Higher-Spin Gauge Theory in One Hour

M.A.Vasiliev

Lebedev Institute, Moscow

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Symmetries

Usual lower-spin symmetries:

- **Relativistic theories: Poincaré and** $(A)dS$ **symmetry:** P_a **and** M_{ab}

- **SUSY:** $P_a, M_{ab} \longrightarrow P_a, M_{ab}, \mathrm{Q}_\alpha\,, \qquad \alpha = 1,2,3,4$

- **Inner symmetries:** $[T_i\,, (P_a, M_{ab})] = 0$

- **Conformal (super)symmetries:** $P_a, M_{ab} \longrightarrow P_a, M_{ab}, D, K^a$

- **Is it possible to go to higher HS symmetries?**

$P_a, M_{ab} \longrightarrow P_a, M_{ab}, P_{a_1 \ldots a_k} \ldots$

HS gauge theory: theory of maximal symmetries

Fronsdal Fields

All $m = 0$ HS fields are gauge fields **C. Fronsdal 1978**

$\varphi_{a_1 \dots a_s}$ is a rank s symmetric tensor obeying $\varphi^{c}{}_{c}{}^{b}{}_{b a_5 \dots a_s} = 0$

Gauge transformation:

$$\delta \varphi_{a_1 \dots a_s} = \partial_{(a_1} \varepsilon_{a_2 \dots a_s)} \,, \qquad \varepsilon^{b}{}_{b a_3 \dots a_{s-1}} = 0$$

Field equations: $G_{a_1 \dots a_s}(x) = 0$ $\quad G_{a_1 \dots a_s}(x)$ **: Ricci-like tensor**

$$G_{a_1 \dots a_s}(x) = \Box \varphi_{a_1 \dots a_s}(x) - s \partial_{(a_1} \partial^{b} \varphi_{a_2 \dots a_s b)}(x) + \frac{s(s-1)}{2} \partial_{(a_1} \partial_{a_2} \varphi^{b}{}_{a_3 \dots a_s b)}(x)$$

Action

$$S = \int_{M^d} \left(\frac{1}{2} \varphi^{a_1 \dots a_s} G_{a_1 \dots a_s}(\varphi) - \frac{1}{8} s(s-1) \varphi_{b}{}^{b a_3 \dots a_s} G^{c}{}_{c a_3 \dots a_s}(\varphi) \right)$$

No-go and the Role of $(A)dS$

In 60th it was argued (Weinberg, Coleman-Mandula) that HS symmetries cannot be realized in a nontrivial local field theory in Minkowski space

In 70th it was shown by Aragone and Deser that HS gauge symmetries are incompatible with GR if expanding around Minkowski space

Green light: AdS **background with** $\Lambda \neq 0$ Fradkin, MV, 1987

In agreement with no-go statements the limit $\Lambda \to 0$ **is singular**

HS Holography

AdS_4 HS theory is dual to $3d$ vectorial conformal models

Klebanov-Polyakov (2002), Sezgin–Sundell (2002); Giombi and Yin (2009)

HS symmetry in AdS_{d+1}: maximal symmetry of a d-dimensional free conformal field(s)=singletons, usually, scalar and/or spinor.

Symmetries of KG equation in Minkowski space

Shaynkman, MV 2001 $3d$; Shapovalov, Shirokov 1992, Eastwood 2002 $\forall d$

Construction simplifies at $d = 3$ within spinor formalism

$3d$ Lorentz algebra: $o(2,1) \sim sp(2,R) \sim sl_2(R)$.

Unfolded massless equations of the form

$$\left(\frac{\partial}{\partial x^{\alpha\beta}} + \frac{\partial^2}{\partial y^\alpha \partial y^\beta}\right) C(y|x) = 0\,, \qquad C(y|x) = \sum_{n=0}^{\infty} C^{\alpha_1 \dots \alpha_{2n}}(x) y_{\alpha_1} \dots y_{\alpha_{2n}}$$

are invariant under $\delta C(y|x) = \epsilon(y, \frac{\partial}{\partial y}|x) C(y|x)$

$$\epsilon(y, \frac{\partial}{\partial y}|x) = \exp\left[-x^{\alpha\beta}\frac{\partial^2}{\partial y^\alpha \partial y^\beta}\right] \epsilon_{gl}(y, \frac{\partial}{\partial y}) \exp\left[x^{\alpha\beta}\frac{\partial^2}{\partial y^\alpha \partial y^\beta}\right]$$

$\epsilon_{gl}(y, \frac{\partial}{\partial y})$ describes global HS transformations

NonAbelian HS Algebra

$3d$ Conformal HS symmetry = AdS_4 HS symmetry

HS gauge fields: $\omega(Y|x)$ **1986**

$Y_A = (y_\alpha, \bar{y}_{\dot\alpha})$, $\alpha, \dot\alpha = 1, 2$ **two-component spinor indices**

$$\omega(Y|x) = \sum_{n,m=0}^{\infty} \frac{1}{2n!m!} \omega_{\alpha_1 \dots \alpha_n, \dot\alpha_1 \dots \dot\alpha_m}(x) y^{\alpha_1} \dots y^{\alpha_n} \bar{y}^{\dot\alpha_1} \dots \bar{y}^{\dot\alpha_m}$$

HS curvature and gauge transformation

$$R(Y|x) = d\omega(Y|x) + \omega(Y|x) * \wedge \omega(Y|x)$$

$$\delta\omega(Y|x) = D\epsilon(Y|x) = d\epsilon(Y|x) + [\omega(Y|x)\,, \epsilon(Y|x)]_*$$

$$[y_\alpha\,, y_\beta]_* = 2i\varepsilon_{\alpha\beta}\,, \qquad [\bar{y}_{\dot\alpha}\,, \bar{y}_{\dot\beta}]_* = 2i\varepsilon_{\dot\alpha\dot\beta}$$

Star product is nonlocal in Y^A !

$$(f * g)(Y) = f(Y) \exp\,[i \overleftarrow{\partial_A} \overrightarrow{\partial_B} C^{AB}] g(Y)$$

Global symmetry of bosonic HS theory **Fradkin, MV 1986, MV 1988**

Properties of HS Algebras

Let T_s be a homogeneous polynomial of degree $2(s-1)$

$$[T_{s_1}\,, T_{s_2}] = T_{s_1+s_2-2} + T_{s_1+s_2-4} + \cdots + T_{|s_1-s_2|+2}\,.$$

Once spin $s > 2$ appears, the HS algebra contains an infinite tower of higher spins: $[T_s, T_s]$ gives rise to T_{2s-2} as well as T_2 of $o(3,2) \sim sp(4)$.

Usual symmetries: spin-$s \leq 2$ $u(1) \oplus o(3,2)$: maximal finite-dimensional subalgebra of $hu(1,0|4)$. $u(1)$ is associated with the unit element.

HS symmetries do not commute with space-time symmetries

$$[T^a\,, T^{HS}] = T^{HS}\,, \qquad [T^{ab}\,, T^{HS}] = T^{HS}$$

HS transformations map gravitational fields (metric) to HS fields:

Riemann geometry is not appropriate for HS theory:

concept of local event may become illusive!

HS Gauge Theory and Quantum Gravity

HS symmetry is in a certain sense maximal relativistic symmetry. Hence, it cannot result from spontaneous breakdown of a larger symmetry: HS symmetries are manifest at ultrahigh energies above any scale including Planck scale

- **HS gauge theory should capture effects of Quantum Gravity: restrictive HS symmetry versus unavailable experimental tests**
- **Lower-spin theories as low-energy limits of HS theory: lower-spin symmetries: subalgebras of HS symmetry**
- **String Theory as spontaneously broken HS theory?!** $(s > 2, m > 0)$

Space-Time and Spin

Space-time M **is where symmetry** $G = O(d-1,2)$ **acts**

Spin s**: different** G**-modules** V_s **where fields** $\phi^A(x)$ **are valued.**

V_s **contain ground (primary) fields** $\phi^A(x)$ **along with their derivatives** $\partial_{n_1} \dots \partial_{n_k} \phi^A(x)$ **(descendants)**

HS vertices contain higher derivatives Bengtsson, Bengtsson, Brink (1983), Berends, Burgers and H. Van Dam (1984), (1985), Fradkin, MV; Metsaev,...

HS symmetries Fradkin, MV 1986 **are infinite dimensional extesions of** G

Infinite towers of spins $\Rightarrow$ **infinite towers of derivatives.**

How (non)local is HS gauge theory?

HS Multiplets

Infinite set of spins $s = 0, 1/2, 1, 3/2, 2 \ldots$

$\omega_{\alpha_1 \ldots \alpha_n, \dot\beta_1 \ldots \dot\beta_m}$ **and** $C_{\alpha_1 \ldots \alpha_n, \dot\beta_1 \ldots \dot\beta_m}$ **with all** $n \geq 0$ **and** $m \geq 0$.

Generating functions $\omega(Y|x)$ **and** $C(Y|x)$: **unrestricted functions of commuting spinor variables** $Y = (y_\alpha, \bar y_{\dot\alpha})$

$$A(Y|x) = \sum_{n,m=0}^{\infty} \frac{1}{2n!m!} A_{\alpha_1 \ldots \alpha_n, \dot\alpha_1 \ldots \dot\alpha_m}(x) y^{\alpha_1} \ldots y^{\alpha_n} \bar y^{\dot\alpha_1} \ldots \bar y^{\dot\alpha_m}$$

Gauge one-forms $\omega_{\alpha_1 \ldots \alpha_n, \dot\beta_1 \ldots \dot\beta_m}$, $n + m = 2(s-1)$

$s = 1:$ $\omega(x) = dx^\nu \omega_\nu(x)$

$s = 2:$ $\omega_{\alpha\dot\beta}(x)$, $\omega_{\alpha\beta}(x)$, $\bar\omega_{\dot\alpha\dot\beta}(x)$

$s = 3/2:$ $\omega_\alpha(x)$, $\bar\omega_{\dot\alpha}(x)$

Frame-like fields: $|n-m| = 0$ **(bosons) or** $|n-m| = 1$ **fermions**

Auxiliary Lorentz-like fields: $|n-m| = 2$ **(bosons)**

Extra fields: $|n-m| > 2$

Zero-forms $C(Y|x)$: **matter fields and higher derivatives of massless fields**

Free Field Unfolded Massless Equations

The full unfolded system for free massless bosonic fields is 1989

$$\star \qquad R_1(y,\bar y\,|\,x) = \frac{i}{4}\left(\eta \overline{H}^{\dot\alpha\dot\beta}\frac{\partial^2}{\partial\bar y^{\dot\alpha}\partial\bar y^{\dot\beta}}\, C(0,\bar y\,|\,x) + \bar\eta H^{\alpha\beta}\frac{\partial^2}{\partial y^{\alpha}\partial y^{\beta}}\, C(y,0\,|\,x)\right)$$

$$\star\star \qquad \tilde{\mathrm{D}}_0 C(y,\bar y\,|\,x) = 0$$

$$R_1(y,\bar y\,|\,x) := D_0^{ad}\omega(y,\bar y\,|\,x) \qquad D_0^{ad} := D^L - e^{\alpha\dot\beta}\left(y_\alpha\frac{\partial}{\partial\bar y^{\dot\beta}} + \frac{\partial}{\partial y^\alpha}\bar y_{\dot\beta}\right)$$

$$\tilde{\mathrm{D}}_0 = D^L + e^{\alpha\dot\beta}\left(y_\alpha\bar y_{\dot\beta} + \frac{\partial^2}{\partial \mathbf{y}^\alpha\partial\bar{\mathbf{y}}^{\dot\beta}}\right) \qquad D^L := \mathrm{d}_x - \left(\omega^{\alpha\beta}y_\alpha\frac{\partial}{\partial y^\beta} + \bar\omega^{\dot\alpha\dot\beta}\bar y_{\dot\alpha}\frac{\partial}{\partial\bar y^{\dot\beta}}\right)$$

$$H^{\alpha\beta} := e^{\alpha}{}_{\dot\alpha}e^{\beta\dot\alpha}\,, \qquad \overline{H}^{\dot\alpha\dot\beta} := e_\alpha{}^{\dot\alpha}e^{\alpha\dot\beta}$$

$\star\star$ implies that higher-order terms in y and $\bar y$ describe higher-derivative descendants of the primary HS fields

Zero-Form Sector

Equations on the gauge invariant zero-forms C

$$C(Y;K|x)=\sum_{n,m=0}^{\infty}\frac{1}{2n!m!}C_{\alpha_1\dots\alpha_n,\dot\alpha_1\dots\dot\alpha_m}(x)y^{\alpha_1}\dots y^{\alpha_n}\bar y^{\dot\alpha_1}\dots\bar y^{\dot\alpha_m}$$

decompose into independent subsystems associated with different spins

Spin-s zero-forms are $C_{\alpha_1\dots\alpha_n,\dot\alpha_1\dots\dot\alpha_m}(x)$ with

$$n-m=\pm 2s$$

Perturbative unfolded equations

$$\mathrm{d}_x C=\sigma_- C+\text{lower-derivative and nonlinear terms}$$

$$\sigma_-:=\mathrm{e}^{\alpha\dot\beta}\frac{\partial^2}{\partial \mathrm{y}^\alpha\partial\bar{\mathrm{y}}^{\dot\beta}},\qquad \sigma_-^2=0$$

$C_{\alpha_1\dots\alpha_n,\dot\alpha_1\dots\dot\alpha_m}(x)$ contain $\frac{n+m}{2}-\{s\}$ space-time derivatives of the spin-s dynamical fields. Presence of zero-forms C in the HS vertices may induce infinite towers of derivatives and, hence, non-locality.

HS Vertices

Diffeomorphisms without Riemannian geometry: Cartan formalism of differential forms with field equations in the unfolded form

$$\mathrm{d}_x\omega = -\omega * \omega + \Upsilon(\omega,\omega,C) + \Upsilon(\omega,\omega,C,C) + \dots\,,$$

$$\mathrm{d}_x C = -[\omega, C]_* + \Upsilon(\omega, C, C) + \dots$$

The problem: consistent non-linear corrections 1988 in the local frame

The vertices can be put into the form

$$\Upsilon(\Phi,\Phi,\dots) = F(Q^i, P^{nm}; \bar{Q}^j, \bar{P}^{kl})\Phi(Y_1)\dots\Phi(Y_n)|_{Y_i=0}$$

with $\Phi = \omega,\ C$ and some non-polynomial functions $F(Q^i, P^{nm}; \bar{Q}^j, \bar{P}^{kl})$ of the Lorentz-covariant combinations

$$Q^i := y^\alpha \frac{\partial}{\partial y_i{}^\alpha}\,, \qquad P^{ij} := \frac{\partial}{\partial y_i^\alpha}\frac{\partial}{\partial y_{j\alpha}}\,, \qquad \bar{Q}^i := \bar{y}^{\dot\alpha}\frac{\partial}{\partial \bar{y}_i{}^{\dot\alpha}}\,, \qquad \bar{P}^{ij} := \frac{\partial}{\partial \bar{y}_i{}^{\dot\alpha}}\frac{\partial}{\partial \bar{y}_{j\dot\alpha}}$$

The fundamental problem: find a proper class of functions $F(Q^i, P^{nm}; \bar{Q}^j, \bar{P}^{kl})$ guaranteeing spin-locality (minimal non-locality) of the HS theory

Unfolded Dynamics

First-order form of differential equations

$$\dot{q}^i(t) = \varphi^i(q(t)) \qquad \text{initial values:} \quad q^i(t_0)$$

 # degrees of freedom = # of dynamical variables

Unfolded dynamics: multidimensional generalization

$$\frac{\partial}{\partial t} \to d\,, \qquad q^i(t) \to W^\Omega(x) = dx^{n_1} \wedge \ldots \wedge dx^{n_p} W^\Omega_{n_1 \ldots n_p}(x)$$

$$dW^\Omega(x) = G^\Omega(W(x))\,, \qquad d = dx^n \partial_n$$

$G^\Omega(W)$: **function of "supercoordinates"** W^Ω

$$G^\Omega(W) = \sum_{n=1}^{\infty} f^\Omega{}_{\Lambda_1 \ldots \Lambda_n} W^{\Lambda_1} \wedge \ldots \wedge W^{\Lambda_n}$$

Covariant first-order differential equations

$d > 1$: **Compatibility conditions**

$$G^\Lambda(W) \wedge \frac{\partial G^\Omega(W)}{\partial W^\Lambda} \equiv 0$$

Properties

- General applicability
- Manifest (HS) gauge invariance under the gauge transformation

$$\delta W^\Omega = d\varepsilon^\Omega + \varepsilon^\wedge \frac{\partial G^\Omega(W)}{\partial W^\wedge},$$

  gauge parameter $\varepsilon^\Omega(x)$ is a $(p_\Omega - 1)$-form.
- Invariance under diffeomorphisms

  Exterior algebra formalism
- Interactions: nonlinear deformation of $G^\Omega(W)$
- Degrees of freedom are in 0-forms $C^i(x_0)$ at any $x = x_0$ (as $q(t_0)$)

  infinite-dimensional module dual to the space of single-particle states

realized as a space of functions of auxiliary variables (like $C(y, \bar{y})$ instead of phase space coordinates in the Hamiltonian approach

Unfolded dynamics provides a tool to control unitarity in presence of higher derivatives

Space-Time Metamorphoses

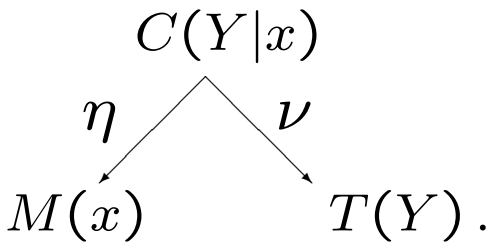
Independence of ambient space-time: geometry is encoded by $G^\Omega(W)$

Key observation: unfolded equation makes sense in any space-time

$$dW^\Omega(x) = G^\Omega(W(x))\,, \quad x \to X = (x,y)\,, \quad d_x \to = d_X = d_x + d_y\,, \quad d_y = dy^u \frac{\partial}{\partial y^u}$$

X-dependence is reconstructed in terms of fields $W^\Omega(X_0) = W^\Omega(x_0, y_0)$ at any X_0 To take $W^\Omega(x_0, y_0)$ in space M_X with coordinates X_0 is the same as to take $W^\Omega(x_0)$ in the space $M_x \in M_X$ with coordinates x

Unfolding as a Covariant Twistor Transform



MV 1404.1948
N.Misuna 2404.13212

$W^{\Omega}(Y|x)$ are functions on the "correspondence space" C.

Space-time M : coordinates x. Twistor space T : coordinates Y.

Unfolded equations: Penrose transform mapping functions on T to solutions of field equations in M.

Holographic duality: different space-times M for the same twistor space

Locality and Non-Locality

arXiv: 2208.02004

Equations of motion in perturbatively local field theory $E_{A_0,s_0}(\partial,\phi)=0$

$$E_{A_0,s_0}(\partial,\phi)=\sum_{k=0,l=1}^{\infty} a^{n_1\dots n_k}_{A_0\,A_1\dots A_l}(s_0,s_1,\dots,s_l)\partial_{n_1}\dots\partial_{n_k}\phi^{A_1}_{s_1}\dots\phi^{A_l}_{s_l}$$

have a finite # of non-zero coefficients $a^{n_1\dots n_k}_{A_0\dots A_l}$ **at any order** l.

s_0 **is the spin of the field on which the linearized equation is imposed**

HS theory involves infinite towers of Fronsdal fields of all spins.

$a^{n_1\dots n_k}_{A_0\dots A_l}$ **may take an infinite # of values.**

It makes sense to distinguish between Gelfond, MV 2018

local: finite number of derivatives at any order

$$a^{n_1\dots n_k}_{A_0\dots A_l}(s_0,s_1,\dots s_l)=0 \quad \text{at } k>k_{max}(l)$$

spin-local: finite number of derivatives for any finite subset of fields

$$a^{n_1\dots n_k}_{A_0\dots A_l}(s_0,s_1,s_2,\dots s_l)=0 \quad \text{at } k>k_{max}(s_0,s_1,s_2,\dots s_l)$$

non-local: infinite number of derivatives for a finite subset of fields at some order.

Compact Spin-Locality

The simplest option: replacement of the class of local field theories with the finite # of fields by spin-local models with infinite # of fields. Spin-local-compact vertices in addition obey

$$a^{n_1\dots n_k}_{A_0\,A_1\dots A_l}(s_0, s_1, \dots, s_k + t_k, \dots, s_l) = 0 \quad t_k > t_k^0 \quad \forall k$$

non-compact otherwise.

Compactness is in the space of spins, not in space-time

Both types of vertices in HS theory:

Cubic HS vertices $\omega * \omega$ built from HS gauge potentials are spin-local-compact: spins s_0, s_1, s_2 obey the triangle inequalities $s_0 \leq s_1 + s_2$ etc.

Vertices associated with the conserved currents built from gauge invariant field strength are spin-local non-compact. These include conserved currents of any integer s_0 built from two spin-zero fields ($s_1 = s_2 = 0$).

Nonlinear System via Doubling of Spinors

How to find nonlinear corrections to HS equations? The efficient trick

MV 1992 reduces the problem to De Rham cohomology with respect to additional spinor variables $Z^A = (z^\alpha, \bar z^{\dot\alpha})$ in presence of Klein operators K

$$\omega(Y;K|x) \longrightarrow W(Z;Y;K|x)\,, \qquad C(Y;K|x) \longrightarrow B(Z;Y;K|x)\,, \quad Y^A = (y^\alpha, \bar y^{\dot\alpha})$$

Some of the nonlinear HS equations

$$\begin{cases} \mathrm{d}_x W + W\star W = 0 \\ \mathrm{d}_x B + W\star B - B\star W = 0 \\ \mathrm{d}_x S + W\star S + S\star W = 0 \qquad\qquad \textbf{1992} \\ \mathrm{S}\star\mathrm{B} - \mathrm{B}\star\mathrm{S} = 0 \\ \mathrm{S}\star\mathrm{S} = \mathrm{i}(\theta^{\mathrm{A}}\theta_{\mathrm{A}} + \eta\theta^\alpha\theta_\alpha \mathrm{B}\star\mathrm{k}\star\kappa + \bar\eta\bar\theta^{\dot\alpha}\bar\theta_{\dot\alpha}\mathrm{B}\star\mathrm{k}\star\bar\kappa) \end{cases}$$

determine Z_A-dependence in terms of "initial data" $\omega(Y;K|x)$ and $C(Y;K|x$

$S(Z;Y;K|x) = \theta^A S_A(Z;Y;K|x)$ is a connection along Z^A ($\theta^A \equiv dZ^A$)

Klein operators $K = (k, \bar k)$ generate chirality automorphisms

$$kf(A) = f(\tilde A)k\,, \qquad A = (a_\alpha\,, \bar a_{\dot\alpha}) : \qquad \tilde A = (-a_\alpha\,, \bar a_{\dot\alpha})$$

Inner Klein operators: $\kappa = \exp iz_\alpha y^\alpha\,, \bar\kappa = \exp i\bar z_{\dot\alpha}\bar y^{\dot\alpha}\,, \kappa\star f = \widetilde{f}\star\kappa\,, \qquad \kappa\star\kappa = 1$

Dynamical content is in the d-independent twistor sector

Perturbative Analysis

Vacuum solution

$$B_0 = 0\,, \qquad S_0 = \theta^A Z_A\,, \qquad W_0 = \frac{1}{2} w^{AB}(x) Y_A Y_B$$

$$\mathrm{d}_x W_0 + W_0 \star W_0 = 0\,, \qquad w^{AB} : AdS_4$$

$$[\mathrm{S_0}\,, \mathrm{f}]_\star = -2\mathrm{i}\mathrm{d}_\mathrm{Z}\mathrm{f}\,, \qquad \mathrm{d}_\mathrm{Z} = \theta^\mathrm{A} \frac{\partial}{\partial \mathrm{Z}^\mathrm{A}}$$

First-order fluctuations

$$B_1 = C(Y)\,, \qquad S = S_0 + S_1\,, \qquad W = W_0(Y) + W_1(Y) + W_0(Y) C(Y)$$

Order-n equations containing S have the form

$$\mathrm{d}_Z U_n(Z;Y|dZ) = V[U_{<n}](Z;Y|\theta) \qquad \mathrm{d}_Z V[U_{<n}](Z;Y|\theta) = 0$$

can be solved by shifted homotopy with shift parameters Q

$$U_n(Z;Y|\theta) = \mathrm{d}_Z^* V[U_{<n}](Z;Y|\theta) + \mathrm{h(Y)} + \mathrm{d}_Z \epsilon(Z;Y|\theta)$$

$$\mathrm{d}_Z^* V(Z;Y|\theta) = (Z^A - Q^A) \frac{\partial}{\partial \theta^A} \int_0^1 \frac{dt}{t} V(tZ + (1-t)Q; Y|t\theta)$$

Interpretation

The contracting homotopy freedom encodes:

All possible gauge choices in d_z-exact forms $\mathrm{d}_z\epsilon(Z;Y|dZ)$

All possible choices of field variables in d_z cohomology $\mathrm{h}(\mathrm{Y})$

Any unfolded HS system is associated with one or another solution to the nonlinear HS system.

How to single out the proper (e.g., minimally nonlocal) frames?

Spin-local limit: $\beta \to -\infty$ with $Q_A = \beta \frac{\partial}{\partial Y^A}$

Didenko, Gelfond, Korybut, MV 1909.04876

Projectively compact vertex was obtained by hand 1605.02662 but so far has not been reached by a systematic homotopy method

Differential Homotopy

Homotopy and shift parameters t^a are treated as coordinates of some manifold $\mathcal{M}$ with the total differential

$$\mathsf{d} := \mathsf{d}_Z + \mathsf{d}_t\,, \qquad \mathsf{d}_Z := \theta^A \frac{\partial}{\partial Z^A}\,, \qquad \mathsf{d}_t := dt^a \frac{\partial}{\partial t^a}\,,$$

Equations to be solved at every perturbation order still have the form

$$\mathsf{d} f(Z,t,\theta,dt) = g(Z,t,\theta,dt)\,, \qquad \mathsf{d} g(Z,t,\theta,dt) = 0\,.$$

Fundamental Ansatz

Lower-order computation yields expressions of the remarkable form

$$f_\mu = \int_{p_i^2 r_i^2 u^2 v^2 \tau\sigma\beta\rho} \mu(\tau,\sigma,\beta,\rho,u,v,p,r)\mathrm{d}\Omega^2 \mathcal{E}(\Omega) G(g(r))\,,$$

where $\mu(\tau,\sigma,\beta,\rho,\ldots)$ **is demanded to have compact support in** τ,σ,β,ρ

$$\mathrm{d}\Omega^2 := \mathrm{d}\Omega^\alpha \mathrm{d}\Omega_\alpha$$

$$\Omega_\alpha(\tau,t) := \tau z_\alpha - (1-\tau)(p_\alpha(\sigma) - \beta v_\alpha + \rho(y_\alpha + p_{+\alpha} + u_\alpha))$$

$$p_{+\alpha} := \sum_{i=1}^{l} p_{i\alpha}\,, \qquad p_\alpha(\sigma) = \sum_{i=1}^{k} p_{i\alpha}\sigma_i$$

$$\mathcal{E}(\Omega) := \exp i\Big(\Omega_\beta(y^\beta + p_+^\beta + u^\beta) + u_\alpha v^\alpha - \sum_{i=1}^{l} p_{i\alpha} r_i^\alpha - \sum_{k\geq j>i\geq 1} p_{i\beta} p_j^\beta\Big)\,,$$

$$G_l(g) := g_1(r_1)\ldots g_l(r_l)k\,,$$

$g_i(y)$ **are some functions of** y_α **(e.g.,** $C(y)$ **or** $\omega(y)$**).**

Homology Map

Fundamental Ansatz has the remarkable property that

$$\mathrm{d}\Big(\mathrm{d}\Omega^2 \exp i\Big(\Omega_\beta(y^\beta + p_+^\beta + u^\beta) + u_\alpha v^\alpha - \sum_{i=1}^{l} p_{i\alpha} r_i^\alpha - \sum_{k\geq j>i\geq 1} p_{i\beta} p_j^\beta\Big)\Big) = 0$$

$(\mathrm{d}\Omega)^3 = 0$ **since the one-forms** $\mathrm{d}\Omega^\alpha$ **are anticommuting and** $\alpha = 1, 2$.

As a result d **effectively acts only on** μ

$$\mathrm{d} f_\mu = f_{\mathrm{d}\mu}\,,$$

mapping homological problem in terms of spinors Z^A **to that on** $\mu(t_i)$.

Great advantage of this formalism is that there is no more need to use the Schouten identity: the only formula where it manifests itself is the homology map.

The problem takes purely geometric form on $\mathcal{M}$

$$\mathrm{d}\mu_f \cong \mu_g$$

That $\mathrm{d}\mu_g \cong 0$ **implies** $\mu_g \cong \mathrm{d}h_g$ **allowing to set**

$$\mu_f = h_g$$

Projectively-Compact Spin-Local Vertex

Final result

$$\mathcal{D}C = J^{\eta}_{pc} + J^{\bar{\eta}}_{pc}$$

$$J^{\eta}_{pc} = \frac{i\eta}{4}\int_{u^2v^2\tau\rho\beta\sigma_1\sigma_2} d^2u d^2v \mathbb{D}(\tau)\mathbb{D}(\sigma_1+\rho)\mathbb{D}(\sigma_2-(1-\beta-\rho))\vartheta(-\sigma_1)\vartheta(\sigma_2)\mathrm{d}\Omega^2\tilde{\mu}(\beta)\mathcal{E}(\Omega)$$
$$\Big[P(\beta-1,\sigma_\omega,\sigma_1,\sigma_2,1-\beta)\omega(r_\omega,\bar{y};K)\bar{*}C(r_1,\bar{y};K)\bar{*}C(r_2,\bar{y};K)$$
$$+P(\beta-1,\sigma_1,\sigma_\omega,\sigma_2,1-\beta)C(r_1,\bar{y};K)\bar{*}\omega(r_\omega,\bar{y};K)\bar{*}C(r_2,\bar{y};K)$$
$$+P(\beta-1,\sigma_1,\sigma_2,\sigma_\omega,1-\beta)C(r_1,\bar{y};K)\bar{*}C(r_2,\bar{y};K)\bar{*}\omega(r_\omega,\bar{y};K)\Big]\Big|_{r_{\omega,C_i}=0}^{k},$$

$$P_k(\sigma_l,\sigma_{l+1},\ldots,\sigma_{l+k}) := \vartheta(\sigma_{l+k}-\sigma_{l+k-1})\ldots\vartheta(\sigma_{l+1}-\sigma_l), \qquad \mathbb{D}(a) := da\delta(a)$$

J^{η}_{pc} **is both spin-local and projectively-compact containing an overall factor of** $y^{\alpha}p_{\omega\alpha}$. **To reach projective compactness at higher orders the prexponent factors have to be of the form**

$$\prod_i \mathbb{D}(\sigma_{C_i}+\rho+\ldots)$$

Self-Dual HS Theory

The dynamically nontrivial equation

$$\mathrm{S}\star\mathrm{S}=\mathrm{i}(\theta^{\mathrm{A}}\theta_{\mathrm{A}}+\eta\theta^{\alpha}\theta_{\alpha}\mathrm{B}\star\mathrm{k}\star\kappa+\bar{\eta}\bar{\theta}^{\dot{\alpha}}\bar{\theta}_{\dot{\alpha}}\mathrm{B}\star\mathrm{k}\star\bar{\kappa})$$

contains two coupling constants that are complex conjugated in Minkowski signature but real in Euclidean and maximally split signatures

In 1992 the model with $(\eta=0)\bar{\eta}=0$ was identified with (anti)self-dual HS theory.

Lower-order spin-local vertices of the SDHS theory were found

Gelfond, Didenko, Koribut, MV 1909.04876, 2009.02811

An interesting new scheme leading to spin-local SD vertices in all orders was proposed by Didenko 2209.01966

and partly independently by Sharapov and Skvortsov 2209.01796, 2209.1544

Coxeter HS Theory

To extend HS theory towards String theory more copies of oscillators are needed. A naive generalization is inconsistent.

In the Coxeter HS theory all fields W, S, B depend on p sets of variables like Y^n_A, Z^n_A, dZ^A_n $(A = 1, \dots 4,\ n = 1, \dots p)$, idempotents I_n and Klein operators $\widehat{K}_v$ associated with all root vectors of $\mathcal{C}$

$$K_v q^n_\alpha = R_v{}^n{}_m q^m_\alpha K_v\,, \qquad R_{v_a} x^i = x^i - 2 v^i_a \frac{(v_a, x)}{(v_a, v_a)}\,, \qquad R^2_{v_a} = I\,.$$

Coxeter HS equations are 2018

$$S * S = -i \left(dZ^{An} dZ_{An} + \sum_i \sum_{v \in \mathcal{R}_i} F_{i*}(B) \frac{dZ^\alpha_n v^n dZ_{\alpha\, m} v^m}{(v, v)} * \widehat{\kappa}_v \right)$$

Further extension is via multiparticle HS algebras $U(hs(...))$ rich enough to generate String spectrum 2013

Conclusion

HS gauge theories contain gravity along with infinite towers of other fields with various spins including ordinary matter fields: singlet scalar!

Infinite-dimensional HS symmetry

HS theories exist in various dimensions.

Unbroken HS symmetries demand AdS background

HS vertices contain higher derivatives.

Customary concepts of Riemann geometry are not applicable: study of exact solutions is very instructive:

BH-like solutions Didenko, MV 2009, Iazeola, Sundell 2010

Other solutions Didenko, Falyahov 2025

One of the central problems is the mechanism of spontaneous breakdown of HS symmetries

Coxeter HS theory as a step to String Theory Tarusov, Ushakov, MV 2025, 2026

The new differential homotopy approach is designed to figure out the actual level of non-locality of the HS theory. It is geometric: cohomology problem on polyhedra in the space $\mathcal{M}$ of homotopy parameters.

MV 2023, Kirakosyants, Valerev, MV 2025

Recent progress in SDHS theory as a sector of HS theory

To do: higher orders and exact solutions in HS theories of various kind

Other directions

Massive HS fields: Zinoviev

Supersymmetric HS theory Buchbinder, Ivanov, Zaigraev; Misuna

Light-cone approach Metsaev, Ponomarev...

BRST approach Buchbinder, Fedoruk, Reshetnyak, ...