COSMOLOGICAL PERTURBATIONS IN STEGR

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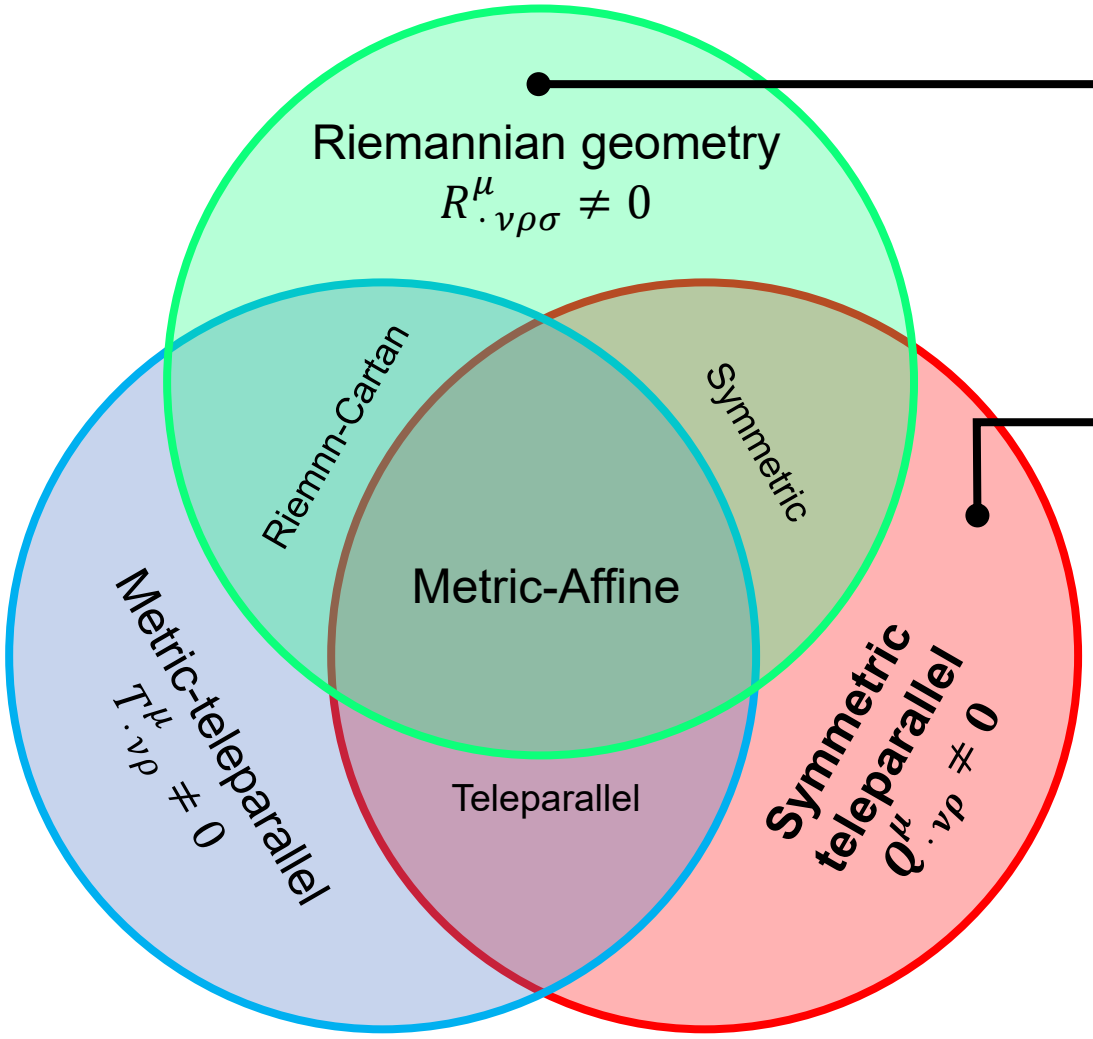
MOTIVATION

Global goal: try to describe dark matter and dark energy as specific behaviour of gravity, not as new form of matter. Or prove that it's not possible for metric-affine case.

Current tasks:

- Find components of Einstein's equation for cosmological perturbations in Friedmann's metric with conformal time.
- Find components of Einstein's equation for cosmological perturbations in general Friedmann's metric.

CLASSICAL & MODIFIED STEGR



Hilbert Einstein's Action

$$S = \int d^4x\sqrt{-g}\cdot\hat{\mathfrak{R}}$$

$\hat{\mathfrak{R}}$ - Richie's scalar

$\hat{\mathfrak{R}} \to \mathfrak{Q}$ - nonmetricity scalar

STEGR action

$$S = -\int d^4x\sqrt{-g}\cdot\mathfrak{Q}$$

$$\mathfrak{Q} = \frac{1}{4}Q_{\alpha\mu\nu}Q^{\alpha\mu\nu} - \frac{1}{2}Q_{\alpha\mu\nu}Q^{\mu\alpha\nu} - \frac{1}{4}Q_{\mu}Q^{\mu} + \frac{1}{2}Q_{\mu}\tilde{Q}^{\mu}$$

$\mathfrak{Q} \to \mathcal{Q}$ - modified nonmetricity scalar

Modified STEGR action

$$S = -\int d^4x\sqrt{-g}\mathcal{Q}$$

$$\mathcal{Q} = \frac{a_1}{4}Q_{\alpha\mu\nu}Q^{\alpha\mu\nu} - \frac{a_2}{2}Q_{\alpha\mu\nu}Q^{\mu\alpha\nu} - \frac{a_3}{4}Q_{\mu}Q^{\mu} + \frac{a_4}{2}\tilde{Q}_{\mu}\tilde{Q}^{\mu} + \frac{a_5}{2}Q_{\mu}\tilde{Q}^{\mu}$$

Variation by metric $g_{\mu\nu}$

$G^{\mu\nu}$ - Einstein's tensor

EINSTEIN'S EQUATION

$$G^{\mu\nu} = \varkappa T^{\mu\nu}$$

Einstein's tensor
$G^{\mu\nu} = G^{\mu\nu}(g_{\mu\nu})$

Perturbed metrics

- Minkovsky's
- Friedmann's in conformal time
- General Friedmann's

Types of perturbations

- Scalar ϕ, ψ, σ
- Vector $V_i,\ c_i$
- Tensor h_{ij}

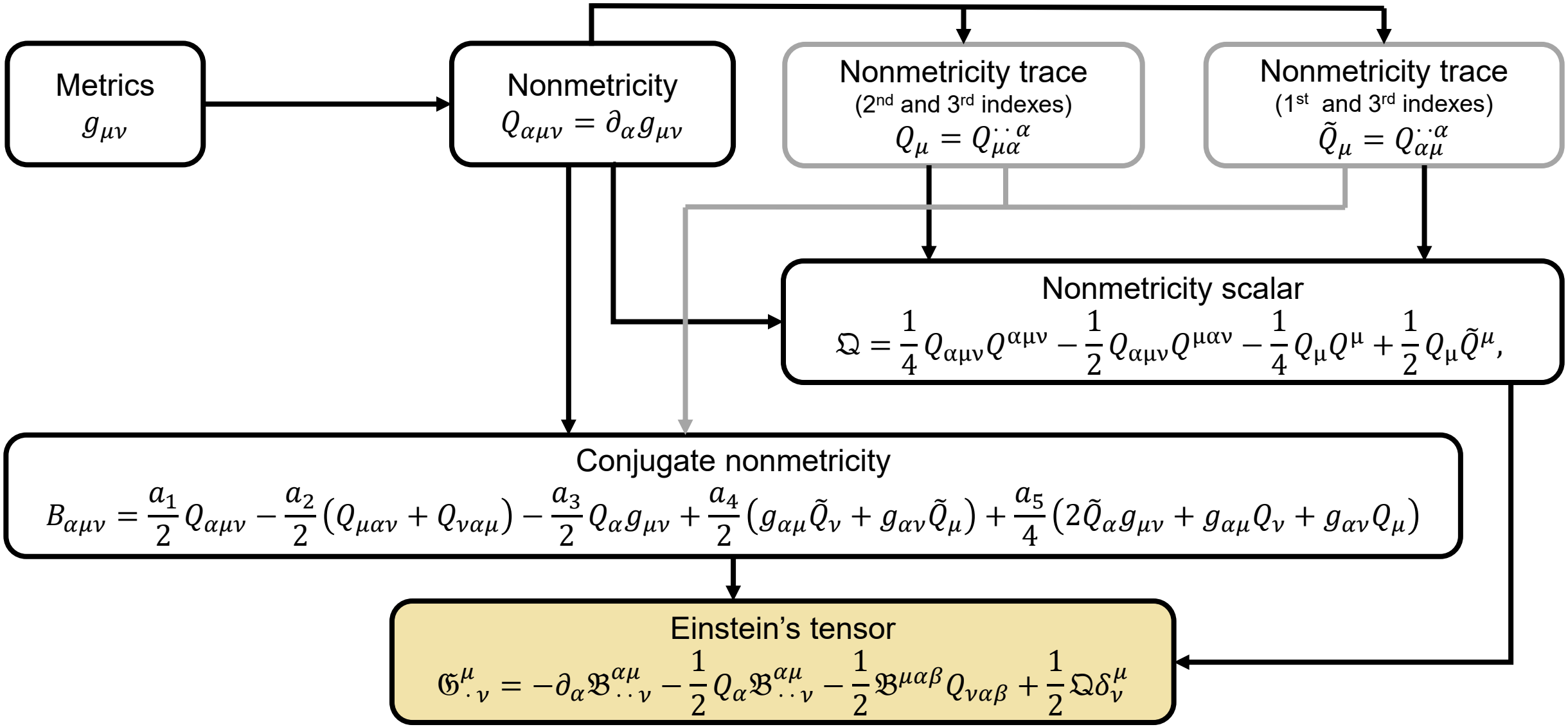
Energy-momentum tensor for vacuum
$T^{\mu\nu} = 0$

Energy-momentum tensor for ideal liquid
$T^{\mu}_{\nu} = (\varepsilon + p)U_{\nu}U^{\mu} - p\delta^{\mu}_{\nu}$

Perturbed energy-momentum tensor for ideal liquid

$$T^0_0 = \varepsilon + \delta\varepsilon,$$
$$T^0_i = (\varepsilon + p)(\partial_i V + V_i),$$
$$T^i_0 = (\varepsilon + p)(u_i - v_i - \partial_i V - V_i),$$
$$T^i_j = -(p + \delta p)\delta_{ij}.$$

ALGORITHM TO CALCULATE EINSTEIN'S TENSOR



FRIEDMANN'S METRIC IN CONFORMAL TIME

COMPONENTS OF METRIC

$$g_{00} = a^2(1 + 2\phi),$$
$$g_{0i} = g_{0i} = a^2(\partial_i V + V_i),$$
$$g_{ij} = a^2\left(-\delta_{ij} + 2\psi\delta_{ij} - 2\partial_{ij}^2\sigma - \partial_i c_j - \partial_j c_i - h_{ij}\right).$$

TENSOR PERTURBATIONS

The only non-zero components of Einstein's equation are the spatial components. Concluding only on tensorial changes, we obtain:

$$\frac{a_1}{2a^2}\left\{\ddot{h}_{ij} + 2H\dot{h}_{ij} - \Delta h_{ij}\right\} = 0,$$

This is the equation for gravitational waves in the modified STEGR

VECTOR PERTURBATIONS

Mixed components:

$$-\frac{H}{a^2}\left(\dot{v}_i(a_1 - a_2 + a_4) + a_4\Delta c_i\right) - \frac{1}{2a^2}\left(\ddot{v}_i(a_1 - a_2 + a_4) + \Delta\dot{c}_i(a_4 - a_2) - a_1\Delta v_i\right) = \varkappa(\varepsilon + p)V_i.$$

Space components:

$$-\frac{H}{a^2}\left(a_1\partial_{(i}\dot{c}_{j)} + a_2\partial_{(i}v_{j)}\right) - \frac{1}{2a^2}\left\{a_1\partial_{(i}\ddot{c}_{j)} + a_2\partial_{(i}\dot{v}_{j)} + (a_2 - a_1)\Delta\partial_{(i}v_{j)} - a_4\partial_{(i}v_{j)}\right\} = 0$$

FRIEDMANN'S METRIC IN CONFORMAL TIME

SCALAR PERTURBATIONS

Time components:

$$\left((2\mathfrak{b}_1+4\mathfrak{b}_2)\dot{H}+(8\mathfrak{b}_1+10\mathfrak{b}_2)H^2\right)\phi+2a_5\Delta\psi-\{(a_1-a_3)\phi+(a_3-a_5)(3\psi-\Delta\sigma)\}$$
$$-(a_5-a_2+a_4)\{2\ddot{\phi}-\Delta\dot{V}\}-\{2(a_1-a_3+2a_5-2a_2+2a_4)\dot{\phi}+2(a_1-a_2-5a_3+a_4+4a_5)(\Delta\dot{\sigma}-3\dot{\psi})+$$
$$+(a_1-a_2-4a_3-a_4+2a_5)\Delta V\}H=\varkappa a^2\delta\varepsilon$$

Mixed:

$$(\varepsilon+p)V$$
$$=\{(a_1-a_2+a_4)\dot{V}+(a_1-3a_2-4a_3+a_4+4a_5)\phi-(3a_1-3a_2-12a_3+5a_4+12a_5)\psi$$
$$+\ (a_1-a_2-4a_3-a_4+2a_5)\Delta V\}H+\left(2a_5\dot{\psi}-\left(\frac{a_1-a_2+a_4}{2}\right)V-(a_5-a_2+a_4)(\dot{\phi}-\dot{\psi}+\Delta\dot{\sigma})\right)$$

Space:

$$-\{(3a_3-a_1)\ddot{\psi}+(a_5-a_3)\ddot{\phi}+a_3\Delta\phi-(3a_3-a_1-a_5)\Delta\psi+(a_3-a_5)\Delta^2\sigma-\Delta(a_3\ddot{\sigma}+a_5\dot{V})\}$$
$$+2\phi\delta_{ij}(\mathfrak{b}_1\dot{H}-\mathfrak{b}_2H^2)-\{2(a_1-a_2-3a_3+a_4+2a_5)\dot{\phi}+(2a_1-6a_3)\dot{\psi}$$
$$+\Delta(2a_3\dot{\sigma}+(a_1-a_2-4a_3+a_4+2a_5)V)\}H=-\varkappa\delta p$$

$$a_5\phi-(3a_5-2a_2+2a_4)\psi-a_1\ddot{\sigma}-(a_2-a_4)\dot{V}-(2a_2-a_1-a_5-2a_4)\Delta\sigma-2H(a_1\dot{\sigma}+a_2V)=0$$

GENERAL FRIEDMANN'S METRIC

COMPONENTS OF METRIC

$$g_{00} = N^2(1+2\phi),$$
$$g_{0i} = g_{0i} = a^2(\partial_i V + V_i),$$
$$g_{ij} = a^2\left(-\delta_{ij} + 2\psi\delta_{ij} - 2\partial_{ij}^2\sigma - \partial_i c_j - \partial_j c_i - h_{ij}\right).$$

TENSOR PERTURBATIONS

The only non-zero components of Einstein's equation are the spatial components. Concluding only on tensorial changes, we obtain:

$$\frac{a_1}{2N^2}\left\{\ddot{h}_{ij} + (3H - M)\dot{h}_{ij} - \left(\frac{N}{a}\right)^2 \Delta h_{ij}\right\} = 0,$$

This is the equation for gravitational waves in the modified STEGR

VECTOR PERTURBATIONS

Mixed components:

$$N^{-2}[\frac{1}{2}\Delta(a_2 c_i + a_1 V_i - a_4(c + (3H - M)c_i) - \frac{1}{2}(a_1 - a_2 + a_4)\left(\frac{a}{N}\right)^2 (V_i + (5H - 3M)V_i)] = \varkappa(\varepsilon + p)V_i.$$

Space components:

$$N^{-2}\left[\left(\frac{N}{a}\right)^2 (a_1 - a_2 + a_4)\Delta\partial_{(i}c_{j)} + a_4\partial_{(i}V_{j)} - a_1\left(\partial_{(i}c_{j)} + (3H - M)\partial_{(i}c_{j)}\right) - a_2\left(\partial_{(i}V_{j)} + (3H - M)\partial_{(i}V_{j)}\right)\right] = 0$$

SUMMERY

General:

- We derived the action for the modified STEGR;
- By varying the action metric for the modified STEGR, we obtained the generalized Einstein tensor for this theory.

For Friedmann's metric in conformal time:

- We obtained the components of the Einstein tensor for vector and tensor cosmological perturbations in the modified STEGR;
- Using conformal transformations, we obtained the components of the Einstein tensor for scalar cosmological perturbations;
- Using the perturbed stress-energy tensor, we obtained the components of the equation of motion for all types of cosmological perturbations.
- We verified the vacuum equations for cosmological perturbations for the case of Einstein parameters.

For general Friedmann's metric:

- We obtained the components of the Einstein tensor for all types of cosmological perturbations in the modified STEGR;

CONCLUSION

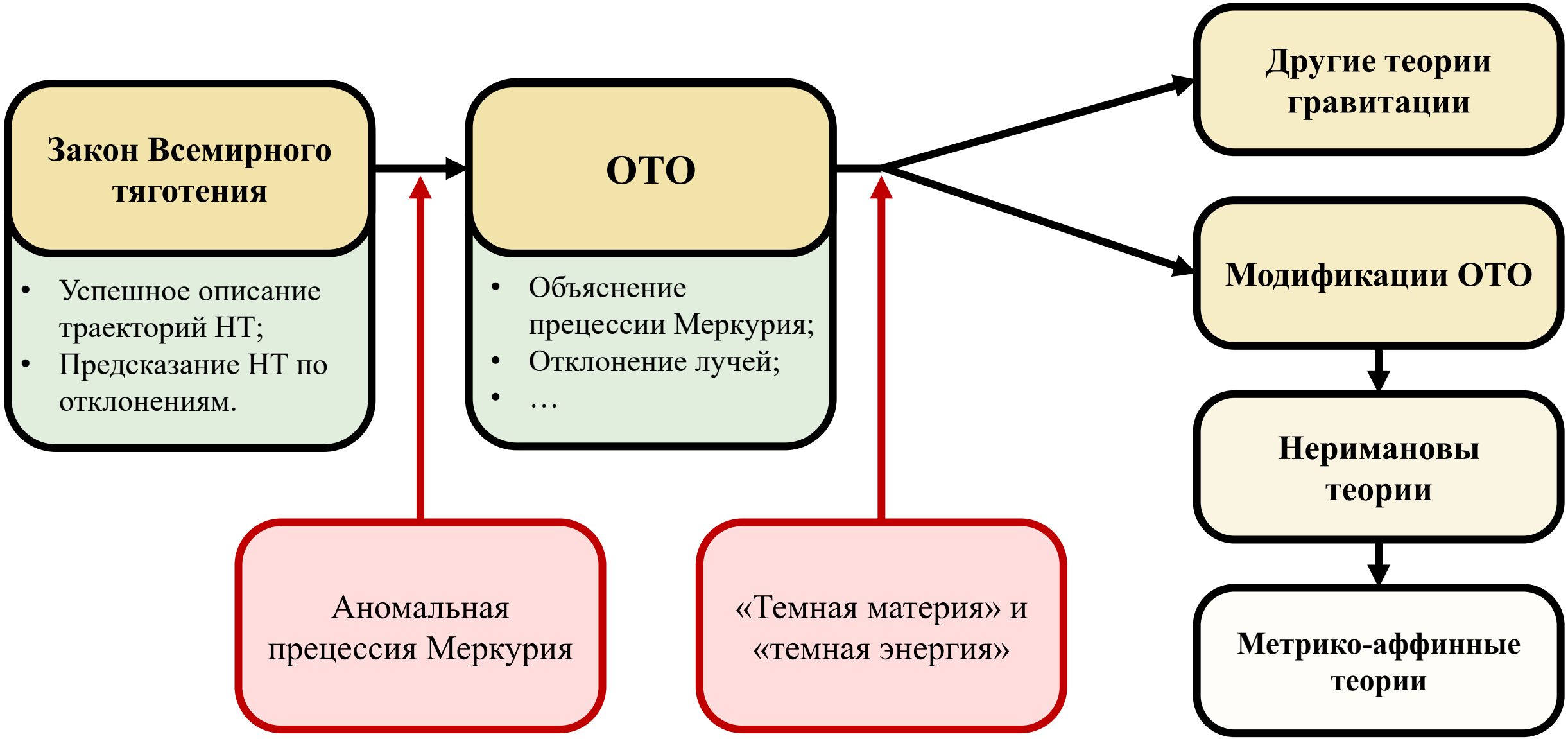
- We derived the full linear structure of cosmological perturbations in the most general quadratic nonmetricity theory (modified STEGR).
- In the tensor sector, we obtained the standard wave equation with a modified effective coupling, confirming the absence of additional tensor degrees of freedom at linear order.
- In the vector sector, the structure of the equations indicates restoration of diffeomorphism invariance at the perturbative level.
- The scalar sector requires further consistency analysis, but the Einstein limit is correctly recovered for the specific choice of parameters.
- These results support the consistency of quadratic nonmetricity gravity at the level of linear cosmological perturbations.

Further work includes:

- analysis of dynamical degrees of freedom,
- stability conditions,
- confrontation with cosmological observations.

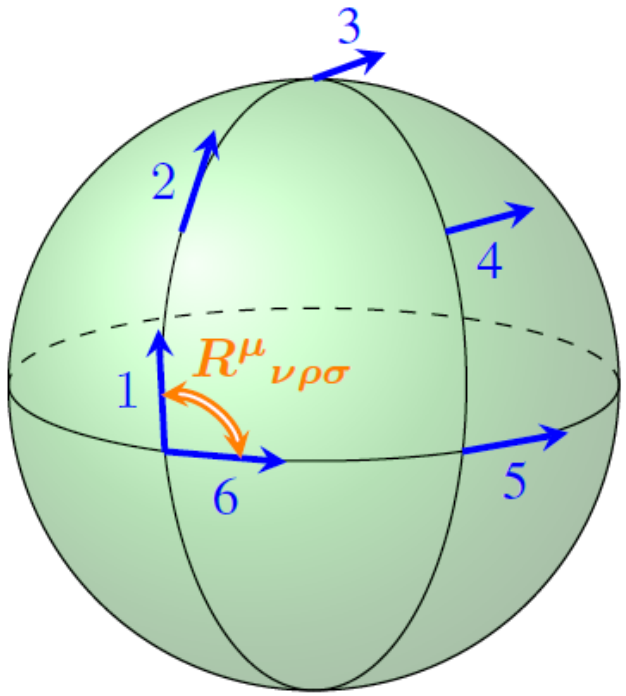
THANK YOU FOR YOUR ATTENTION!

ТЕОРИИ ГРАВИТАЦИИ



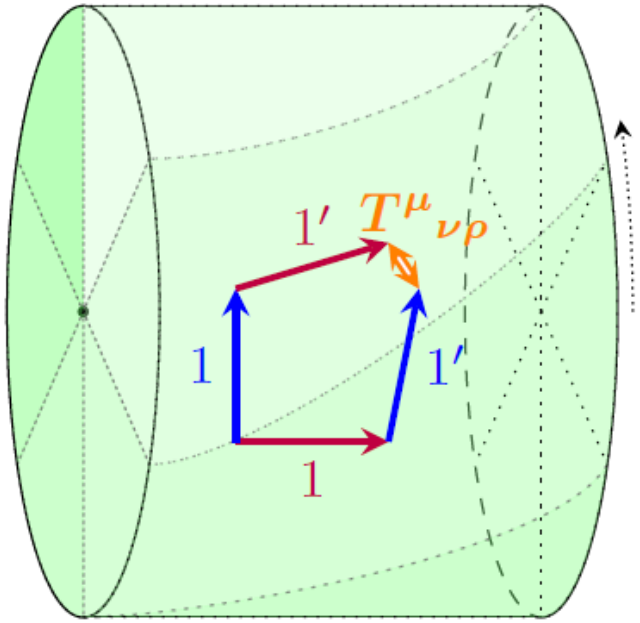
МЕТРИКО-АФФИННАЯ ГЕОМЕТРИЯ

КРИВИЗНА



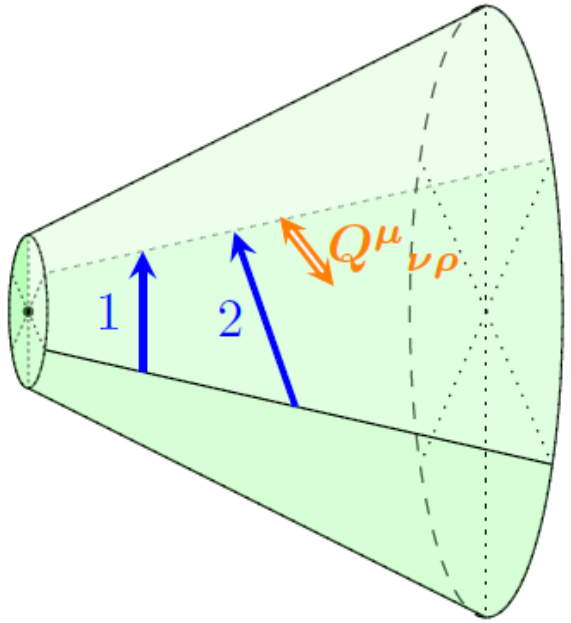
$R^{\mu}_{\cdot\nu\rho\sigma}$ – ***тензор кривизны***

КРУЧЕНИЕ



$T^{\mu}_{\cdot\nu\rho}$ – ***тензор кручения***

НЕМЕТРИЧНОСТЬ

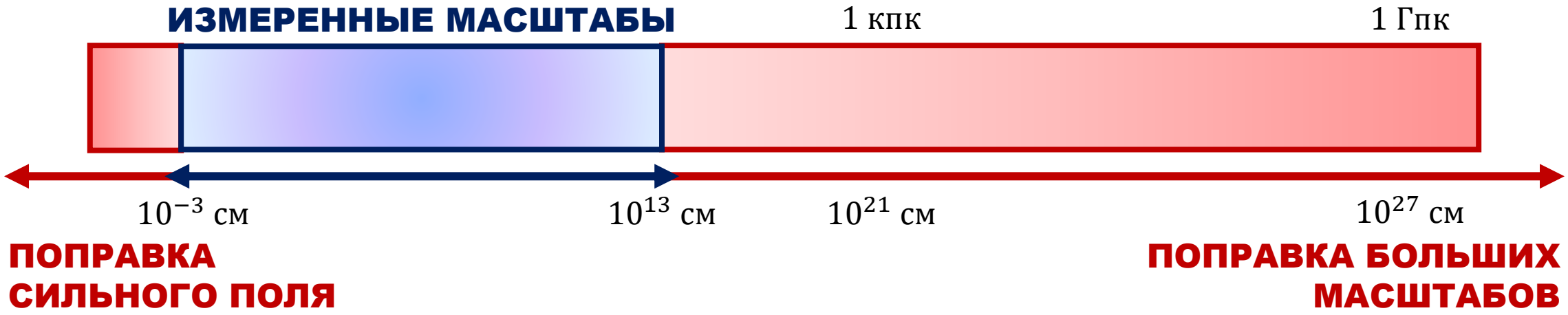


$Q^{\mu}_{\cdot\nu\rho}$ – ***тензор неметричности***

Bahamonde S. et al. Teleparallel gravity: from theory to cosmology //Reports on Progress in Physics. – 2023. – Т. 86. – №. 2. – С. 026901.

ЗАЧЕМ МОДИФИЦИРОВАТЬ ОТО?

1. Гравитация не исследована на всех уровнях
2. ОТО нуждается в корректировке в режимах сильной гравитации
3. Противоречия между ранними и поздними наблюдениями за Вселенной
4. Темная материя
5. Темная энергия
6. Невозможность использовать квантовую теорию поля



Shankaranarayanan S., Johnson J. P. Modified theories of gravity: Why, how and what? //General Relativity and Gravitation. – 2022. – Т. 54. – №. 5. – С. 44.

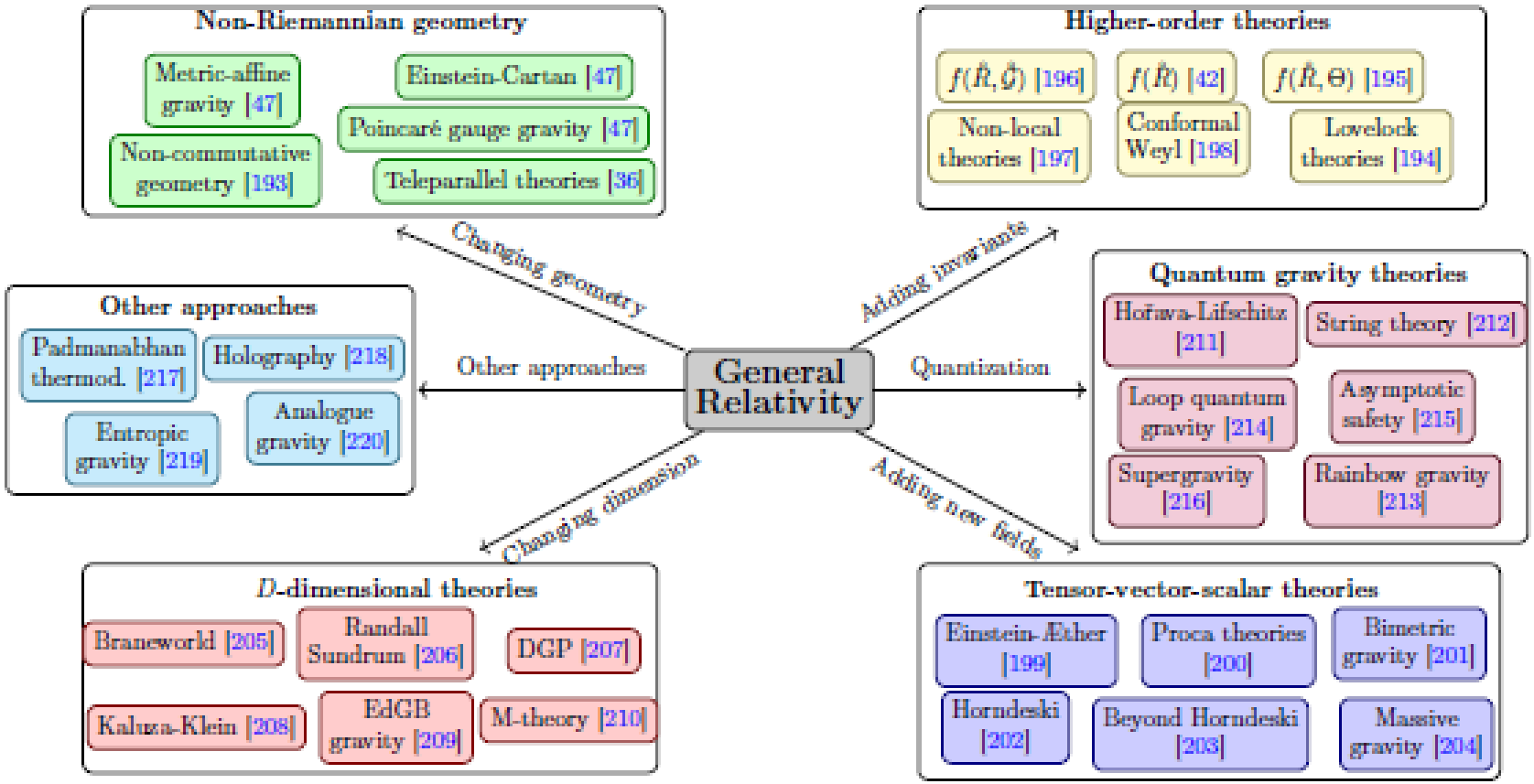
ДРУГИЕ ТЕОРИИ ГРАВИТАЦИИ

Table 1. Comparison of Selected Gravitational Theories

Theory	*Metric*	*Other Fields*	*Free Elements*	*Status*
Newtonian (1687)[1]	Nonmetric	Potential	None	Nonrelativistic theory
Nordstrom (1913)[1,2]	Minkowski	Scalar	None	Fails to predict observed light detection
Einstein's General Relativity (1915)[1, 2]	Dynamic	None	None	Viable
Belifante-Swihart (1957)[2]	Nonmetric	Tensor	*K* parameter	Contradicted by Dicke-Braginsky experiments
Brans-Dicke (1961)[1-3] Generic Scalar	Dynamic	Scalar	ω parameter	Viable for ω > 500
Tensor (1970)[2]	Dynamic	Scalar	2 free functions	Viable
Ni (1970)[1, 2]	Minkowski	Tensor, Vector, and Scalar	One parameter, 3 functions	Predicts unobserved preferred-frame effects
Will-Nordtvedt (1972)[2]	Dynamic	Vector	None	Viable
Rosen (1973)[2]	Fixed	Tensor	None	Contradicted by binary pulsar data
Rastall (1976)[2]	Minkowski	Tensor, vector	None	Viable
VMT (1977)[2]	Dynamic	Scalar	2 parameters	Viable for a wide range of the parameters
MOND (1983)[4]	Nonmetric	Potential	Free function	Nonrelativistic theory

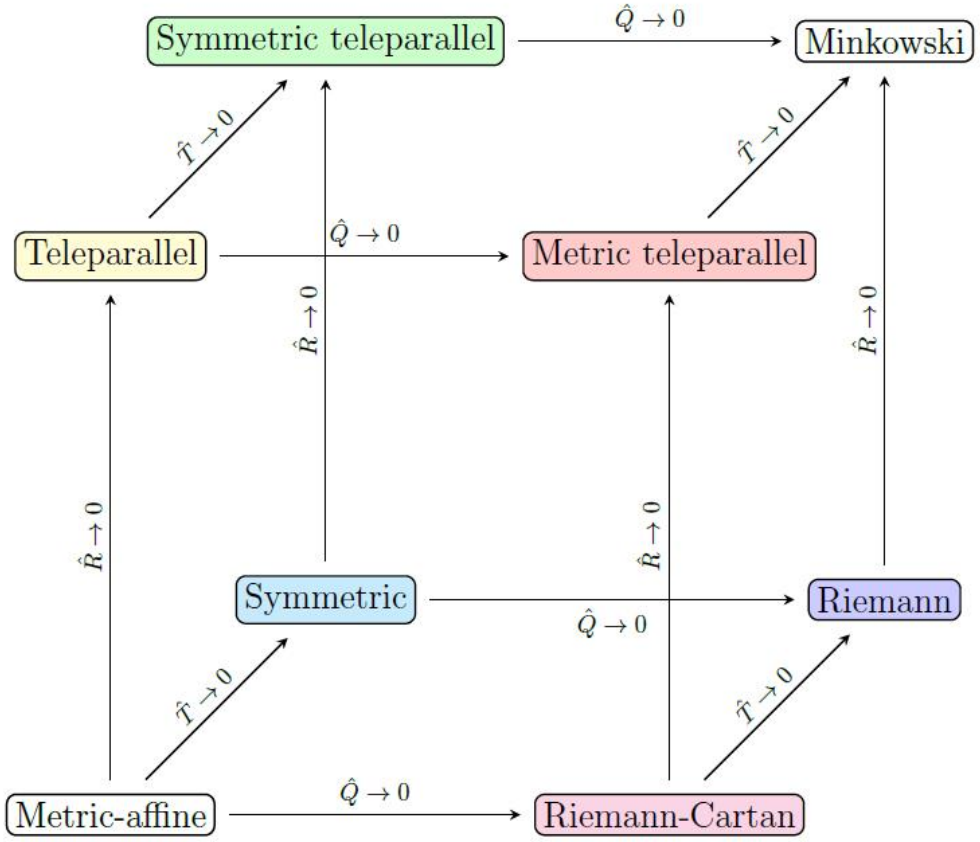
Jacob D. Bekenstein. GRAVITATIONAL THEORIES: https://ned.ipac.caltech.edu/level5/ESSAYS/Bekenstein/bekenstein.html

КАК МОЖНО МОДИФИЦИРОВАТЬ ОТО?



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МЕТРИКО-АФФИИНЫЙ ПОДХОД



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