Quantization of totally antisymmetric tensor spinor field in the AdS space

I.L. Buchbinder

BLTP, JINR, Dubna

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Goals

Quantization procedure for constructing effective action in gauge theories with linearly dependent generators (first stage of reducibility).

Covariant quantization of totally antisymmetric tensor spinor field in AdS_d space

Based on A. Barvinsky, I.B, V. Krykhtin, D. Nesterov, Phys.Rev. D, 2025; Phys.Rev. D, 2026.

Plan

- Motivations
- Model of totally antisymmetric tensor spinor field and problem of its quantization
- General gauge theories
- Faddeev-Popov quantization procedure
- Adaptation of Faddeev-Popov procedure for first-stage reducible gauge theories
- Quantization of second rank totally antisymmetric tensor spinor theory
- Summary

Motivations

- Gauge fields are one of main ingredient of any approaches to unified theories of fundamental interactions. Therefore, the development of methods for quantizing various gauge theories, in particular, the derivation of the correct functional integral for such theories, is one of important general trend of quantum field theory.
- The correct functional integral for theories of the Yang-Mills type was simultaneously constructed long enough ago by Faddeev and Popov on the one hand and De Witt on the other hand. Faddeev-Popov quantization method is extremely simple and intuitively clear and is based on the factorization procedure of gauge group volume from naive functional integral.
- Some later, general methods for quantizing practically arbitrary gauge theories were developed (canonical BFV (Batalin-Fradkin-Vilkovisky) method and covariant BV (Batalin-Vilkovisky) method).

Motivations

- The development of superstring theory has led to the construction of a variety of supersymmetric gauge field theories, in particular, supergravity in various dimensions. All such theories contain the antisymmetric fields with gauge structure of a non-Yang-Mills type and a functional integral cannot be written on the base of Faddeev-Popov prescription. First example of such theories is Ogievetsky-Polubarinov model of notoph. Also there are the gauge models with constrained parameters. Chalenge for methods of quantization!
- Quantization of such theories can in principle be carried out in the framework of general methods, however, such general methods are complicated enough since they involve a large number of auxiliary and ghosts fields, the exclusion of which in concrete theories requires a certain amount of work. Many theories that are not of the Yang-Mills type are Abelian, and as noted in the basic Batalin-Vilkovisky paper, "quantizing such theories within our method is the same as cracking nuts with a forge press."
- It seems reasonable to develop a simple and efficient method for quantizing non - Yang-Mills type gauge theories by adapting some way the Faddeev-Popov quantization procedure.
- Recently proposed theory of totally antisymmetric tensor spinor field as an example of applying quantization procedure under consideration

Model of totally antisymmetric tensor spinor field and problem of its quantization

- Theory of totally antisymmetric tensor spinor filed $\psi_{\mu_1\mu_2\ldots\mu_p} = \psi_{\{\mu_1\mu_2\ldots\mu_p\}} = \psi_{\mu[p]}$ (fermionic p-form theory, I.L.B, V.A. Krykhtin, L.L. Ryskina, 2009; Yu.M. Zinoviev, 2009).

$$S[\psi,\bar{\psi}] = \int d^dx\sqrt{|g|}\big(\bar{\psi}_{\nu_1\nu_2\ldots\nu_p} i\gamma^{\nu_1\nu_2\ldots\nu_p\rho\mu_1\mu_2\ldots\mu_p} D_\rho - m\big)\psi_{\mu_1\mu_2\ldots\mu_p}$$

  Theory is defined only on AdS space.

- Consistency conditions: AdS_d space, $d < 2p$. Here $D_\mu = \nabla_\mu \pm \frac{i}{2}\sqrt{r}\gamma_\mu$, $r > 0$ is a modulus of AdS curvature.
- In massless case this is reducible gauge theory with $p-1$-stage reducible gauge transformations:

$$\delta\psi_{\mu[p]} = D_\mu\lambda_{\mu[p-1]},\ \delta\lambda_{\mu[p-1]} = D_\mu\lambda_{\mu[p-2]}, \ldots, .$$

Model of totally antisymmetric tensor spinor field and problem of its quantization

Quantization problem

For simplicity, one considers the case of $p = 2$.

- Gauge transformations:

$$\delta\psi_{[\mu\nu]} = D_{[\mu}\lambda_{\nu]},\ \delta\lambda_\nu = D_\nu\lambda,\ \delta\lambda = 0$$

- Covariant gauge fixing condition for $p = 2$:
  $\chi_\mu = \gamma^\nu\psi_{\mu\nu} + \frac{1}{d}\gamma_\mu\gamma^{\alpha\beta}\psi_{\alpha\beta}, \gamma^\mu\chi_\mu = 0$. (Gauge condition is degenerate).
- Faddeev-Popov matrix is degenerate. Faddeev-Popov prescription does not work. Quantization problem!

Quantization: A. Barvinsky, I.B, V. Krykhtin, D. Nesterov (p=2, PRD 2025); (arbitrary p, PRD 2026).

General gauge theory

General gauge theory is defined in terms of De Witt condensed notations by the following conditions:

- Set of fields φ^i
- Action functional $S[\varphi]$
- Infinitesimal (gauge) transformations $\delta\varphi^i = R^i{}_\alpha[\varphi] f^\alpha$
- Condition of gauge invariance $S_{,i}[\varphi] R^i{}_\alpha[\varphi] = 0$

Notations: all continuous and discrete indices included into i and α, $R^i{}_\alpha[\varphi]$ are the gauge generators, f^α are the gauge parameters, $S_{,i} = \frac{\delta S}{\delta\varphi^i}$. On repeating indices the integration and summation are carried out.

$$R_\alpha[\varphi] = R^i{}_\alpha[\varphi] \frac{\delta}{\delta\varphi^i}$$

$$[R_\alpha[\varphi], R_\beta[\varphi]] = F^\gamma{}_{\alpha\beta}[\varphi] R_\gamma[\varphi] + X^{ij}{}_{\alpha\beta}[\varphi] S_{,j}[\varphi] \frac{\delta}{\delta\varphi^i}$$

Structure functions $F^\gamma{}_{\alpha\beta}[\varphi] = F^\gamma{}_{[\alpha\beta]}[\varphi]$. $X^{ij}{}_{\alpha\beta} = X^{(ij)}{}_{[\alpha\beta]}$. Gauge algebra is called closed if $X^{ij}{}_{\alpha\beta} = 0$. In opposite case it is called open.
The generators are called linearly dependent if there exists the functional $Z^\alpha[\varphi]$ such as $R^i{}_\alpha Z^\alpha[\varphi] = 0$.
Reducible gauge theory.

Structure of reducible gauge theories

- If to consider the generators $R^i_\alpha[\varphi]$ as matrices with indices i and α, then the relation $R^i{}_\alpha Z^\alpha[\varphi] = 0$ means that functional Z^α is a null-vector of these matrices.
- In principle, there can be several such null-vectors, $R^i{}_\alpha[\varphi] Z^\alpha{}_a[\varphi] = 0$, $\alpha = 1, \dots m_0$, $a = 1 \dots m_1, m_1 < m_0$.
- In principle, these null-vectors can have their own null-vectors, $Z^\alpha{}_{a_1} Z^{a_1}{}_{a_2} = 0$ and etc. Further we will consider the case when the generators have only one null-vector. The corresponding theories are called first stage reducibility theories.
- Quantization of reducible gauge theories requires developing special methods.

Faddeev-Popov quantization procedure

Faddeev-Popov quantization procedure is a simple enough and intuitively clear method to obtain the correct functional integral for gauge theories with closed gauge algebra and independent gauge generators (Yang-Mills type theories).

Procedure consists of several steps.

- It is useful to use the notion of a gauge group orbit. Let φ_0 is a some gauge field. Orbit of this field is a set of all fields that are obtained from given field φ_0 with help of gauge transformations with all gauge parameters f.
- Set of all fields φ can be represent as a set of all non-intersecting orbits $\{\varphi\}$ and all gauge transformations f. Due to gauge invariance, $S[\varphi] = S[\{\varphi\}]$.
- Consider formally a functional integral

$$\int \mathcal{D}\varphi e^{iS[\varphi]} = \int \mathcal{D}f \int \mathcal{D}\{\varphi\} e^{iS[\{\varphi\}]} = V \times \int \mathcal{D}\{\varphi\} e^{iS[\{\varphi\}]},$$

  where V is a formal volume of gauge group.
- Correct functional integral should have the form $\int \mathcal{D}\{\varphi\} e^{S[\{\varphi\}]}$. However such a form is usefulness since integral over orbits is ill-defined. The problem arises of how to rewrite the integral over the orbits as an integral over the initial fields. This problem was solved in the work of Faddeev and Popov in 1967 in context of Yang-Mills theory.

Faddeev-Popov quantization procedure

- First, one introduces gauge fixing functions $\chi^\alpha[\varphi]$ and the functional $\Delta[\varphi]$ by the equation

$$\Delta[\varphi]\int \mathcal{D}f\delta[\chi[\varphi^f]] = 1,$$

  where $\varphi^f = \varphi + \delta\varphi$. One can prove that functional $\Delta[\varphi]$ is gauge invariant, $\Delta[\varphi^f] = \Delta[\varphi]$.
- Functional $\Delta[\varphi]$ is calculated in the form of functional determinant (Faddeev-Popov determinant) $\Delta[\varphi] = DetQ^\alpha{}_\beta[\varphi]$ with Faddeev-Popov matrix $Q^\alpha{}_\beta[\varphi] = \chi^\alpha{}_{,i}[\varphi]R^i{}_\beta[\varphi]$.
- Consider a naive integral and insert the unit in the form $\Delta[\varphi]\int \mathcal{D}f\delta[\chi[\varphi^f]] = 1$. It leads to

$$\int \mathcal{D}\varphi e^{iS[\varphi]} = \int \mathcal{D}\varphi e^{iS[\varphi]} \underbrace{\Delta[\varphi]\int \mathcal{D}f\delta[\chi[\varphi^f]]}_{1}.$$

- Change of variables $f \to f^{-1}$, use invariance integral over group under group shifts. Integral over group is factorized and one gets:

$$\int \mathcal{D}\varphi e^{iS[\varphi]} = V \times \int \mathcal{D}\varphi e^{iS[\varphi]}\Delta[\varphi]\delta[\chi[\varphi]].$$

Faddeev-Popov quantization procedure

Final result: correct functional integral for gauge theories under consideration is postulated in the form

$$\int \mathcal{D}\varphi e^{iS[\varphi]}\Delta[\varphi]\delta[\chi[\varphi]].$$

Comments

1. Procedure is simple enough and intuitively clear. However, this procedure involves many ill-defined operations. Therefore, the final result needs justification.

   In particular:
2. It is assumed that the gauge transformations form a group. It can be justified for theories with closed gauge algebra and independent generators.
3. Faddeev-Popov determinant vanishes for reducible gauge theories where $R^i{}_\alpha Z^\alpha = 0$

   $$Q^\alpha{}_\beta Z^\beta = \chi^\alpha{}_{,i} R^i{}_\beta Z^\beta = 0.$$

   Faddeev-Popov method is not applicable for theories with linearly dependent generators. Such theories require a special consideration.
4. It is reasonable to try to adapt the Faddeev-Propov procedure for gauge theories with dependent generators such that preserve the simplicity of the original Faddeev-Popov method.

Delta-function for first-stage reducible gauge conditions

Motivation: Faddeev-Popov quantization procedure uses the delta-function of gauge fixing condition. How can such a delta function be introduced in the reducible gauge theory? Consider first-stage reducible theory, $R^i{}_\alpha[\varphi]Z^\alpha[\varphi] = 0$. Number of gauge conditions in conventional gauge theories is equal to a number of independent gauge parameters, that is to values of index α. However, in the reducible theories, the gauge parameters themselves are subject to their gauge transformations with their parameters and a number of independent parameters less then a number of values of index α. In general case it is natural to use the gauge conditions with index α. But then, we should assume that the gauge fixing conditions $\chi^\alpha[\varphi]$ satisfy at least the relation $\bar{Z}_\alpha[\varphi]\chi^\alpha[\varphi] = 0$ with some functional $\bar{Z}_\alpha$. In this case the delta-function is ill defined, $\delta[\chi^\alpha[\varphi]] \sim \delta(0)$. It is clear that correct delta-function of reduced gauge condition requires a independent definition related to factorization of gauge group volume from naive expression $\delta[\chi^\alpha[\varphi]]$. Next we consider the theory of first stage of reducibility where we need the only functional $\bar{Z}_\alpha$.

Adaptation of Faddeev-Popov procedure for factorization of gauge volume in reducible gauge theories was invented in the work of I.L.B, S.M. Kuzenko, 1988 at quantization of the reducible $\mathcal{N} = 1$ superfield gauge models. Here we discuss formulation in general gauge theory.

Delta-function for reducible gauge conditions

- Consider naive expression

$$\delta[\chi] = \int D\pi e^{i\pi_\alpha \chi^\alpha}$$

- Integral is ill-defined since the integrand is invariant under gauge transformations of π with gauge parameter ξ

$$\pi_\alpha \to \pi^\xi_\alpha = \pi_\alpha + \bar{Z}_\alpha \xi,\ \pi^\xi_\alpha \chi^\alpha = \pi_\alpha \chi^\alpha.$$

- To construct a correct integral for delta-function, we again use the Faddeev-Popov procedure to factorize the volume of gauge group. To do that, one introduces the gauge fixing condition for π_α in the form $\sigma[\pi] = \pi_\alpha \sigma^\alpha$ with some function σ^α.
- Now one uses the Faddeev-Popov procedure. Introduce the functional $\bar{\Delta}_1$ by equation

$$\bar{\Delta}_1 \int D\xi \delta[\sigma[\pi^\xi]] = 1,\ \ \bar{\Delta}_1 = Det(\bar{Z}_\alpha \sigma^\alpha) = Det\bar{Q}.$$

Delta-function for first-stage reducible gauge conditions

- After that, the naive integral for delta-function takes the form

$$\int D\pi e^{i\pi_\alpha \chi^\alpha} = \int D\pi e^{i\pi_\alpha \chi^\alpha} \underbrace{\bar{\Delta}_1 \int D\xi \delta[\sigma[\pi^\xi]]}_{1} =$$

$$\int D\xi \int D\pi e^{i\pi_\alpha \chi^\alpha} Det\bar{Q}\delta[\sigma[\pi]] = V_1 \times Det\bar{Q} \int Dc\delta[\chi^a + \sigma^\alpha c]$$

  where V_1 is a volume of gauge transformation group of the variables π_α.

- As a result, one defines the delta-function of reducible gauge conditions in the form

$$\hat{\delta}[\chi] = Det\bar{Q} \int Dc\delta[\chi^a + \sigma^\alpha c].$$

- One can prove that $\hat{\delta}[\chi]$ does not depend on σ^α.

Construction of functional integral for first-stage reducible gauge theory

Next, to construct the functional integral, one uses the correct delta function $\hat{\delta}[\chi]$. Following the Faddeev-Popov procedure, we should introduce the functional Δ by the equation

$$\Delta \int Df \hat{\delta}[\chi[\varphi^f]] = 1.$$

However, here we immediately meet a new trouble. The integral over Df is ill-defined in reducible gauge theories. The matters is that any functional $\Phi[\varphi^f]$ is gauge invariant under the transformations of the parameters:
$f^\alpha : f^\alpha \to (f^{f_1})^\alpha = f^\alpha + Z^\alpha f_1$

$$(\varphi^{f^{f_1}})^i = \varphi^i + R^i{}_\alpha f^\alpha + R^i{}_\alpha Z^\alpha f_1 = \varphi^i + R^i{}_\alpha f^\alpha = (\varphi^f)^i,$$

where the relation $R^i{}_\alpha Z^\alpha = 0$ has been used. Due to gauge invariance of integrand, the integral over f is ill-defined and the functional Δ can not be calculated. The integral over f involves the integral over gauge group with parameters f_1 which should be factorized. Like in conventional gauge theory, for factorization of gauge group volume one applies again the Faddeev-Popov procedure.

Construction of functional integral for first-stage reducible gauge theory

The factorization is realized as follows

1. Introduce the gauge fixing condition for f in the form $\omega[f] = \omega_\alpha f^\alpha$ with some function ω_α.
2. Define the functional Δ_1 by the equation

$$\Delta_1 \int Df_1 \delta[\omega[f^{f_1}]] = 1, \;\; \Delta_1 = Det(\omega_\alpha Z^\alpha) = DetQ_1.$$

3. Consider the naive integral

$$\int Df \Phi[\varphi^f] = \int Df \Phi[\varphi^f] \Delta_1 \int Df_1 \delta[\omega[f^{f_1}]] =$$

$$\int Df_1 \int Df \Delta_1 \delta[\omega[f] \Phi[\varphi^f] = V_1 \times \int D\mu_f \Phi[\varphi^f]$$

4. Integration measure of f to integrate the gauge invariant functionals of f

$$D\mu_f = Df DetQ_1 \delta[\omega[f^]]$$

Construction of functional integral for first-stage reducible gauge theory

Now apply the Faddeev-Popov procedure to initial integral of $e^{iS[\varphi]}$.

- Introduce the functional Δ by the equation

$$1 = \Delta \int D\mu_f \hat{\delta}\chi[\varphi^f] =$$

$$\Delta\, Det\bar{Q}\, DetQ_1 \int Df\, dc\, \delta[\omega_a f^a]\, \delta[\chi^\alpha + Q^\alpha{}_\beta f^\beta + \sigma^\alpha c]$$

Here $Q^\alpha{}_\beta$ is a naive (degenerate) Faddeev-Popov matrix, $Q^\alpha{}_\beta = \chi^a{}_{,i} R^i{}_\beta$, $Q^\alpha{}_\beta Z^\beta = 0$, $\bar{Z}_\alpha Q^\alpha{}_\beta = 0$.

- After some calculations

$$\Delta = \frac{DetF}{Det\bar{Q}\, DetQ_1}, \quad F^\alpha{}_\beta = Q^\alpha{}_\beta + \sigma^\alpha \omega_\beta,$$

where $F^\alpha{}_\beta$ is a correct non-degenerate Faddeev-Popov matrix.

- Factorization of gauge group volume

$$\int D\varphi e^{iS[\varphi]} = \int D\varphi e^{iS[\varphi]} \Delta \int D\mu_f \delta[\chi[\hat{\varphi}^f]] =$$

$$V \times \int D\varphi\, Dc\, e^{iS[\varphi]}\, \delta[\chi^\alpha + \sigma^a c]\, \frac{DetF}{DetQ_1}.$$

Quantization of second rank antisymmetric tensor spinor theory

- Model

$$S[\bar{\psi},\psi] = \int d^d x \sqrt{-g} \bar{\psi}_{\mu_1\mu_2} \gamma^{\mu_1\mu_2\rho\nu_1\nu_2} i D_\rho \psi_{\nu_1\nu_2},$$

$$D_\rho = \nabla_\rho \pm i \frac{r^{\frac{1}{2}}}{2} \gamma_\rho.$$

- Gauge transformations

$$\delta\psi_{\mu_1\mu_2} = D_{\mu_1}\lambda_{\mu_2} - D_{\mu_2}\lambda_{\mu_1},\ \delta\bar{\psi}_{\mu_1\mu_2} = \bar{D}_{\mu_1}\bar{\lambda}_{\mu_2} - \bar{D}_{\mu_2}\bar{\lambda}_{\mu_1}.$$

- Gauge transformations themselves are gauge invariant under the first-stage gauge transformations with spinor parameters

$$\delta\lambda_\mu = D_\mu\lambda,\ \ \delta\bar{\lambda}_\mu = \bar{D}_\mu\bar{\lambda}.$$

- Relation with the general scheme: fermionic gauge fleld $\varphi^i \to (\psi_{\mu_1\mu_2}, \bar{\psi}_{\mu_1\mu_2})$, fermionic parameters $f^\alpha \to (\lambda_\mu, \bar{\lambda}_\mu)$, $f \to (\lambda, \bar{\lambda})$
- Although the model is fermionic, its quantization completely corresponds to general scheme, taking into account that the fields and parameters are anticommuting and all relevant functional integrals are the Berezin type with the corresponding functional determinants.

- Gauge fixing functions: $f^\alpha \to (\chi_\mu, \bar{\chi}_\mu)$

$$\chi_\mu = \gamma^\nu \psi_{\mu\nu} + \frac{1}{d}\gamma_\mu \gamma^{\nu_1\nu_2}\psi_{\nu_1\nu_2}, \;\; \gamma^\mu \chi_\mu = 0..$$

- General quantization scheme leads to partition function

$$Z = \frac{\Delta_0^2}{\Delta_1^2}\int D\psi_{\mu\nu} D\bar{\psi}_{\mu\nu} D\psi D\bar{\psi} \delta[\chi_\mu(\psi_{\mu\nu}) + \gamma_\mu \psi]\delta[\bar{\chi}_\mu(\bar{\psi}_{\mu\nu}) + \gamma_\mu \bar{\psi}] e^{S[\psi_{\mu\nu}, \bar{\psi}_{\mu\nu}]}$$

- Linear change of variables to gamma-irreducible or gamma-transversal fields $\Psi_{\mu\nu}, \bar{\Psi}_{\mu\nu}$:

$$\psi_{\mu\nu} = \Psi_{\mu\nu} + \frac{1}{d-2}(\gamma_\mu \Psi_\nu - \gamma_\nu \Psi_\mu) + \gamma_{\mu\nu}\Psi,$$

$$\gamma^\mu \Psi_{\mu\nu} = 0, \;\; \gamma^\mu \Psi_\mu = 0.$$

(Similar for $\bar{\psi}_{\mu\nu}$.)

Quantization of second rank antisymmetric tensor spinor theory

- Partition function takes the form

$$Z = \frac{\Delta_0^2}{\Delta_1^2} \int D\Psi_{\mu\nu} D\bar{\Psi}_{\mu\nu} D\Psi D\bar{\Psi} e^{i\tilde{S}[\Psi_{\mu\nu},\Psi,\bar{\Psi}_{\mu\nu},\bar{\Psi}]},$$

$$\tilde{S}[\Psi_{\mu\nu},\Psi,\bar{\Psi}_{\mu\nu},\bar{\Psi}] = -\int d^d x \{\bar{\Psi}^{\mu\nu}\gamma^\rho i\nabla_{(2)\,\rho}\Psi_{\mu\nu} + \bar{\Psi}\gamma^\rho i\nabla_{(0)\,\rho}\Psi\}.$$

- The operators $\gamma^\mu \nabla_{(p)\,\mu} = i\gamma^\mu\nabla_\mu \pm \frac{r^{\frac{1}{2}}}{2}(d-2p)$ acts on gamma-irreducible fermionic p-forms.
- Final result for partition function

$$Z = \frac{\Delta_2 \Delta_0^3}{\Delta_1^2},$$

$$\Delta_p = Det(i\gamma^\mu \nabla_{(p)\,\mu}), \;\; p = 0, 1, 2, \ldots.$$

Summary

- Final result: correct functional integral for first-stage reducible gauge theories

$$Z = \int D\varphi\, Dc\, e^{iS[\varphi]}\, \delta[\chi^\alpha + \sigma^a c]\, \frac{DetF}{DetQ_1}.$$

- Method can directly be generalized to arbitrary-finite-stage reducible bose-fermi gauge theories by multiple use of the Faddeev-Popov procedure.
- Final result for partition function in the fermionic 2-form theory in AdS_d space:

$$Z_2 = \frac{\Delta_2 \Delta_0^3}{\Delta_1^2}.$$

- Hypothesis for partition function for arbitrary fermionic p-form theory in AdS_d space:

$$Z_p = \frac{\Delta_p \Delta_{p-2}^3 \Delta_{p-4}^5 \cdots}{\Delta_{p-1}^2 \Delta_{p-3}^4 \cdot \cdots}$$

Proved in A. Barvinsky, I.B, V. Krykhtin, D. Nesterov, PRD 2026, arXiv:2507.09312 [hep-th] in the framework of the BV theory.

Summary

Open questions:

- *Calculation of the functional determinants Δ_p on the AdS_d space.*
- *Quantization of the massive fermionic p-form theory in the AdS_d space.*
- *Calculation of the partition function for massive fermionic p-form theory in the AdS_d space.*
- *Supersymmetric extension of the fermionic p-form theory in the AdS_d space, its quantization and deriving effective action.*
- *Canonical quantization of the fermionic p-form theory in the AdS_d space.*
- *Quantum equivalence classically dual fermionic models.*

Thank you very much!