\mathbf{x}) = \frac{\xi_1(\mathbf{x}) + \xi_2(\mathbf{x})}{2}$, expression (5) is rewritten as

$$\begin{aligned} \langle\hat{\mathcal{O}}[\hat{\phi}(\mathbf{x})]\rangle_t = \int \mathcal{D}\Phi(\mathbf{x})\mathcal{D}\Pi(\mathbf{x})\mathrm{W}_0[\Phi,\Pi]\,\hat{\mathcal{O}}\left[\phi\Big|_{\text{on-shell}}\right], \\ \mathrm{W}_0[\Phi,\Pi] = \int \mathcal{D}\beta(\mathbf{x})\,\exp\left(-i\int d^{D-1}\mathbf{x}\beta(\mathbf{x})\Pi(\mathbf{x})\right)\,\langle\Phi + \frac{\beta}{2}|\,\hat{\rho}_0\,|\Phi - \frac{\beta}{2}\rangle\,. \end{aligned} \tag{7}$$

The on-shell conditions

$$\begin{aligned} \mathrm{EOM}\left[\phi\Big|_{\text{on-shell}}\right] = 0, \quad \phi(t_0,\mathbf{x})\Big|_{\text{on-shell}} = \Phi(\mathbf{x}), \quad \partial_t\phi(t_0,\mathbf{x})\Big|_{\text{on-shell}} = \frac{1}{\sqrt{|g|}g^{00}}\Pi(\mathbf{x}), \\ \phi(t,\mathbf{x})\Big|_{\text{on-shell}} = -\int d^{D-1}\mathbf{y}\left[\sqrt{|g(t_0,\mathbf{y}|}g^{00}(t_0,\mathbf{y})\partial_t G_R(t,\mathbf{x}|t_0,\mathbf{y})\Phi(\mathbf{y}) + G_R(t,\mathbf{x}|t_0,\mathbf{y})\Pi(\mathbf{y})\right] \end{aligned} \tag{8}$$

New approach to quench dynamics I

Now we take the system out-of-equilibrium by the action of quench operator:

$$\hat{\rho}_Q(t_0) = \hat{Q}\hat{\rho}_0\hat{Q}^\dagger, \quad \hat{Q} = e^{i\alpha\hat{V}[\hat{\phi}(\mathbf{x})]} \tag{9}$$

The modification of the Wigner functional can be represented as

$$\mathrm{W}_Q[\Phi,\Pi] = \mathcal{Q}\left[\Phi, \frac{\delta}{\delta\Pi}\right]\mathrm{W}_0[\Phi,\Pi], \tag{10}$$

where we introduce the functional quench operator

$$\mathcal{Q}\left[\Phi, \frac{\delta}{\delta\Pi}\right] = \exp\left\{-i\alpha\left(\hat{V}\left[\Phi - \frac{i}{2}\frac{\delta}{\delta\Pi}\right] - \hat{V}\left[\Phi + \frac{i}{2}\frac{\delta}{\delta\Pi}\right]\right)\right\}. \tag{11}$$

Then, after functional integration by parts, the general post-quench evolution

$$\boxed{\langle\hat{\mathcal{O}}[\hat{\phi}(\mathbf{x})]\rangle_t^Q = \int \mathcal{D}\Phi(\mathbf{x})\mathcal{D}\Pi(\mathbf{x})\mathrm{W}_0[\Phi,\Pi]\,\mathcal{Q}\left[\Phi, -\frac{\delta}{\delta\Pi}\right]\hat{\mathcal{O}}\left[\phi\Big|_{\text{on-shell}}\right]} \tag{12}$$

New approach to quench dynamics II

For the simplest case of a Gaussian initial perturbation

$$\hat{V}[\hat{\phi}(\mathbf{x})] = \int d^{D-1}\mathbf{x}\sqrt{|g(t_0,\mathbf{x})|}\hat{\phi}^2(\mathbf{x}),$$
$$\mathcal{Q}\left[\Phi, -\frac{\delta}{\delta\Pi}\right] = \exp\left(2\alpha\int d^{D-1}\mathbf{x}\,\Phi(\mathbf{x})\frac{\delta}{\delta\Pi(\mathbf{x})}\right). \tag{13}$$

Using (12), we can derive the following closed-form expression for the exact post-quench dynamics:

$$iG_K^Q(t,\mathbf{x}|t,\mathbf{y}) \equiv \langle\hat{\phi}(\mathbf{x})\hat{\phi}(\mathbf{y})\rangle_t^Q = iG_K(t,\mathbf{x}|t,\mathbf{y}) -$$
$$-2\alpha\int d^{D-1}\xi\sqrt{|g(t_0,\xi)|}\Big[iG_K(t,\mathbf{x}|t_0,\xi)\,G_R(t,\mathbf{y}|t_0,\xi) + iG_K(t,\mathbf{y}|t_0,\xi)\,G_R(t,\mathbf{x}|t_0,\xi)\Big] +$$
$$+8\alpha^2\int d^{D-1}\xi_1 d^{D-1}\xi_2\sqrt{|g(t_0,\xi_1)|}\sqrt{|g(t_0,\xi_2)|}iG_K(t_0,\xi_1|t_0,\xi_2)G_R(t,\mathbf{x}|t_0,\xi_1)G_R(t,\mathbf{x}|t_0,\xi_2) \tag{14}$$

It would be more convenient to treat the integrals in the momentum space (if the spatial slices are homogeneous):

$$iG_K^Q(t,\mathbf{x}|t,\mathbf{y}) = iG_K(t,\mathbf{x}|t,\mathbf{y}) - 4\alpha I_1(t,t_0,|\mathbf{x}-\mathbf{y}|) + 8\alpha^2 I_2(t,t_0,|\mathbf{x}-\mathbf{y}|) \tag{15}$$

Flat space-time example I

To test our approach for global quench dynamics, consider a free massive scalar field in flat spacetime:

$$
\begin{aligned}
iG_K(\mathbf{k}|t,t_0) &= \frac{1}{2\sqrt{k^2+m^2}}\cos\left(\sqrt{k^2+m^2}(t-t_0)\right),\\
G_R(\mathbf{k}|t,t_0) &= -\frac{\theta(t-t_0)}{\sqrt{k^2+m^2}}\sin\left(\sqrt{k^2+m^2}(t-t_0)\right).
\end{aligned}
\tag{16}
$$

The corresponding post-quench dynamics after a gaussian perturbation of the initial state is

$$
\begin{aligned}
iG_K^Q(t,\mathbf{x}|t,\mathbf{y}) &= \underbrace{\frac{1}{4\pi^2}\frac{m}{r}K_1(mr)+\frac{\alpha^2}{\pi^2}K_0(mr)}_{iG_K^{\text{post-quench}}(t,\varkappa|t,\mathbf{y})}+\text{tail}(t,r)\\
\text{tail}(t,r) &\sim \frac{1}{(mt)^{3/2}}\cos\left(2mt+\delta\right)
\end{aligned}
\tag{17}
$$

The result is qualitatively the same as for the $m(t)$ quench protocol!

Flat space-time example II

For the massless field, we obtain the exact solution

$$iG_K^Q(t,\mathbf{x}|t,\mathbf{y}) = \frac{1}{4\pi^2 r^2} + \frac{\alpha}{4\pi^2 r}\log\left[\frac{2t+r}{2t-r}\right] + \frac{\alpha^2}{\pi^2}\left\{\log\left[\frac{\sqrt{4t^2-r^2}}{r}\right] + \frac{t}{r}\log\left[\frac{2t+r}{2t-r}\right]\right\} \quad (18)$$

But...

- This two-point function does NOT equilibrate at late times!
- This result differs from the $m(t)$ quench protocol in $D = 4$ dimensions: similar non-relaxating behavior is known in $D = 2$ and $D = 3$.

This feature will be studied in more detail in future work.

Quantum quench in the expanding Poincarè patch I

Consider the case of the expanding Poincare patch of de Sitter spacetime as the background geometry:

$$ds^2 = \frac{1}{\eta^2}\left(d\eta^2 - d\mathbf{x}^2\right), \quad \eta = e^{-t},$$

$$iG_K(\mathbf{k}|t_1,t_2) = \mathrm{Re}\left\{f_k(\eta_1)f_k^*(\eta_2)\right\}, \quad G_R(\mathbf{k}|t_1,t_2) = -2\theta(t_1-t_2)\,\mathrm{Im}\left\{f_k(\eta_1)f_k^*(\eta_2)\right\}, \tag{19}$$

$$f_k(\eta) = \eta^{\frac{D-1}{2}}\frac{\sqrt{\pi}}{2}H_\nu^{(1)}(k\eta), \quad \nu = \sqrt{\frac{(D-1)^2}{4} - m^2}.$$

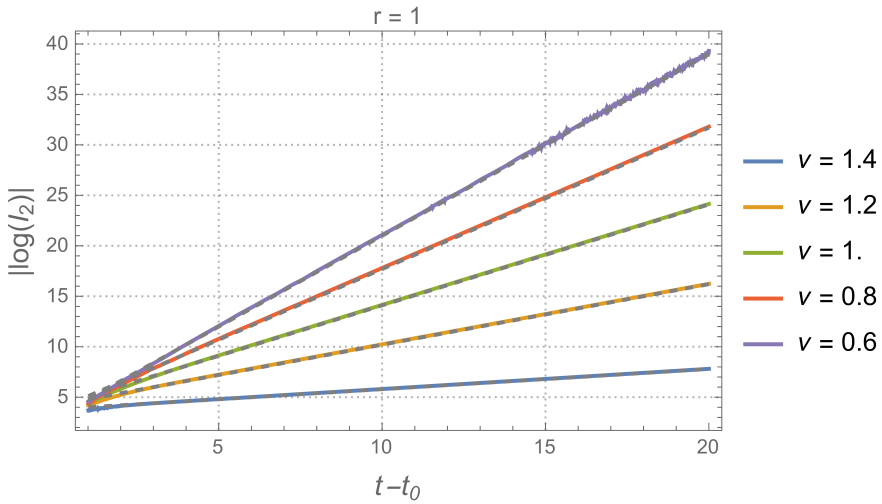
1. If $m < m_{cr} = \sqrt{2}$:

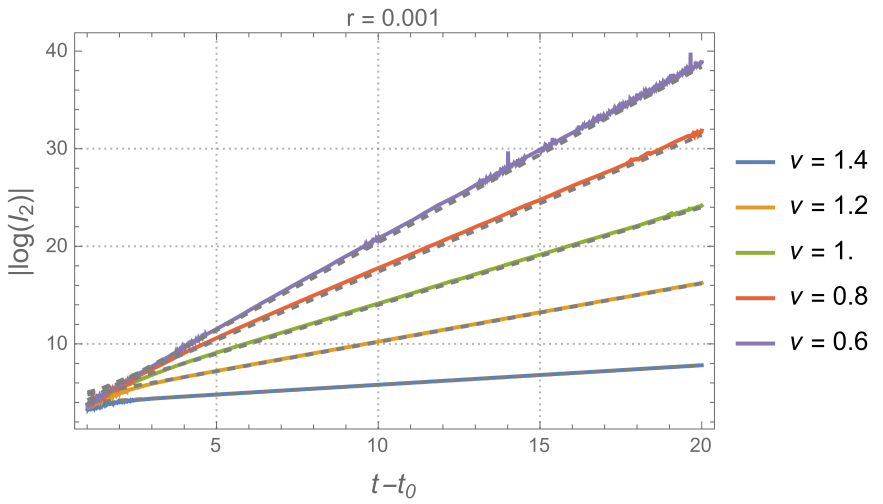
$$iG_K^Q = iG_K + i\Delta_Q G_K \sim a\left(\frac{\eta}{r}\right)^{3-2\nu} + b\left(\frac{\eta}{\eta_0}\right)^{3-2\nu}. \tag{20}$$

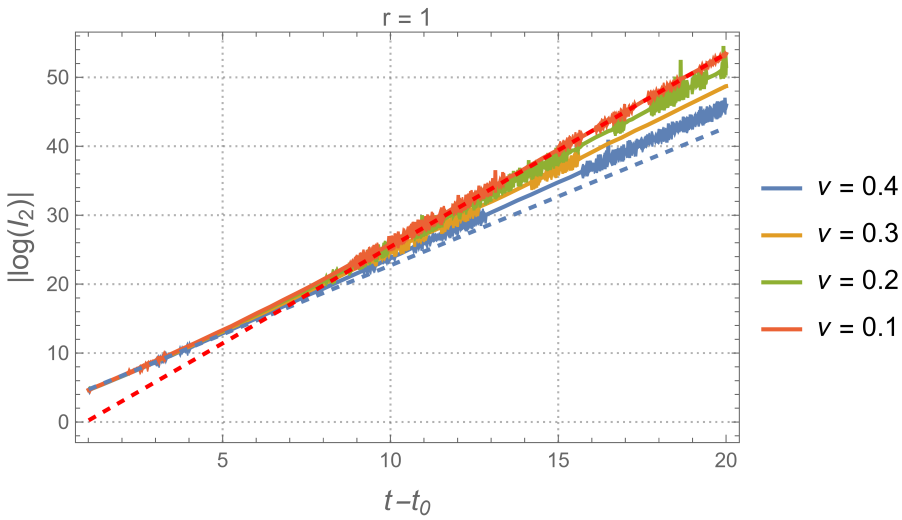
2. If $m > m_{cr}$:

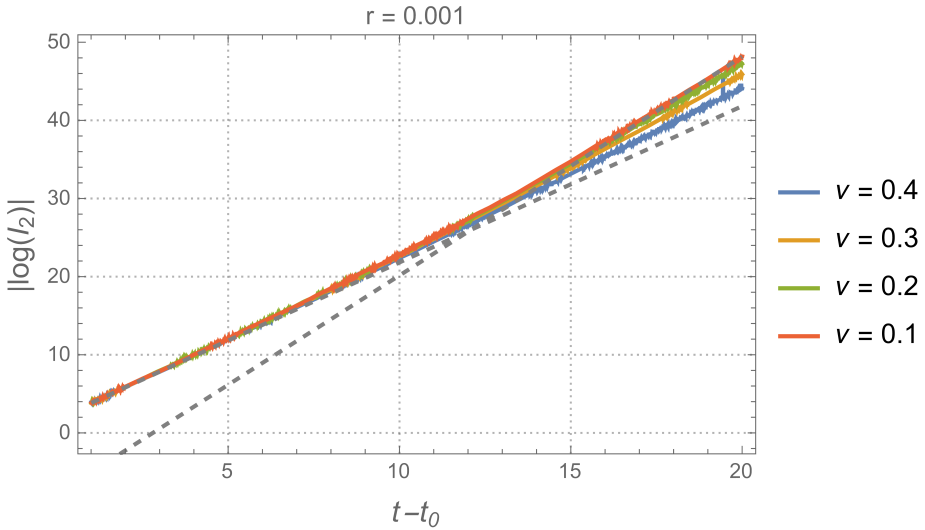
$$iG_K^Q = iG_K + i\Delta_Q G_K \sim a\left(\frac{\eta}{r}\right)^{3-2\nu} + \begin{cases} b\left(\frac{\eta}{\eta_0}\right)^2, & \eta \gtrsim r, \\ b\left(\frac{\eta}{\eta_0}\right)^{3-2\nu}, & \eta \lesssim r. \end{cases} \tag{21}$$

Quantum quench in the expanding Poincarè patch II







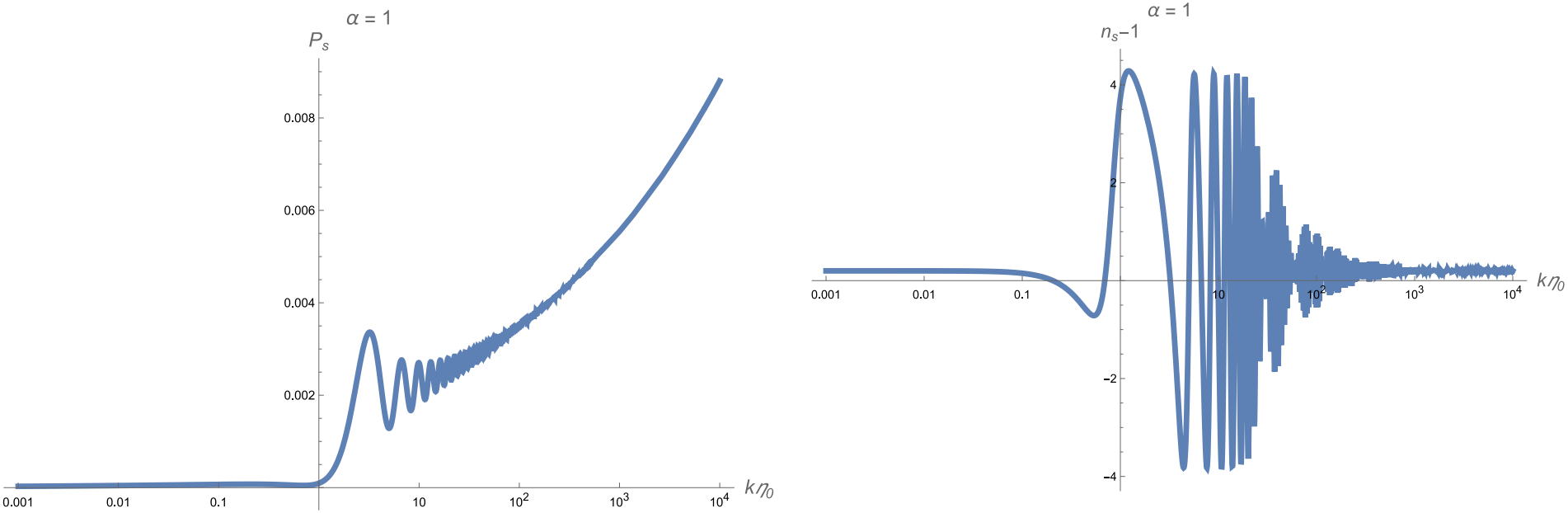


The numerical plots of $|\log(I_2)|$ as a function of time $t - t_0$. The dashed lines show the analytical estimate.

Quantum quench in the expanding Poincarè patch III

Another application of nonequilibrium dynamics on the de Sitter background that is relevant for the early Universe is the study of the scalar power spectrum

$$\mathcal{P}_s(k) = \frac{k^3}{2\pi^2} iG_K^Q(\mathbf{k}|t,t), \quad n_s(k) - 1 = \frac{d\log\mathcal{P}_s}{d\log k} \tag{22}$$



Power spectrum $\mathcal{P}_s$ and spectral index $n_s - 1$ as the functions of the scale $k\eta_0$ for $\alpha = 1$.

Conclusion

1. We have shown that for massive fields, the new approach yields qualitatively identical results for the relaxation rate after a global "Gaussian" quench compared to the standard approach.

2. The method can be easily generalized to include non-homogeneous ($\alpha \to \alpha(\mathbf{x})$) and non-Gaussian ($\hat{V}[\phi] \sim \hat{\phi}^n$) initial distributions

3. It is applicable in any spacetime for which the corresponding In-In Green's functions are known.

4. Using this class of states, we should reinvestigate critical post-quench dynamics, relaxation of local composite operators $\hat{T}_{\mu\nu}$, possible modifications of power spectra $\mathcal{P}_s(k)$, and secularly growing loop corrections.

Thank you for your time and attention!



Appendix A:

$$
\begin{aligned}
I_1 &= \int \frac{d^{D-1}\mathbf{k}}{(2\pi)^{D-1}} e^{i\mathbf{k}(\mathbf{x}-\mathbf{y})} \sqrt{|g(t_0)|}\, iG_K(\mathbf{k}|t,t_0) G_R(\mathbf{k}|t,t_0) \\
I_2 &= \int \frac{d^{D-1}\mathbf{k}}{(2\pi)^{D-1}} e^{i\mathbf{k}(\mathbf{x}-\mathbf{y})} \sqrt{|g(t_0)|}\sqrt{|g(t_0)|}\, iG_K(\mathbf{k}|t,t_0) G_R(\mathbf{k}|t,t_0) G_R(\mathbf{k}|t,t_0)
\end{aligned}
\tag{23}
$$