Old and New Problems in Cosmology

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Problems of Modern Mathematical Physics

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Outline

1. Cosmology in Brief
2. Old Problems in Cosmology
3. New Problems in Cosmology

Outline

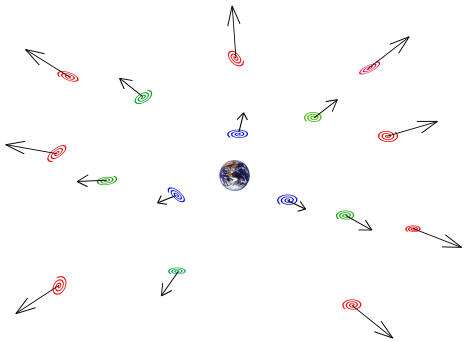
1 Cosmology in Brief

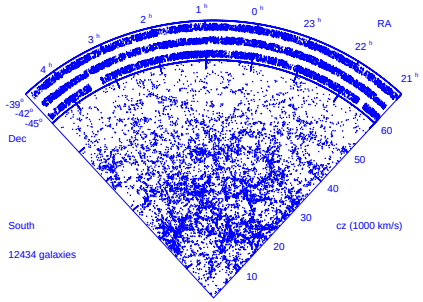
2 Old Problems in Cosmology

3 New Problems in Cosmology

Conclusions from observations

The Universe is homogeneous, isotropic, hot and expanding...





Conclusions

- interval between events gets modified

$$ds^2 = c^2\,dt^2 - a^2(t)\,d\mathbf{x}^2$$

in GR expansion is described by the Friedmann equation

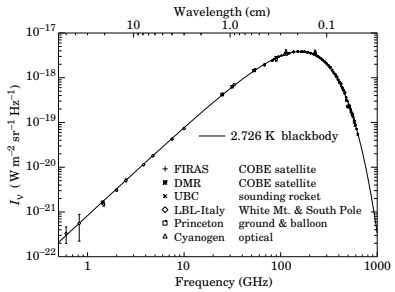
$$\left(\frac{\dot{a}}{a}\right)^2 = H^2(t) = \frac{8\pi}{3}\,G\rho^{\text{energy}}_{\text{density}}$$

$$\rho^{\text{energy}}_{\text{density}} = \rho_{\text{radiation}} + \rho_{\text{matter}} + \dots$$

- in the past the matter density was higher, our Universe was "hotter" filled with electromagnetic plasma

$$\rho_{\text{matter}} \propto 1/a^3(t)\,,\ \ \rho_{\text{radiation}} \propto 1/a^4(t)\,,\ \ \rho_{\text{curvature}} \propto 1/a^2(t)$$

certainly known back to $T \sim 1\,\text{MeV} \sim 10^{10}\,\text{K}$



Einstein equations for homogeneous Universe

$T_{\mu\nu}$: macroscopic description

$T_{\mu\nu} = (p+\rho)u_\mu u_\nu - g_{\mu\nu}p$

$\frac{1}{2}\int d^4x\sqrt{-g}T_{\mu\nu}\delta g^{\mu\nu}$

ideal fluid with $\rho(t)$ and $p(t)$

in the comoving frame $u^0 = 1, \mathbf{u} = 0$ (almost) always works

$$T^\nu_\mu = diag(\rho, -p)$$

$$ds^2 = dt^2 - a^2(t)\gamma_{ij}dx^i dx^j \ ,$$

$$S_{EH} = -\frac{1}{16\pi G}\int d^4x\sqrt{-g}R \ : \quad R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 8\pi G T_{\mu\nu}$$

$$(00): \quad \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi}{3}G\rho$$

both expansion and contraction $t \to -t$

Friedmann equation $(00):\quad \left(\frac{\dot a}{a}\right)^2 = \frac{8\pi}{3}G\rho$

$$\nabla_\mu T^{\mu 0} = 0 \;\longrightarrow\; \dot\rho + 3\frac{\dot a}{a}(\rho + p) = 0$$

the equation of state

$$p = p(\rho)$$

many-component fluid,
in case of thermal equilibrium

other equations

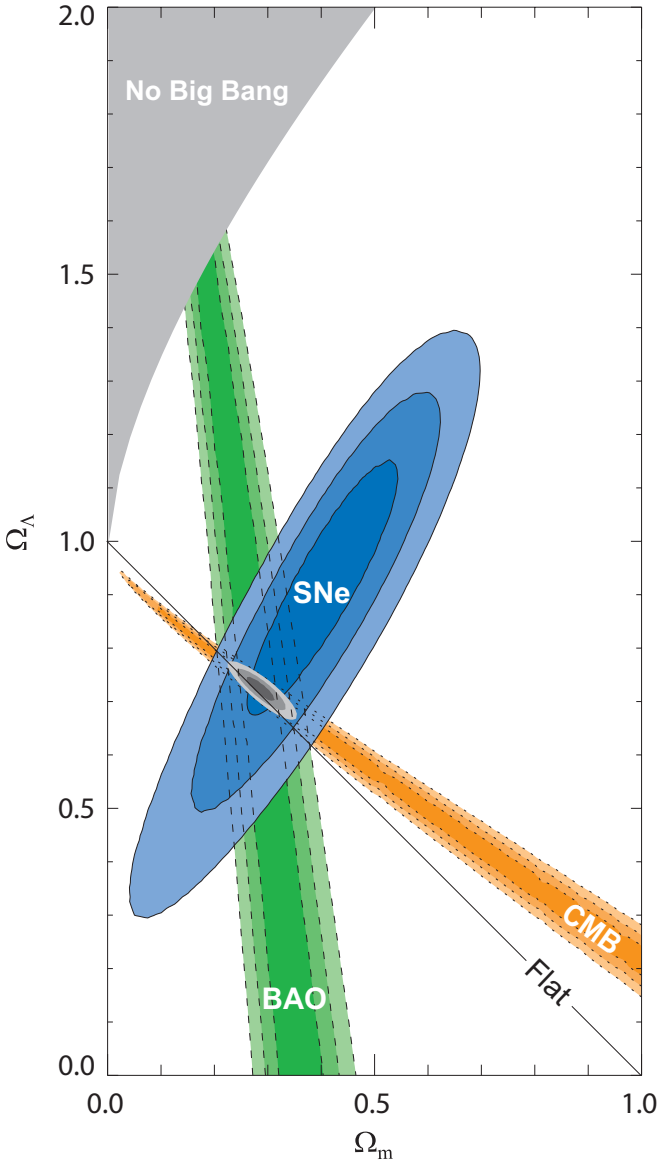
$$-3d(\ln a) = \frac{d\rho}{p+\rho} = d(\ln s)$$

entropy of cosmic primordial plasma is conserved in a comoving frame

$$sa^3 = \text{const}$$

usefull: for any decouled component $\quad n_X/s = \text{const}$

Cosmological data suggest . . . DM, DE, flatness, etc



$$\left(\frac{\dot{a}}{a}\right)^2 = H^2(t) = \frac{8\pi}{3} G \rho^{\text{energy}}_{\text{density}}$$

$$\rho^{\text{energy}}_{\text{density}} = \rho_{\text{radiation}} + \rho^{\text{ordinary}}_{\text{matter}} + \rho^{\text{dark}}_{\text{matter}} + \rho_\Lambda$$

$$\rho_{\text{radiation}} \propto 1/a^4(t) \propto T^4(t)\,, \quad \rho_{\text{matter}} \propto 1/a^3(t)$$

$$\rho_\Lambda = \text{const}, \quad 1/a^2(t) \propto \rho_{\text{curvature}} = 0$$

$$\frac{3H_0^2}{8\pi G} = \rho^{\text{energy}}_{\text{density}}(t_0) \equiv \rho_c \approx 0.53 \times 10^{-5} \frac{\text{GeV}}{\text{cm}^3}$$

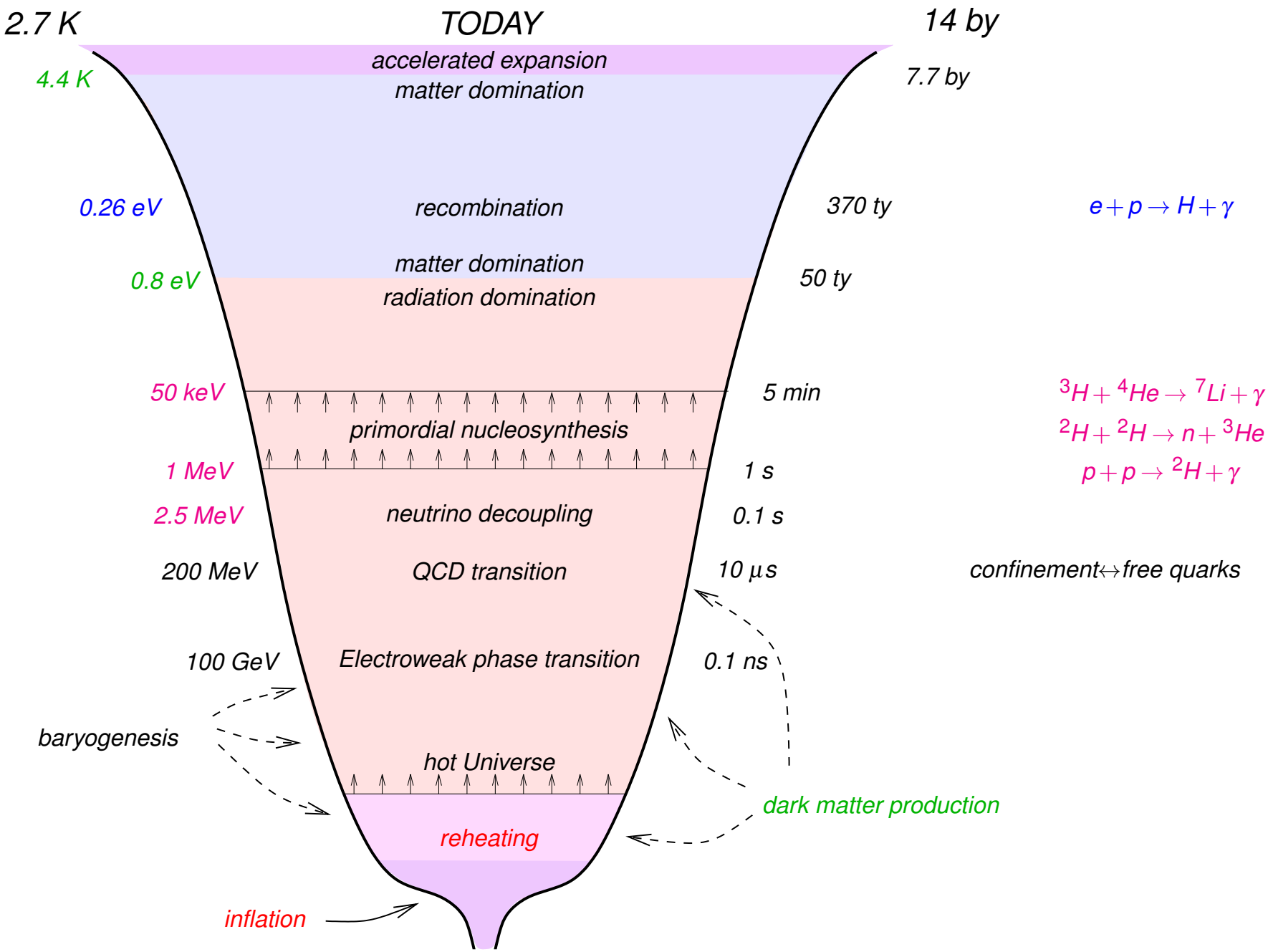
radiation: $\Omega_\gamma \equiv \frac{\rho_\gamma}{\rho_c} = 0.5 \times 10^{-4}$

Baryons (H, He): $\Omega_B \equiv \frac{\rho_B}{\rho_c} = 0.05$

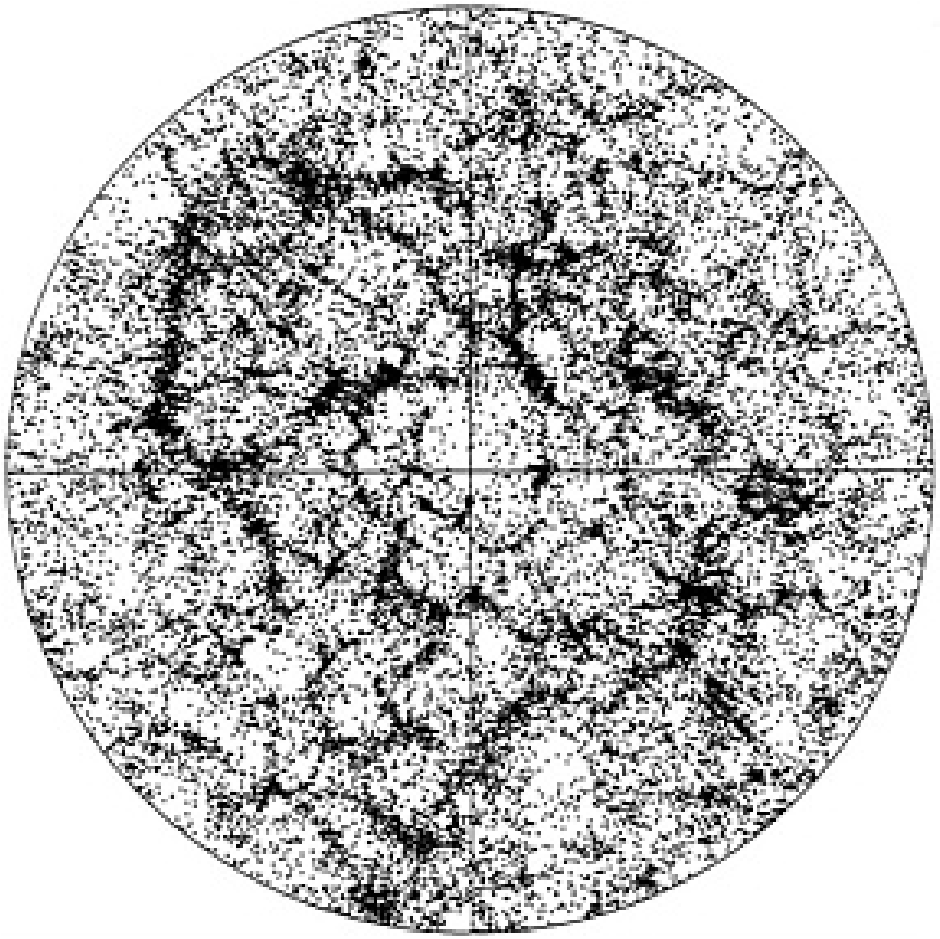
Neutrino: $\Omega_\nu \equiv \frac{\sum \rho_{\nu_i}}{\rho_c} < 0.01$

Dark matter: $\Omega_{DM} \equiv \frac{\rho_{DM}}{\rho_c} = 0.27$

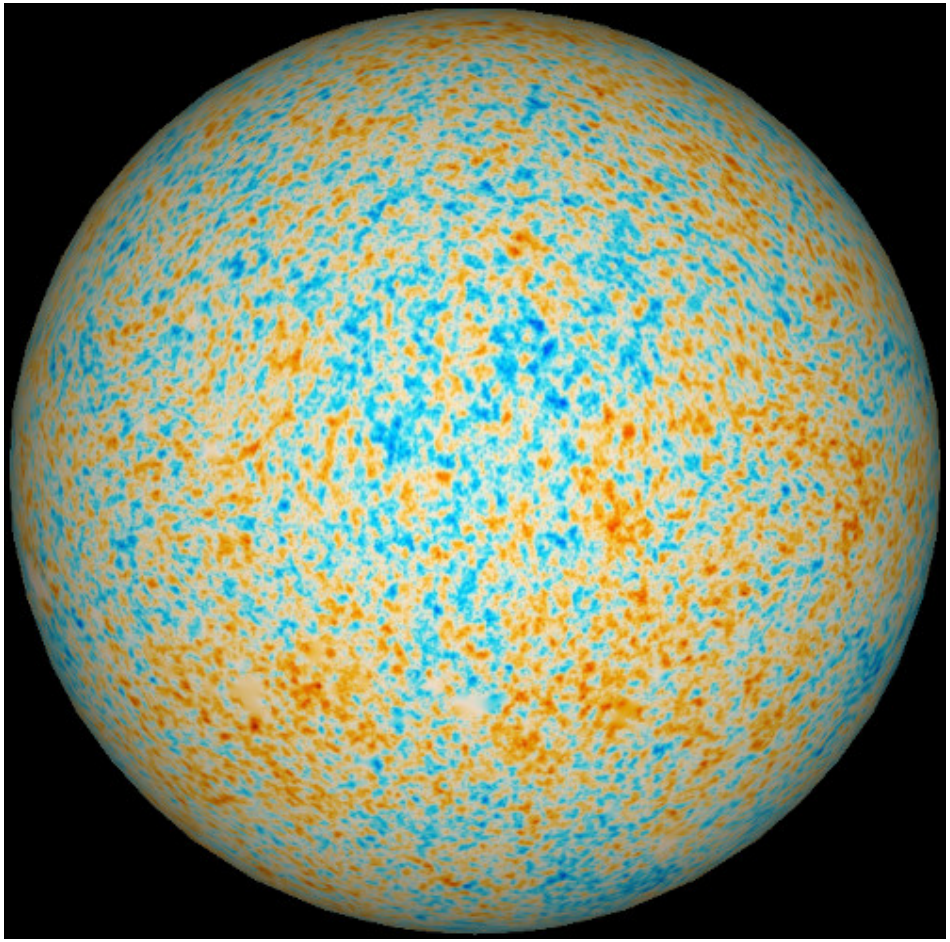
Dark energy: $\Omega_\Lambda \equiv \frac{\rho_\Lambda}{\rho_c} = 0.68$


2.7 K
TODAY
14 by
accelerated expansion
4.4 K
matter domination
7.7 by
0.26 eV
recombination
370 ty
$e+p \to H+\gamma$
matter domination
0.8 eV
radiation domination
50 ty
50 keV
5 min
$^3H+^4He \to ^7Li+\gamma$
primordial nucleosynthesis
$^2H+^2H \to n+^3He$
1 MeV
1 s
$p+p \to ^2H+\gamma$
2.5 MeV
neutrino decoupling
0.1 s
200 MeV
QCD transition
10 μs
confinement↔free quarks
100 GeV
Electroweak phase transition
0.1 ns
baryogenesis
hot Universe
dark matter production
reheating
inflation

Inhomogeneous Universe



Large Scale Structure



CMB anisotropy

Small inhomogeneities in the expanding Universe

matter perturbations (perfect fluid approximation)

$$T^0_0 \to \rho(t) + \delta\rho(\eta, \mathbf{x}), \quad T^0_i \to \partial_i v(\eta, \mathbf{x}), \quad T^i_j \to \delta p(\eta, \mathbf{x})$$

gravitational perturbations (scalar and tensor modes)

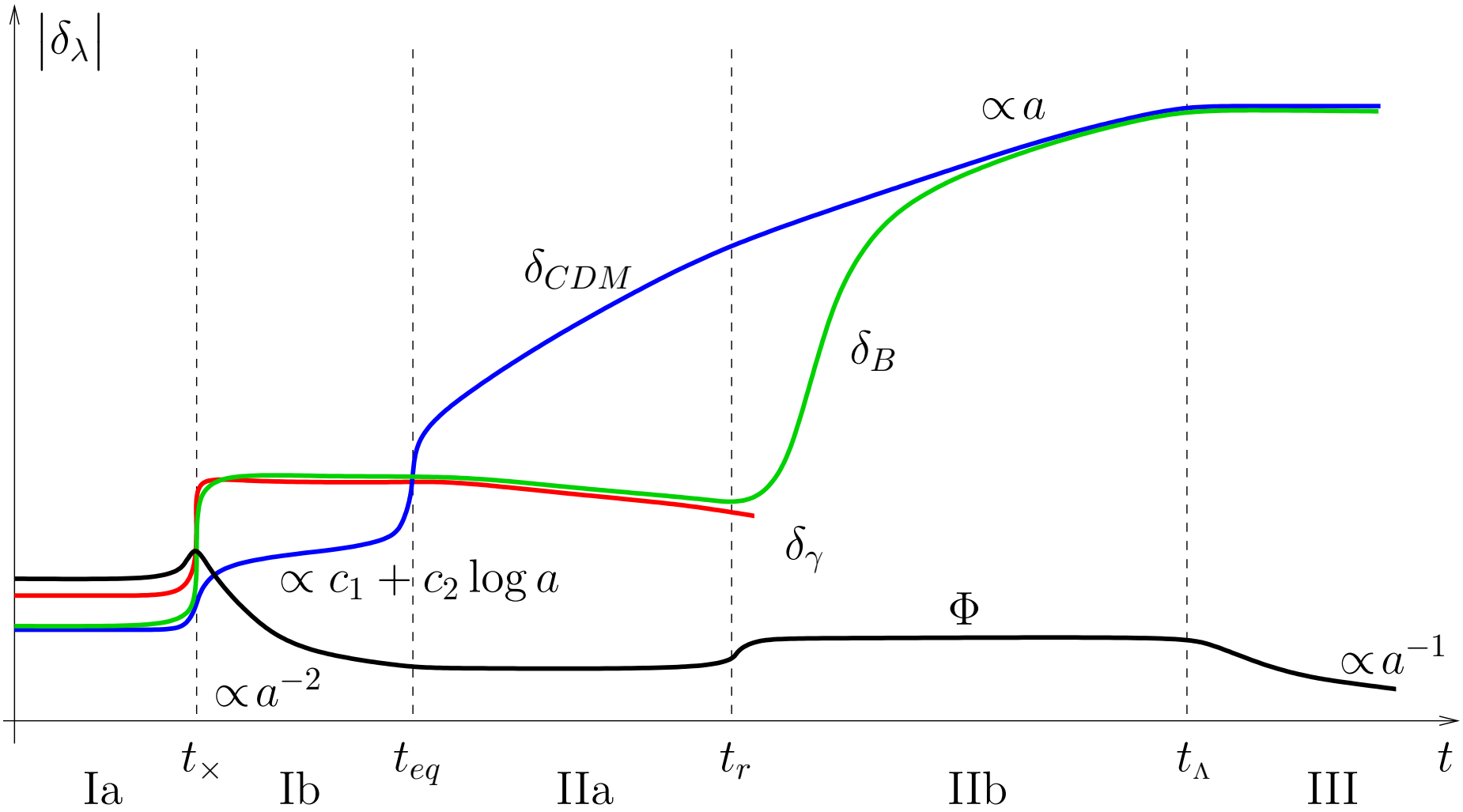
$$ds^2 = a^2(\eta)\left[(1+2\Phi(\eta,\mathbf{x}))\,d\eta^2 - (1+2\Psi(\eta,\mathbf{x}))\,\mathbf{dx}^2 - h^{TT}_{ij}(\eta,\mathbf{x})dx^i dx^j\right]$$

Equations for linear perturbations, $\delta\rho/\rho \equiv \delta \ll 1$, $\Phi \ll 1$, etc

$$R_{\mu\nu} + \frac{1}{2}Rg_{\mu\nu} = 8\pi G T_{\mu\nu} \quad \to \quad \dots$$

$$\nabla_\mu T^{\mu\nu} = 0 \quad \to \quad \dots$$

Subhorizon modes ($k/a > H$) at various stages



These inhomogeneities (matter perturbations)

originate from the initial matter density (scalar) perturbations

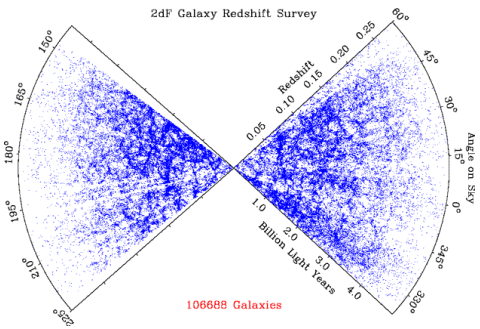
$\delta\rho/\rho \sim \delta T/T \sim 10^{-4}$, which are

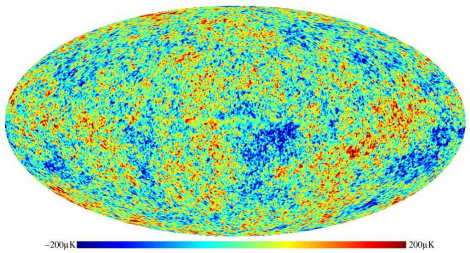
adiabatic $$\delta\left(\frac{n_B}{s}\right) = \delta\left(\frac{n_{DM}}{s}\right) = \delta\left(\frac{n_L}{s}\right)$$

Gaussian $$\left\langle \frac{\delta\rho}{\rho}(\mathbf{k}) \frac{\delta\rho}{\rho}(\mathbf{k}') \right\rangle \propto \left(\frac{\delta\rho}{\rho}(\mathbf{k})\right)^2 \times \delta(\mathbf{k}+\mathbf{k}')$$

flat spectrum $$\left\langle \left(\frac{\delta\rho}{\rho}(\mathbf{x})\right)^2 \right\rangle = \int_o^\infty \frac{\mathbf{dk}}{\mathbf{k}} \mathscr{P}_S(\mathbf{k}) \qquad \mathscr{P}_S(\mathbf{k}) \approx \text{const}$$

LSS and CMB $$\mathscr{P}_S \equiv A_S \times \left(\frac{k}{k_*}\right)^{n_s-1} \qquad A_S \approx 2.5 \times 10^{-9}, \quad n_S \approx 0.97$$





Standard cosmological model $ds^2 = dt^2 - a^2(t)dx^2$

$$\left(\frac{\dot{a}}{a}\right)^2 \equiv H^2 = H_0^2\left[\Omega_\Lambda + (\Omega_{DM}+\Omega_B+\Omega_{\nu,m\neq 0})\left(\frac{a_0}{a}\right)^3 + (\Omega_\gamma + \Omega_{\nu,m=0})\left(\frac{a_0}{a}\right)^4\right]$$

- $l_{s,rec} \sim l_{H,rec}/\sqrt{3} \to \quad H_0 = h\cdot 100\,\text{km/s/Mpc} \quad h = 0.67, \quad \Longrightarrow \quad \rho_0 = 5\,\text{GeV/m}^3$
- $T_\gamma = 2.735\,\text{K}, \quad \Longrightarrow \quad \Omega_\gamma \sim 10^{-5}$
- $N_\nu = 3.03.., \quad \Omega_\nu h^2 = \frac{\sum m_\nu}{93.12\,\text{eV}}, \quad \sum m_\nu < 0.2\,\text{eV} \quad \Longrightarrow \quad \Omega_{\nu,\neq 0}, \Omega_{\nu,0} \sim 10^{-5}$?
- $\Omega_B = 4.5\% \quad \Longrightarrow \quad \eta_B \equiv n_B/n_\gamma = 6\times 10^{-10}$
- $\Omega_{DM} = 27.5\%$
- $\Omega_\Lambda = 68\% \quad \Longrightarrow$ flat space
- adiabatic, gaussian matter perturbations

$$\left\langle\left(\frac{\delta\rho}{\rho}\right)^2\right\rangle \sim A_S\int\frac{dk}{k}\left(\frac{k}{k_*}\right)^{n_S-1}$$

  with $A_S = 3\times 10^{-9}$ and $n_S = 0.97$
- no tensor perturbations, $r \equiv A_T/A_S < 0.05$
- reionization at $z_{rei} \equiv a_0/a = 8$

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List of long-standing problems

- What is DE?
- What is DM?
- BAU ?
- Why the Universe is flat, homogeneous and isotropic?
- What is the source of perturbations?
- Li^7 in BBN...
- Cores vs Cusps in the galaxy centers...
- Lack of dwarf galaxies...

Dark Matter properties from cosmology: $p = 0$

(If) particles:

1. stable on cosmological time-scale
   requires new (almost) conserved quantum number
2. produced in the early Universe
   some time before RD/MD-transition ($T = 0.8\,\text{eV}$)
3. nonrelativistic particles long before RD/MD-transition ($T = 0.8\,\text{eV}$)
   (either Cold or Warm, $v_{RD/MD} \lesssim 10^{-3}$)
   Otherwise no small-size structures, like dwarf galaxies: $l_{fs} = a\int v(t)dt/a(t)$
   smoothed out by free streaming
   If were in thermal equilibrium: $M_X \gtrsim 1$ keV
4. (almost) collisionless $p = 0$, $v_{\text{sound}} = 0$
5. (almost) electrically neutral CMB distortion
6. all matter inhomogeneities (perturbations) are adiabatic:

$$\delta\left(\frac{n_B}{n_{DM}}\right) = \delta\left(\frac{n_B}{n_\gamma}\right) = \delta\left(\frac{n_\nu}{n_\gamma}\right) = 0$$

Dark Matter properties from astrophysics

1. stable on cosmological time-scale
2. (almost) collisionless — to form ellipsoidal halos
3. (almost) electrically neutral — to be Dark
4. stability of globular stellar clusters — $M_X \lesssim 10^3 M_\odot \approx 10^{61}$ GeV
   otherwise too strong tidal forces
5. confinement in a galaxy: — quantum physics!

   de Broglie wavelength: $\lambda = 2\pi/(M_X v_X) < l_{\text{galaxy}}$, — for bosons

   in a galaxy
   $v_X \sim 0.5 \cdot 10^{-3}$ $\longrightarrow$ $M_X \gtrsim 3 \cdot 10^{-22} \to 10^{-20}$ eV

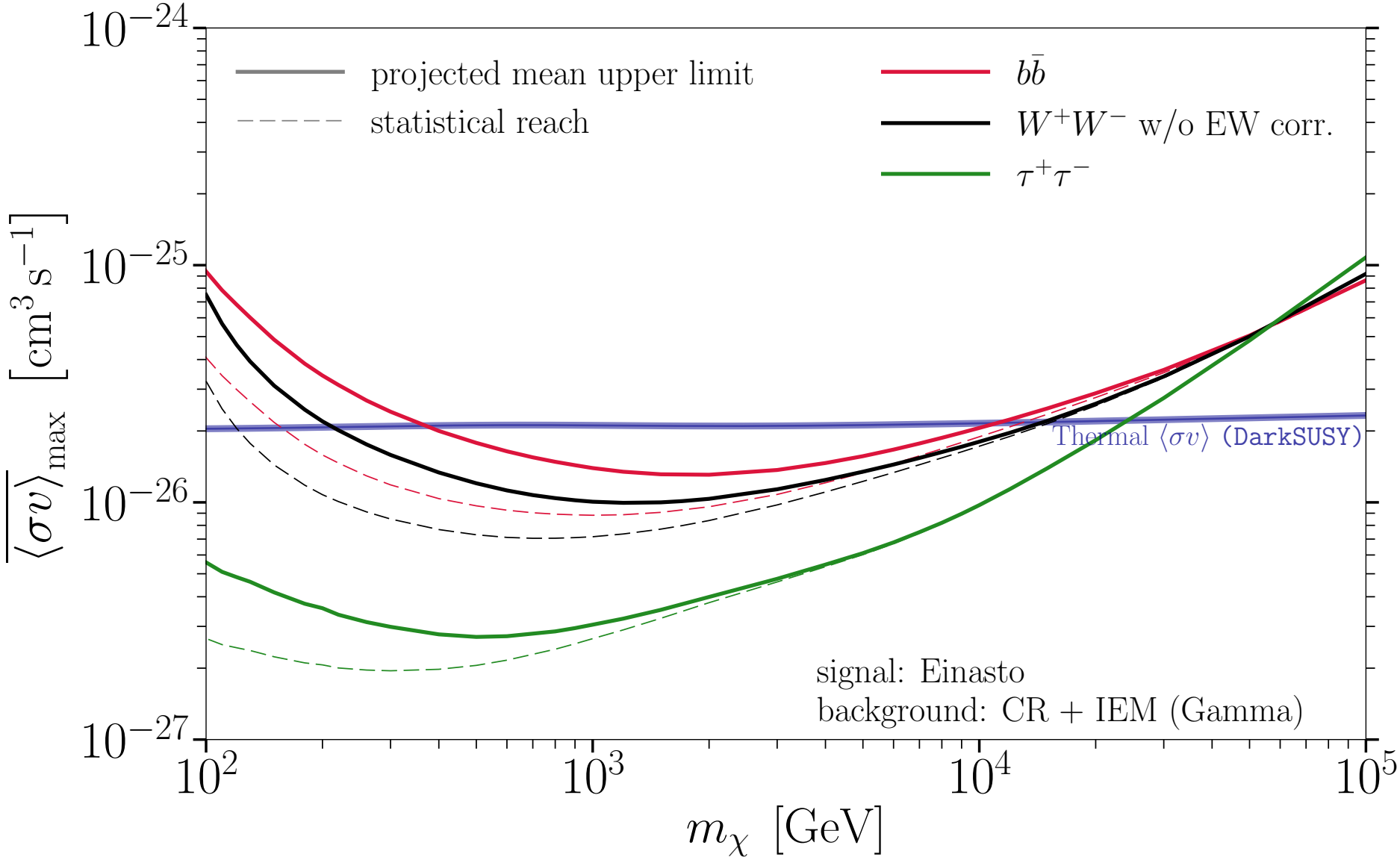
   for fermions

   Pauli blocking: $M_X \gtrsim 0.75 \to 1.5$ keV

$$f(\mathbf{p},\mathbf{x}) = \frac{\rho_X(\mathbf{x})}{M_X} \cdot \frac{1}{\left(\sqrt{2\pi} M_X v_X\right)^3} \cdot e^{-\frac{\mathbf{p}^2}{2M_X^2 v_X^2}}\Big|_{\mathbf{p}=0} \leq \frac{g_X}{(2\pi)^3}$$

Next generation: CTA

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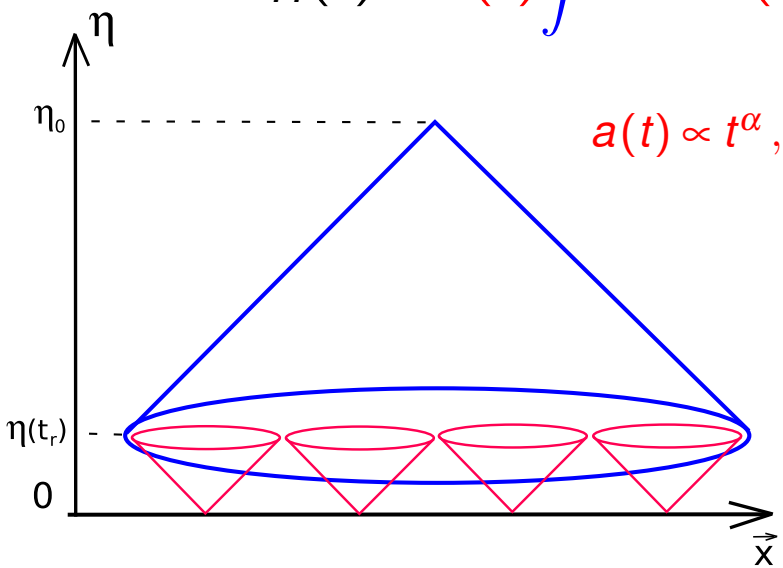
Horizon problem $l_H(t)$

a distance covered by photon emitted at $t = 0$

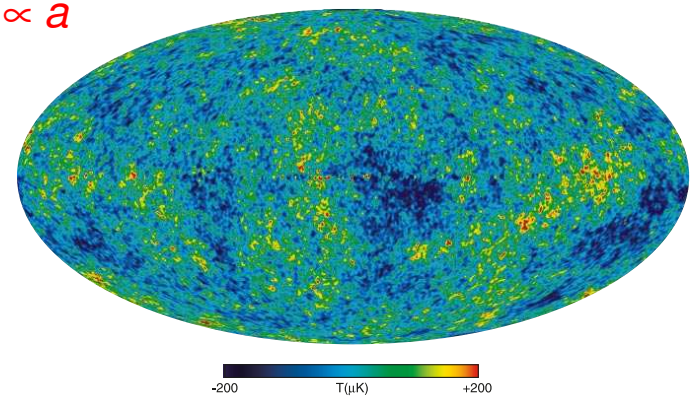
size of the causally connected part, that is the visible part of the Universe ("inside horison")

$$ds^2 = dt^2 - a^2(t)\,dx^2 = a^2(\eta)\left(d\eta^2 - dx^2\right) \qquad ds^2 = 0$$

$$l_H(t) = a(t)\int dx = a(t)\int d\eta = a(t)\int_0^t \frac{c\,dt'}{a(t')} \propto t \quad \propto 1/H(t)$$



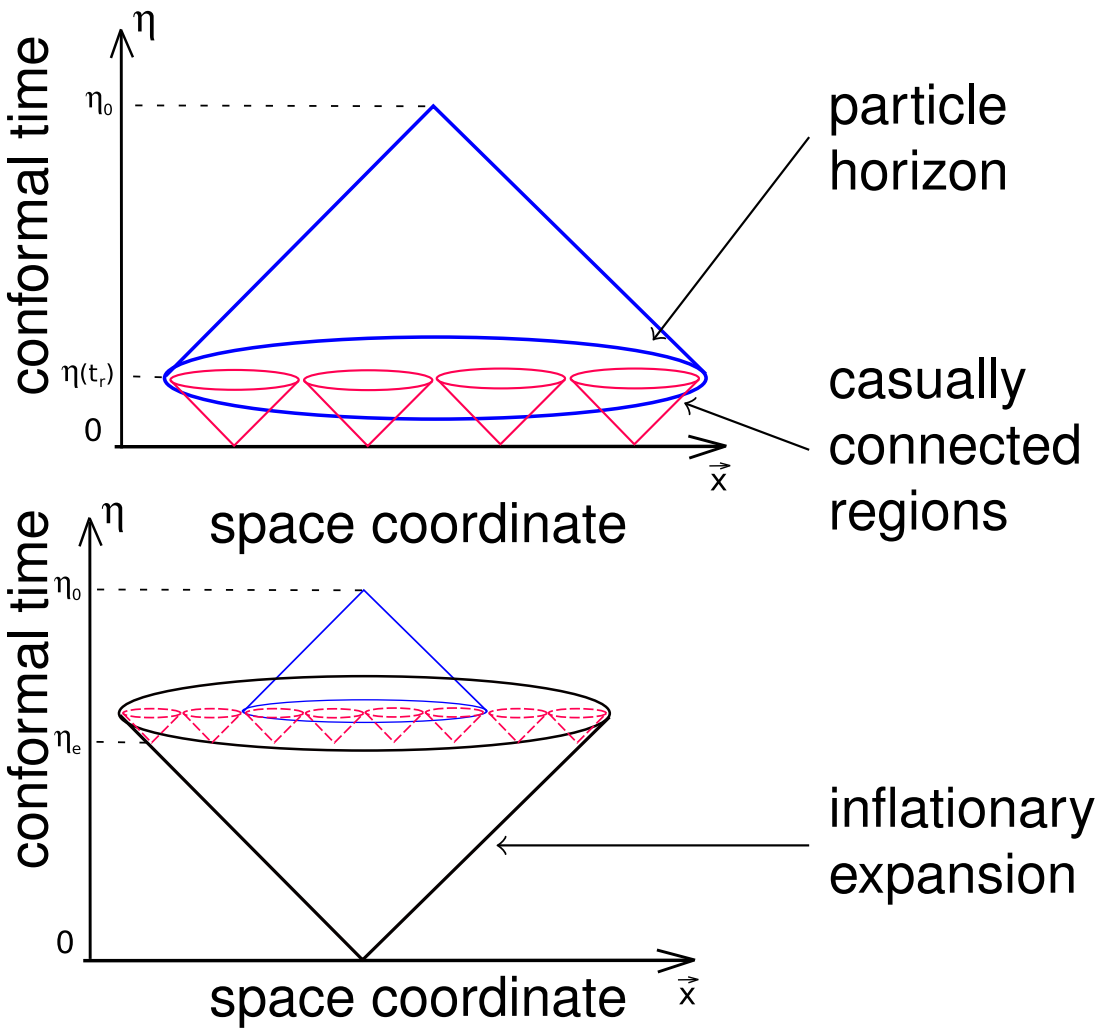
$a(t) \propto t^\alpha$, $0 < \alpha < 1$, $L_{phys} \propto a$

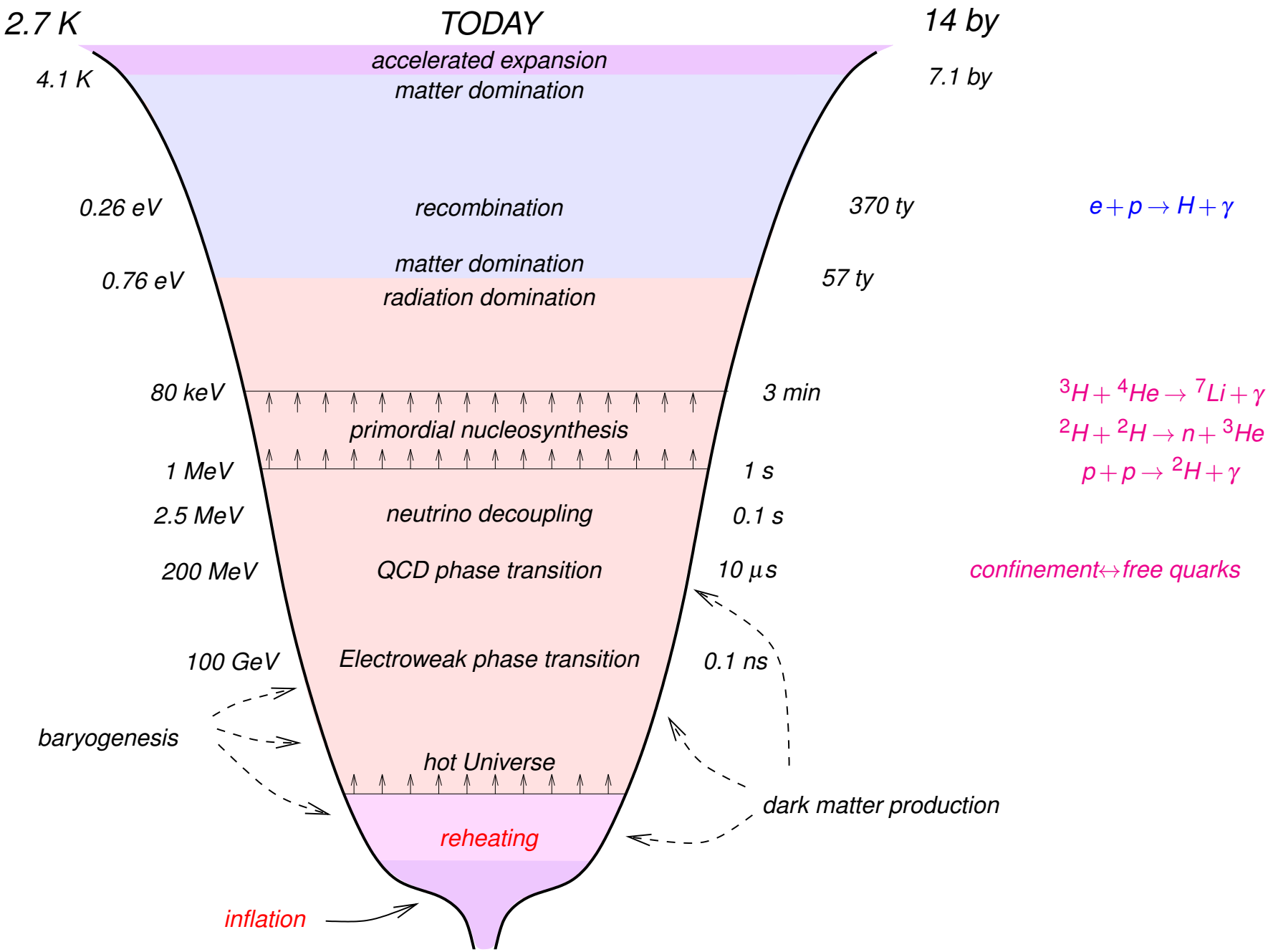


$$l_{H_0}/l_{H,r}(t_0) \sim l_{H_0}/l_{H,r}(t_r)\,a(t_r)/a_0 \sim H_r/H_0\,a(t_r)/a_0 \sim \sqrt{1+z_r} \simeq 30$$

Inflationary solution of Hot Big Bang problems

- no initial singularity in dS space
- all scales grow exponentially, including the radius of the 3-sphere
  the Universe becomes exponentially flat
- any two particles are at exponentially large distances
  no heavy relics
  no traces of previous epochs!
- no particles in post-inflationary Universe
  to solve entropy problem we need post-inflationary reheating




2.7 K
TODAY
14 by
accelerated expansion
4.1 K
matter domination
7.1 by
0.26 eV
recombination
370 ty
$e+p \to H+\gamma$
matter domination
0.76 eV
radiation domination
57 ty
80 keV
3 min
$^3H+^4He \to ^7Li+\gamma$
primordial nucleosynthesis
$^2H+^2H \to n+^3He$
1 MeV
1 s
$p+p \to ^2H+\gamma$
2.5 MeV
neutrino decoupling
0.1 s
200 MeV
QCD phase transition
10 μs
confinement↔free quarks
100 GeV
Electroweak phase transition
0.1 ns
baryogenesis
hot Universe
dark matter production
reheating
inflation

Chaotic inflation at large fields: graceful exit

If $V(\phi)$ dominates by chance

Chaotic inflation, A.Linde (1983), A.Linde (1984)

$$\ddot{\phi} - \Delta\phi/a^2 + 3H\dot{\phi} + V'(\phi) = 0$$

for power-law potential at $\phi > M_{Pl}$

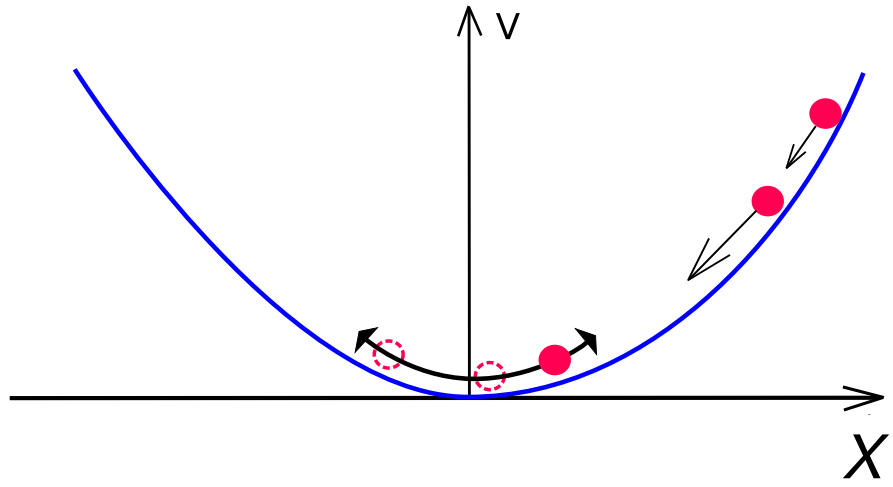
$$V \simeq \text{const}$$

"slow roll" solution

$$H^2 = \frac{8\pi}{3M_P^2} V(\phi) \ , \quad a(t) \propto e^{Ht}$$

valid while slow roll conditions

$$M_P^2 \frac{V''}{V} \ll 1 \ , \quad M_P^2 \frac{V'^2}{V^2} \ll 1$$



Inflaton must couple
to Standard Model fields
to reheat the Universe
after inflation

Quantum perturbations at inflation

$$\ddot{\varphi}_k + 3H\dot{\varphi}_k + \frac{k^2}{a^2}\varphi_k = 0$$

conformal time, rescaled field: $dt \equiv a(\eta)d\eta, \quad \phi \equiv \chi/a(\eta)$:

$$a = -\frac{1}{H\eta}, \quad \frac{k}{a} = H \quad \longrightarrow \quad k\eta = 1$$

$$\chi_k'' + \left(k^2 - \frac{2}{\eta^2}\right)\chi_k = 0$$

classical solutions

$$\chi_k = C_1 \times e^{ik\eta}\left(1 + \frac{i}{k\eta}\right) + C_2 \times e^{-ik\eta}\left(1 - \frac{i}{k\eta}\right)$$

Unexpected bonus: generation of perturbations

- Quantum fluctuations of wavelength λ of a free massless field φ have an amplitude of $\delta\varphi_\lambda \simeq 1/\lambda$
- In the expanding Universe: $\lambda \propto a$

inflation: $l_H \sim 1/H = \text{const}$, so modes "exit horizon"
Ordinary stage: $l_H \sim 1/H \propto t$, $l_H/\lambda \nearrow$, modes "enter horizon"

Evolution at inflation

- inside horizon: $\lambda < l_H$

$\lambda \propto a \Rightarrow \qquad \delta\varphi_\lambda \propto 1/\lambda \propto 1/a$



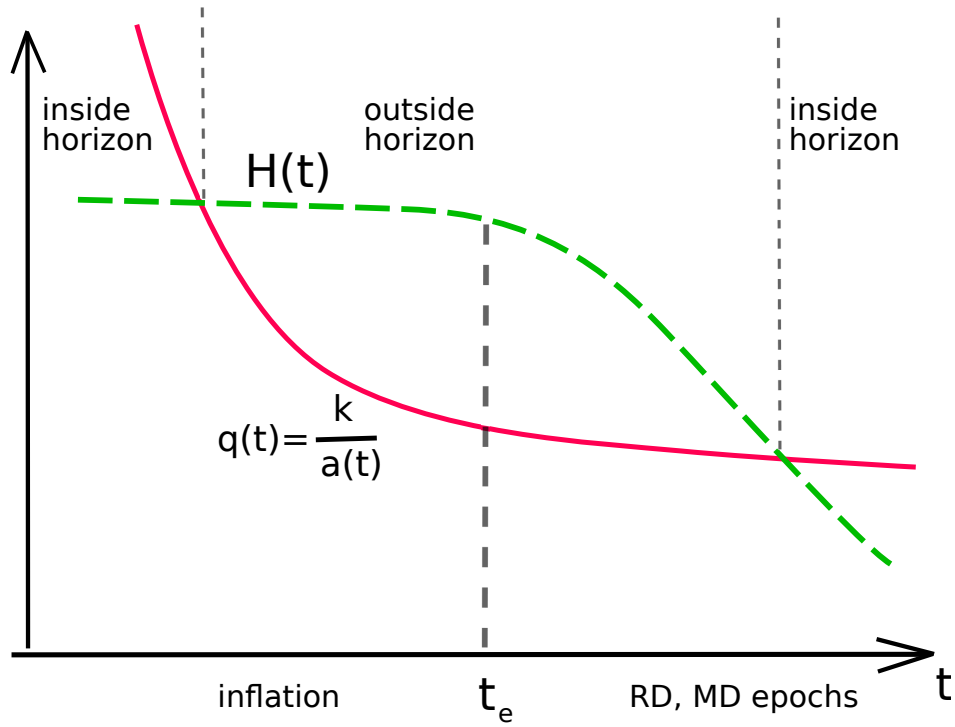
- outside horison: $\lambda > l_H$

$\lambda \propto a \Rightarrow \qquad \delta\varphi_\lambda = \text{const} = H_{infl}$!!!



- got "classical" fluctuations:

$$\delta\varphi_\lambda = \delta\varphi_\lambda^{quantum} \times e^{N_e}$$



Transfer to matter perturbations: simple models

Illustration: Local delay(advance)δt in evolution due to impact of $\delta\phi$ of all modes with $\lambda > H$:

$$\delta\phi = \dot{\phi}_c\,\delta t\,, \quad \delta\rho \sim \dot{\rho}\,\delta t$$

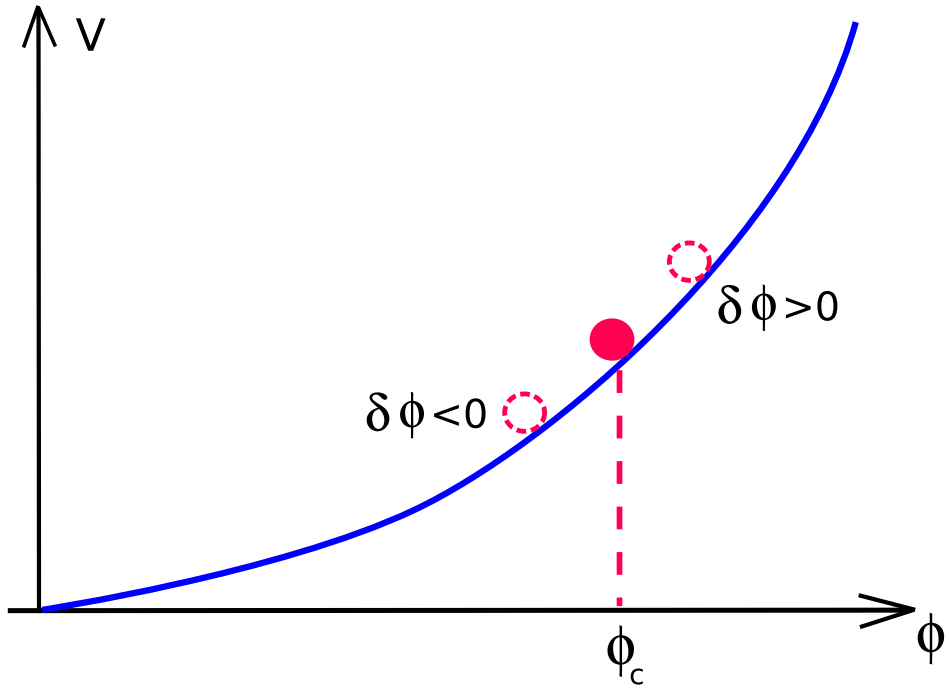
at the end of inflation $\dot{\rho} \sim -H\rho$, then

$$\frac{\delta\rho}{\rho} \sim \frac{H}{\dot{\phi}_c}\,\delta\phi$$

Hence, $\delta\rho/\rho$ is also gaussian.
Power spectrum of scalar perturbations

$$\mathscr{P}_{\mathscr{R}}(k) = \left(\frac{H^2}{2\pi\,\dot{\phi}_c}\right)^2\,,$$

everything is calculated at $t = t_k : H = k/a$

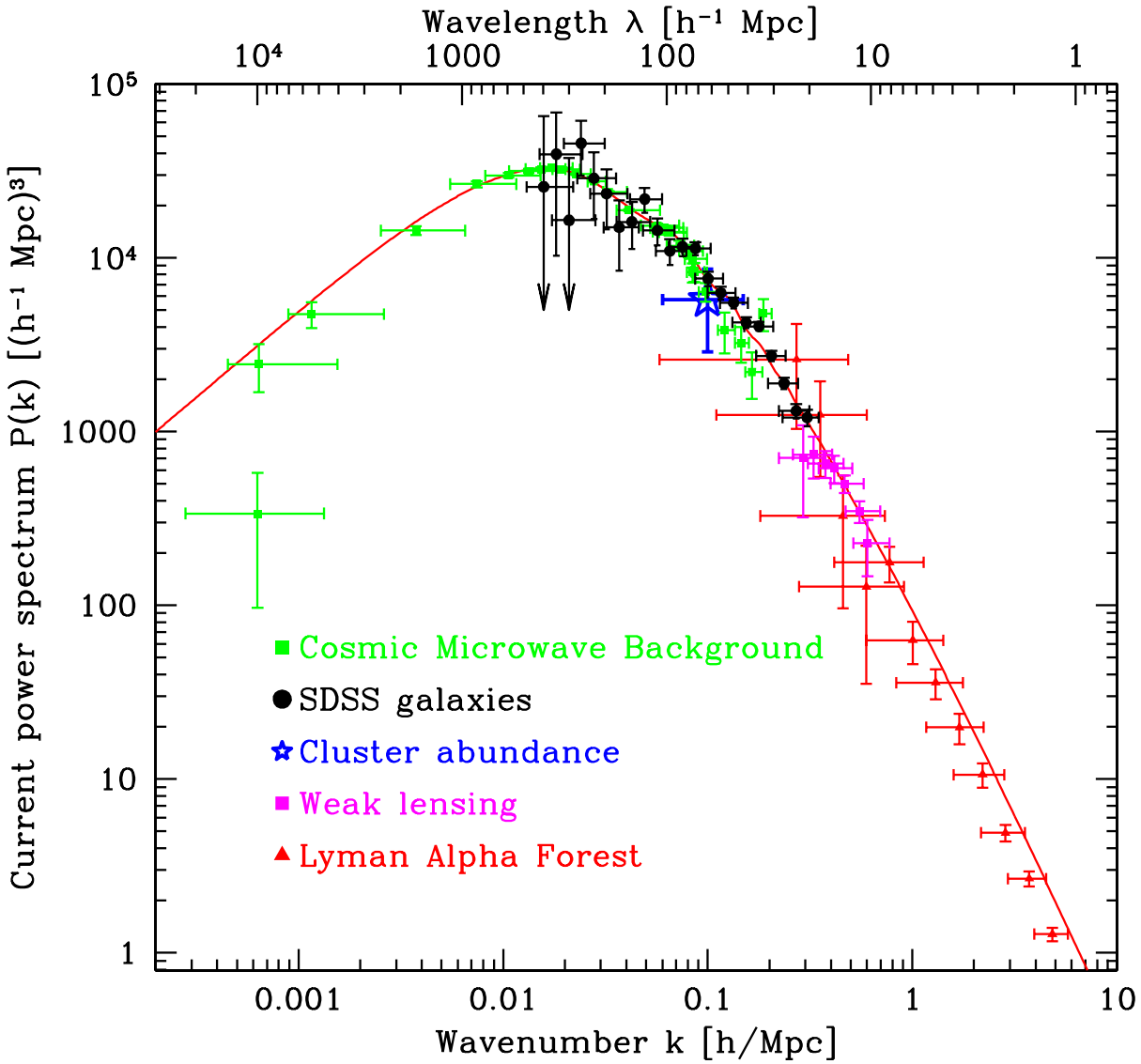


Analogously for the tensor perturbations: each of the two polarizations of the gravity waves solves the free scalar field equation!

$$\mathscr{P}_T(k) = \frac{16}{\pi}\,\frac{H_k^2}{M_{Pl}^2}$$

To the leading order no k-dependence: both spectra are “flat” (scale-invariant)!

Actually we observe rather narrow range



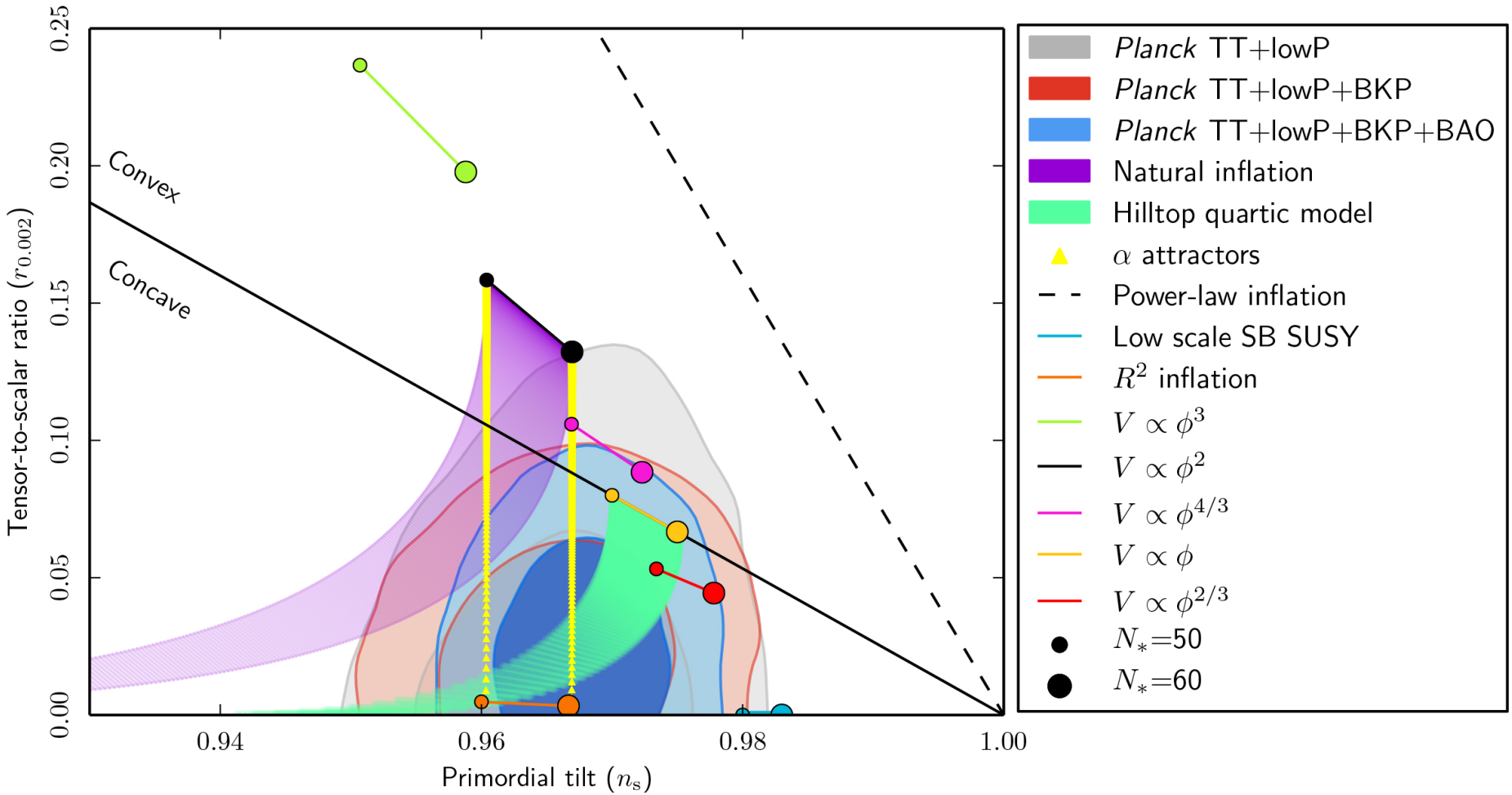
Observable range:

$$\frac{k_{max}}{k_{min}} \sim 10^5$$

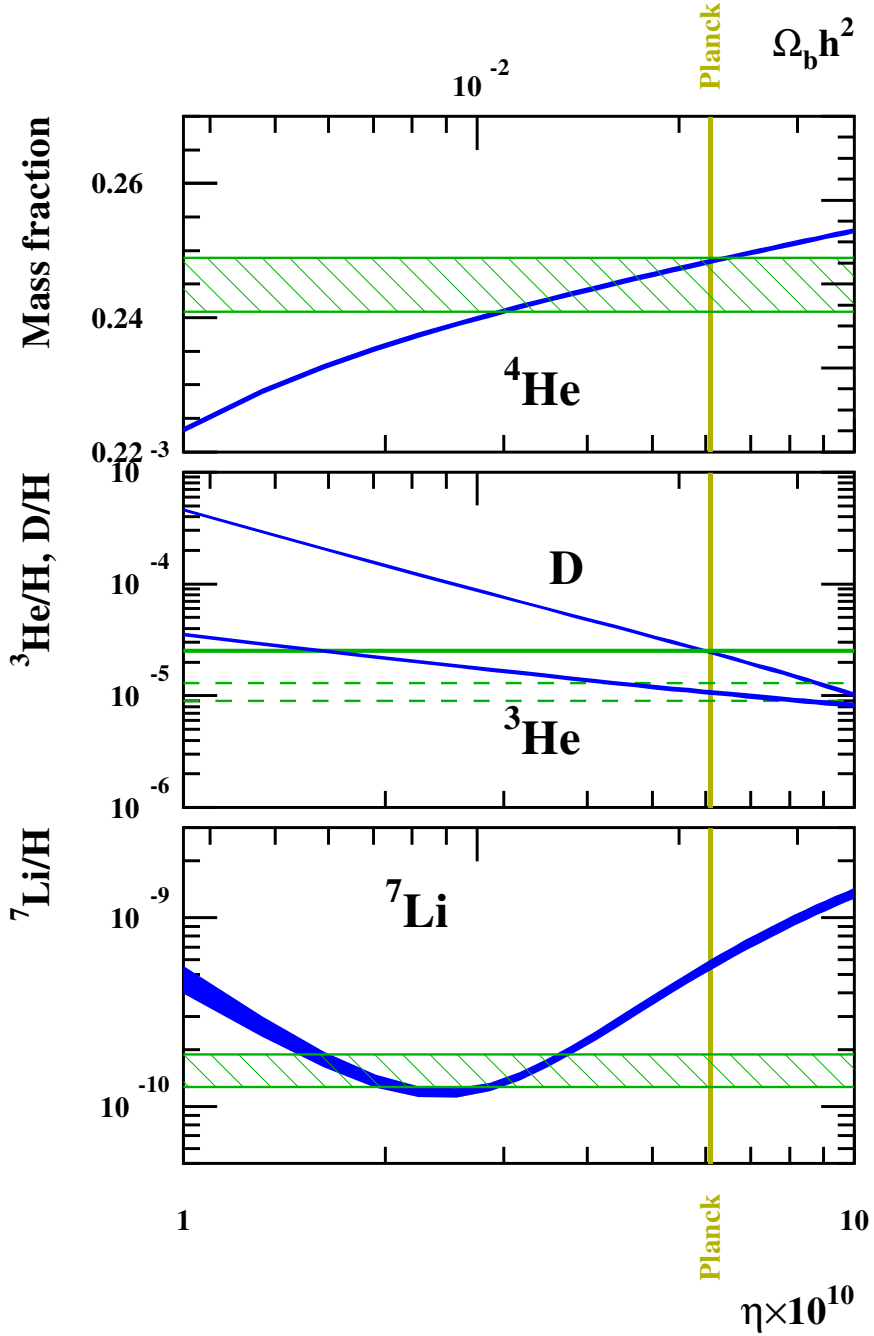
$$\Delta N_e \simeq 10$$

Small scales cannot describe:
for a long time in nonlinear regime

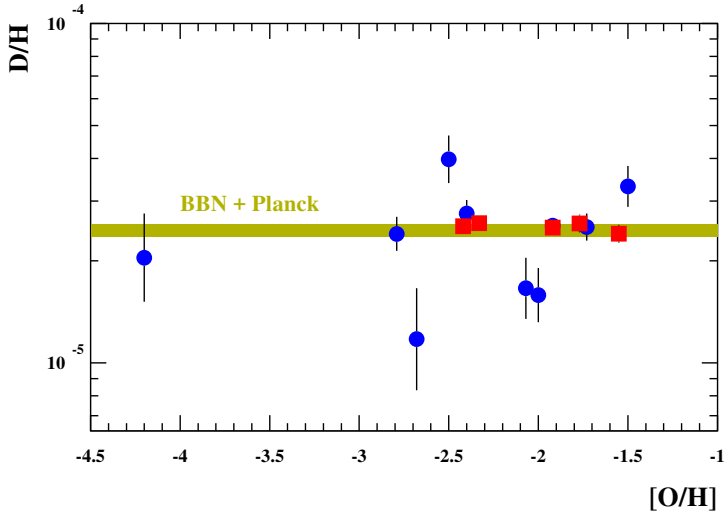
Planck 2015 bounds on tensor perturbations



$$r = \frac{A_T}{A_S} \propto \frac{\dot{\phi}^2}{H^2 M_{Pl}^2} \propto \left(\frac{V'}{V}\right)^2 \ll 1$$

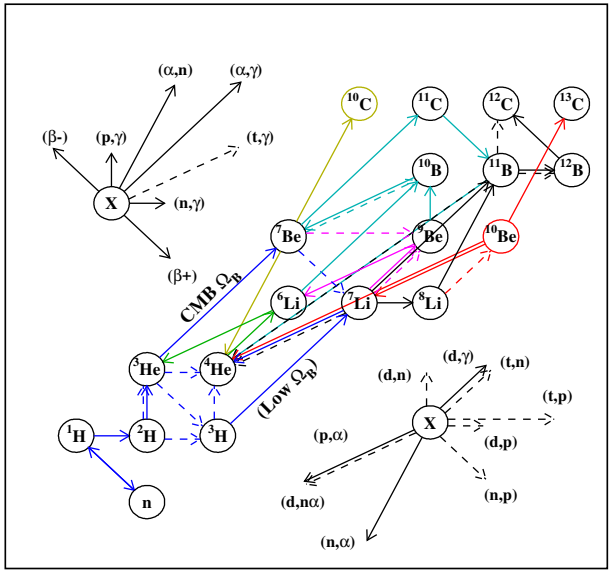


1707.01004 Measurement of $\eta_B = n_B/n_\gamma$ at $T \sim 1$ MeV



Lack of Lithium. . .

Exotics needed?



Outline

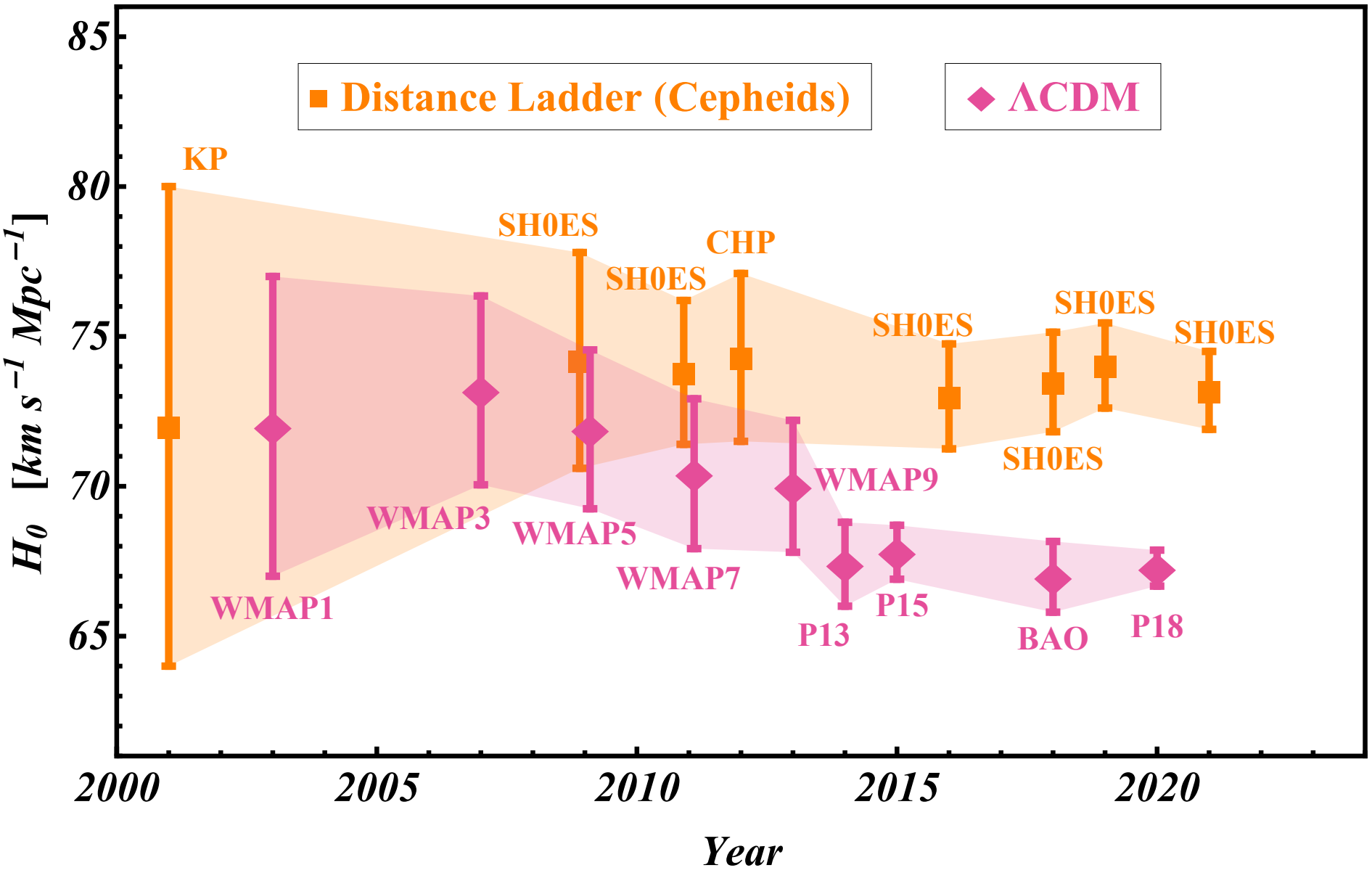
1 Cosmology in Brief

2 Old Problems in Cosmology

3 New Problems in Cosmology

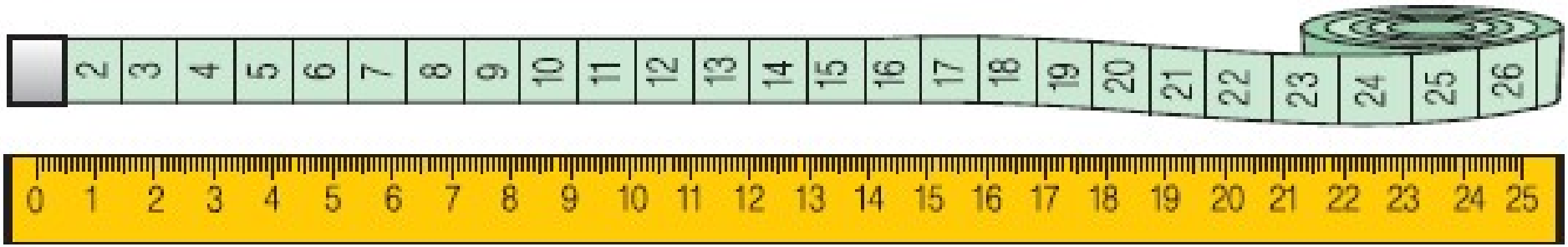
New Problems in Cosmology

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Distance Ladder (Cepheids)
ΛCDM
KP
SH0ES
CHP
SH0ES
SH0ES
SH0ES
SH0ES
SH0ES
WMAP1
WMAP3
WMAP5
WMAP7
WMAP9
P13
P15
BAO
P18
85
80
75
70
65
H_0 [km s^{-1} Mpc^{-1}]
2000
2005
2010
2015
2020
Year

2105.05208

Distance ladder... from observer



MEASURE



- size of the Earth orbit

$$2\pi R = v \times 1\,\text{year}$$

- distance through angle (annual paralax)

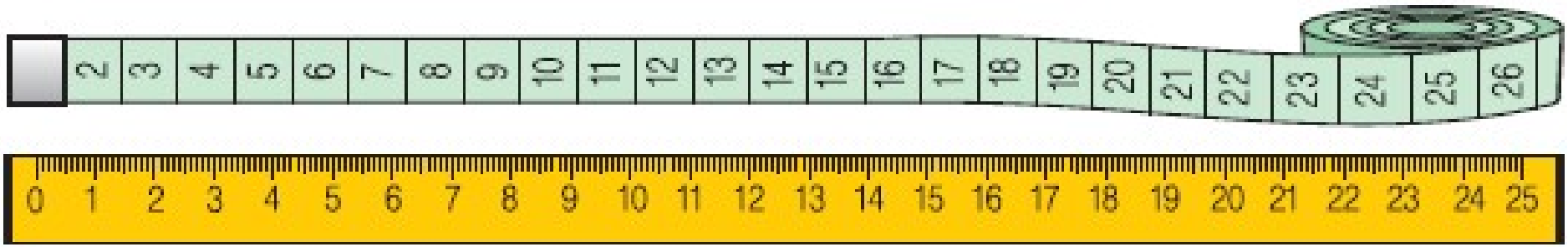
$$L = \theta \times 2R$$

- luminosity F via brightness J and distance

$$J = \frac{F}{4\pi L^2}$$

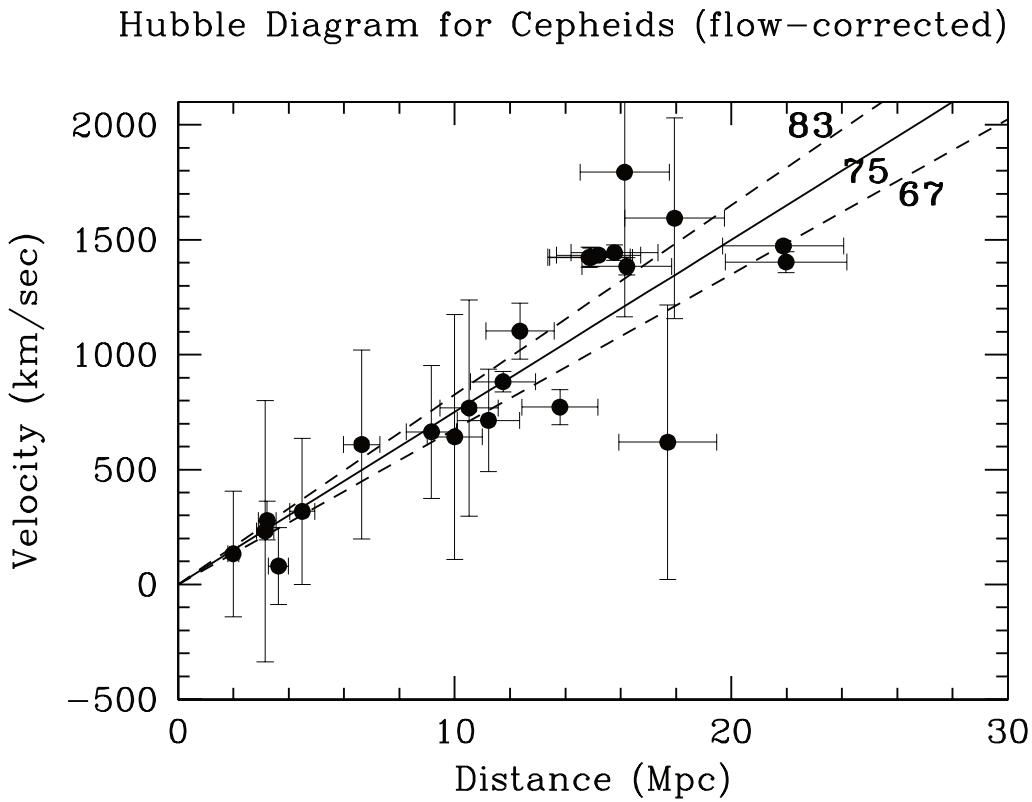
- find a relation between F and other observable (frequency of bursts, decay rate, ...) – the standard candle is defined !

Distance ladder... from observer

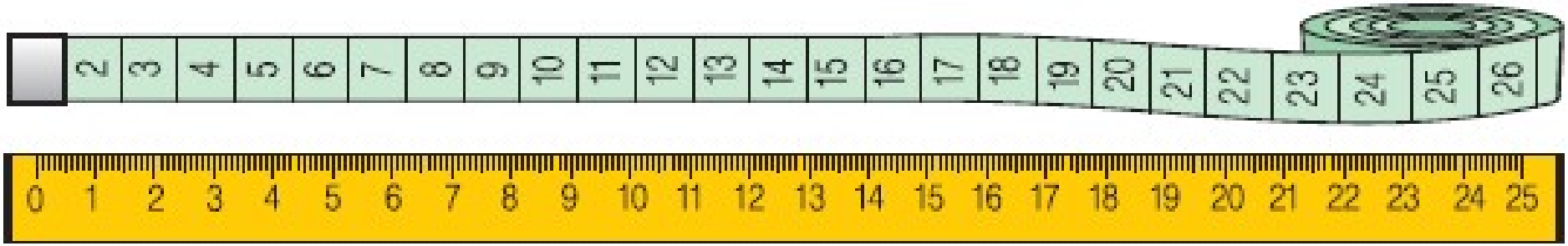


Measure luminosity with brightness and distance





Distance ladder... from cefeids to supernovae



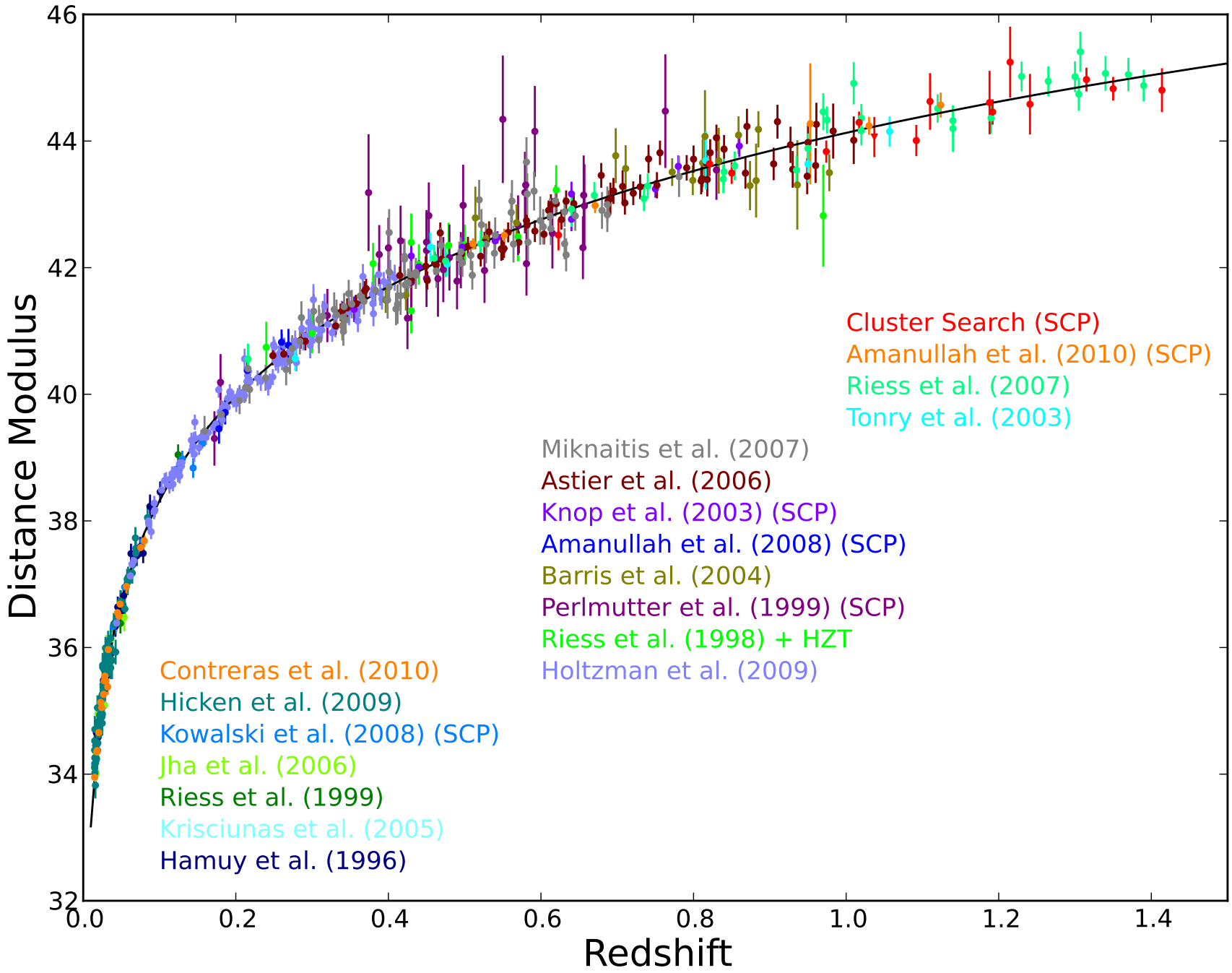
Measure the brightness J of the object of luminosity F – "standard candles"

$$J = \frac{F}{4\pi L^2} \longrightarrow J = \frac{F}{4\pi L^2 (1+z)(1+z)} \qquad \frac{\text{energy}}{\text{time} \times \text{area}}$$



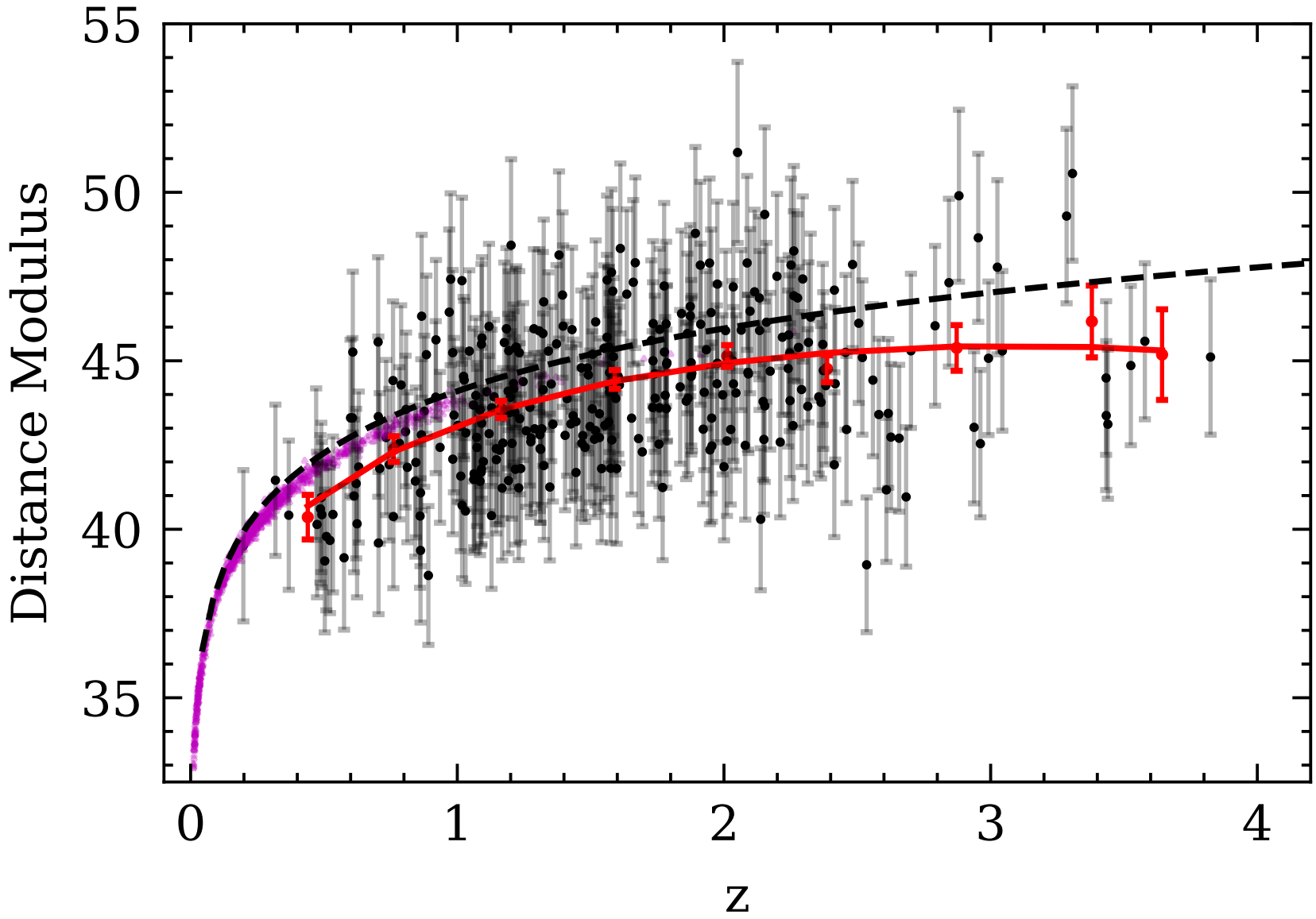
In the expanding Universe it gets modified

$$J = \frac{F}{4\pi d_L{}^2}\,, \qquad \text{where} \qquad d_L \equiv (1+z) \times d_A = (1+z) \int_0^z \frac{dz'}{H(z')}$$

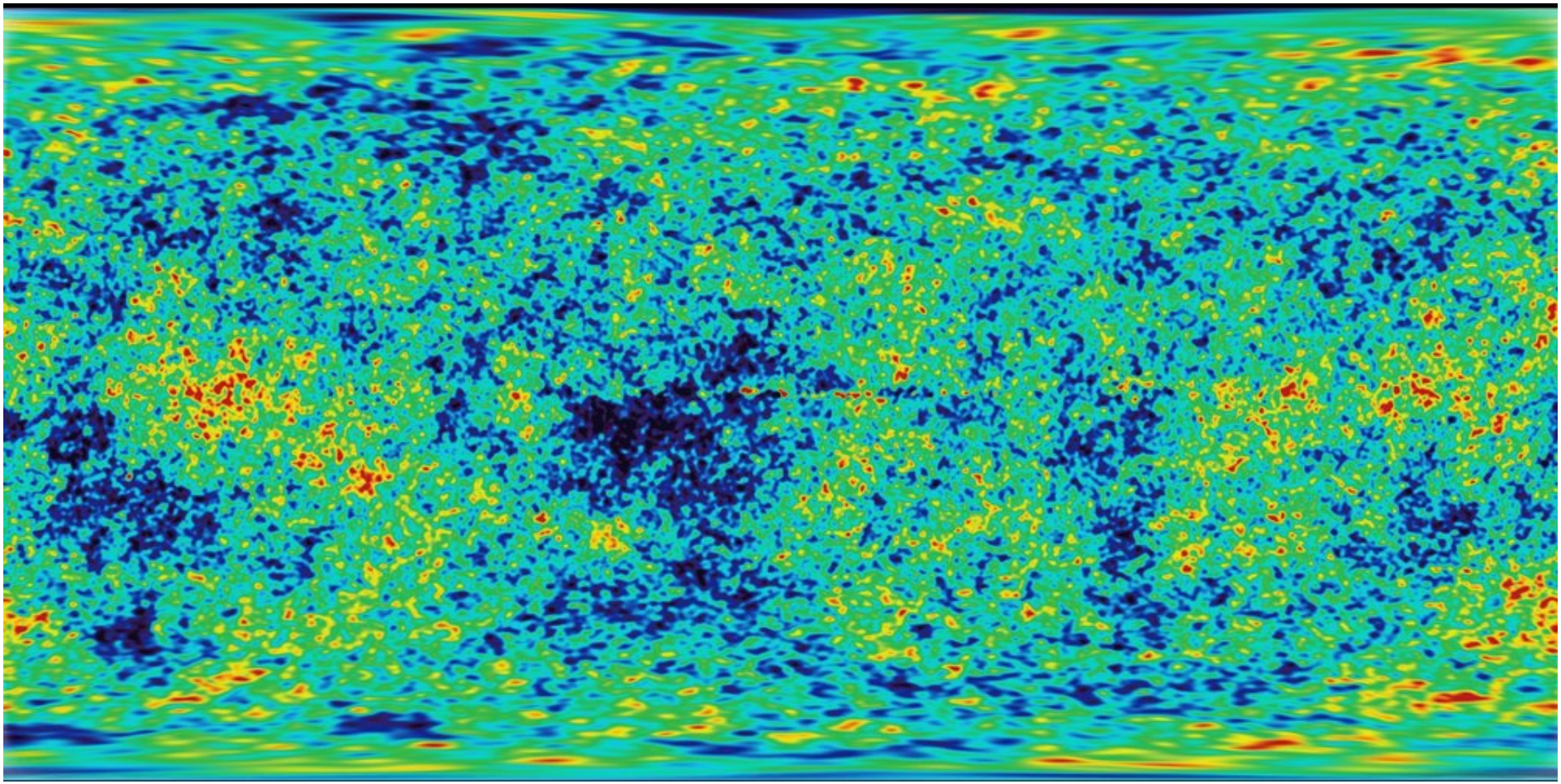

Distance Modulus
Redshift
46
44
42
40
38
36
34
32
0.0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
Cluster Search (SCP)
Amanullah et al. (2010) (SCP)
Riess et al. (2007)
Tonry et al. (2003)
Miknaitis et al. (2007)
Astier et al. (2006)
Knop et al. (2003) (SCP)
Amanullah et al. (2008) (SCP)
Barris et al. (2004)
Perlmutter et al. (1999) (SCP)
Riess et al. (1998) + HZT
Holtzman et al. (2009)
Contreras et al. (2010)
Hicken et al. (2009)
Kowalski et al. (2008) (SCP)
Jha et al. (2006)
Riess et al. (1999)
Krisciunas et al. (2005)
Hamuy et al. (1996)

Next step is quasars

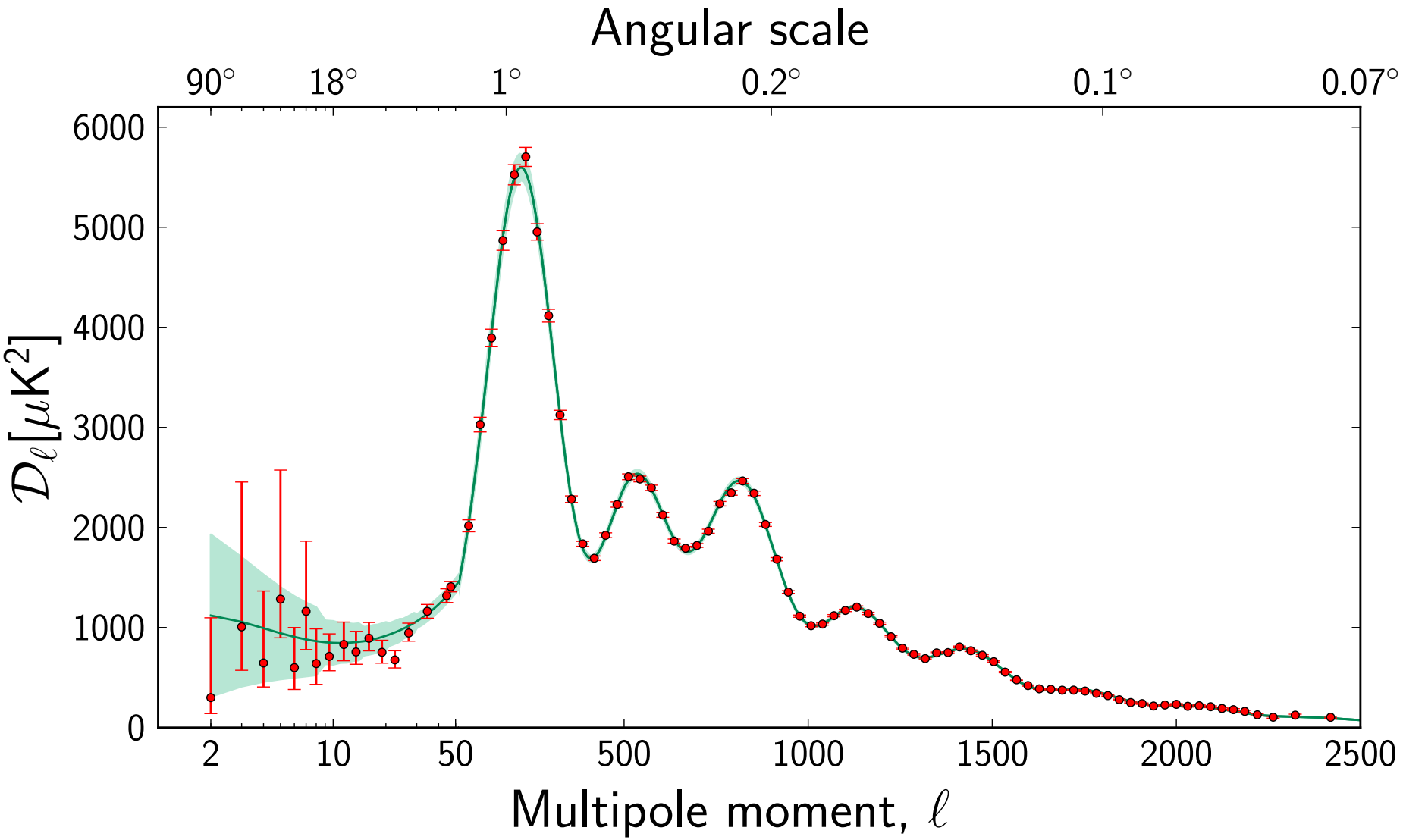
2110.03671



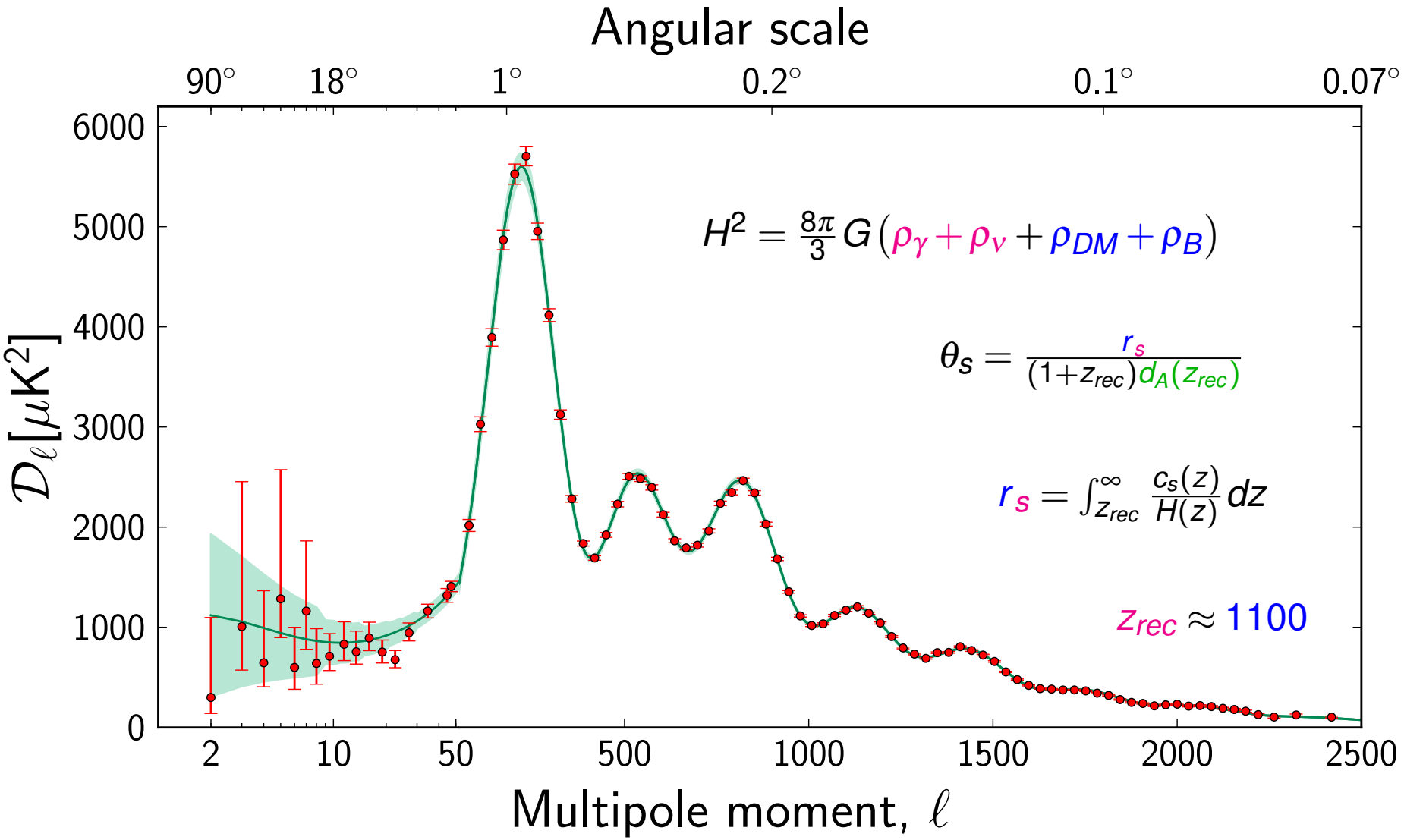
CMB map



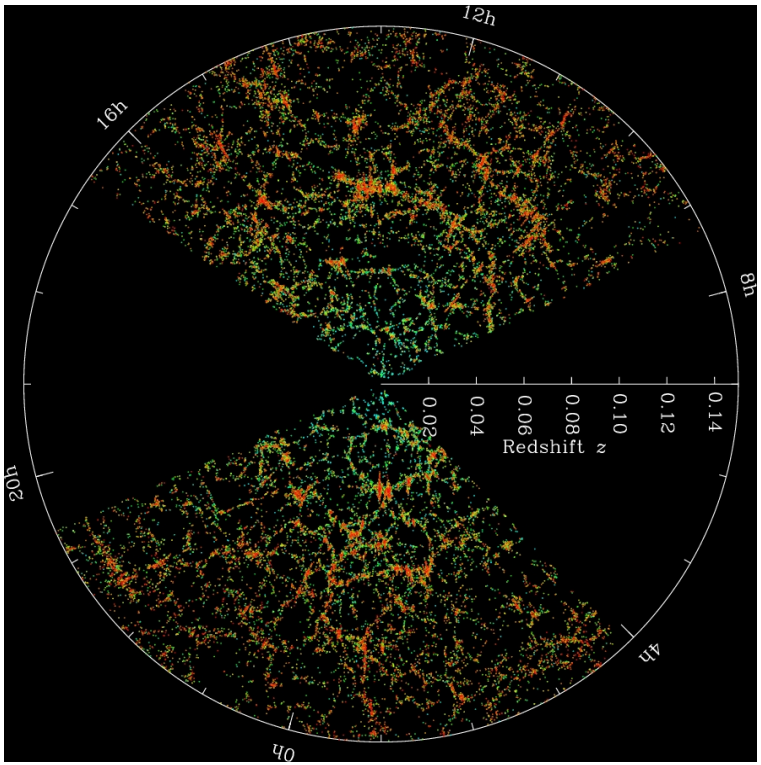
fit to ΛCDM model $\theta_{rec}, \Omega_{DM}h^2, \Omega_B h^2, z_{rei}, A_S, n_s$



The way to infer the Hubble constant

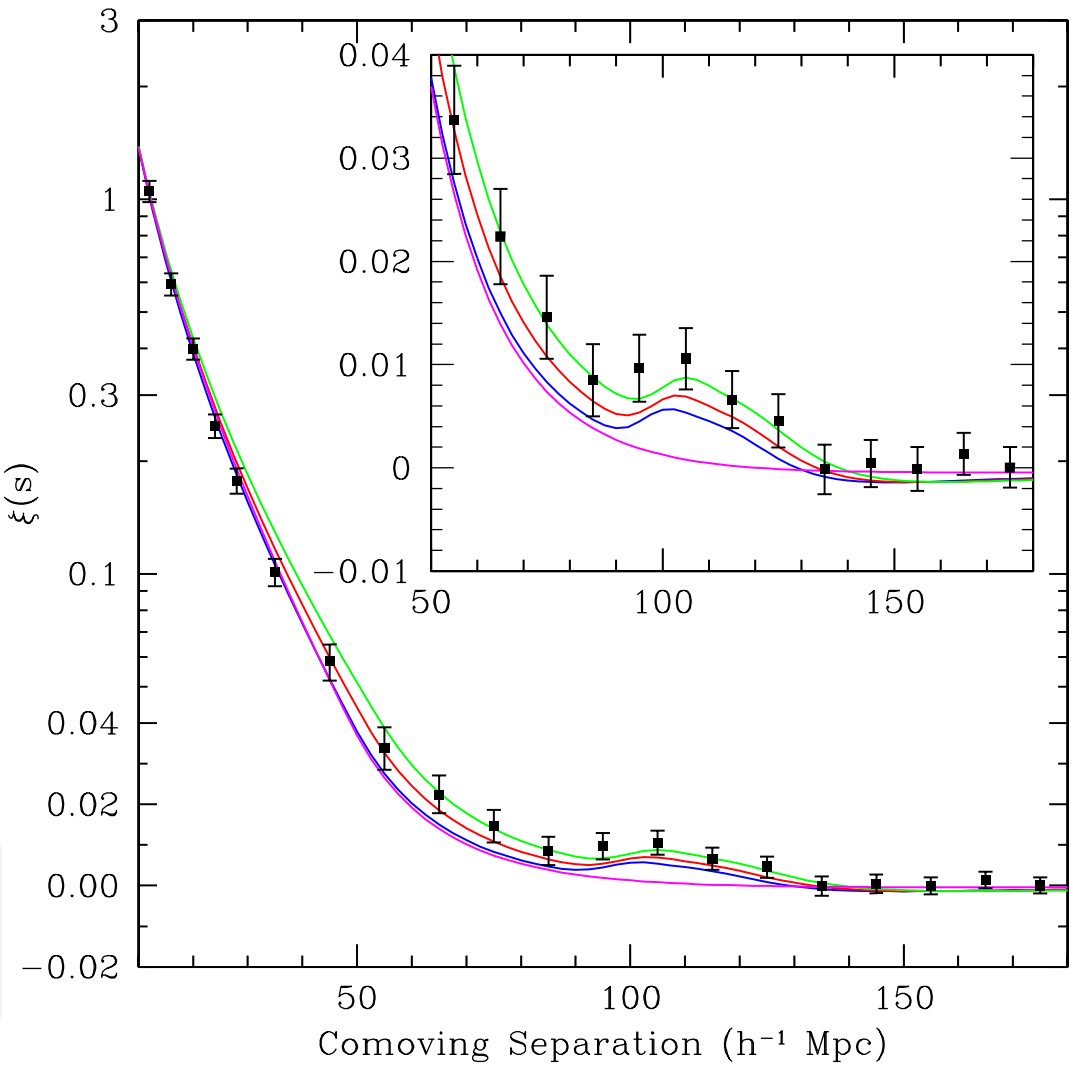


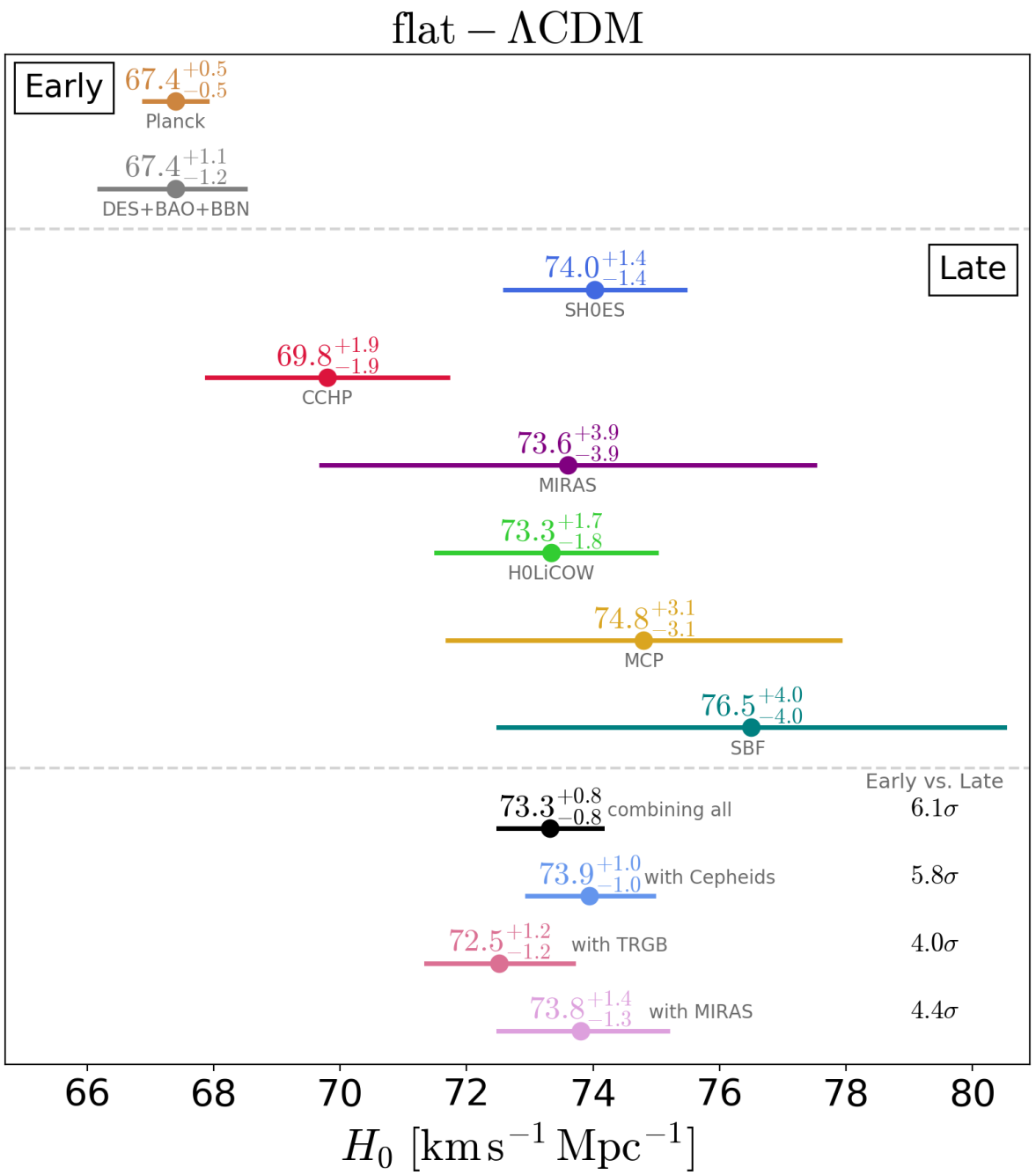
Baryon Acoustic Oscillations (BAO), Sakharov



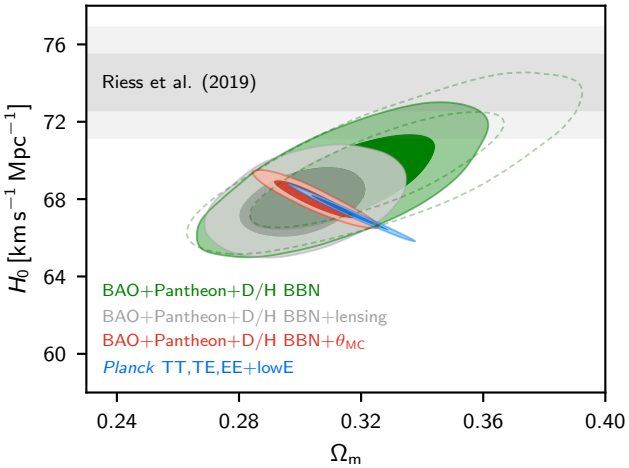
peak at $r_s \approx 147\,\mathrm{Mpc}$

$$\xi_z(s) \propto \frac{\langle n_z(\vec{x})\, n_z(\vec{x}+\vec{z})\rangle_{\vec{x}}}{n_z\, n_z}$$





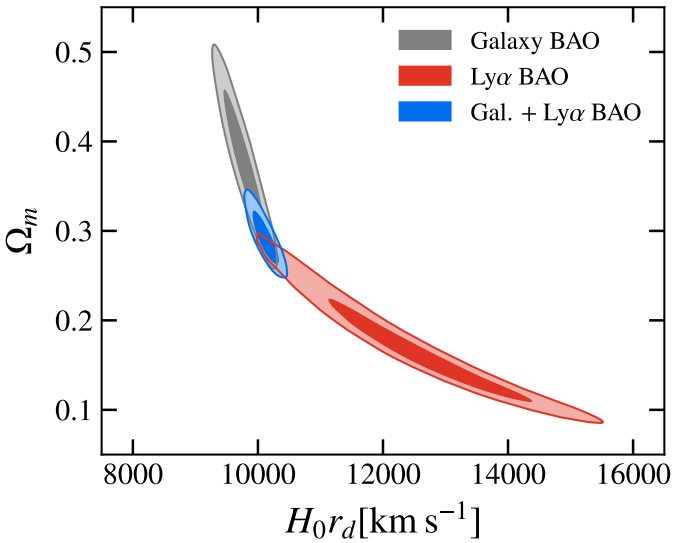
1807.06209

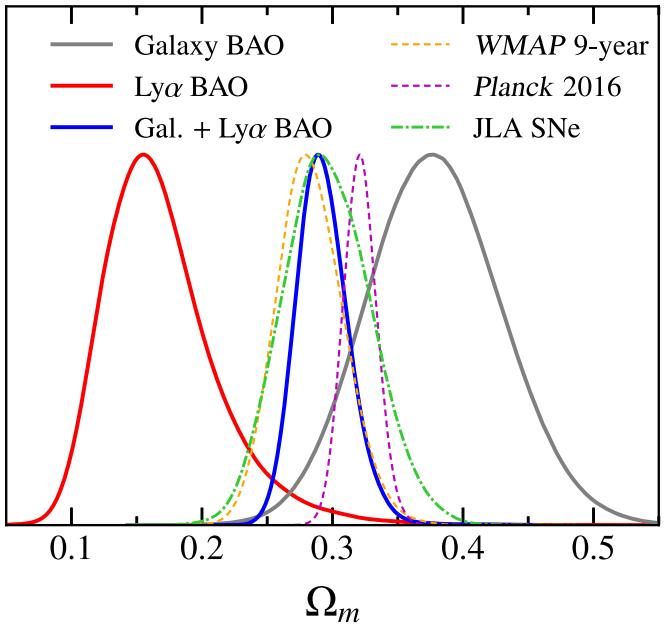


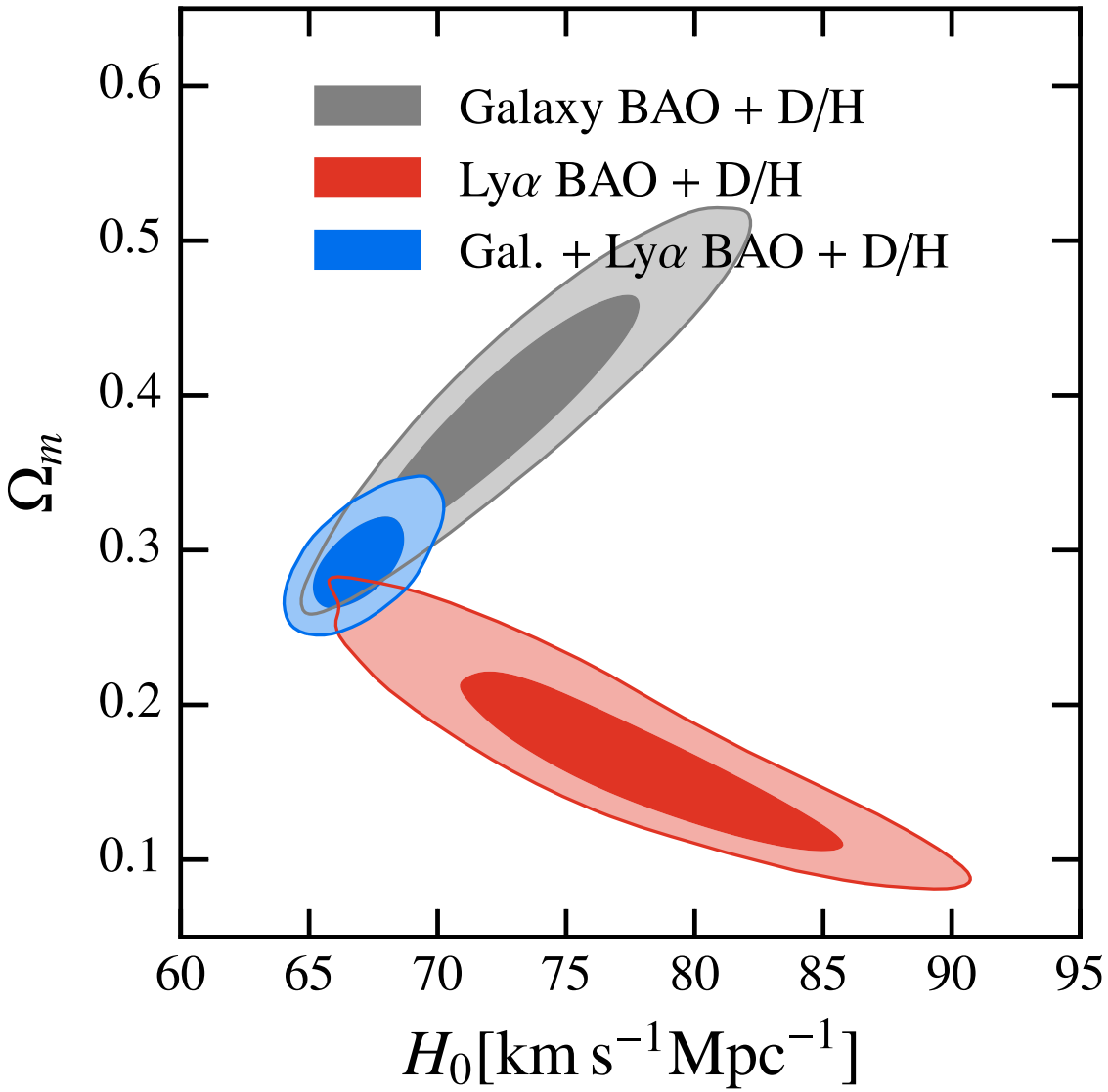
1907.10625

Impact of BAO: Galaxies vs Ly-α

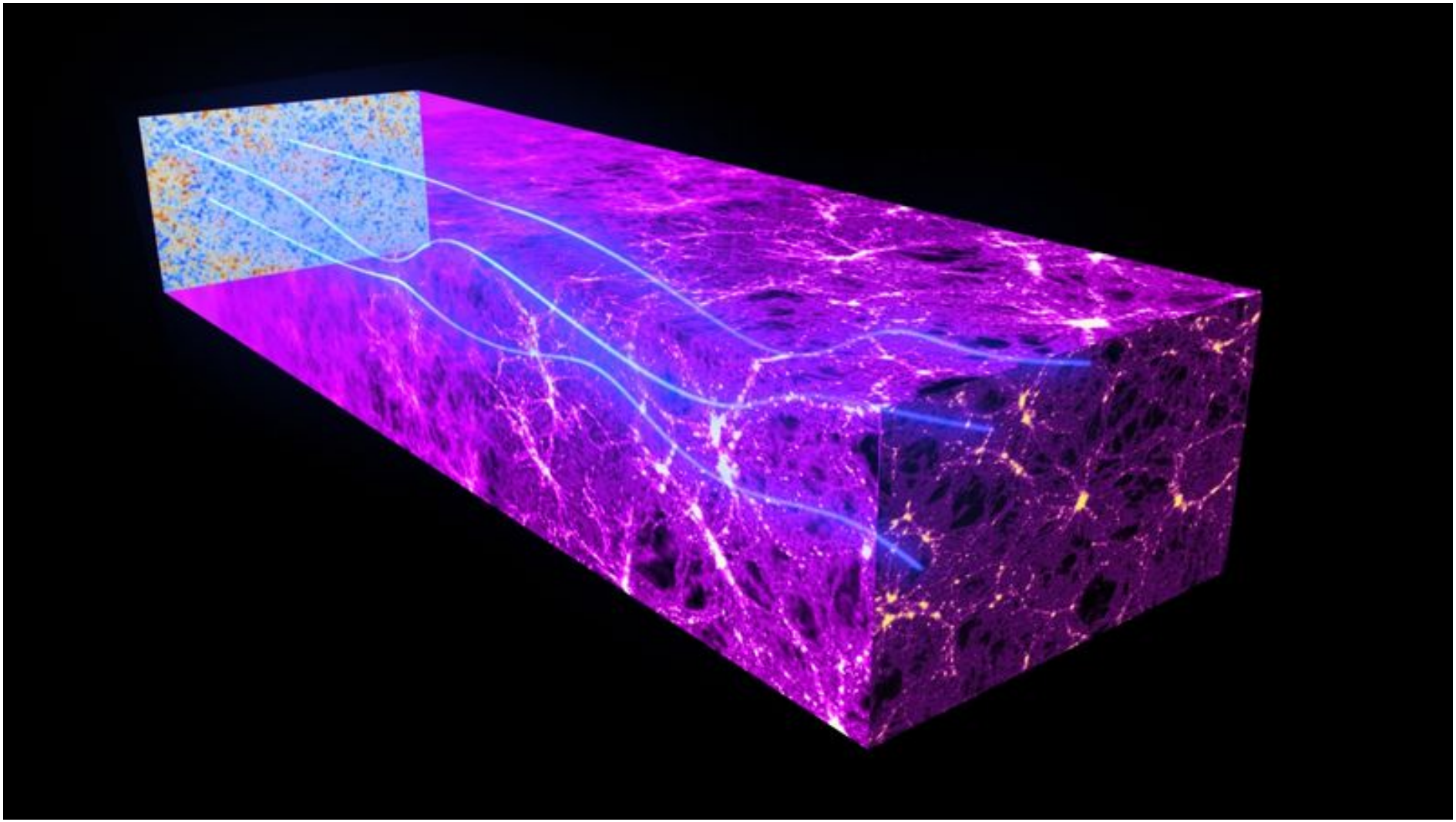
1707.06547





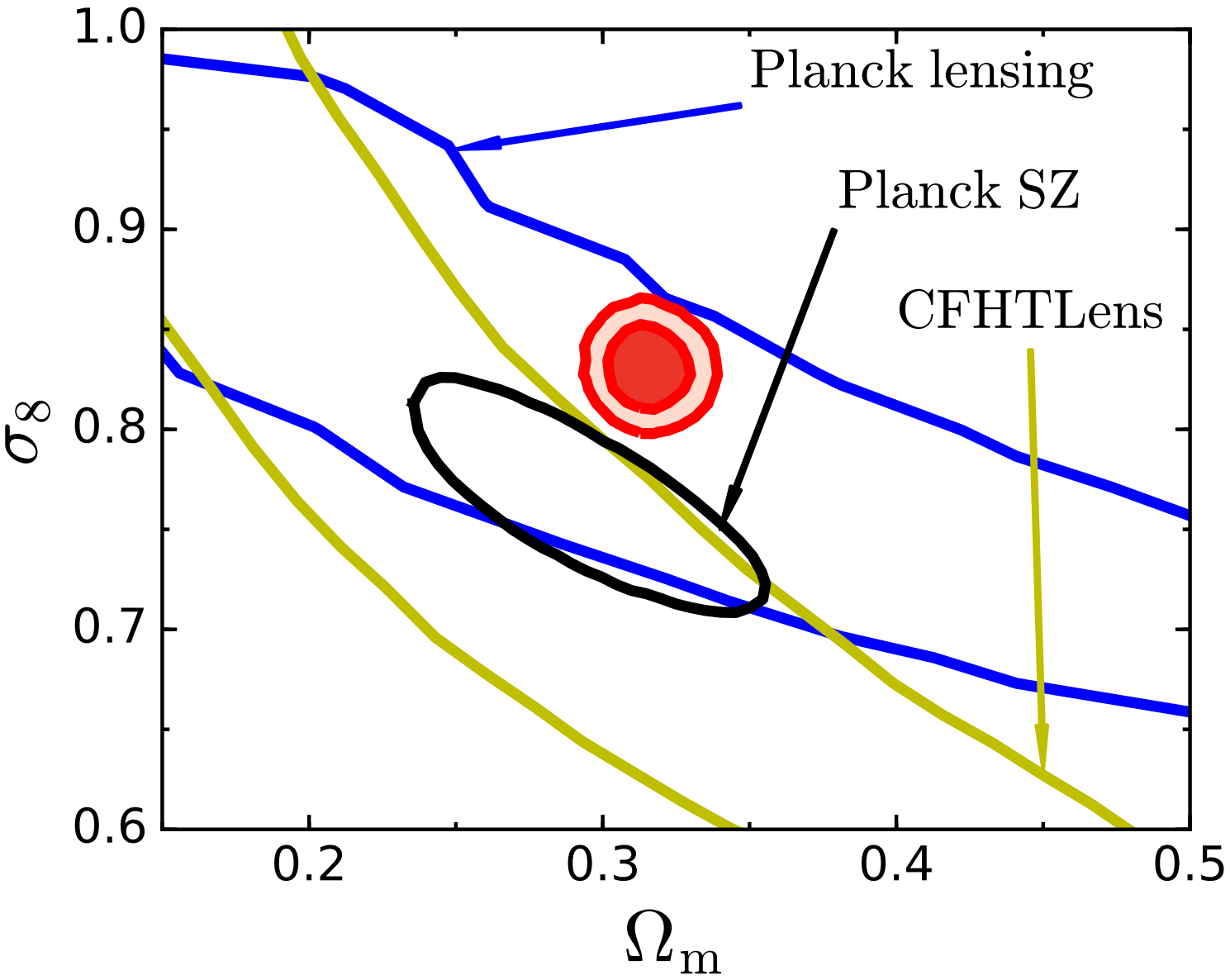


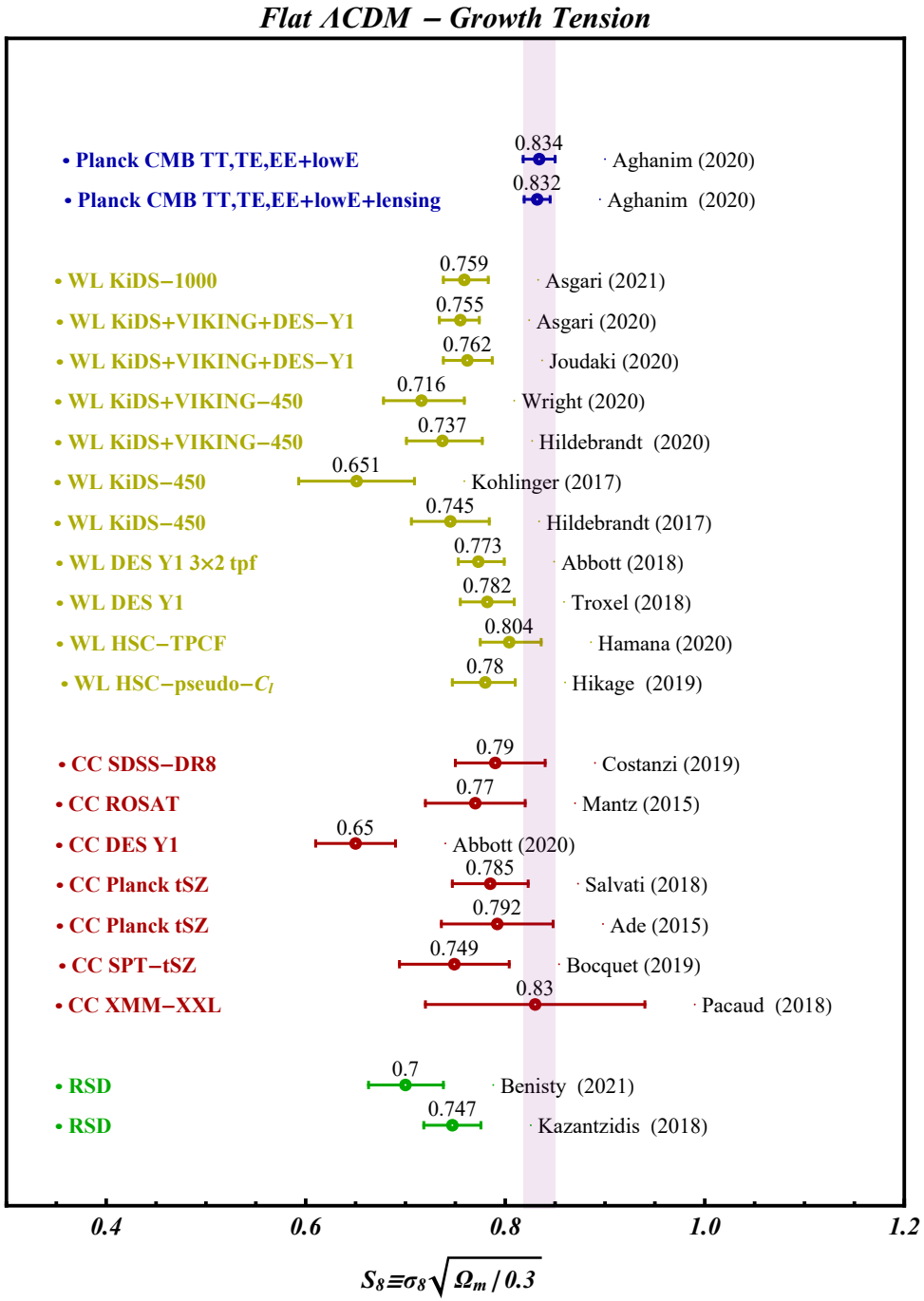
Matter in expanding Universe



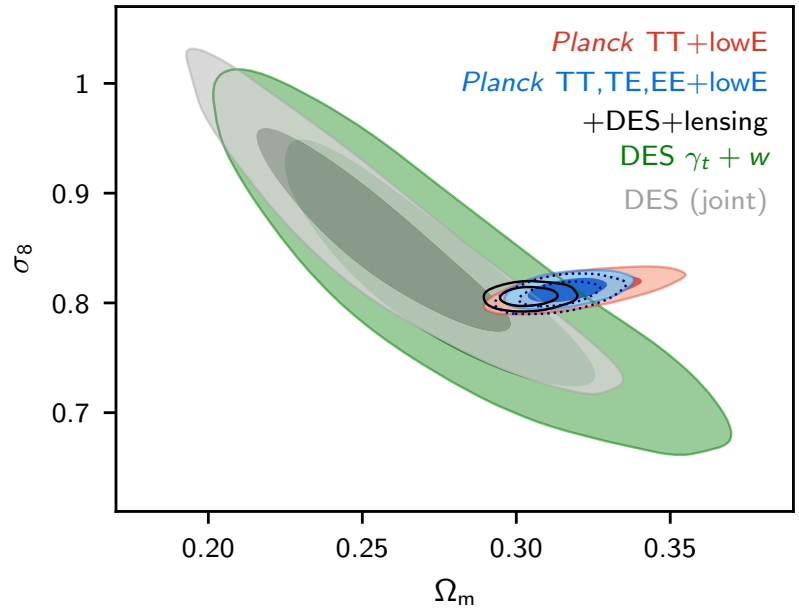
Lensing vs clusters

1801.07348





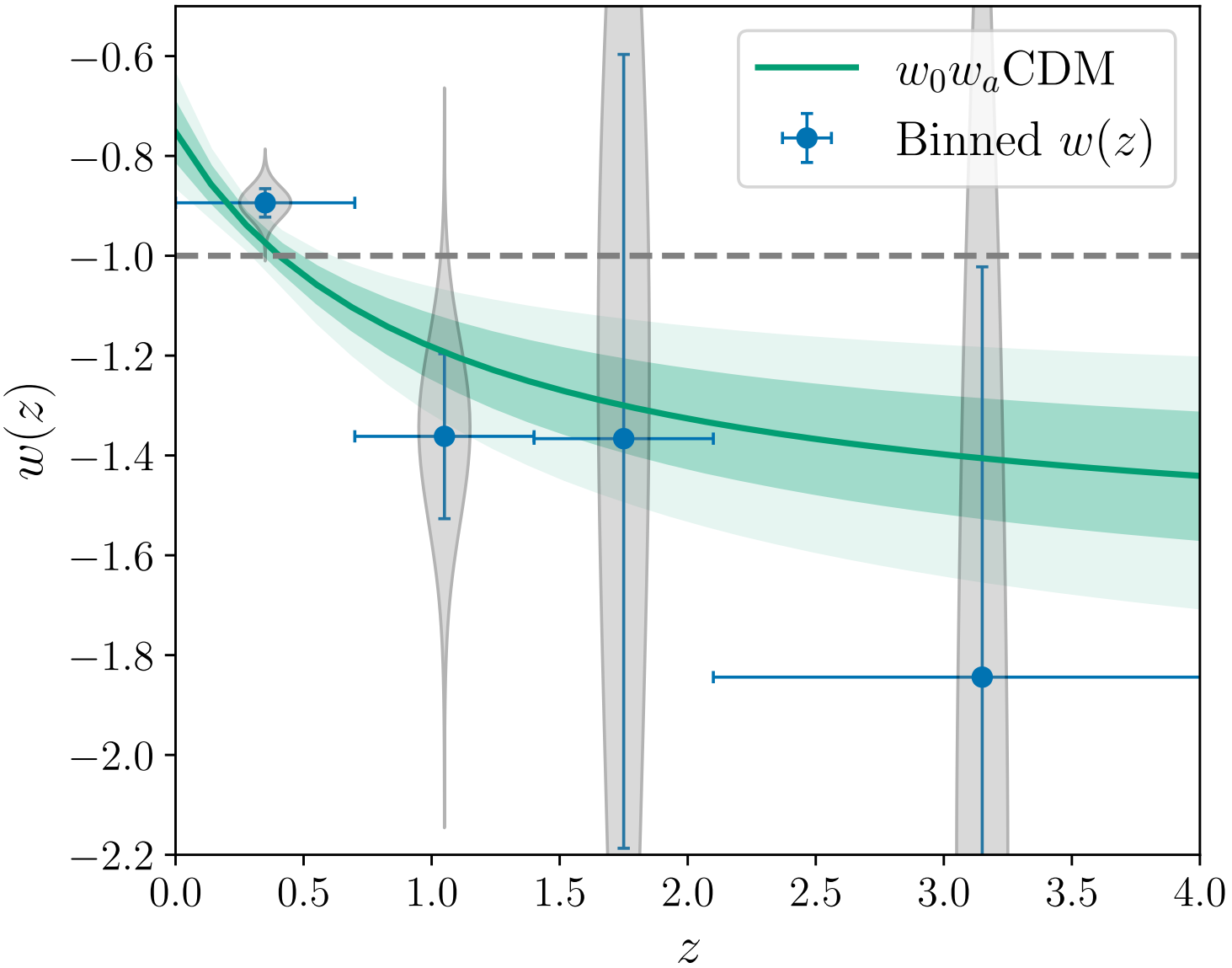
2105.05208



1807.06209

Results of DESI: DE evolution $p = \omega(z)\rho$

2503.14738



$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi}{3}\rho$

$\dot{\rho} + 3H(\rho + p) = 0$

$\omega = -1$: $p = -\rho$ gives $\rho = \text{const}$

$\omega > -1$ gives $\rho \searrow$

$\omega < -1$ gives $\rho \nearrow$

$\omega = 0$ is an oscillator: $\langle E_k \rangle = \langle E_p \rangle$

dust, non-relativistic particles,

dark matter

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Dark Matter in clusters

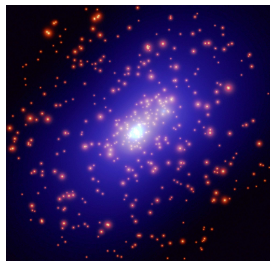
X-rays from hot gas in clusters

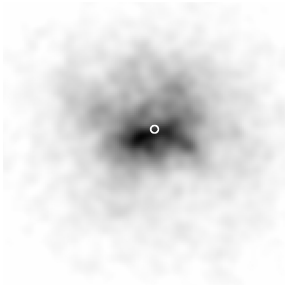
$$\frac{dP}{dR} = -\mu n_e(R) m_p \frac{GM(R)}{R^2}\,, \quad M(R) = 4\pi \int_0^R \rho(r) r^2 dr\,, \quad P(R) = n_e(R) T_e(R)$$

galaxies in clusters — virial theorem

$$U + 2E_k = 0$$

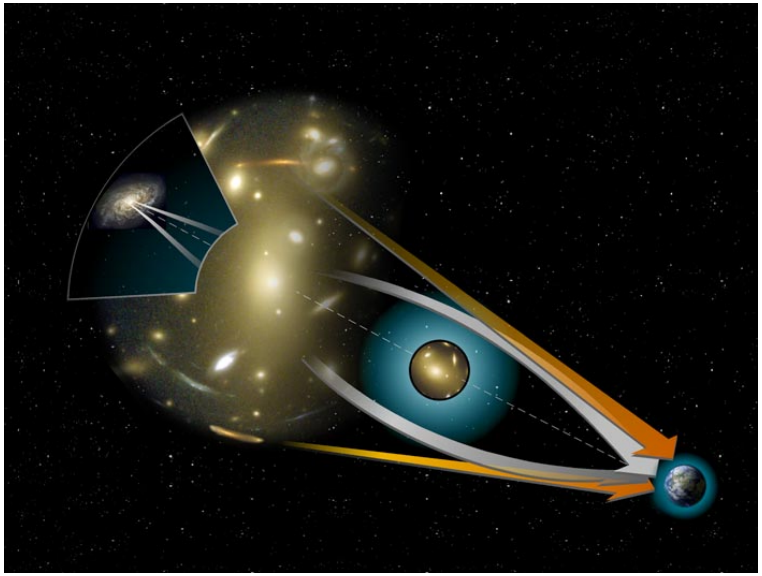
$$3M\langle v_r^2 \rangle = G\frac{M^2}{R}$$



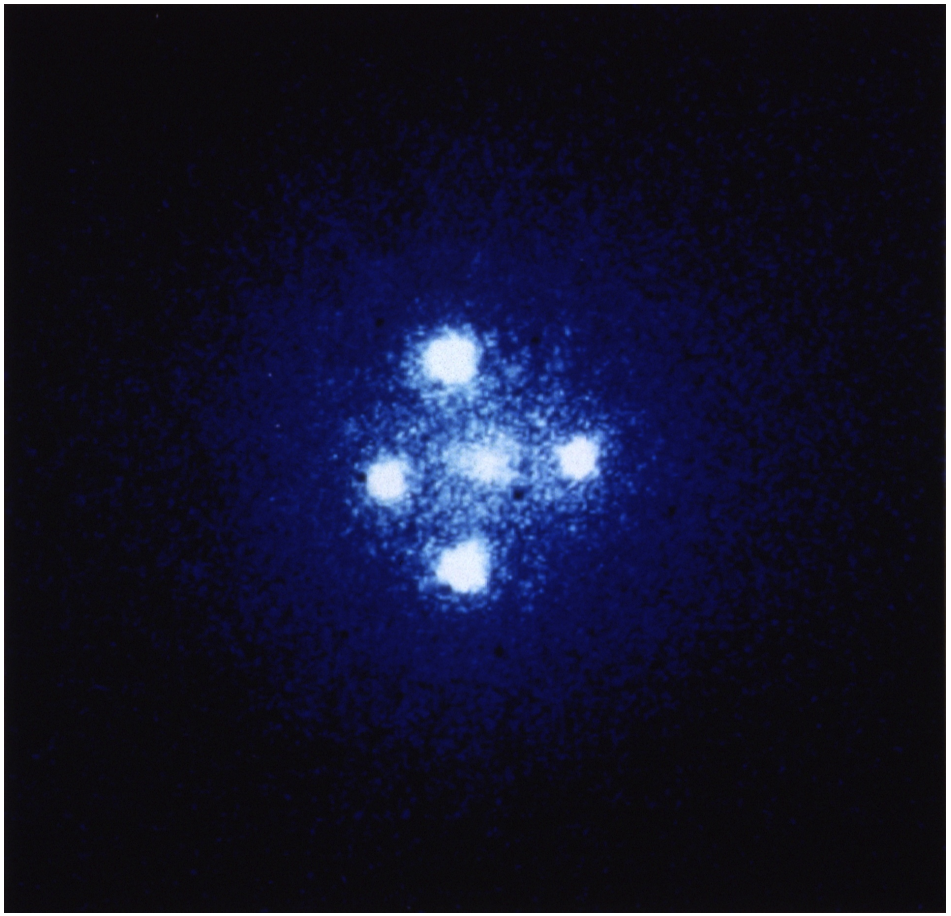


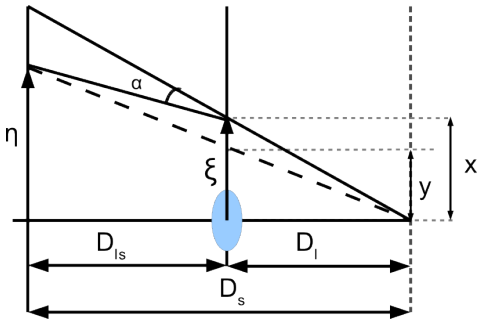
Milky Way: Virgo infall

Gravitational lensing in GR: $\alpha = 4GM/(c^2 b)$



Einstein Cross





$$\vec{\eta} = \frac{D_s}{D_l}\vec{\xi} - D_{ls}\,\vec{\alpha}\left(\vec{\xi}\right)$$

common lens with specific refraction coefficient

$$\vec{\alpha}\left(\vec{\xi}\right) = \frac{4G}{c}\int \frac{\vec{\xi}-\vec{\xi}'}{\left|\xi-\vec{\xi}'\right|^2}\, d^2\xi' \int \rho\left(\vec{\xi}', z\right) dz$$

source: quasar $D_s = 2.4$ Gpc

lens: galaxy $D_l = 120$ Mpc

Dark Matter in clusters

gravitational lensing $\rho_B \approx 0.25\rho_{DM}$

