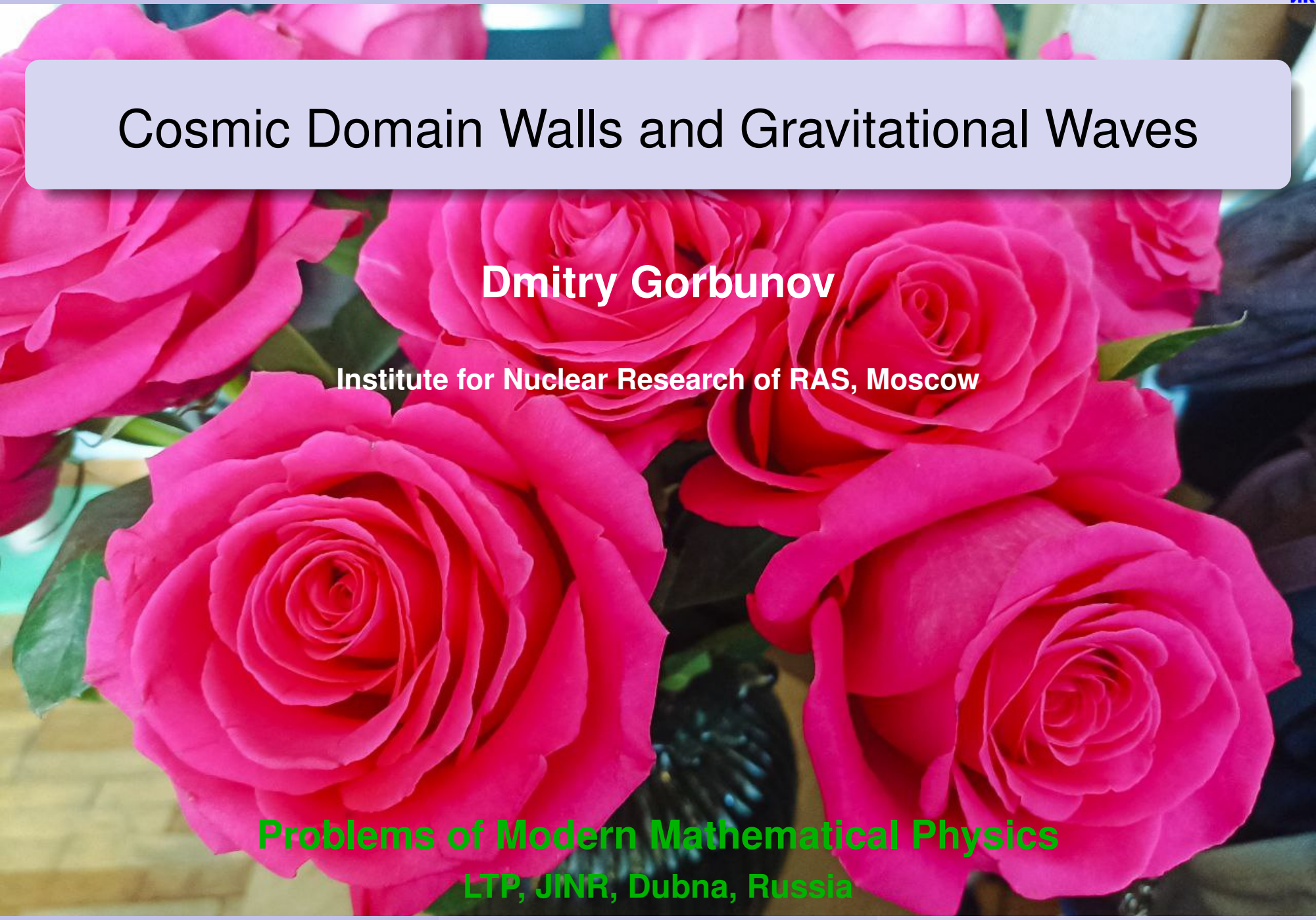
Cosmic Domain Walls and Gravitational Waves



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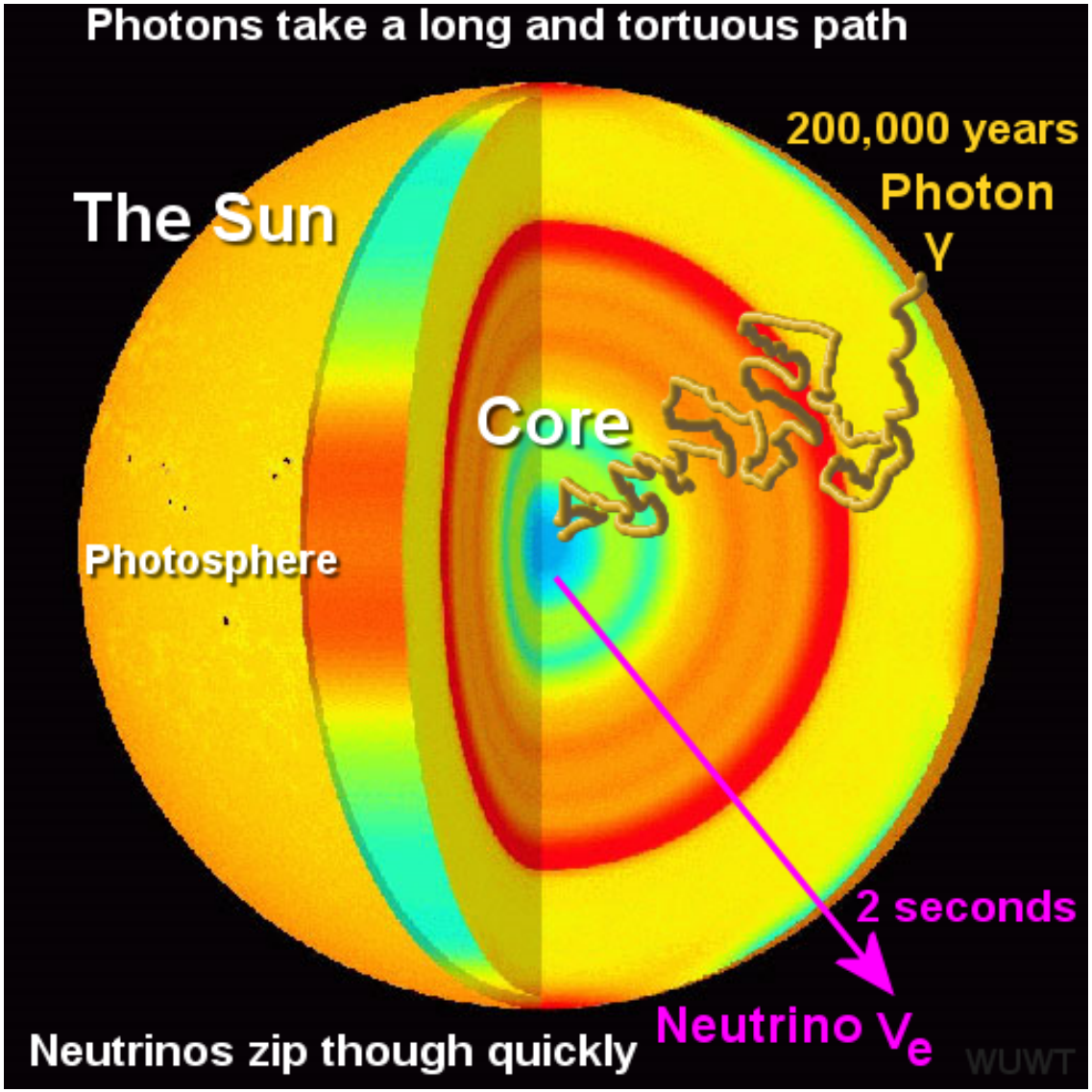
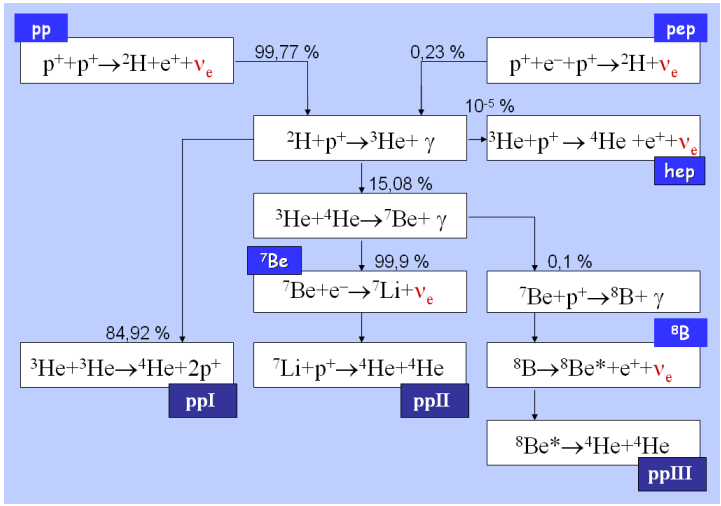
Problems of Modern Mathematical Physics

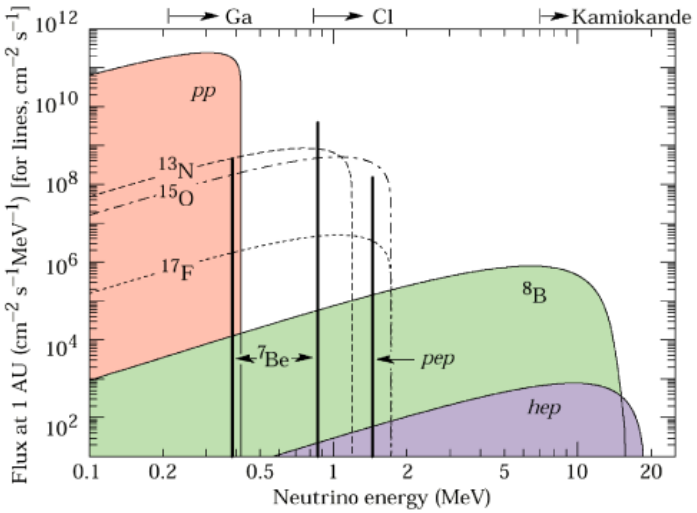
LTP, JINR, Dubna, Russia



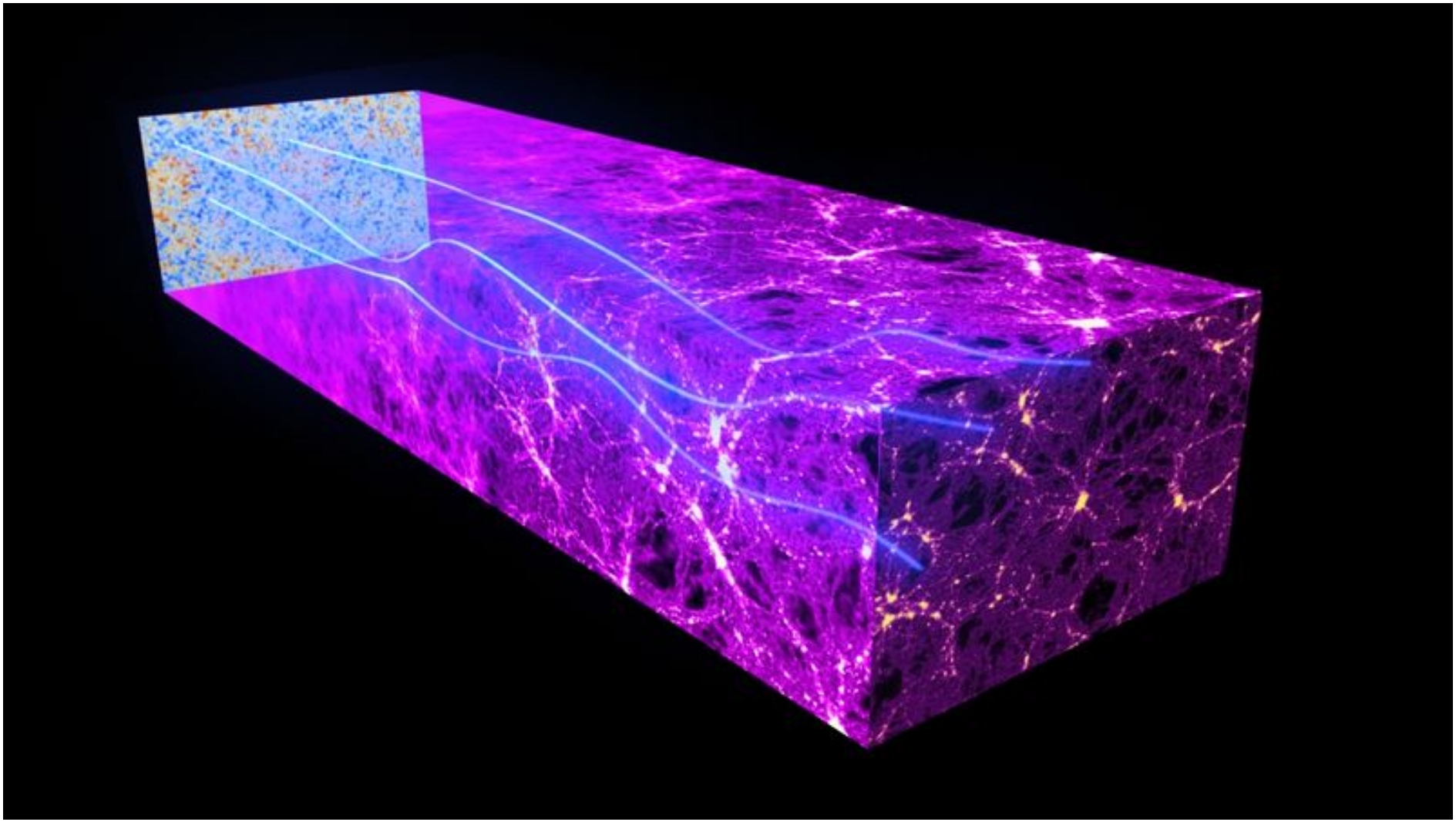
Solar neutrinos: fusion $p + p \rightarrow D + e^+ + \nu_e, \ldots$

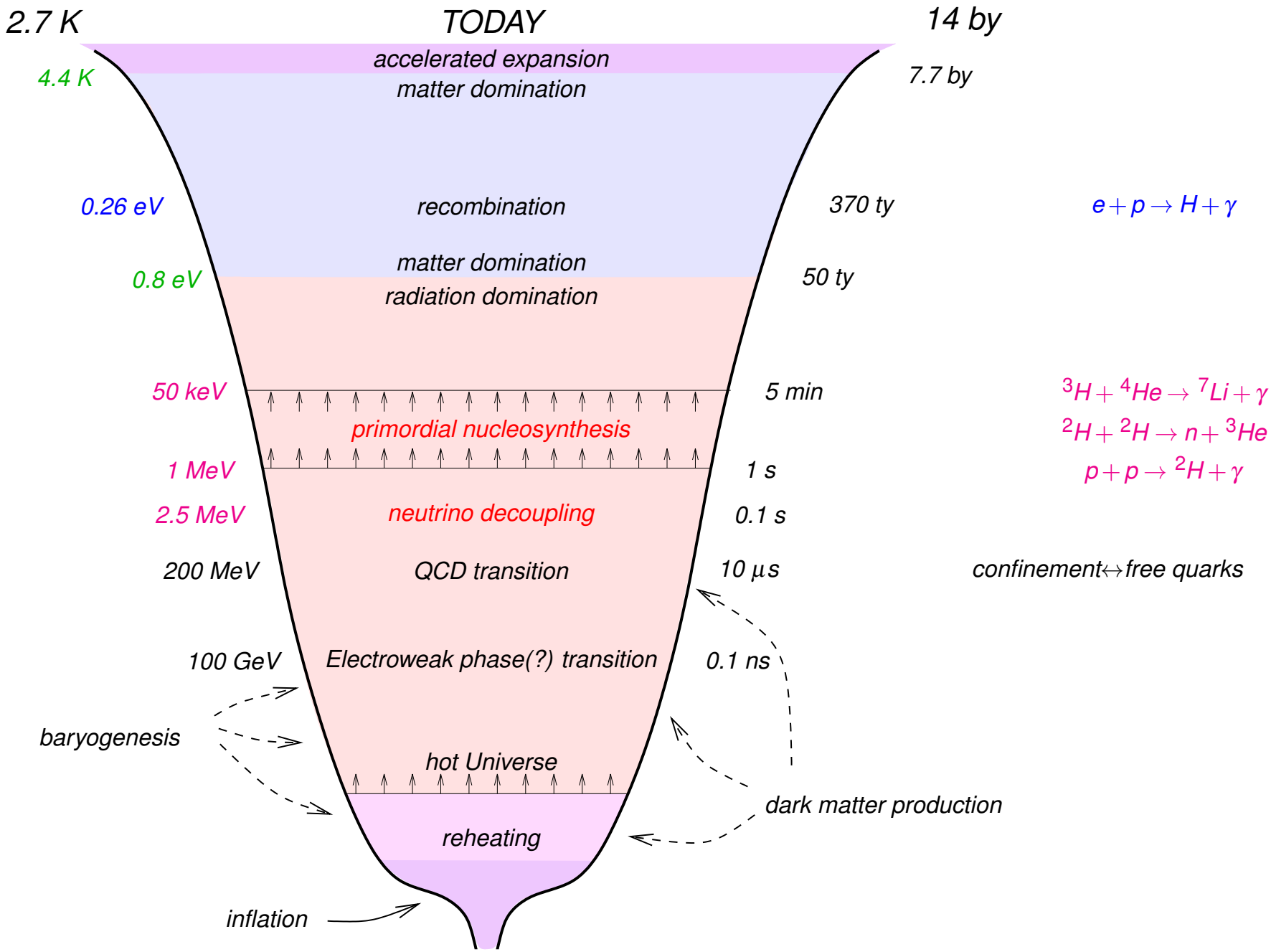






Universe: the surface of last scattering $e + p \to H + \gamma$

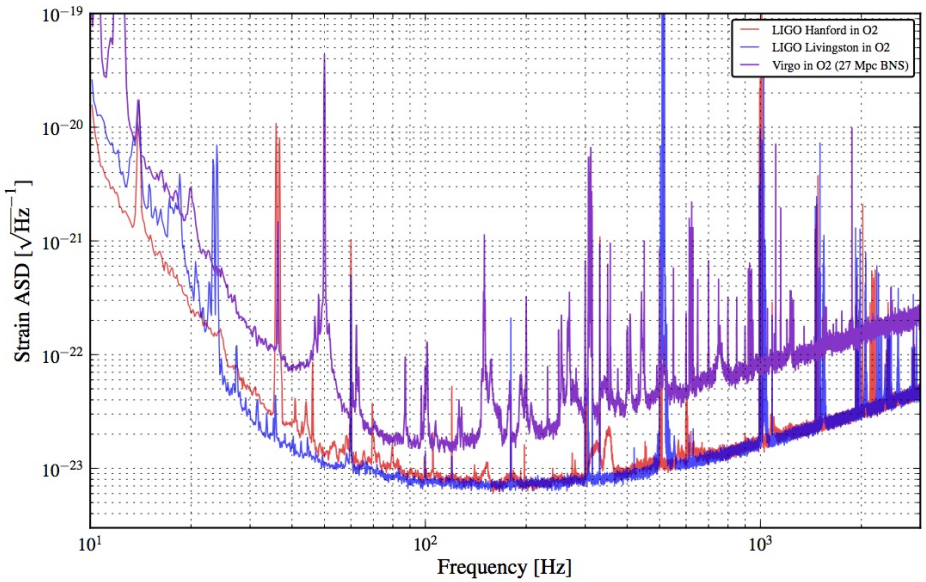


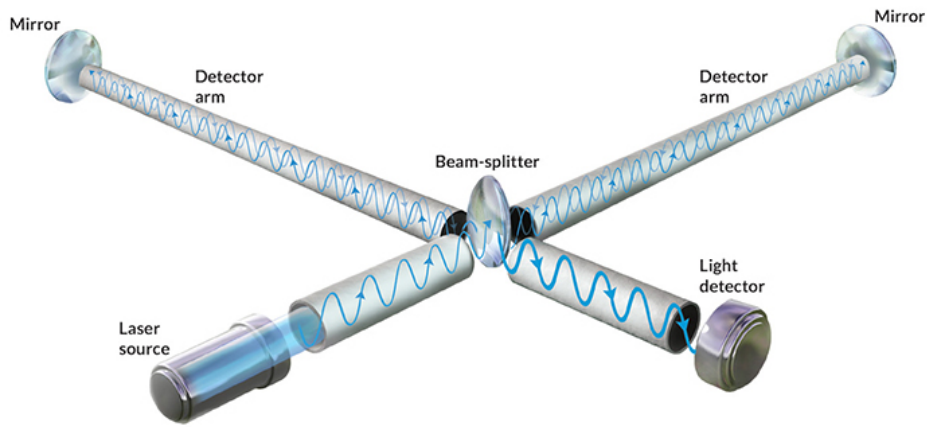

2.7 K
TODAY
14 by
accelerated expansion
4.4 K
matter domination
7.7 by
0.26 eV
recombination
370 ty
$e+p \to H+\gamma$
matter domination
0.8 eV
radiation domination
50 ty
50 keV
5 min
$^3H+^4He \to ^7Li+\gamma$
primordial nucleosynthesis
$^2H+^2H \to n+^3He$
1 MeV
1 s
$p+p \to ^2H+\gamma$
2.5 MeV
neutrino decoupling
0.1 s
200 MeV
QCD transition
10 μs
confinement↔free quarks
100 GeV
Electroweak phase(?) transition
0.1 ns
baryogenesis
hot Universe
dark matter production
reheating
inflation

Nobel prize in 2017: discovery of GW



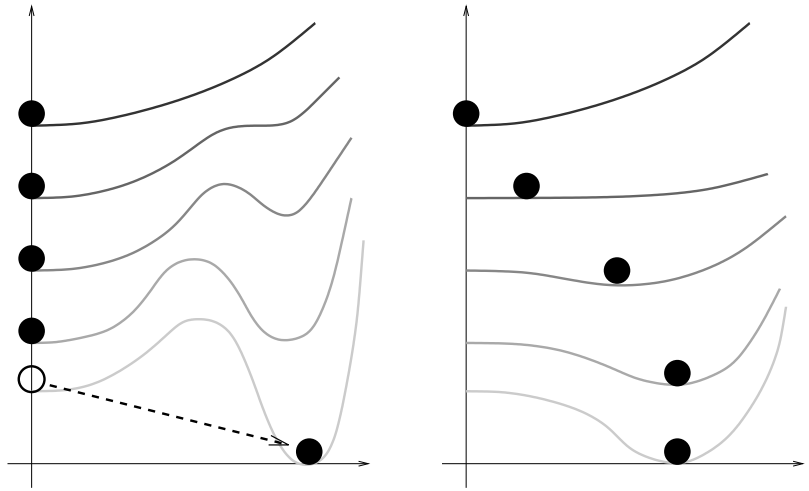








More symmetry !!

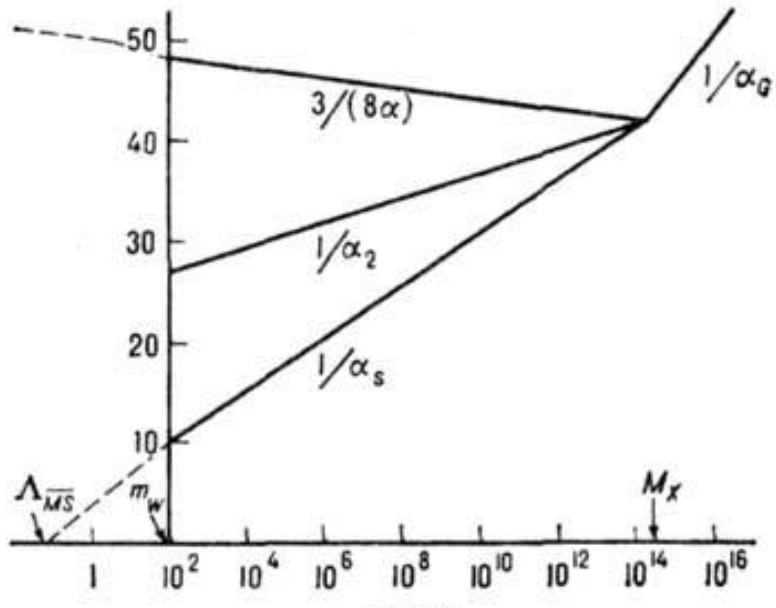


- Electroweak unification
  (transition at $T \sim 100\,\text{GeV}$)

$$U(1)_{em} \to SU(2)_L \times U(1)_Y$$

- Grand Unification ?
  (transition(s) in the early Universe)

$$SU(3)_c \times SU(2)_L \times U(1)_Y \to SU(5)$$
$$SU(3)_c \times SU(2)_L \times U(1)_Y \to$$
$$\to SU(3)_c \times SU(2)_R \times SU(2)_L \times U(1)_Y$$
$$\to SO(10)$$



- Any semisimple group inevitably leads to formation of topological defects:
  – monopoles
  – strings
  – domain walls

Topological defects are dangerous for cosmology !

Naturally overclose the hot Universe

- free monopoles vs radiation

$$\rho_{mon} = M_{mon} \times n_M \propto \frac{1}{a^3(t)} \quad \text{vs} \quad \rho_{rad} \propto \frac{1}{a^4(t)}$$

- free strings (gas of straight strings) vs radiation

$$\rho_{strings} \propto \frac{1}{a^2(t)} \quad \text{vs} \quad \rho_{rad} \propto \frac{1}{a^4(t)}$$

- free domain walls (gas of straight domain walls) vs radiation

$$\rho_{walls} \propto \frac{1}{a(t)} \quad \text{vs} \quad \rho_{rad} \propto \frac{1}{a^4(t)}$$

But the configurations are highly non-linear

They naturally annihilate via various processes

- free monopoles form monopoloniums and annihilate
- free strings intersect, form small closed strings which shrink and disappear leading to the scaling solution: fixed number of long strings inside the horizon

$$\rho_{str-sc} \simeq N\frac{M_s}{l_H^3} \simeq N\frac{\sigma_s l_H}{l_H^3} \propto H^2 \propto \rho_{tot}$$

strings are harmless

- free domain walls get crumpled and shrivel

domain wall network appoaches scaling regime

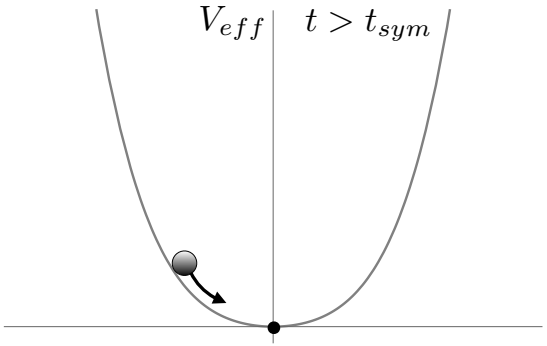
$$\rho_{walls-sc} \simeq N\frac{M_s}{l_H^3} \simeq N\frac{\sigma_s l_H^2}{l_H^3} \propto H\searrow \quad \text{while} \quad \rho_{tot} \propto H^2 \searrow\searrow$$

cosmic domain wall network is generically dangerous

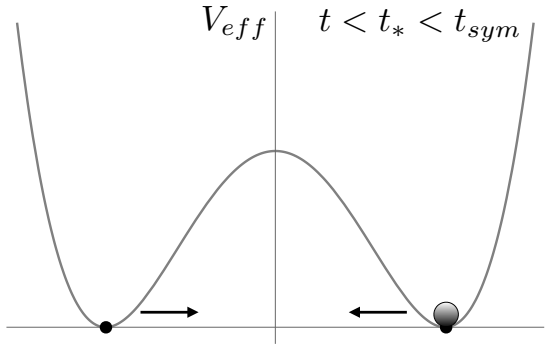
hence, must be destroyed

Production of cosmic domain walls: breaking Z_2-symmetry

$t < t_b$



$t > t_b$



$\rightarrow$

$$\mathscr{L} = \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - \frac{\lambda}{4} \left(\phi^2 - \eta^2 \right)^2$$

Straight domain wall in Minkowsky is an analytical solution, kink.

Its width is $\delta_{wall} = \frac{\sqrt{2}}{\sqrt{\lambda}\eta}$, its tension is $\sigma_{wall} = \frac{2\sqrt{2\lambda}\eta^3}{3}$

Simulation of breaking Z_2-symmetry in expanding Universe with CosmoLattice code

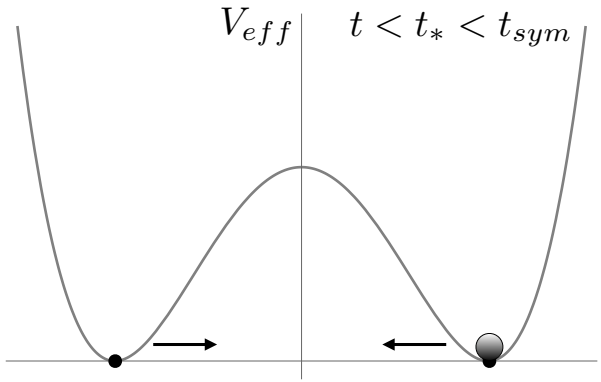
$ds^2 = a^2(\tau)\left(d\tau^2 - \mathbf{dx}^2\right)$ radiation domination start at $\tau = 1$

$$\langle\phi(\mathbf{k})\phi(\mathbf{q})\rangle = A(k)\delta(\mathbf{k}+\mathbf{q}) \quad \langle\dot{\phi}(\mathbf{k})\dot{\phi}(\mathbf{q})\rangle = B(k)\delta(\mathbf{k}+\mathbf{q})$$

with $\phi = 0 + \delta\phi(\mathbf{x}, t)$

1) vacuum fluctuations:

$$A(k) = \frac{\Theta(k - k_{cut})}{16\pi^3 k}, \qquad B(k) = \frac{k}{16\pi^3}\cdot\Theta(k - k_{cut}) .$$

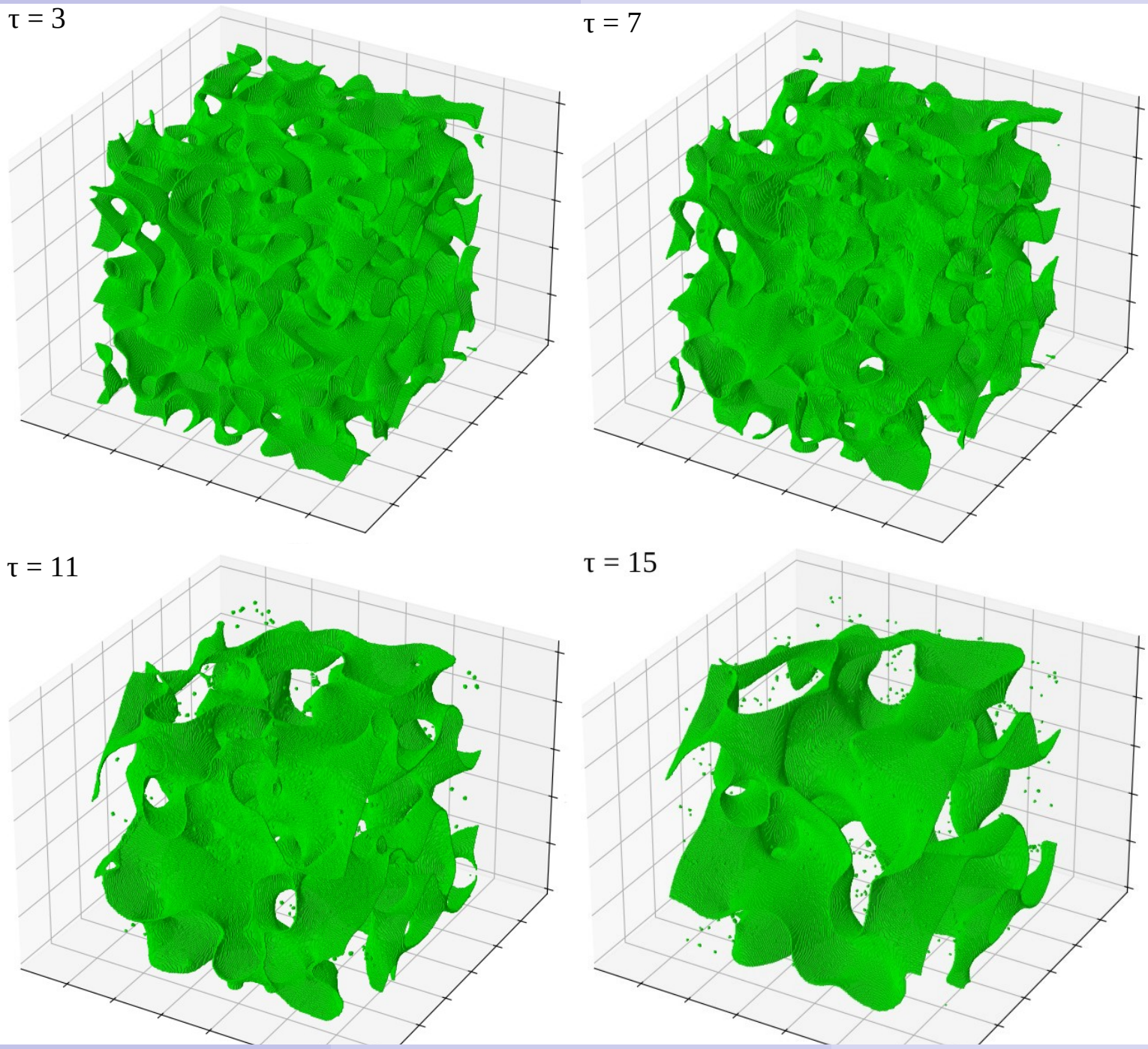


2) thermal fluctuations:

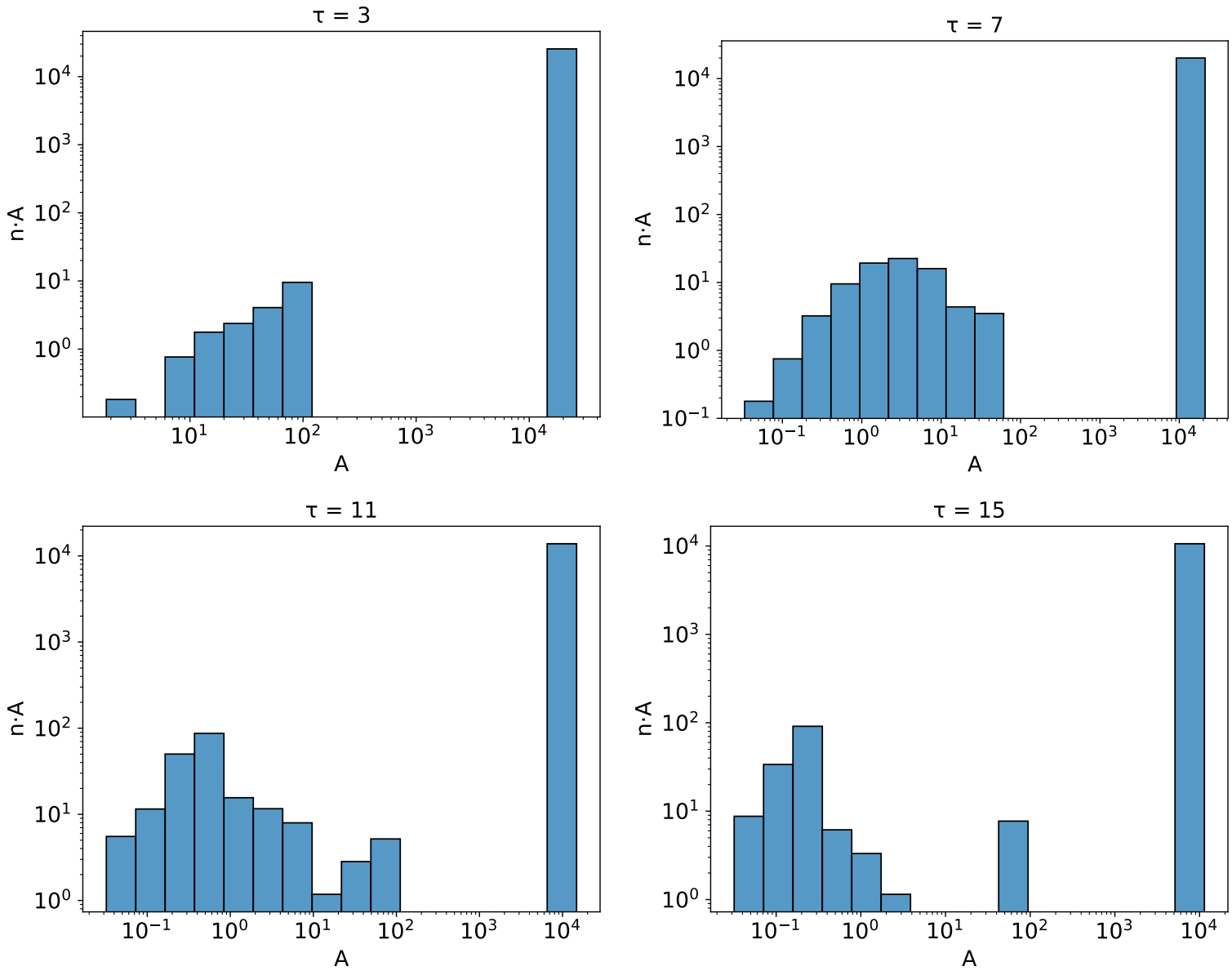
$$A(k) = \frac{1}{8\pi^3\cdot k\left(e^{\frac{k}{T}} - 1\right)}, \qquad B(k) = \frac{k}{8\pi^3\cdot\left(e^{\frac{k}{T}} - 1\right)},$$

numerical normalization: in terms of $\sqrt{\lambda}\eta$ 2406.17053

$$\mathscr{L} = \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - \frac{\lambda}{4}\left(\phi^2 - \eta^2\right)^2$$



2406.17053



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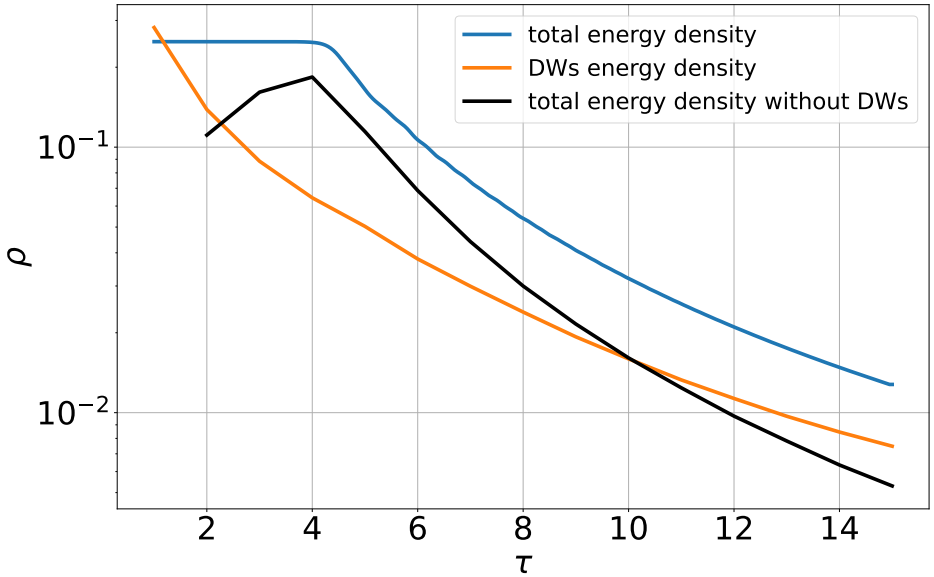
Budget of scalar energy density

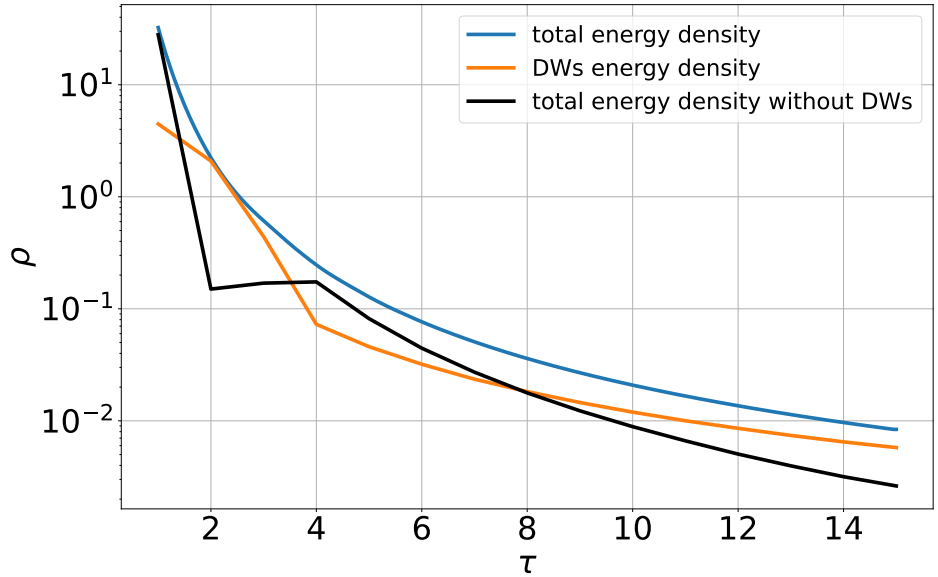
2406.17053

radiation domination: $a \propto \tau$, $l_H \propto \tau^2$

vacuum perturbations



thermal perturbations



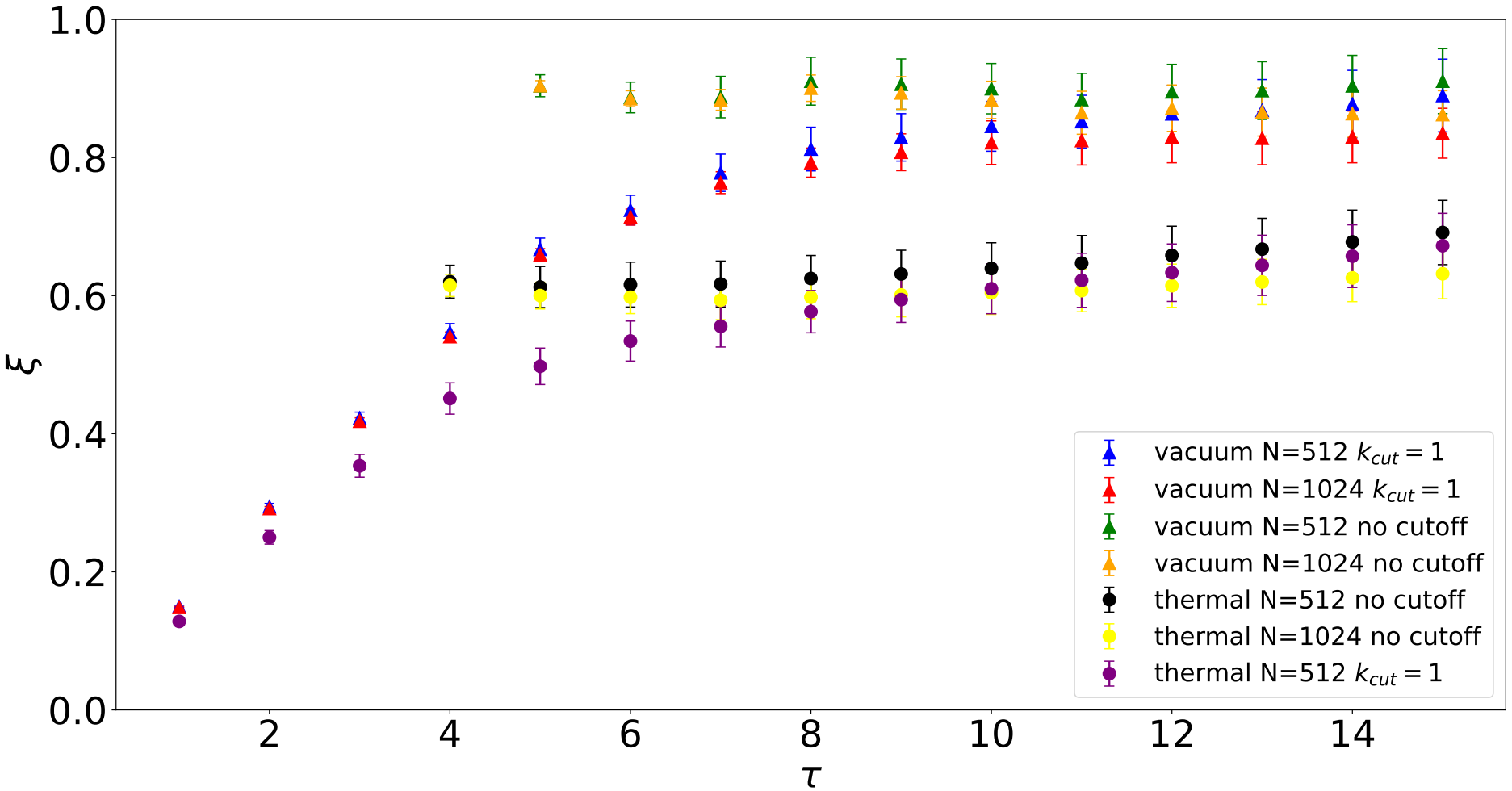
DW energy density: $\rho \propto \frac{1}{l_H} \propto \frac{1}{\tau^2}$

scalar field energy density: $\rho \propto \frac{1}{a^3} \propto \frac{1}{\tau^3}$

Scaling of a DM network: CosmoLattice simulation,

$$\rho_{dw} = \frac{\sigma_{dw} A}{a(t) V} \propto \frac{1}{l_H} \propto \frac{1}{t}, \quad \xi \equiv \frac{At}{a(t)V} \to 1.2$$

2406.17053, 2509.25367



$$\rho_{dw} = \frac{\sigma_{dw} A}{a(t) V} \propto \frac{1}{l_H} \propto \frac{1}{t}, \quad \xi \equiv \frac{At}{a(t)V}$$

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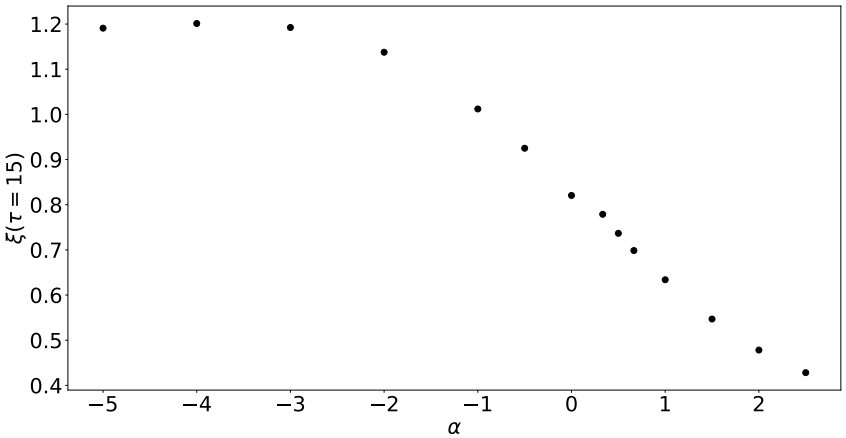
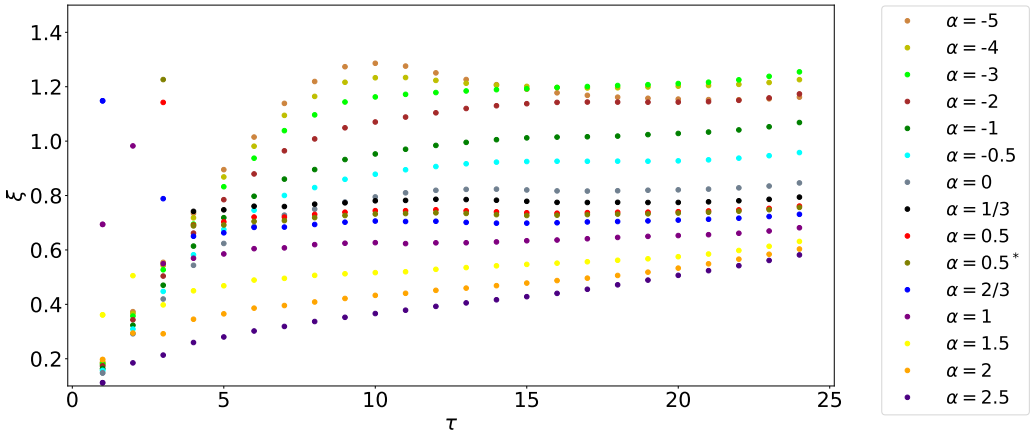
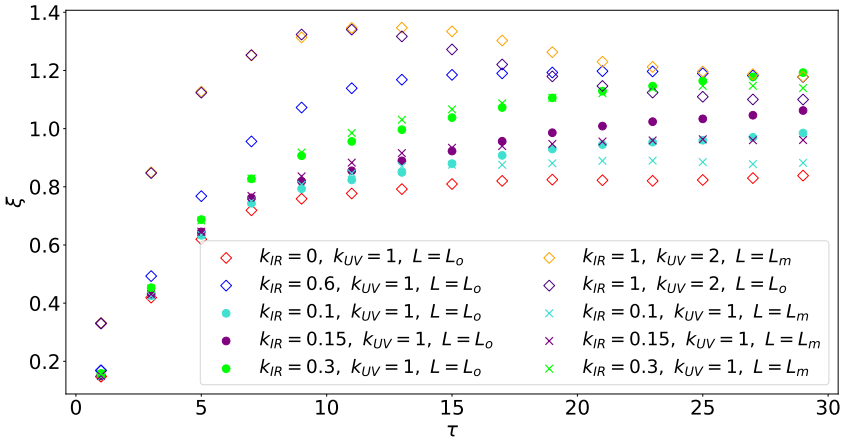
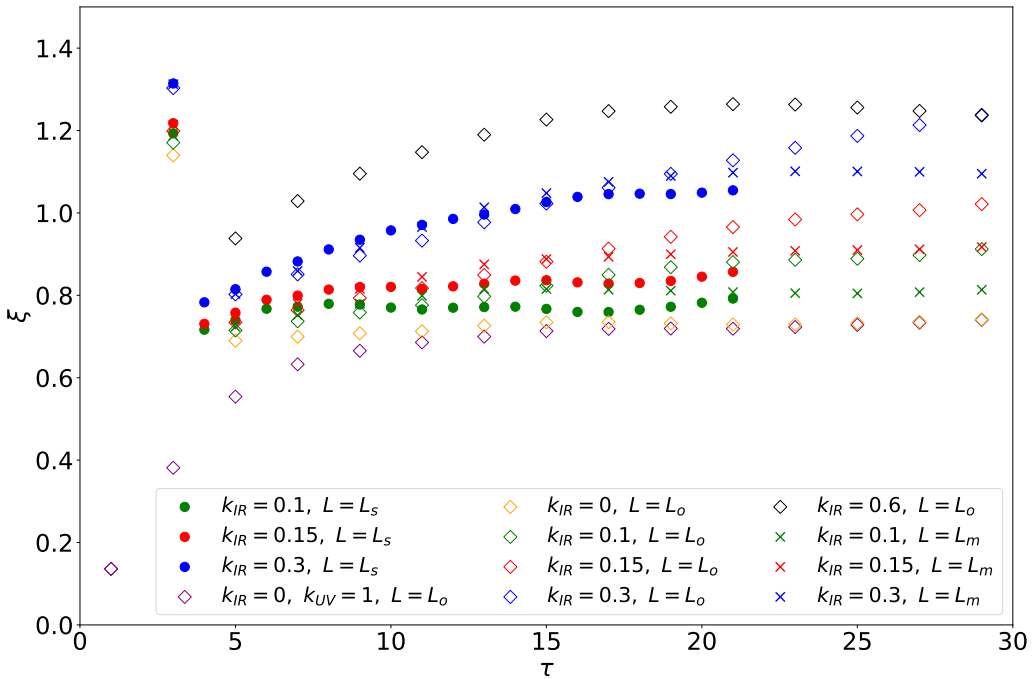




$$A(k) = \frac{\exp\left(-\frac{k_{IR}^2}{k^2}\right)\cdot\theta(k_{UV}-k)}{k\left(e^{\frac{k}{T}}-1\right)^{\alpha}}, \qquad B(k) = \frac{k\cdot\exp\left(-\frac{k_{IR}^2}{k^2}\right)\cdot\theta(k_{UV}-k)}{\left(e^{\frac{k}{T}}-1\right)^{\alpha}}.$$





Microscopic processes in the expanding Universe

A competition between scattering, decays, etc and expansion

kinetic equations for particle number densities $n_X \propto a^{-3}$

$$\frac{dn_X}{dt} + 3Hn_X = \sum(production - destruction)$$

$$\dot{\rho}_{DW} + H\rho_{DW} = -\Gamma\rho_{DW}^2$$

$\Gamma = 0$ describes a gas of non-interacting DWs:

$$\rho_{DW} = \rho_0 \times \frac{a_0}{a}$$

Assuming $a \propto t^\alpha$, we get — scaling for $\alpha < 1$

$$\rho_{DW} = \left(\frac{1}{\rho_0}\frac{a}{a_0} + \frac{\Gamma t_0}{1-\alpha}\left(\frac{t}{t_0} - \frac{a}{a_0}\right)\right)^{-1} \propto \frac{1}{t} \propto \frac{1}{l_H} \propto H$$

If DWs dominate, we restore the non-interacting regime — then $\alpha > 1$

$$H^2 \propto \rho \propto \frac{1}{a} \propto \frac{1}{t^2}$$

The DWs stretch and “leave the horizon”, rather than ”enter the horizon”

Scaling solution

- Non-linear dynamics + causality leads to a self-similarity see e.g. 2406.17053

$$\frac{\langle \rho(x,\tau)\rho(y,\tau)\rangle}{\langle \rho^2(0,\tau)\rangle} = f\left(\frac{|x-y|}{\tau}\right)$$

- But the microscopic processes are not determined:
  - DW hitting the heavy particles in the bulk ?
  - rescattering of waves on the DW and particle emittion into the bulk ?
  -
- But who cares, they must be destroyed anyway. . .

Domain walls produce Gravitational Waves

In expanding Universe $ds^2 = a^2(\tau)\left(d\tau^2 - dx^2\right)$

$$h''_{ij}(\mathbf{k},\tau) + \frac{2a'}{a} h'_{ij}(\mathbf{k},\tau) + k^2 h_{ij}(\mathbf{k},\tau) = 16\pi G_N a^2 \Pi_{ij}(\mathbf{k},\tau)\,.$$

where all the tensors are transverse and traceless

and the source is the scalar field inhomogeneties

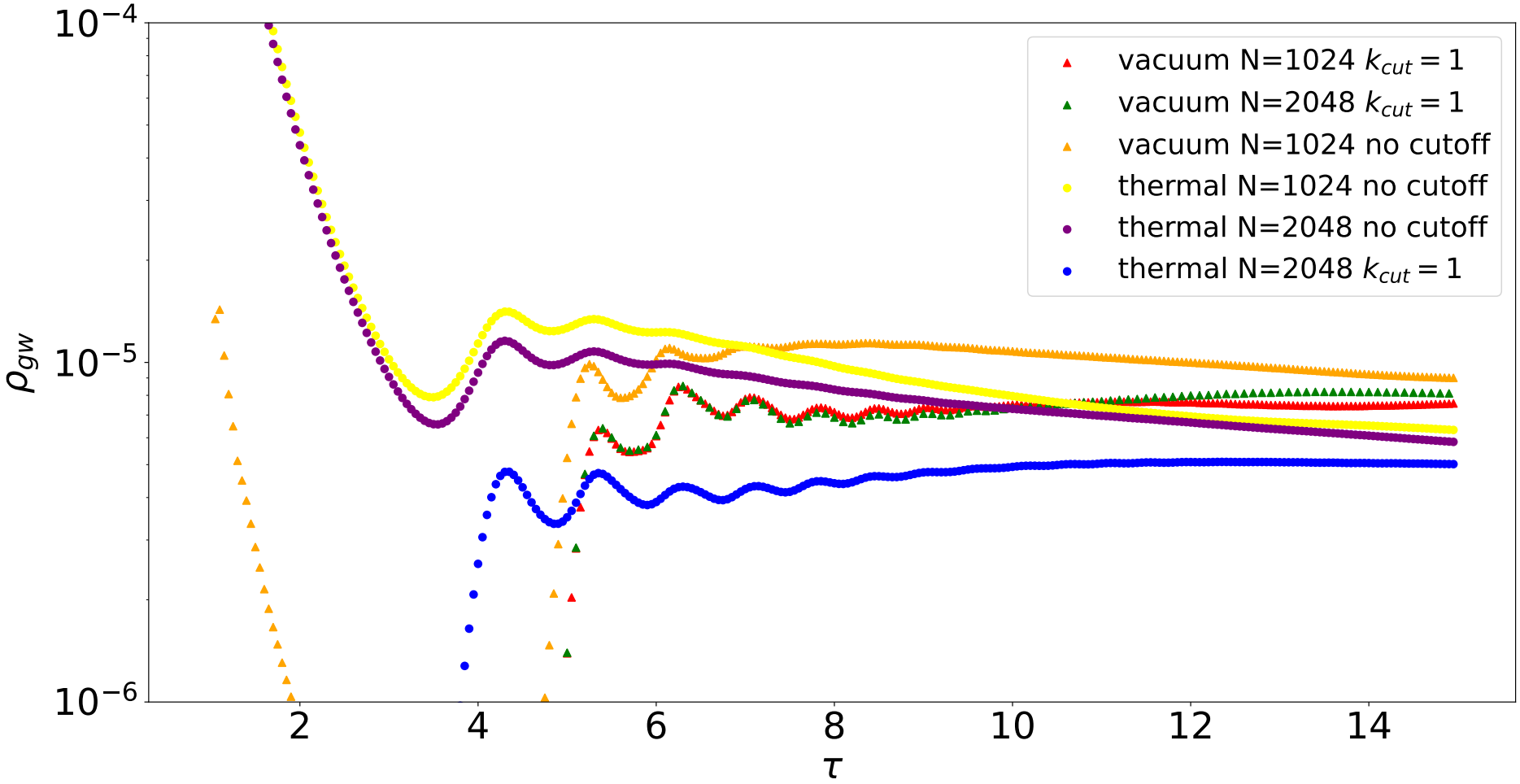
$$\Pi_{ij} \to T_{kl} \sim \partial_k \phi \partial_l \phi$$

With DW network in the scaling regime ar RD-stage, $a \propto \tau$:

$$\rho_{GW} \sim \langle \dot{h}_{ij}\, \dot{h}_{ij} \rangle \propto \left(\frac{h'_{ij}}{a} \frac{a'}{a'} \right)^2 \propto \left(a^2 \rho_{DW} \right)^2 = \text{const}$$

Energy density of GW: simulation with CosmoLattice

2406.17053



GW spectra: simulation with CosmoLattice

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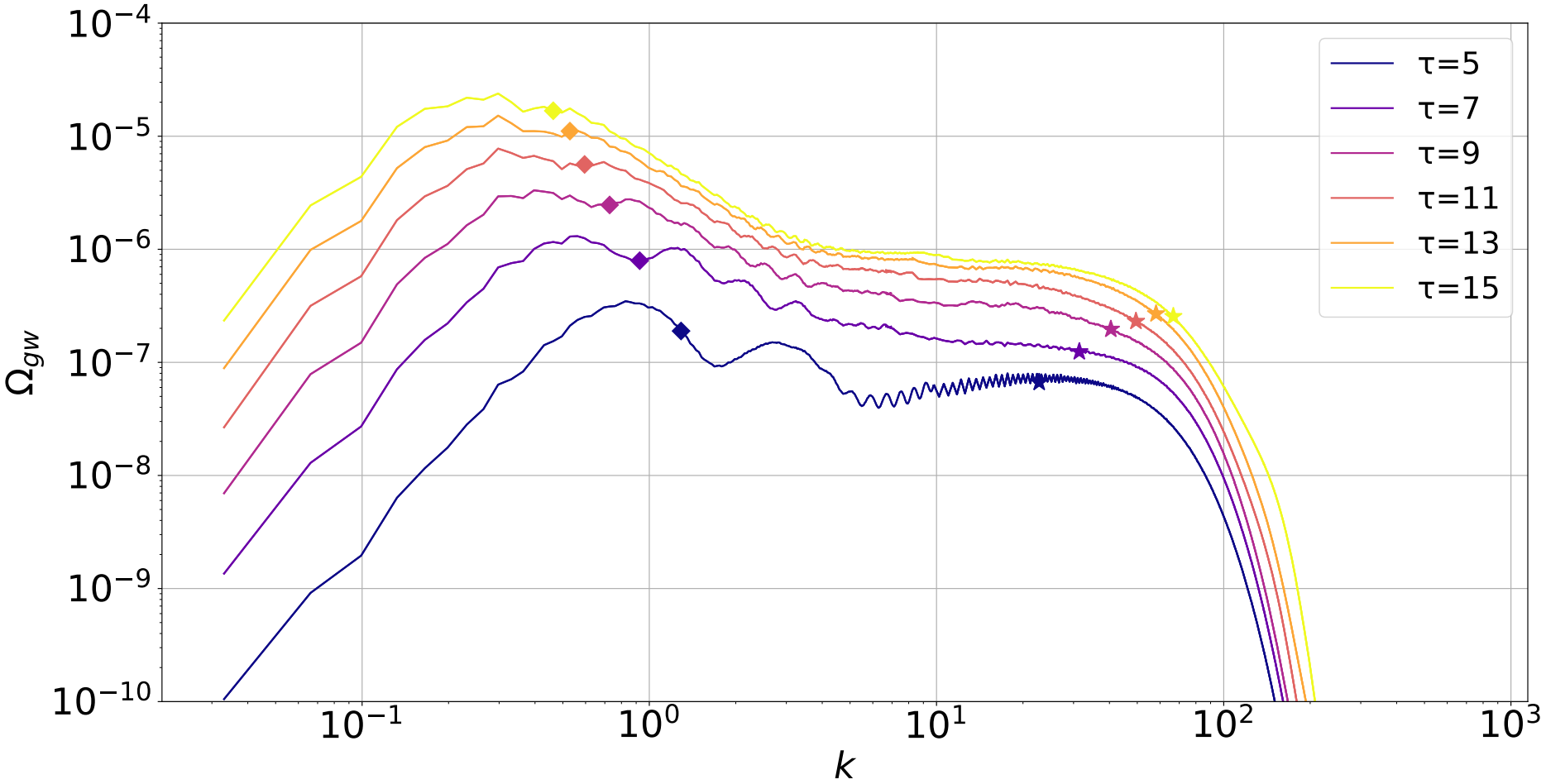


box refers to the current Hubble
$k = 2\pi Ha$

star refers to the domain wall tension
$k = 2\pi a/\delta_{wall}$

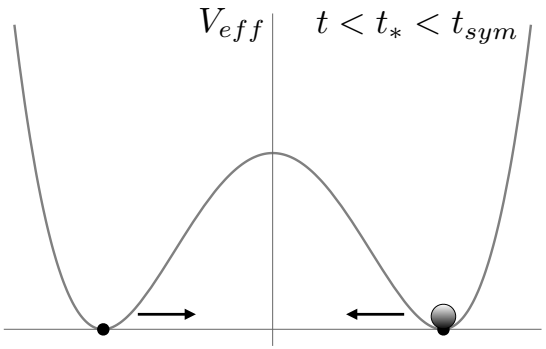
DWs can be destroyed by melting

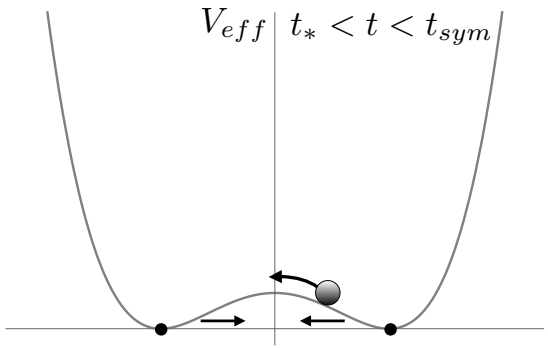
$$\Delta\mathscr{L} = \frac{1}{2} g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + \left(g^2 X^\dagger X - \frac{1}{2}M^2\right)\phi^2 - \frac{\lambda}{4}\phi^4$$

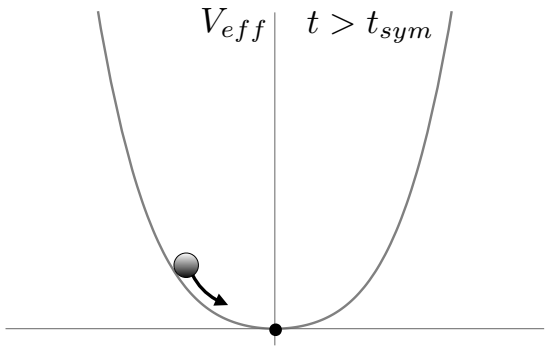
Need non-zero X,
e.g. if it equilibrates in plasma (Higgs ?) changing the potential as:

$$g^2\phi^2 X^\dagger X \;\rightarrow\; g^2\phi^2 T^2/3$$

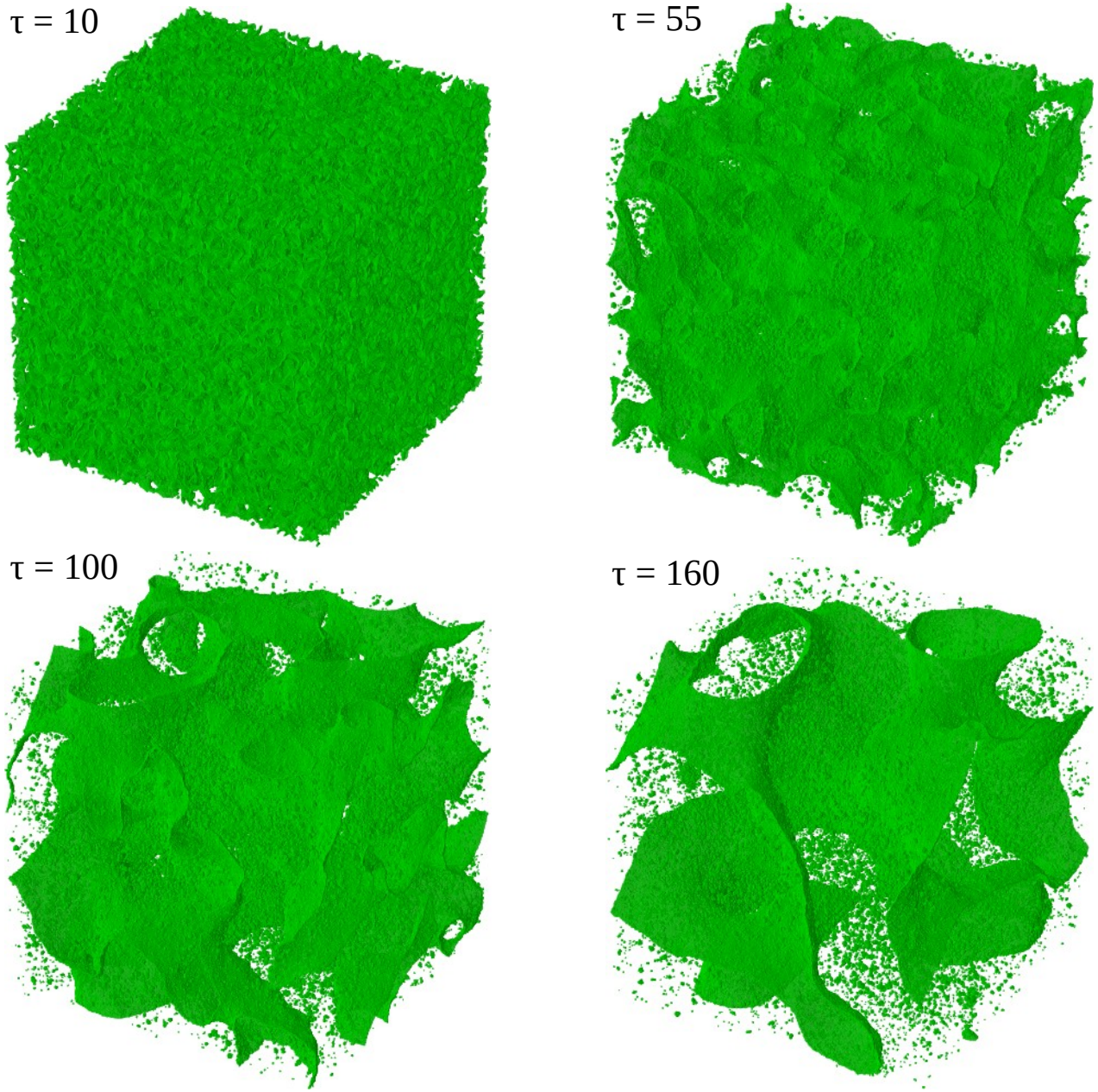
Z_2 symmerty is broken after reheating by the plasma contribution







domain walls with decreasing tension σ_{dw} and width



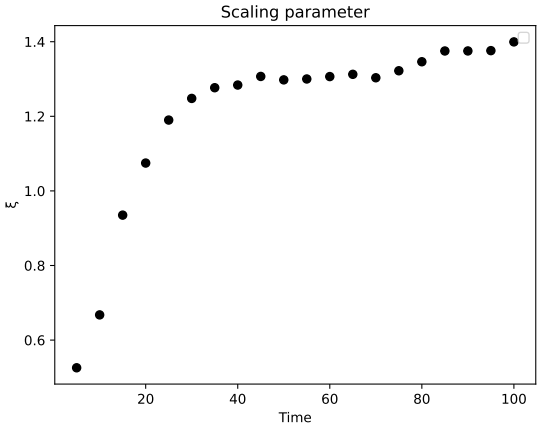
2410.21971

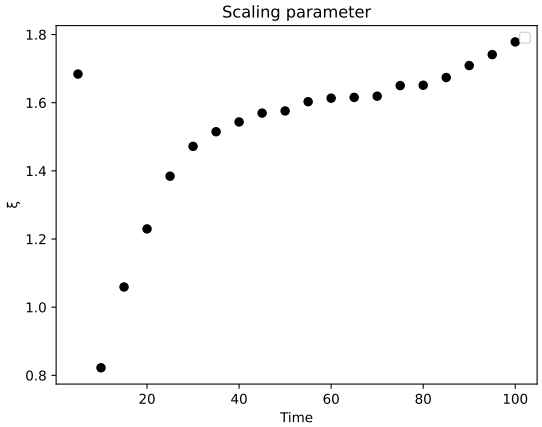
Scaling of melting domain walls

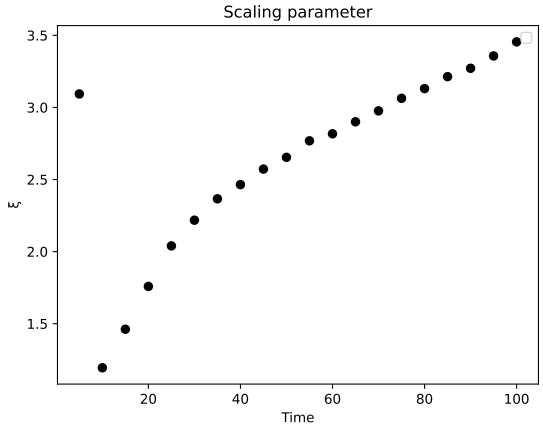
2410.21971

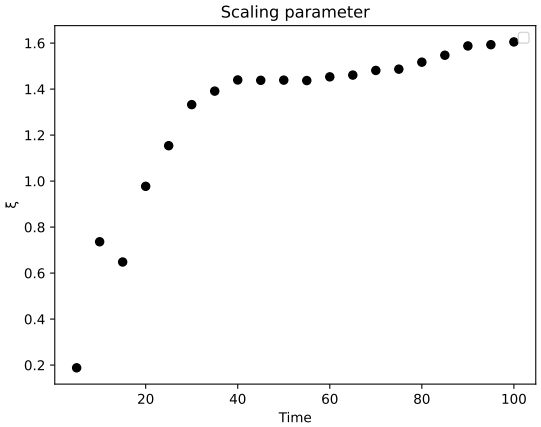
vacuum perturbations

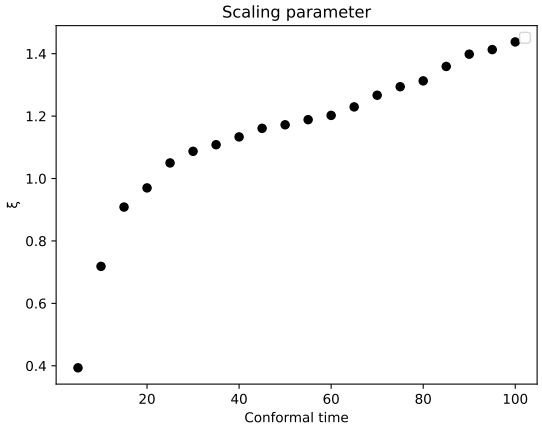
no cuts

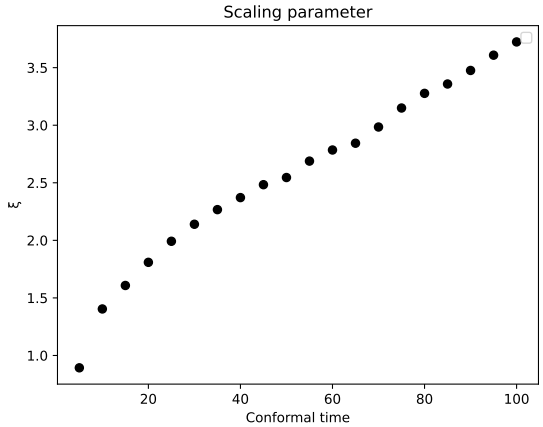












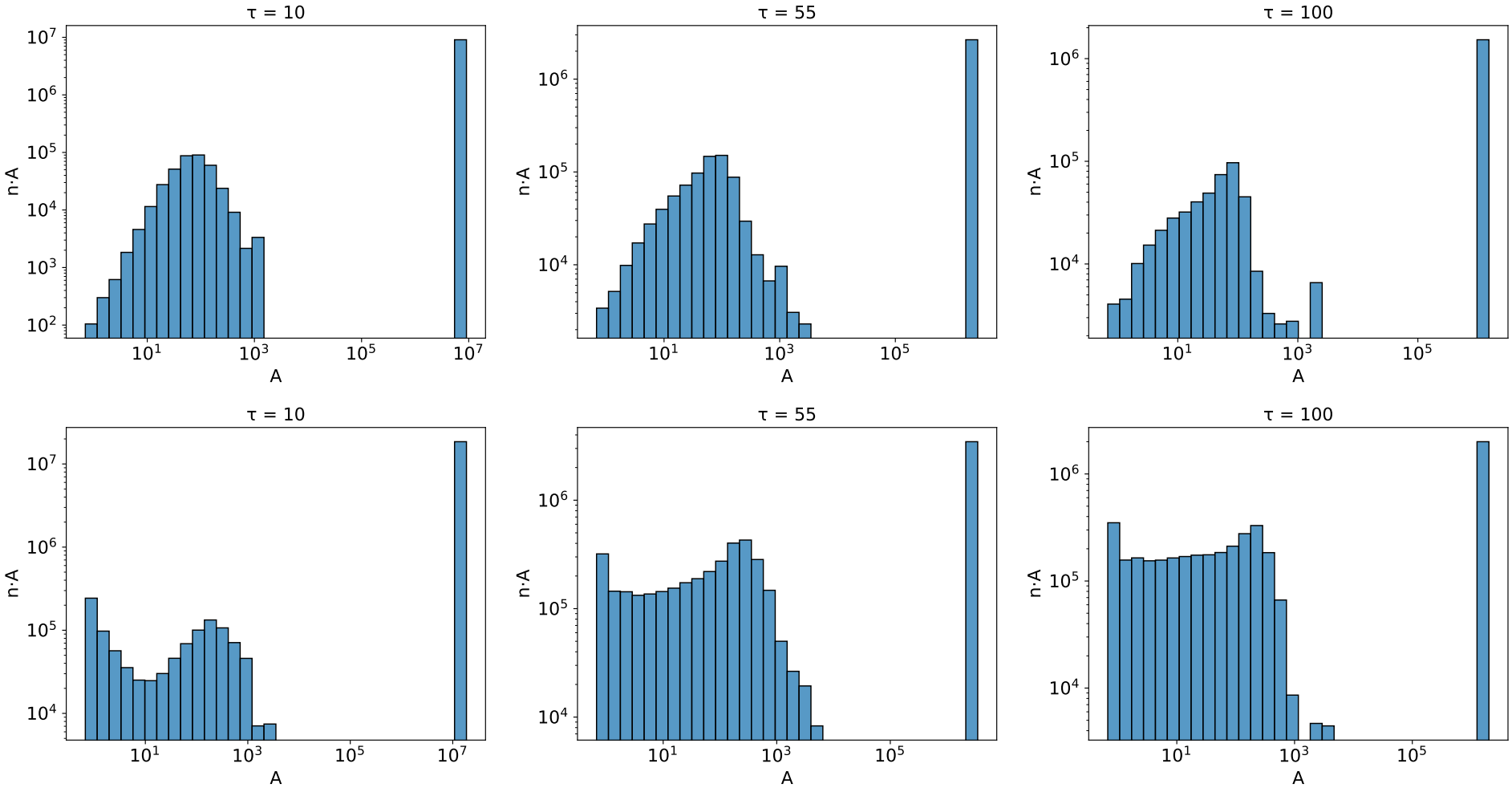
thermal perturbations

no cuts

Spectra of domain walls

2410.21971

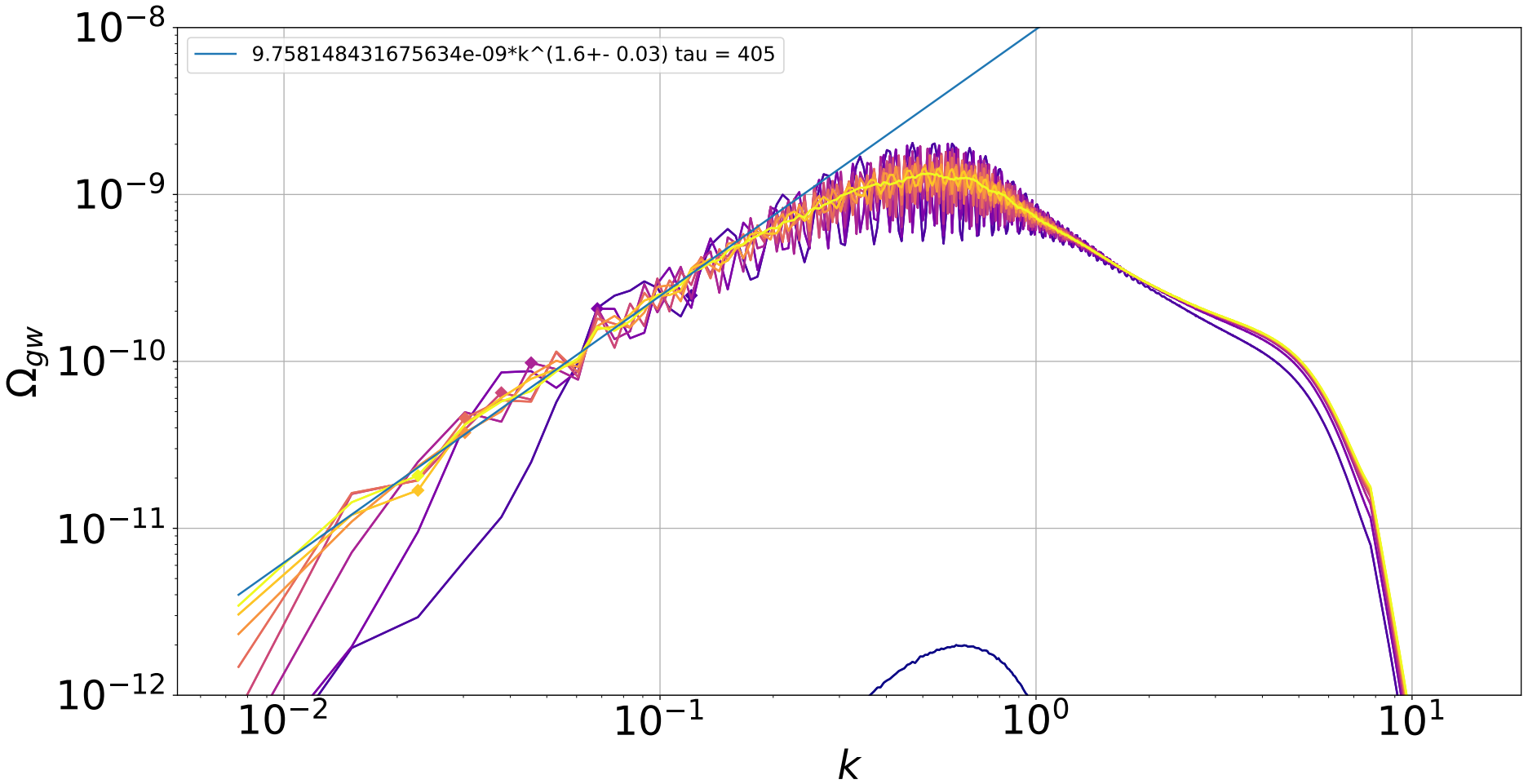
vacuum perturbations



thermal perturbations

GW spectra with vacuum perturbations

2410.21971



production mostly at the network formation...

GW spectra with thermal perturbations

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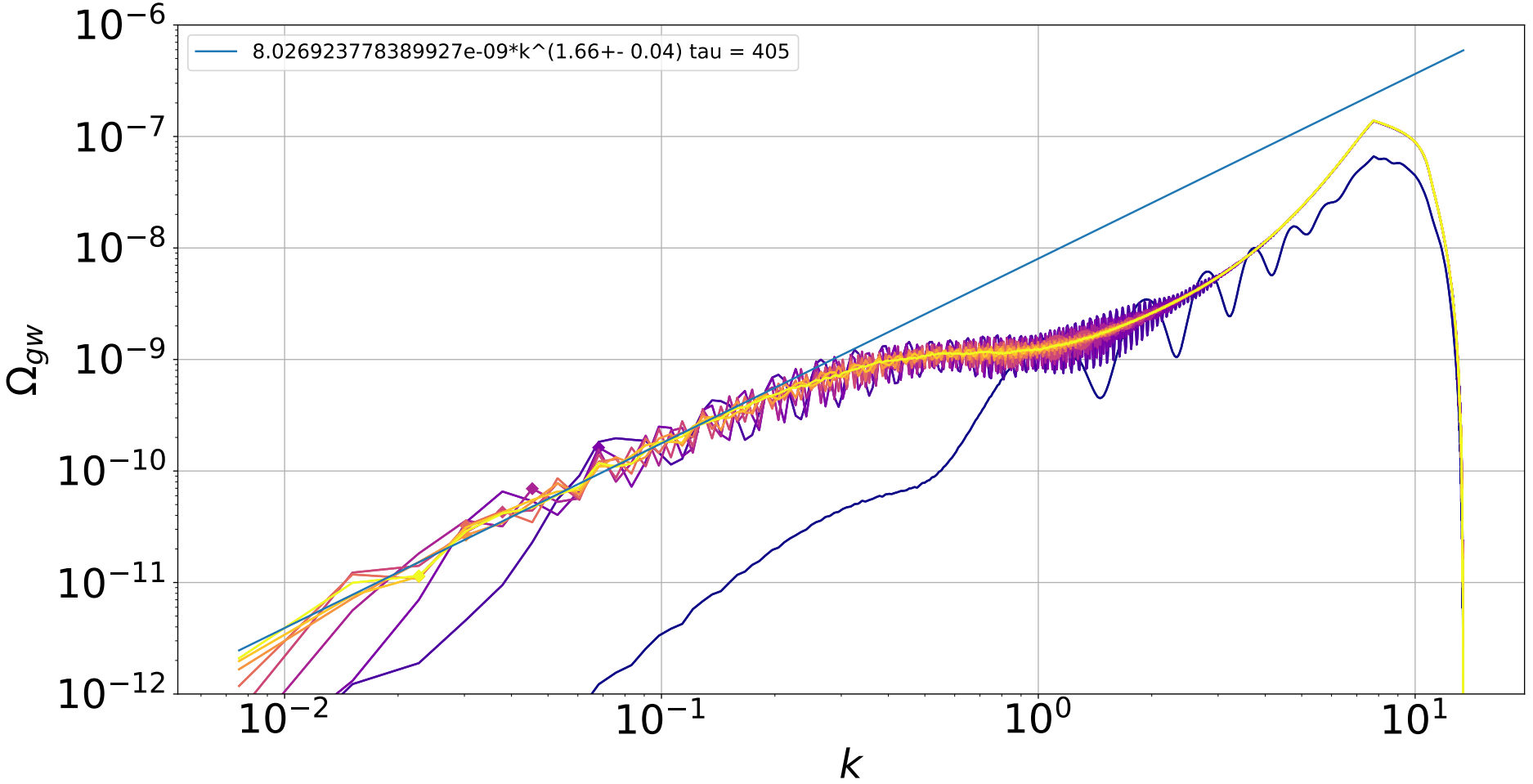


the same IR-tail: $\Omega \propto k^{1.6}$

Analog of a well-known effect in CMB: by Rubakov, Sazhin and Veryaskin

From linear approximation to the geodesic equation...

for tensor perturbations

$$\frac{\delta T}{T}(\mathbf{n},\tau_0) = \frac{1}{2}\int_{\tau_r}^{\tau_0} d\tau\, n_i h_{ij}^{TT\,\prime} n_j\,,$$

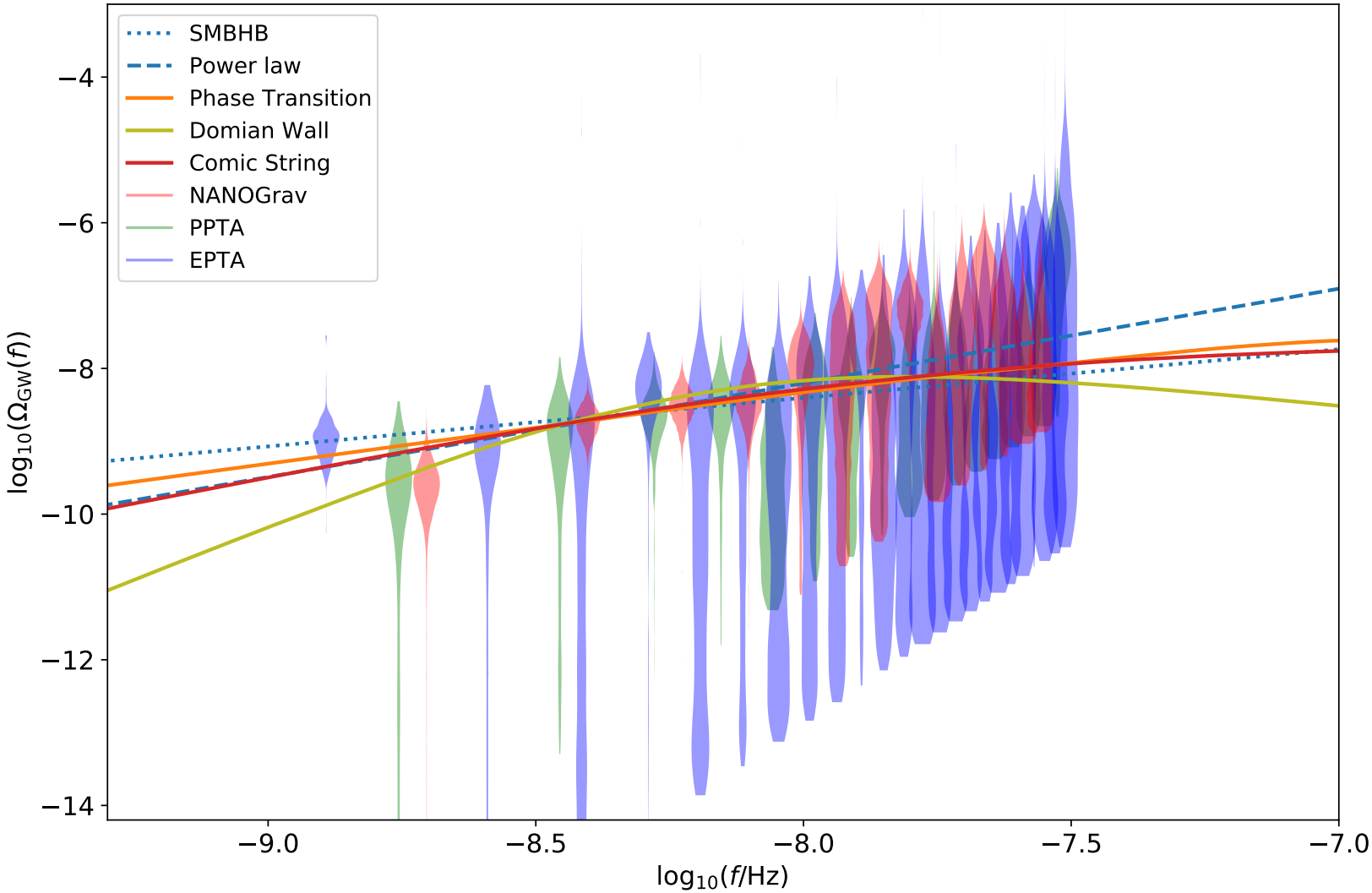
strongest limit on primordial GW and Inflation scale:

$$h_{init} \sim H_{inf}/M_{Pl} \propto \sqrt{\rho_{infl}}$$

Pulsar frequencies are different but stable, and we observe its' time variantion



NANOGrav, T. Klein



$$\Omega_{GW}h^2 = 6.3\cdot 10^{-10}\, A^2 \left(\frac{f}{\mathrm{yr}^{-1}}\right)^d,\quad A=\{6.4^{+4.2}_{-2.7}, 2.9^{+2.6}_{-1.8}, 3.1^{+1.3}_{-0.9}\},\quad d=\{1.8\pm 0.6,\ 0.81^{+0.63}_{-0.73},\ 1.1\pm 0.4\}$$

2307.03141, 2406.02288

DWs with explicitly broken Z_2

$$\mathscr{L} = \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - \frac{\lambda}{4} \left(\phi^2 - v^2 \right)^2 - \varepsilon \phi^3$$

tilted potential induces some pressure...

$$V_b = \varepsilon \phi^3 , \quad V_{bias} = V_b(\phi = v) - V_b(\phi = -v) \approx 2\varepsilon v^3$$

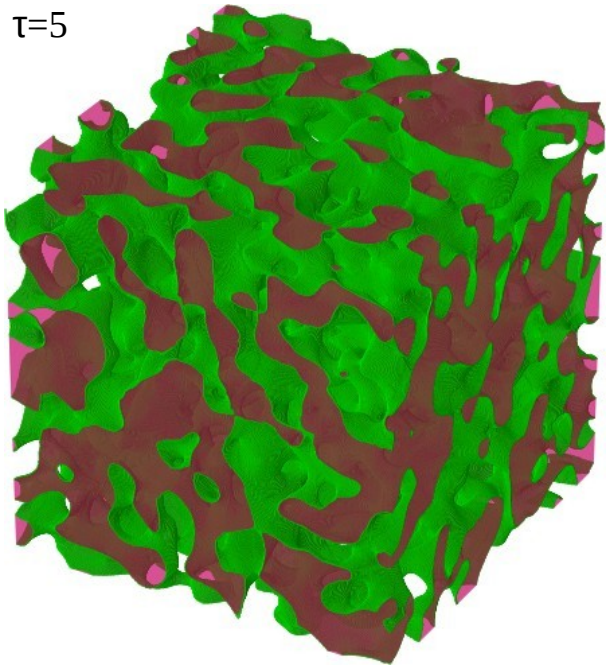
the expected time of DW destruction: e.g. at radiation domination

$$\rho_{DW} \propto \frac{1}{l_H} \propto H \propto \frac{1}{\tau^2}$$

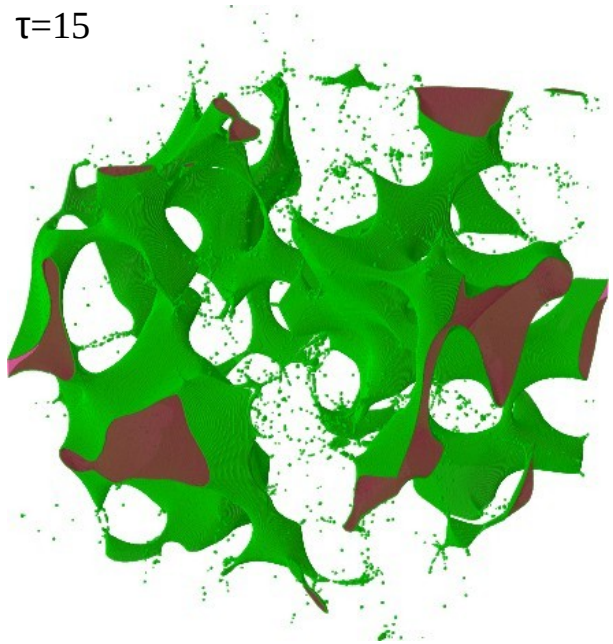
the energy balance

$$\rho_{DW} \sim V_{bias} \quad \rightarrow \tau_{ann} \propto 1/\sqrt{\varepsilon}$$

τ=5



τ=15



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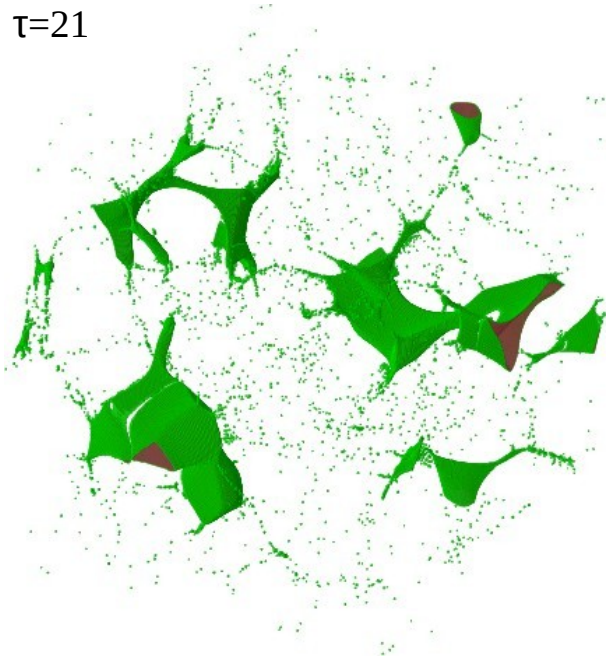
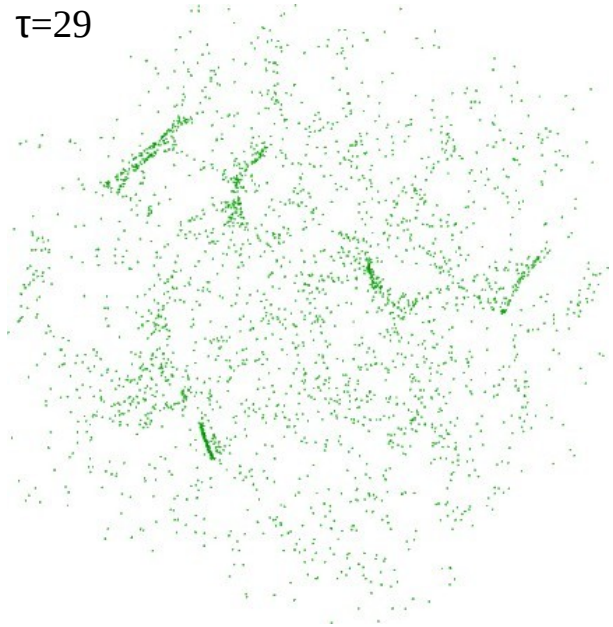
τ=21



τ=29



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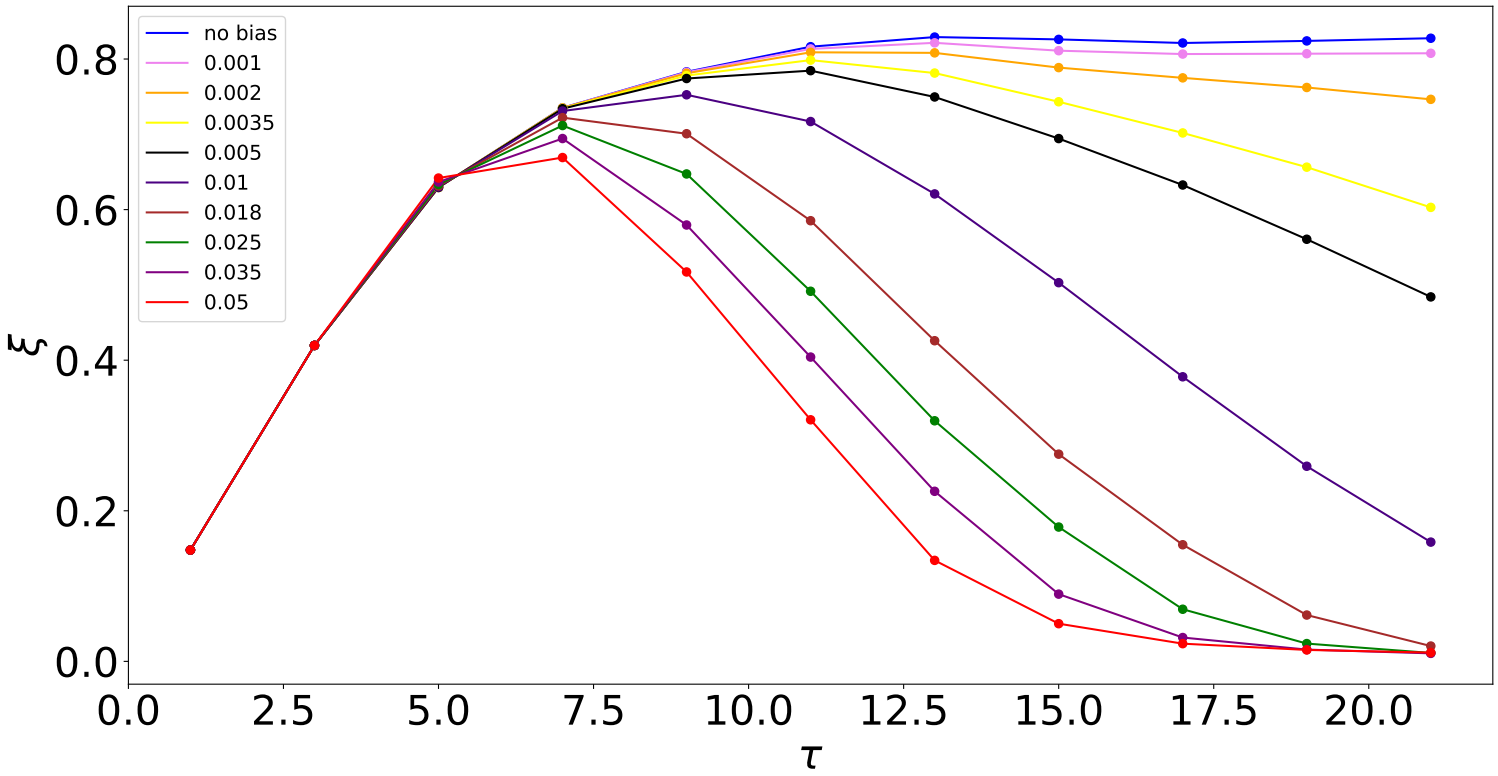
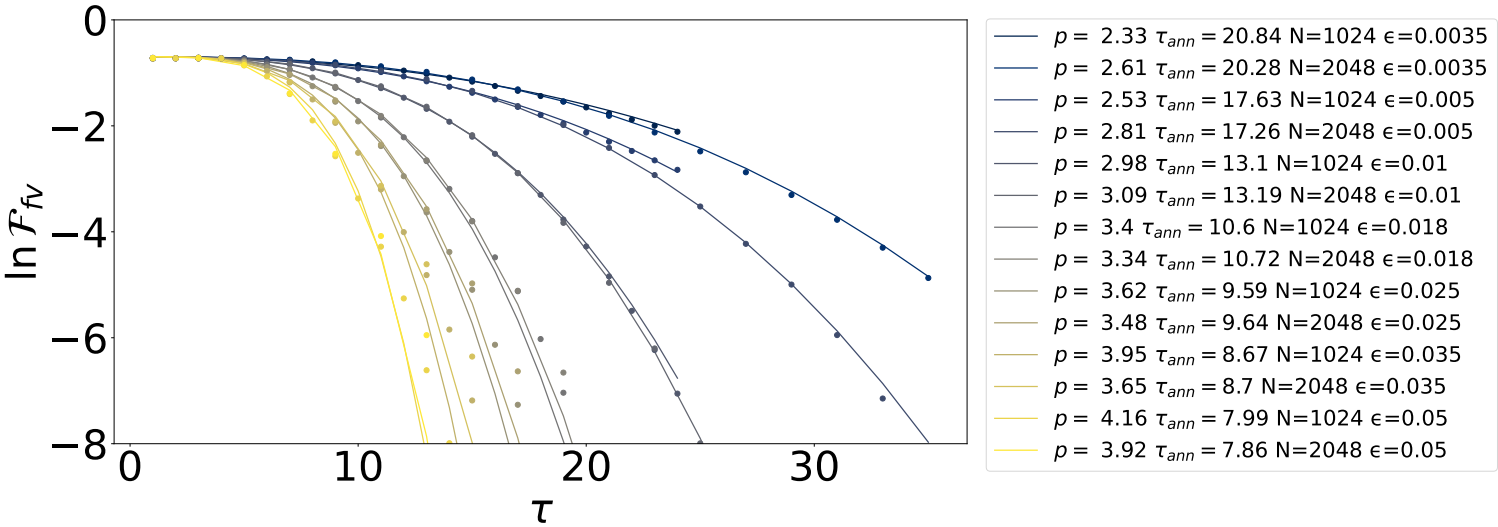


vacuum fraction

$$\mathcal{F}_{fv} = \frac{1}{2} \times \exp\left[-\left(\frac{\tau}{\tau_{ann}}\right)^{p}\right]$$



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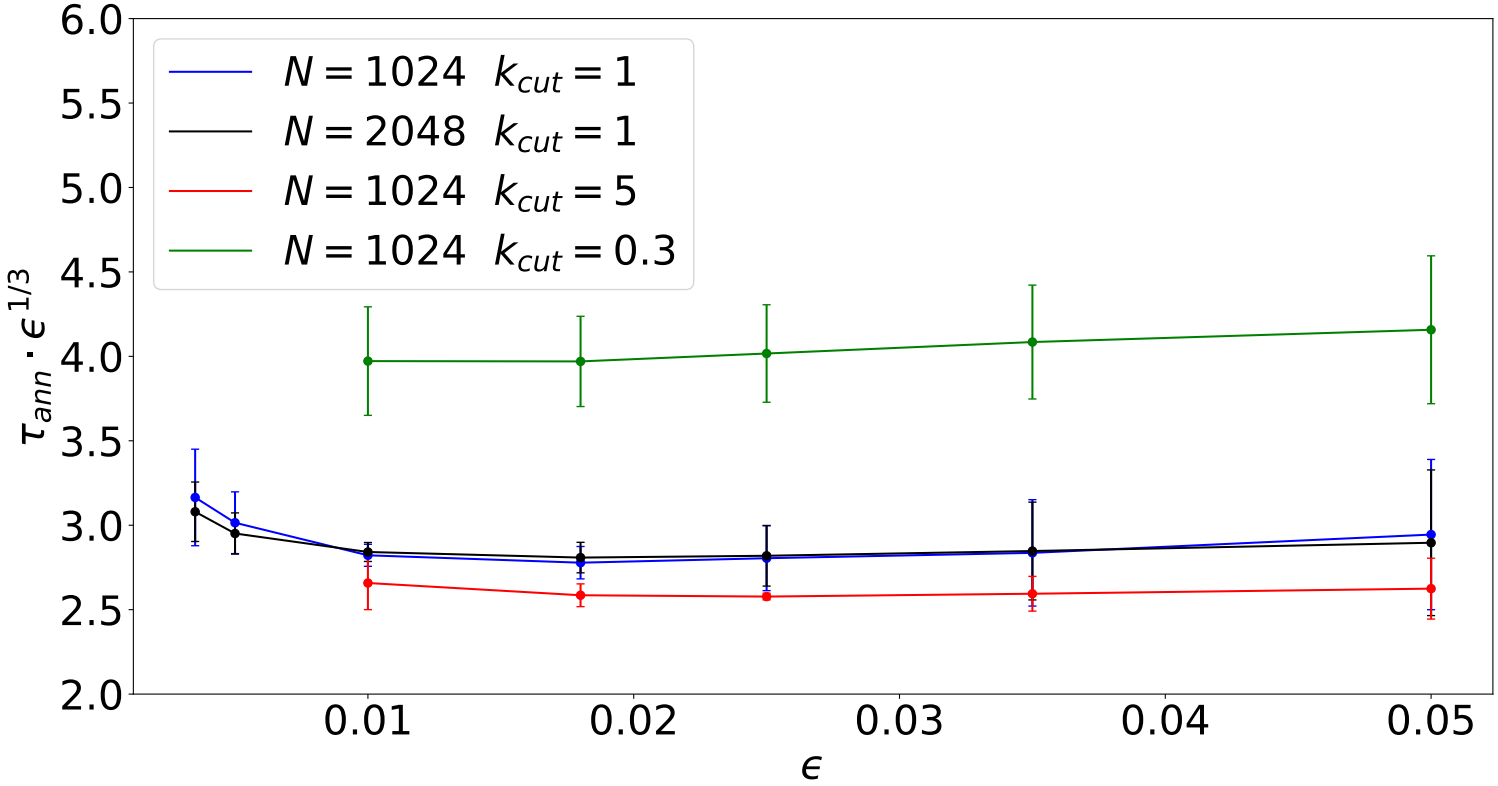
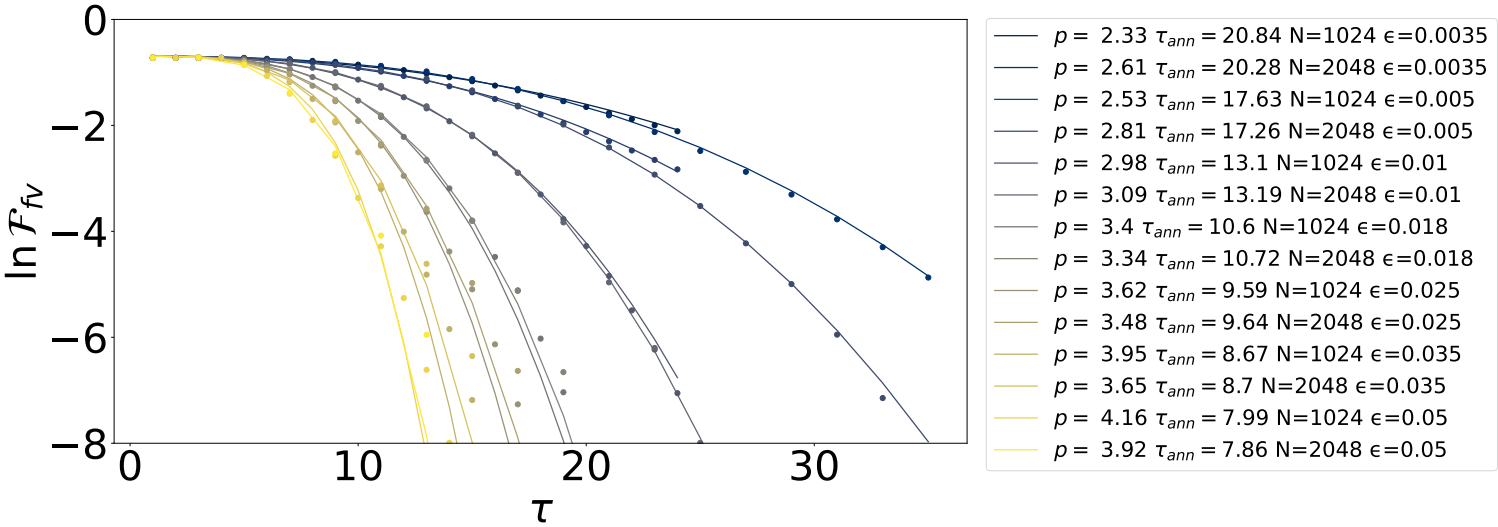


first surprise:

$\tau_{ann} \propto \varepsilon^{-1/3}$

no idea why...



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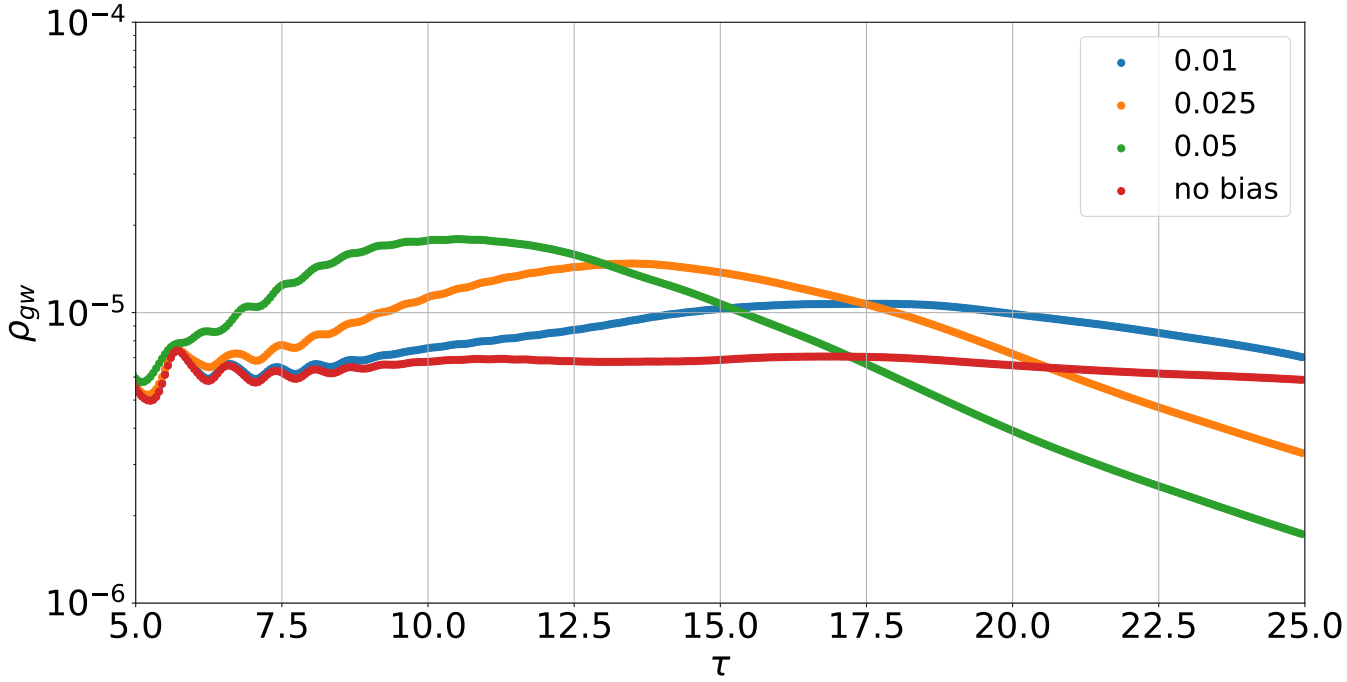

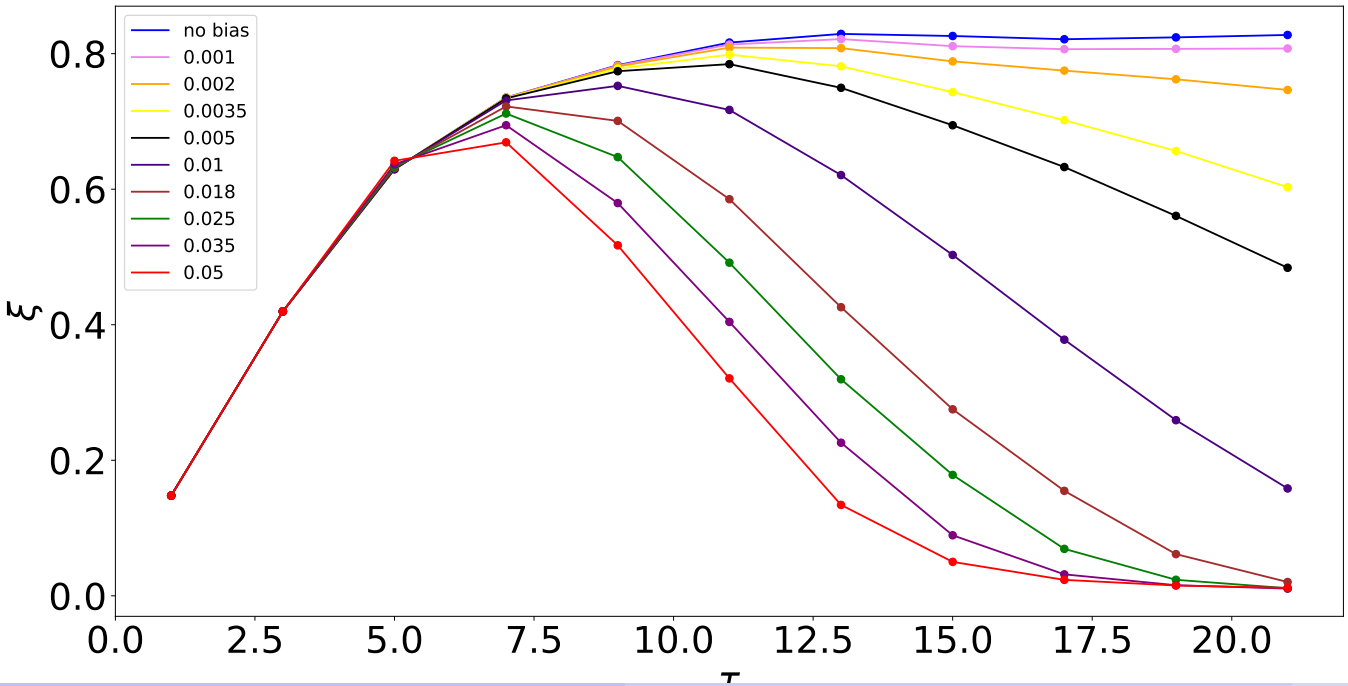

$$\varepsilon = 0.025$$

second surprise:
GW production stops later

$$\tau_{GW} \simeq 2\,\tau_{ann}$$

while naively at RD

$$\Omega_{GW} = \frac{\rho_{GW}}{\rho_{tot}} \propto a_{GW}^4$$

$$\Omega_{GW} \propto \tau_{GW}^4 \propto \frac{1}{\varepsilon^2}$$

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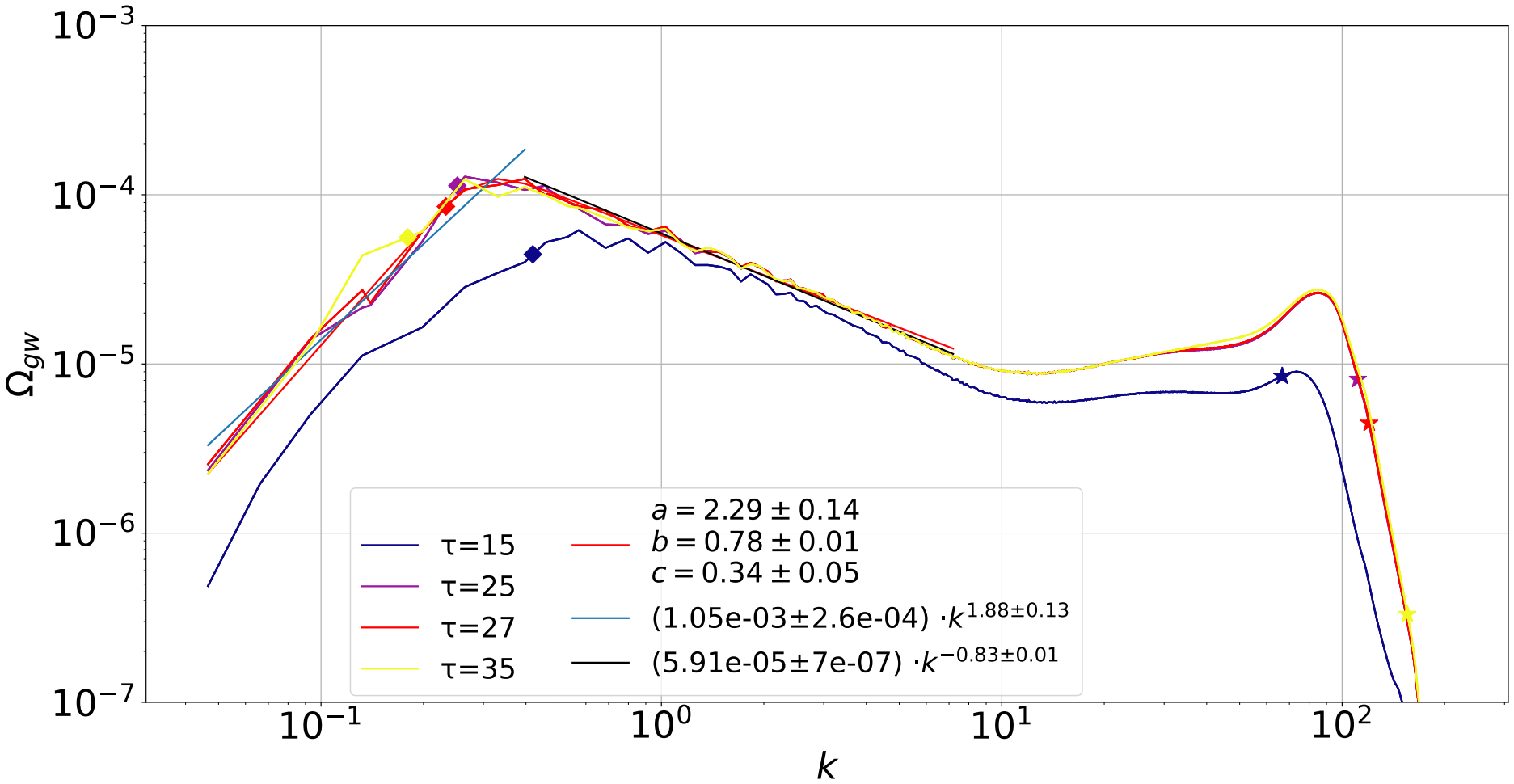
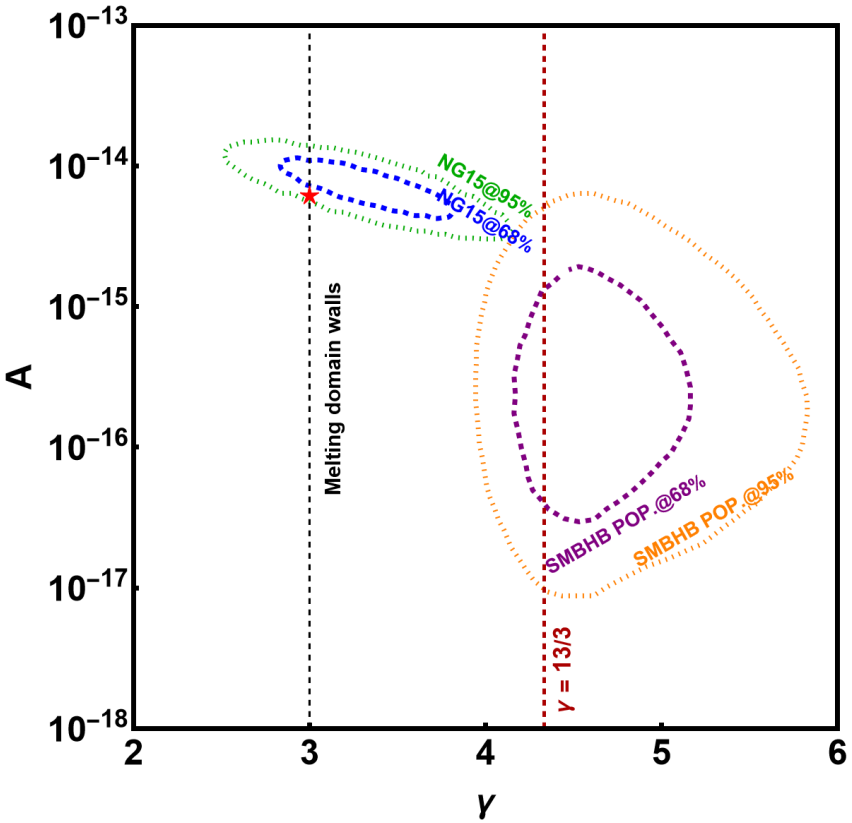
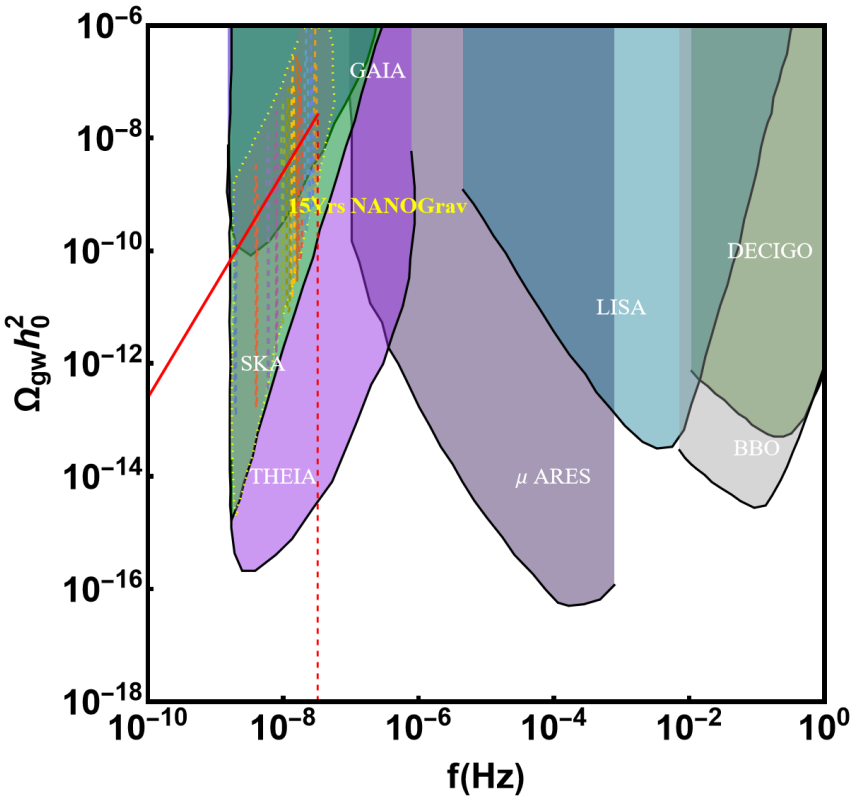


$$f_{peak} \simeq 8\ \text{nHz}\,\lambda^{1/4} \cdot \left(\frac{\varepsilon}{10^{-36} \cdot \lambda v}\right)^{1/3} \cdot \sqrt{\frac{v}{100\ \text{TeV}}} \cdot \left(\frac{100}{g_*(T_{ann})}\right)^{1/12},$$

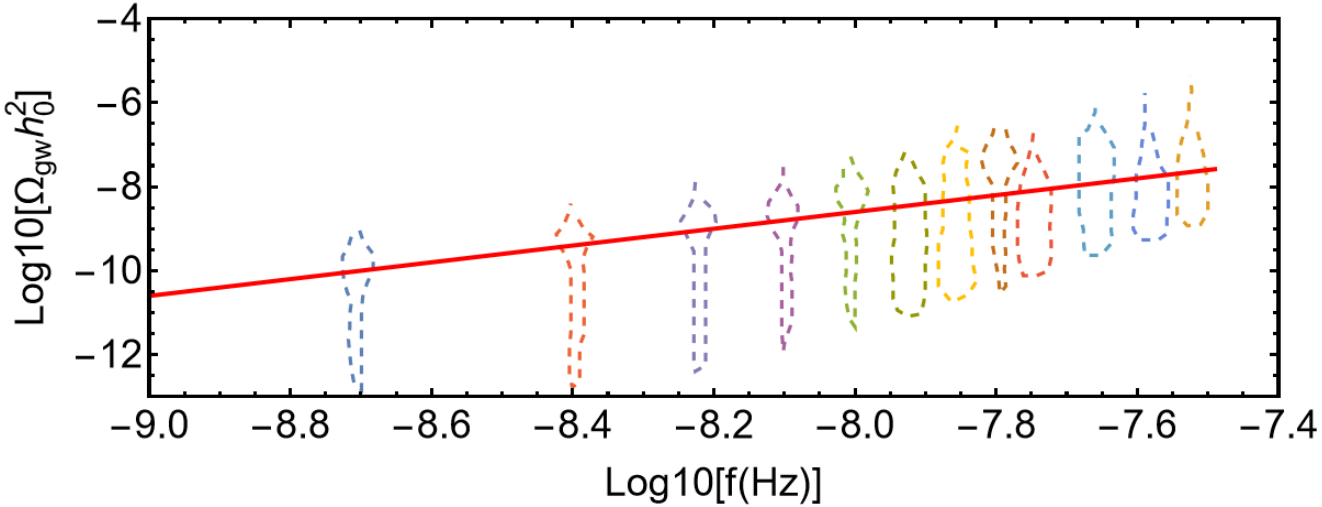
$$\Omega_{gw,peak} h_0^2 \simeq 1 \cdot 10^{-10} \cdot \left(\frac{v}{100\ \text{TeV}}\right)^4 \cdot \left(\frac{10^{-36} \cdot \lambda v}{\varepsilon}\right)^{4/3} \cdot \left(\frac{100}{g_*(T_{ann})}\right)^{1/3}.$$





$$\gamma \equiv 5 - d$$

2307.04582



Conclusion

- What NANOGrav and others observe might be explained by the GW from melting domain walls
- Note: $f = 1$ nHz implies GW production at $T \sim 100$ MeV
- field ϕ may serve as dark matter !! see e.g. 2004.03410, 2104.13722, 2112.12608
- In realistic models its mass is of order 10^{-15}-10^{-11} eV
- That predicts super-radiance: instability of rotating black holes with astrophysical masses 10-$10^5\, M_\odot$ see e.g. 0905.4720
  Might explain why the BH in the observed (GW) binary system are not with $J \approx 1$
- Biased DWs... A signal of $G_{WG} \propto 1/\varepsilon^{3/2}$ instead of $G_{WG} \propto \varepsilon^2$ ... Naturally $\varepsilon \lll 1$, e.g. gravity
- An approach to dynamics of D-branes in $(D+1)+1$ dimensions... ?

Large based on the results from
2004.03410, 2104.13722, 2112.12608, 2307.04582, 2410.21971 , 2406.17053, 2504.07902, 2509.25367

with Evgeny Babichev, Ivan Dankovsky, Sabir Ramazanov, Rome Samanta and Alex Vikman

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