Brane bound states, deformations and OM

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based on the works with:

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Why deformations?

1. Give family of solutions for complicated EOMs. Example: bi-vector

$$G + B = \left((g + b)^{-1} + \beta\right)^{-1}, \tag{1}$$

2. Lead to new interesting theories - non-commutative, non-supersymmetric, non-conformal.
3. Under gauge/gravity duality families of CFT's correspond to families of supergravity backgrounds. Properties of strings on the backgrounds (integrability, supersymmetry etc.) have dual interpretation.

Open/closed map and non-commutativity for strings

Open/closed string map [Arfaei, Sheikh Jabbari, Seiberg, Witten (1998-1999)]

$$S_{OS} = \frac{1}{4\pi\alpha'} \int_\Sigma d^2\sigma \left(g_{\mu\nu}\eta^{ab} + 2\pi\alpha' B_{\mu\nu}\varepsilon^{ab} \right) \partial_a x^\mu \partial_b x^\nu - \int_{\partial\Sigma} A_\mu \partial_\tau x^\mu. \tag{2}$$

boundary conditions along Dp-brane

$$g_{\mu\nu}\partial_{\vec{n}} X^\nu + 2\pi\alpha' B_{\mu\nu}\partial_\tau X^\nu \Big|_{\partial\Sigma} = 0, \tag{3}$$

Correlator for the ends of a string

$$\langle X^\mu(\tau) X^\nu(\tau') \rangle = -G^{\mu\nu} \log(\tau - \tau')^2 + \frac{i}{2}\theta^{\mu\nu}\varepsilon(\tau - \tau'), \tag{4}$$

with open/closed (or Seiberg–Witten) map, further its inverse also called bi-vector deformation

$$G^{-1} + \frac{1}{2\pi\alpha'}\theta = (g + 2\pi\alpha' B)^{-1}, \quad G_s = g_s \sqrt{\frac{\det G}{\det(g + 2\pi\alpha' B)}}. \tag{5}$$

Open/closed map and non-commutativity for strings

$$S_{OS} = \frac{1}{4\pi\alpha'} \int_\Sigma d^2\sigma \left(g_{\mu\nu}\eta^{ab} + 2\pi\alpha' B_{\mu\nu}\varepsilon^{ab} \right) \partial_a x^\mu \partial_b x^\nu - \int_{\partial\Sigma} A_\mu \partial_\tau x^\mu . \tag{6}$$

In the limit $\sqrt{2\pi\alpha'}|\partial F/F| \ll 1$ the effective action governing dynamics of the open string ends is given by the DBI action [Fradkin, Tseytlin (1999)]

$$S_{DBI} = \frac{2\pi}{g_s (4\pi^2\alpha')^{\frac{p+1}{2}}} \int d^{p+1}\xi \sqrt{-\det\left(g + 2\pi\alpha'(B+F) \right)}. \tag{7}$$

where $F = dA$. The Seiberg–Witten map states that this action is equivalent to

$$\hat{S}_{NC} = \frac{2\pi}{G_s (4\pi^2\alpha')^{\frac{p+1}{2}}} \int d^{p+1}\xi \sqrt{-\det\left(G + 2\pi\alpha'\hat{F} \right)}, \tag{8}$$

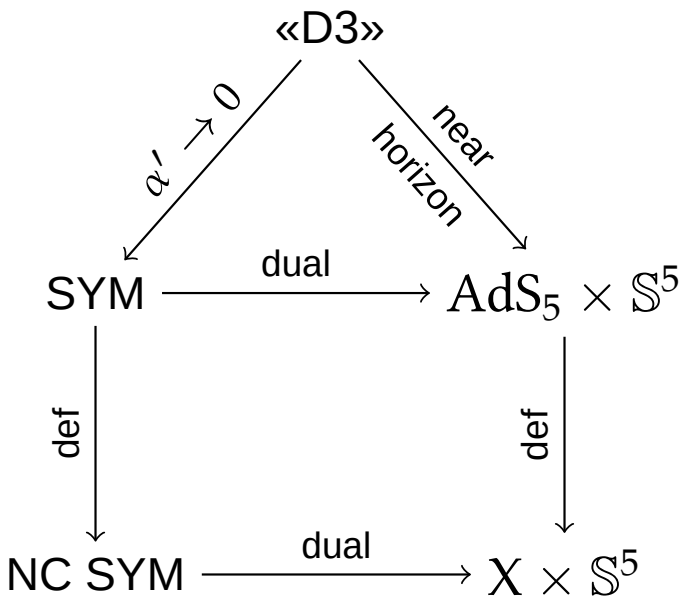
where $\hat{F} = d\hat{A} - [\hat{A}, \hat{A}]_*$ is the non-commutative field strength and $*$ is the standard non-commutative Moyal product defined by θ.

Special limits of open/closed map

1. Zero slope limit with no B-field (low energy) - SYM. Near Dp-brane horizon closed and open string describe the same physics (AdS/CFT) [Aharony, Gubser, Maldacena, Ooguri, Oz (2000)]
2. Zero slope limit with magnetic B-field - $\mathrm{B}_{12} = \mathrm{B}$, the non-commutative Yang–Mills theory.

Formally the limit is taken by rescaling

$$\begin{aligned} &\alpha' = \varepsilon^{\frac{1}{2}}\bar{\alpha}', \qquad \mathrm{g}_{ij} = \varepsilon\bar{\mathrm{g}}_{ij}, \quad \varepsilon \to 0, \\ &\theta \to -\mathrm{B}^{-1}, \quad \mathrm{G}^{-1} \to -(2\pi\alpha')^{-1}\mathrm{B}^{-1}\mathrm{g}\mathrm{B}^{-1} \end{aligned} \tag{9}$$



Special limits of open/closed map

3. Zero slope limit with electric B-field - $\mathrm{B}_{01} = \mathrm{E}$.

$$\mathrm{ds}^2 = \gamma(-(\mathrm{dx}^0)^2 + (\mathrm{dx}^1)^2) + \delta \mathrm{dx}_\perp^2, \tag{10}$$

Naive limit (direct SW map)

$$\begin{aligned} \mathrm{G} &= \left(\varepsilon^2\gamma^2 - \varepsilon(2\pi\alpha')^2\mathrm{E}^2\right)\varepsilon^{-1}\gamma^{-1}\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}, \\ \theta &= \frac{(2\pi\alpha')^2\mathrm{E}\varepsilon}{\varepsilon^2\gamma^2 - \varepsilon(2\pi\alpha')^2\mathrm{E}^2}\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}. \end{aligned} \tag{11}$$

There is $\mathrm{E}_{\mathrm{cr}} = \gamma/2\pi\alpha'$, where open string dynamics becomes essentially different, in particular string ends become separated by a light-like interval and no longer interact, the open string spectrum becomes non-relativistic, no effective field theory, like NCYM above, dynamics essentially stringy (NRST) [Seiberg, Susskind, Toumbas (2000)].
Formal limit $\varepsilon \to 0$ now badly defined.

Interesting limits of open/closed map

Correct way to take the limit and obtain closed string background [Gomis, Ooguri (2001)]

$$\frac{\alpha'}{\alpha'_{\text{eff}}} = \varepsilon \to 0, \quad 1 - \left(\frac{2\pi\alpha' E}{\gamma}\right)^2 = \varepsilon \to 0, \quad 2\pi\alpha'\Gamma^{-1} = 2\pi\alpha'_{\text{eff}}\Gamma_{\text{eff}}^{-1}, \quad \delta = \tilde{\delta}\varepsilon \tag{12}$$

$G = \Gamma\,\mathrm{diag}[-1,1]$. In this limit

$$\theta \to 2\pi\alpha'_{\text{eff}}\Gamma^{-1}\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}. \tag{13}$$

The action then rescales as follows

$$\begin{aligned} S &= \frac{1}{4\pi\alpha'}\int d^2\sigma\left[\gamma(\partial x_{||})^2 + \delta(\partial_\perp)^2 - 2(2\pi\alpha')E\partial x^0\partial x^1\right] \\ &= \frac{1}{4\pi\alpha'_{\text{eff}}}\int d^2\sigma\left[\gamma\varepsilon^{-1}(\partial x_{||})^2 + \tilde{\delta}(\partial x_\perp)^2 - 2\gamma\left(\frac{1}{\varepsilon} - \frac{1}{2}\right)\partial x^0\partial x^1\right]. \end{aligned} \tag{14}$$

The effective closed string background

$$\begin{aligned} ds^2 &= \varepsilon^{-1}\gamma(-(dx^0)^2 + (dx^1)^2) + \tilde{\delta}dx_\perp^2, \\ 2\pi\alpha' B &= \gamma\left(\frac{1}{\varepsilon} - \frac{1}{2}\right)dx^0 \wedge dx^1. \end{aligned} \tag{15}$$

Bi-vector deformations

It is reverse of open/closed map

$$(G + B)^{-1} = (g + b)^{-1} + \beta \tag{16}$$

Sufficient conditions to have a solution of supergravity

$$\begin{aligned}
[k_a, k_b] &= f_{ab}{}^c k_c && \text{(Killing vector algebra)} \\
\beta^{mn} &= k_a{}^m k_b{}^n r^{ab} && \text{(bi-Killing anzats);} \\
r^{b_1[a_1} r^{|b_2|a_2} f_{b_1 b_2}{}^{a_3]} &= 0 && \text{(classical YB equation);} \\
r^{b_1 b_2} f_{b_1 b_2}{}^a k_a{}^m &= I^m = 0 && \text{(unimodularity condition);}
\end{aligned} \tag{17}$$

For particular case of deformation along two spatial commuting Killing vectors such deformation is known to be TsT transformation and on the level of sigma-model described by $T\bar{T}$ transformation [Blair (2020)], also Dp-branes under such TsT transformation is pretty well understood [Imeroni (2008)]

Conclusions I

- There is open closed map for strings
- Electric limit is special and gives NRST
- They can be understood as bivector deformations
- Holographic duality between closed and open string description

Simplest example - KK reduced GR and uni-vectors

The standard General Relativity in $D = 4$

$$S_{GL(4)} = \int d^4x \sqrt{-G} R[G] \tag{18}$$

$$ds^2 = e^{\phi} g_{mn} dx^m dx^n + e^{-\phi} \left(dz + \mathcal{A}_m dx^m \right)^2$$

The Einstein–Maxwell dilaton theory in $D = 3$:

$$S_{EMd} = \int d^3x \sqrt{-g} \left(R[g] - \tfrac{1}{2} \partial_m \phi \, \partial^m \varphi - \tfrac{1}{4} e^{-2\phi} \mathcal{F}^{mn} \mathcal{F}_{mn} \right), \tag{19}$$

has hidden symmetries:

1. global GL(4) (analogue of the global $O(d,d)$);
2. local diffeos modulo the "section condition" $\partial_z = 0$.

Uni-vector deformations

- Bi-vector deformations

$$G + B = \left((g + b)^{-1} + \beta\right)^{-1}, \tag{20}$$

Generates solution for bi-killing deformation with $L_{k_i} = 0$ and CYBE + unimodularity

- Uni-vector deformations

$$\begin{aligned} e^{-\tilde{\varphi}} &= e^{\varphi}\alpha_k\alpha^k + e^{-\varphi}(1 + \mathcal{A}_k\alpha^k)^2\,, \\ \tilde{\mathcal{A}}_m &= e^{\tilde{\varphi}}(e^{\varphi}\alpha_m + e^{-\varphi}\mathcal{A}_m(1 + \mathcal{A}_k\alpha^k))\,, \\ \tilde{g}_{mn} &= e^{-\tilde{\varphi}}(e^{\varphi}g_{mn} + e^{-\varphi}\mathcal{A}_m\mathcal{A}_n) - e^{-2\tilde{\varphi}}\tilde{\mathcal{A}}_m\tilde{\mathcal{A}}_n\,. \end{aligned} \tag{21}$$

Generates solutions if $\alpha^m = \alpha^m(x)$ is a $D = 3$ Killing vector:

$$\mathcal{L}_\alpha\,\varphi = 0, \quad \mathcal{L}_\alpha\,g_{mn} = 0, \quad \mathcal{L}_\alpha\,\mathcal{A}_m = 0, \tag{22}$$

Example

Schwarzschild Black hole: $f(r) = 1 + \frac{r_g}{r}$

$$ds^2 = -f(r)dt^2 + f(r)^{-1}dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2), \quad \varphi = 0, \quad \mathcal{A}_m = 0,$$

Deformation along $\alpha = c\,\partial_\phi$ gives:

$$ds^2 = \sqrt{1 + c^2 r^2 \sin^2\theta}\left[-f(r)dt^2 + f(r)^{-1}dr^2 + r^2\left(d\theta^2 + \frac{\sin^2\theta}{1 + c^2 r^2 \sin^2\theta}d\phi^2\right)\right],$$

$$\mathcal{A} = \frac{c\,r^2 \sin^2\theta}{1 + c^2 r^2 \sin^2\theta}d\phi,$$

$$\tilde{\varphi} = -\frac{\sqrt{3}}{2}\ln\left(1 + c^2 r^2 \sin^2\theta\right), \tag{23}$$

This is not equivalent to the Gibbons–Maeda solution.

Uni-vector and bi-vector comparison

	D-dim Einstein–Maxwell dilaton	10-dim Supergravity
fields:	$g_{mn},\ \mathcal{A}_m,\ \phi$	$g_{mn},\ b_{mn},\ \phi$
parent:	GR in $D+1$	Double Field Theory
coordinates:	$x^M = (x^m, z)$	$x^M = (x^m, \tilde{x}_m)$
section condition:	$\partial_z = 0$	$\tilde{\partial}^m = 0$
hidden symmetry:	$GL(D+1)$	$O(10,10)$
deform. param.:	α^m	$\beta^{mn} = r^{i_1 i_2} k_{i_1}{}^m k_{i_2}{}^n$
conditions:	$L_\alpha = 0$	$L_{k_i} = 0$ CYBE + unimodularity

"Zoo" of poly-vector deformations

All poly-vectors are poly-Killing $\Omega^{m_1\dots m_N} = \rho^{i_1\dots i_N} k_{i_1}{}^{m_1} \dots k_{i_n}{}^{m_N}$

theory	poly-vectors	conditions	extended theory
Einstein-Maxwell-dilaton	α^m	-	GL(D+1) GTR
I/IIA/IIB supergravity	β^{mn}, $\Omega^{m_1\dots m_4}$	CYBE, uni.	O(d,d) DFT, reduced ExFT
heterotic supergravity	β^{mn}, α^m	CYBE*, uni.* commutativity	O(d,d+496) GDFT
11D supergravity	$\Omega^{m_1\dots m_3}$, $\Omega^{m_1\dots m_6}$	gCYBE, uni.	$E_{d(d)}$ ExFT

unimodularity in 11D

$$\rho^{a_1a_2a_3} f_{a_2a_3}{}^{a_4} = 0, \qquad \rho^{[a_1a_2a_3a_4|a_5a_6} f_{a_5a_6}{}^{|a_7]} = 0. \tag{24}$$

GYBE in 11D

$$\rho^{a_1[a_2|a_6|}\rho^{a_3a_4|a_5|} f_{a_5a_6}{}^{a_7]} - \rho^{a_2[a_1|a_6|}\rho^{a_3a_4|a_5|} f_{a_5a_6}{}^{a_7]} - 3 f_{a_5a_6}{}^{[a_1}\rho^{a_2]a_3a_4a_5a_6a_7} = 0, \qquad \rho^{a_1a_2[a_8}\rho^{a_3a_4a_5a_6a_7a_9]} f_{a_8a_9}{}^{a_{10}} = 0. \tag{25}$$

compare with

$$\text{CYBE:} \quad r^{b_1[a_1} r^{|b_2|a_2} f_{b_1b_2}{}^{a_3]} = 0, \quad \text{Uni:} \quad r^{b_1b_2} f_{b_1b_2}{}^{a} = 0 \tag{26}$$

Statements

- Dp-branes (DBI): low energy closed string description of the effective open string
- Dp-F1 bound states: low energy non-relativistic closed string (NRCS) description of NCOS [Harmark (2000)]
- M5-M2 brane bound state: low energy description of space-time «non-commutative» open membranes in the critical C-field limit and dubbed «the OM theory» [Gopakumar, Minwalla, Seiberg, Strominger (2000); Bergshoeff, Berman, Schaar, Sundell (2000)] with open/closed membrane map, that is reproduced by poly-vector deformations.
- The effective way to generate bound states in string and M-theory are poly-vector deformations, turning of which add the corresponding objects to the background (“sedimentation”)

“Sedimentation”

Abelian	α^{m}	β^{mn}	$\Omega^{m_1\dots m_3}$	$\Omega^{m_1\dots m_4}$	$\Omega^{m_1\dots m_6}$	
Generates	?	F1	M2	D3	M5	from AdS

[Barakin, Gubarev, Musaev (2025)]

This is a way to construct bound states. For example, starting with the M5-brane state and performing deformation with

$$\Omega = \gamma\, \partial_0 \wedge \partial_1 \wedge \partial_2, \tag{27}$$

we obtain M5-M2 bound state. The same works for Dp-branes and strings (D3-F1, D1-F1).

Example M2-brane sedimentation

Consider the background generated by a stack of N M2-branes

$$\begin{aligned} ds^2 &= H(r)^{-\frac{2}{3}} dx_{||}^2 + H(r)^{\frac{1}{3}} \left(dr^2 + r^2 d\Omega_7\right), \\ C_{012} &= \pm\left(H(r)^{-1} - 1\right), \end{aligned} \tag{28}$$

where

$$H(r) = 1 + \frac{1}{r^6} \frac{N l_P^6}{6\,\omega_{(7)}}. \tag{29}$$

Perform a tri-vector deformation with

$$\Omega = \gamma\, P_0 \wedge P_1 \wedge P_2. \tag{30}$$

This results in the change of harmonic function

$$\tilde{H}(r) = 1 + \frac{1}{r^6} \frac{N' l_P^6}{6\,\omega_{(7)}}, \quad N' = \frac{N}{1-2\gamma}. \tag{31}$$

Example M5-M2 bound state

Start with a stack of N M5-branes, that is

$$ds^2 = H(r)^{-\frac{1}{3}} dx_{||}^2 + H(r)^{\frac{2}{3}} (dr^2 + r^2 d\Omega_4),$$
$$C_{01\ldots5} = (H(r)^{-1} - 1), \quad H(r) = 1 + \frac{1}{r^3} \frac{N l_P^3}{3\omega_{(4)}} \tag{32}$$

Perform the tri-vector deformation along

$$\Omega = \gamma \partial_0 \wedge \partial_1 \wedge \partial_2, \tag{33}$$

that gives deformed solution

$$\begin{aligned} ds'^2 = {} & H^{-\frac{1}{3}} D^{-\frac{1}{3}} \left(-(dx^0)^2 + (dx^1)^2 + (dx^2)^2\right) \\ & + H^{-\frac{1}{3}} D^{\frac{2}{3}} \left((dx^3)^2 + (dx^4)^2 + (dx^5)^2\right) + H^{\frac{2}{3}} D^{-\frac{1}{3}} (d\rho^2 + \rho^2 d\Omega_4), \\ F_4' = {} & \frac{N l_P^3}{\omega_{(4)}} d\,\mathrm{vol}[\mathbb{S}^4] - \sin\theta d_\rho \left(\frac{DH^{-1}}{\cos\theta} dx^3 \wedge dx^4 \wedge dx^5 - H^{-1} dx^0 \wedge dx^1 \wedge dx^2 \right) \end{aligned} \tag{34}$$

$$H = \frac{1}{\cos^2\theta} \left(H_0 - \sin^2\theta\right) = 1 + \frac{N}{\cos\theta} \frac{l_P^3}{3\omega_{(4)}} \frac{1}{\rho^3}, \quad D^{-1} = \cos^2\theta + \sin^2\theta H^{-1}. \tag{35}$$

Open/closed map for branes equals to poly-vectors

In the limit of a large number of M5-branes for a test M2-brane against their background

[Bergshoeff, Berman, Schaar, Sunde (2000)]

$$S = \frac{1}{3}\int_{\partial\Sigma} d^2\sigma \mathcal{H}_{mnr} X^m \dot{X}^n X'^r, \tag{36}$$

Dirac bracket for $X^m(\sigma)$ (end of M2-brane – loops)

$$\left[X^m(\sigma), X^n(\sigma')\right] = -\frac{1}{h}\frac{\Omega^{mnk} X'^k(\sigma)}{|X(\sigma)|^2}\delta(\sigma - \sigma'). \tag{37}$$

The open membrane metric and non-commutativity parameter read

$$\begin{aligned} G_{mn} &= \left(1 + \frac{1}{3!}c_3^2\right)^{\frac{1}{3}} \left(g_{mn} + \frac{1}{2!}(c_3^2)_{mn}\right), \\ \Omega^{mnk} &= -\left(1 + \frac{1}{3!}c_3^2\right) c^{mnk}, \end{aligned} \tag{38}$$

reproduced by tri-vector deformation rules.

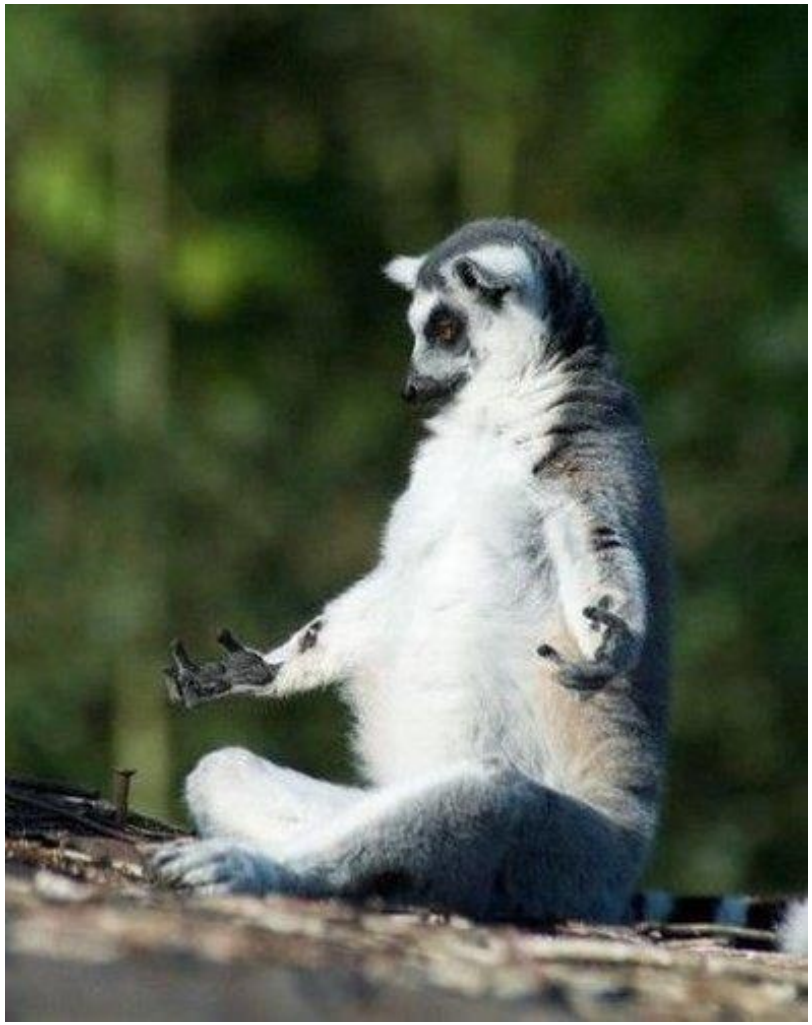
Conclusions II

1. Bi-vector Yang-Baxter deformations generalize up and down
2. Uni-vector and almost-abelian bi-vector Yang-Baxter deformations are equivalent to coordinates transformation in the parent theory
3. Classical Yang–Baxter equation follows from the consistency of the algebra of such transformations
4. Abelian poly-vectors generate boun states of Dp-F1(NRST) & M5-M2 (OM) bound states.

Questions for future

- Uni-vectors and integrability?
- New integrable deformations for 2D sigma models?
- 3D/6D integrability of M2 and M5-branes?
- Fundamental object for the uni-vector deformation?
- Dual to bi-vector – $\beta^{m_1 \dots m_6}$ deformation and NS5 branes?
- Loop non-commutativity, algebras? Nambu mechanics?
- Thermal bound states, Hagedorn behaviour?
- Holographic duals of the deformations?

Thank you for your attention!



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OM!