Gauge invariant SVT generalizations of Horndeski theory

based on papers with S. Mironov and A. Shtennikova

M. Valencia-Villegas

ITMP, MSU and INR, RAS

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Outline

1. Why modified gravity?
   - Issues with non-minimally coupled Scalar–Tensor theories
     - Speed of Gravitational Waves
     - Decay
2. An alternative:

   A class of Scalar–Vector–Tensor theories

1. Introduction:

Why "Modified Gravity"?

Our reference theory, Einstein's gravity (GR)

- generally suffers singularities, if the Null Energy Condition (NEC) holds (Penrose, Hawking theorems),
- ΛCDM (Cosmological Constant Problem, Weinberg (1989)),
- challenges ahead?
  (DESI BAO data, *e.g.* A.G. Adame *et al.*, DESI 2024 VI. JCAP 02 (2025))

$\vdots$

1. Introduction: Horndeski theory

Scalar modification of GR that

- keeps second order equations (No Ghosts),
- does not satisfy the NEC in general,

Unique answer to these conditions: **Horndeski theory** (1974)

Extensively used for

- early and late time cosmology
- compact objects and other modified gravity solutions

There are generalizations: Beyond Horndeski (Gleyzes-Langlois-Piazza-Vernizzi theory) and DHOST (Ben Achour, Crisostomi, Koyama, Langlois, Noui Tasinato (2016) JHEP 12, 100 [arXiv:1608.08135])

1. What is Horndeski theory?

- Take a real scalar field π besides the metric field,
- On top of GR take 4 general Scalar Potentials $G_i(\pi, X)$, $i = 2, 3, 4, 5$, where $X = \nabla_\mu \pi \nabla^\mu \pi$,
- Most general combinations of R and $(\nabla^2 \pi)^p$ with $p \leq 3$ and G_i coefficients, with **second order equations of motion**.

$$\mathcal{L}_\pi = \mathcal{L}_2 + \mathcal{L}_3 + \mathcal{L}_4 + \mathcal{L}_5,$$
$$\mathcal{L}_2 = G_2(\pi, X),$$
$$\mathcal{L}_3 = G_3(\pi, X)\Box\pi,$$
$$\mathcal{L}_4 = G_4(\pi, X)R - 2G_{4X}(\pi, X)\left[(\Box\pi)^2 - \pi_{;mn}\pi^{;mn}\right],$$
$$\mathcal{L}_5 = G_5(\pi, X)G^{mn}\pi_{;mn} + \frac{G_{5X}}{3}\left[(\Box\pi)^3 - 3\Box\pi\pi_{;mn}\pi^{;mn} + 2\pi_{;mn}\pi^{;ml}\pi_{;l}{}^{n}\right],$$

1. What is Horndeski theory?

General issue with non-minimally coupled Horndeski theory and beyond:

- The speed of Gravitational Waves is not constant.
- Gravitational Waves may decay to the scalar perturbation of Dark Energy Creminelli, Lewandowski, Tambalo and Vernizzi (2018) JCAP 12, 025 [arXiv:1809.03484].

In tension with observations

1. Review: The Speed of Perturbations

Consider Horndeski theory + Maxwell

- Take the Perturbed metric

$$\mathrm{d}s^2 = (\eta_{\mu\nu} + \delta g_{\mu\nu})\mathrm{d}x^\mu \mathrm{d}x^\nu$$

with FLRW background

$$\eta_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu = -\mathrm{d}t^2 + a^2\,\delta_{ij}\mathrm{d}x^i \mathrm{d}x^j$$

and metric perturbation,

$$\delta g = a^2\,\left(2\,S_i \mathrm{d}t\,\mathrm{d}x^i + (\partial_i F_j + \partial_j F_i + 2\,h_{ij})\,\mathrm{d}x^i\,\mathrm{d}x^j\right)\,,$$

- The Photon is the perturbation $A_i(t, \vec{x})$

1. Review: The Speed of Perturbations

- Quadratic action for Graviton,

$$\mathcal{S}_{Graviton} = \frac{1}{2}\int \mathrm{d}t\,\mathrm{d}^3x\,a^3\left(\mathcal{G}_\tau\,\dot{h}_{ij}^2 - \frac{\mathcal{F}_\tau}{a^2}\left(\partial_k h_{ij}\right)^2\right)\,, \tag{2}$$

thus, the Speed of Gravitational Waves is,

$$c_g^2 = \frac{\mathcal{F}_\tau}{\mathcal{G}_\tau}\,, \tag{3}$$

- Generally, the quadratic action for the Photon takes the form,

$$\mathcal{S}_{Photon} = \frac{1}{4}\int \mathrm{d}t\,\mathrm{d}^3x\,a\left(\mathcal{G}_A\,\dot{A}_i^2 - \frac{\mathcal{F}_A}{a^2}\left(\partial_k A_i\right)^2\right)\,. \tag{4}$$

For the minimally coupled Photon $\mathcal{F}_A = \mathcal{G}_A = 1$, $c^2 = 1$.
Note that scalar modifications of the vector sector would yield a different Speed of a nonminimal Photon

$$c^2 = \frac{\mathcal{F}_A}{\mathcal{G}_A}\,. \tag{5}$$

1. Speed of GWs: observation vs. theory

- **The event** GW170817 and the electromagnetic counterpart GRB170817A set the speed of GWs (c_g) very close to the speed of light c

$$\left|\frac{c_g}{c} - 1\right| \leq 5 \times 10^{-16}\,, \tag{6}$$

in this talk, I *assume* exactly Luminal GWs, $c_g \equiv c$

- Now, what's with **the theory**? in Horndeski theory on a cosmological FLRW background, the speed of GWs is

$$c_g^2 = \frac{2G_4 - 2\ddot{\pi}XG_{5,X} + XG_{5,\pi}}{2G_4 - 4XG_{4,X} - 2\dot{\pi}XHG_{5,X} - XG_{5,\pi}} \neq \mathbf{1} = \mathbf{c^2}\,, \tag{7}$$

1. Speed of GWs: observation vs. theory

Historically standard approach: Constrain the Scalar couplings by

assuming minimal Photon such that $c_g = c = 1$.

- Consequence of (B): necessarily $G_{4X} = G_5 \equiv 0$

$$c_g^2 = \frac{2G_4 - 2\ddot{\pi}XG_{5,X} + XG_{5,\pi}}{2G_4 - 4XG_{4,X} - 2\dot{\pi}XHG_{5,X} - XG_{5,\pi}} = \mathbf{1} = \mathbf{c^2}\,, \qquad (8)$$

  - We are left with Brans-Dicke type theory $G_4 = G_4(\pi)$.
  - We lose the Fab-Four, in the context of Cosmological constant problem (Charmousis et. al., 2011)

Is there another option?

Yes!

1. Why modified gravity?
    - Issues with non-minimally coupled Scalar–Tensor theories
        - Speed of Gravitational Waves
        - Decay

2. An alternative:

A class of Scalar–Vector–Tensor theories

2. Alternative: Scalar–Vector–Tensor theories (SVT)

- Supplement Horndeski theory with the Scalar—Photon couplings that satisfy

$$\frac{c_g(t)}{c(t)} = 1\,. \tag{9}$$

Note that here we will not assume $c(t) \equiv 1$.

How to obtain
Dark Energy—Photon couplings
such that we see

$$\frac{c_g(t)}{c(t)} = 1 \ ?$$

2. SVT from a Kaluza–Klein reduction of Horndeski Theory

- One possibility: First, take 5D Horndeski theory, just as a tool,
- compactify with the following Ansatz and the Cylinder condition

$$ {}^{(5)}g_{B\,C} = \begin{pmatrix} g_{\mu\,\nu} + A_\mu\, A_\nu & A_\mu \\ A_\nu & 1 \end{pmatrix} , \tag{10} $$

- 4D fields A_μ and g are but components of ${}^{(5)}g$
  - $U(1)$ gauge invariance for the vector in 4D is, as usual, guaranteed by diffeo invariance in 5D
  - **their speeds are** *bound* **to be the same** ...
  - ... But, caveat, we are ignoring the Dilaton $\longrightarrow$ compute speeds and check

(A) Horndeski Theory + DE —Photon couplings

- We obtain

$$\mathcal{L} = \text{Horndeski Theory in 4D} + \mathcal{L}_{4A} + \mathcal{L}_{5A} \tag{11}$$

where the **Scalar—Photon couplings** are

$$\mathcal{L}_{4A} = -\frac{1}{4} G_4(\pi, X)\, F^2 + G_{4,X}\, F_\mu{}^\sigma F_{\nu\sigma}\, \pi^{;\mu}\pi^{;\nu} \tag{12}$$

$$\mathcal{L}_{5A} \quad = \quad G_5(\pi) \left(\frac{1}{8} F^{\mu\nu} F_\mu{}^\rho (-4\, \pi_{;\nu\rho} + g_{\nu\rho} \Box\pi) + \frac{1}{2} F_{\mu\nu} \nabla_\sigma F^{\nu\sigma}\, \pi^{;\mu} \right)$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$.

S. Mironov, A. Shtennikova and **M. V-V** (2024), Phys.Lett.B 858, 139058 [arXiv:2405.02281]

S. Mironov, A. Shtennikova and **M. V-V** (2025), Phys. Rev. D 111, L101501 [arXiv:2412.13460]

2. SVT: Quadratic Action

Why $G_{5,X} = 0$?

- Quadratic action for Graviton and Photon read,

$$\mathcal{S}_{Graviton} = \frac{1}{2}\int \mathrm{d}t\,\mathrm{d}^3x\,a^3\left(\mathcal{G}_\tau\,\dot{h}_{ij}^2 - \frac{\mathcal{F}_\tau}{a^2}\,(\partial_k h_{ij})^2\right)\,, \tag{13}$$

$$\mathcal{S}_{Photon} = \frac{1}{4}\int \mathrm{d}t\,\mathrm{d}^3x\,a\left(\mathcal{G}_A\,\dot{A}_i^2 - \frac{\mathcal{F}_A}{a^2}\,(\partial_k A_i)^2\right)\,, \tag{14}$$

- Simple relation $\mathcal{S}_{Graviton}$ to $\mathcal{S}_{Photon}$,

$$\mathcal{G}_A = \mathcal{G}_\tau + \Delta \tag{15}$$

$$\mathcal{F}_A = \mathcal{F}_\tau\,. \tag{16}$$

2. SVT: Quadratic Action

- Speed of GWs and Light, respectively, on the cosmological medium,

$$c_g^2 = \frac{\mathcal{F}_\tau}{\mathcal{G}_\tau}, \quad c^2 = \frac{\mathcal{F}_A}{\mathcal{G}_A}$$

$$\frac{c_g^2}{c^2} = \frac{\mathcal{F}_\tau}{\mathcal{G}_\tau}\frac{\mathcal{G}_A}{\mathcal{F}_A} = \frac{\mathcal{G}_A}{\mathcal{G}_\tau} = 1 + \frac{\Delta}{\mathcal{G}_\tau}\,, \tag{17}$$

in Horndeski theory with $\Delta = -2\dot{\pi} X H G_{5,X}$

- We get Luminal GWs with $G_{5,X} = 0$:

nonminimal couplings $G_4(\pi, X)$, $G_5(\pi)$ preserve

$$\frac{c_g^2(t)}{c^2(t)} = 1\,. \tag{18}$$

2. SVT: With Dilaton

- We compactified, where does the condition $G_{5X} = 0$ come from? We ignored the Dilaton and broke isotropy

on FLRW

$$\frac{c_g^2}{c^2} = 1 - \frac{2\dot{\pi}\, X\, (H - l)\, G_{5,X}}{\mathcal{G}_\tau}\,, \tag{19}$$

$l = 0$ in our case

(Cross check: with Dilaton (ϕ), $l = \frac{\dot{\phi}}{\phi}$. 5D hom/ isotropy means $l = H$).

2. SVT: GWs $\rightarrow$ DE?

Generalization of SVT to beyond Horndeski

2. SVT: GWs → DE?

- In general, with G_{4X}, $G_5 \neq 0$,

  GWs → DE decay channels **hss** and **hhs**

- Problem holds in Beyond Horndeski theory $\mathcal{L}_{BH}$ (GLPV)

$$\mathcal{L}_{BH} = \text{Horndeski Theory} + \mathcal{L}_{F4} \tag{20}$$
$$\mathcal{L}_{F4} = F_4(\pi, X)\, \epsilon^{\mu\nu\rho\sigma}\, \epsilon^{\mu'\nu'\rho'}{}_{\sigma}\, \pi_{;\nu\nu'}\, \pi_{;\rho\rho'}\, \pi_{;\mu}\, \pi_{;\mu'}$$

- Usual conclusion without DE — Photon couplings:
  Need! $F_4 = G_5 = G_{4X} = 0$

  Creminelli, Lewandowski, Tambalo Vernizzi (2018) JCAP 12, 025 [arXiv:1809.03484]

- Conclusion changes with DE — Photon couplings!

2. SVT of Beyond Horndeski theory (GLPV)

We obtain,

$$\mathcal{L} = \mathcal{L}_{\text{Horndeski in 4D}} + \mathcal{L}_{\text{Beyond Horndeski in 4D}}(F_4(\pi, X)) + \mathcal{L}_{\text{BH}_{4A}} + \mathcal{L}_{5A}\,, \tag{21}$$

$$\mathcal{L}_{\text{BH}_{4A}} = -\frac{G_4}{4}F^2 + \frac{2G_{4,X} + X\,F_4(\pi, X)}{2}(F_{\mu\nu}\pi^\mu)^2\,,$$

$$\mathcal{L}_{5A} = G_5(\pi)\left(\frac{1}{8}F^{\mu\nu}F_\mu{}^\rho(-4\nabla_\nu\nabla_\rho\pi + g_{\nu\rho}\Box\pi) + \frac{1}{2}F_{\mu\nu}\nabla_\sigma F^{\nu\sigma}\nabla^\mu\pi\right)$$

- **Automatically** luminal
- Can suppress GW decay by fixing the X dependance of F_4.

2. DE — Photon couplings: GWs $\nrightarrow$ DE

- GW decay is suppressed if

$$F_4\Big(4G_4 + X(2G_{4,X} + 3G_{5,\pi})\Big) + X\,F_{4,X}\Big(2G_4 + XG_{5,\pi}\Big) = 0$$

$$+4G_{4,X}^2 + 4G_4G_{4,XX} + G_{5,\pi}\Big(4G_{4,X} + 2XG_{4,XX} + G_{5,\pi}\Big) \tag{23}$$

Creminelli, Lewandowski, Tambalo Vernizzi (2018) JCAP 12, 025 [arXiv:1809.03484]

- Without DE — Photon couplings $F_4 = \frac{-2G_{4,X}}{X}$ and $G_5 = 0$ by $c_g = c = 1$.

- With DE — Photon couplings F_4 is free: use this Freedom and satisfy Eqn. (23)!!

$$F_4(\pi, X) = \frac{1}{2X^2}\left(2G_4 - X(4G_{4,X} + G_{5,\pi}) + \frac{4J_4(\pi)}{2G_4 + XG_{5,\pi}}\right), \tag{24}$$

2. More general SVT's with these properties

- An SVT with the same properties can be built for DHOST

  S. Mironov, A. Shtennikova and **M. V-V**, Phys.Rev.D 111 (2025) 10, 10 [arXiv:2412.13460]

- With second order equations of motion the SVT of Horndeski can be generalized

$$S^{(2)}_{vector} = -\int \mathrm{d}t\, \mathrm{d}^3x\, a\, f(\pi, X)\, \left[c_g^{-2}\, \left(\dot{A}_i\right)^2 - \frac{1}{a^2}\, (\partial_i A_j)^2\right]$$

  while one additional free scalar potential $f(\pi, X)$ can be obtained while preserving $\frac{c_g(t)}{c(t)} = 1$. S. Mironov, A. Shtennikova and **M. V-V**, Eur.Phys.J.C 85 (2025) 12, 1378 [arXiv:2509.16850].

  This is the most general theory of this form L. Heisenberg, JCAP 10 (2018) 054 [arXiv:1801.01523], M. A. Gorji, S. Mukohyama, P. Petrov and M. Yamaguchi [arXiv:2509.16526].

Conclusions

- We obtained an extension of Beyond Horndeski theory that
  - automatically propagates luminal Gravitational waves, $\frac{c_g(t)}{c(t)} = 1$
  - suppressed decay into Dark Energy scalar,
  - has nonminimal couplings of the Scalar of Dark Energy to both the Graviton and the Photon
- This is the largest Beyond Horndeski theory (and DHOST) that enjoys this property.
- $G_{5,X}$, F_5 and other DHOST cubic remain ruled out.

Question: can it be done?

2405.02281



and, 2412.13460

Many thanks
for your attention!