On Implementation of Variational Quantum Algorithm on the HybriLIT Quantum Testbed Applied to the Ising Model with a Transverse Field

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MMCP'2026
Dubna, June 22–26, 2026

HybriLIT Quantum Testbed

- is an open educational and research environment deployed on the basis of the "Govorun"supercomputer.
- **Characteristics of the dedicated server:**
  - 2×CPU AMD EPYC 7763 (64 Cores @ 2.45 GHz);
  - 1.94 TB RAM;
  - 8×GPU NVIDIA Tesla A100 SXM4 80 GB HBM2.
- **Advantages of working on the polygon:**
  - two access modes: batch mode and JupyterLab notebooks;
  - user-friendly JupyterLab web interface — no software installation required;
  - pre-configured environments with popular libraries;
  - ability to perform calculations on both CPU and GPU — accelerating computations;
  - visualization of quantum circuits directly in the browser.
- **Simulators:** `Cirq, Qiskit, PennyLane, QuTiP, QuantNN, DWave.`

The scheme used: `Cirq + qsim`

1. Cirq (Google Quantum AI) — a Python library for creating and running quantum circuits on simulators and real quantum processors. Supports up to 30 qubits.
2. qsim (Google Quantum AI) — integrated with Cirq, written in C++, supports vectorization and multithreading. Supports up to 40 qubits.

Variational Quantum Algorithms

VQA (gate based) are hybrid quantum-classical algorithms, which employ a short-depth quantum circuit to efficiently evaluate a cost function depended on the parameters of a quantum gate sequence, and then leverage classical optimizers to minimize this cost function.

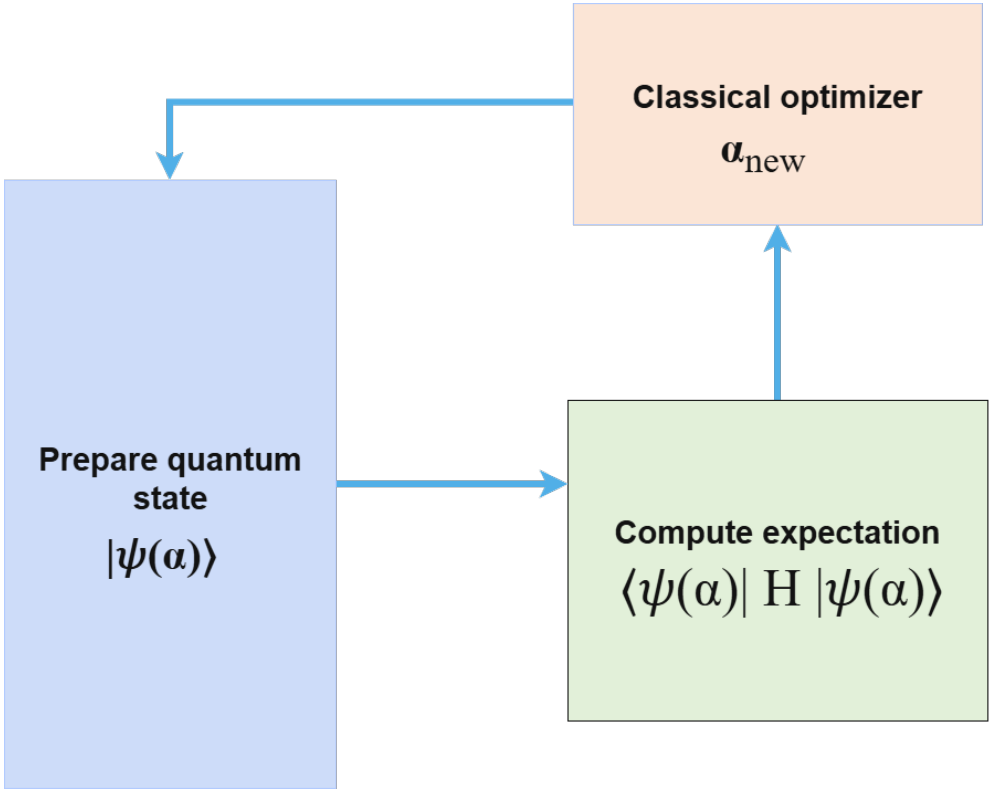
Quantum Variational Principle (Rayleigh–Ritz)

for any parametrized trial wave-function (state vector) $|\psi(\alpha)\rangle$, $\alpha = (\alpha_1, \ldots, \alpha_n)^T$

$$\langle\psi(\alpha)|\,\mathcal{H}\,|\psi(\alpha)\rangle \geq E_{Ground}.$$

The expectation value of the Hamiltonian plays the role of the cost function.

General Scheme of Variational Quantum Algorithms



Computation Loop

Quantum Computer: Prepare $|\psi(\alpha)\rangle$ and measure observables

Classical Computer: Update parameters α to minimize the cost Hamiltonian expectation value

Presentation of lattice spin configuration by a quantum register state

Spin Configuration

A spin configuration is a bitstring $z = z_1 z_2 \dots z_n$, where $z_i = \pm 1$ is the spin orientation at site i.

Mapping to Qubit States

The value of the variable z_i corresponds to the measurement outcome of the Pauli-Z operator on the i-th qubit of the quantum register in the computational basis:

$$|0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \leftrightarrow z = +1 \text{ and } |1\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \leftrightarrow z = -1,$$

i.e. z_i are eigenvalues of $Z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ for the bases $|0\rangle$ and $|1\rangle$.

$$\text{bitstring } z = z_1 z_2 \dots z_n \longleftrightarrow \text{ register state } |z\rangle = |z_1 z_2 \dots z_n\rangle.$$

There are 2^n computational basis states representing all spin configurations on the lattice.

Hamiltonian for Ising model with a transverse field

Ising Hamiltonian with Nearest-Neighbour Interaction

$$\mathcal{H} = -J \sum_{\langle i,j \rangle} Z^{(i)} Z^{(j)} - h \sum_{i} X^{(i)}$$

$Z^{(i)} = \mathbb{I} \otimes \cdots \otimes Z \otimes \cdots \otimes \mathbb{I}$ Pauli operator Z on i-th position acts on i-th qubit

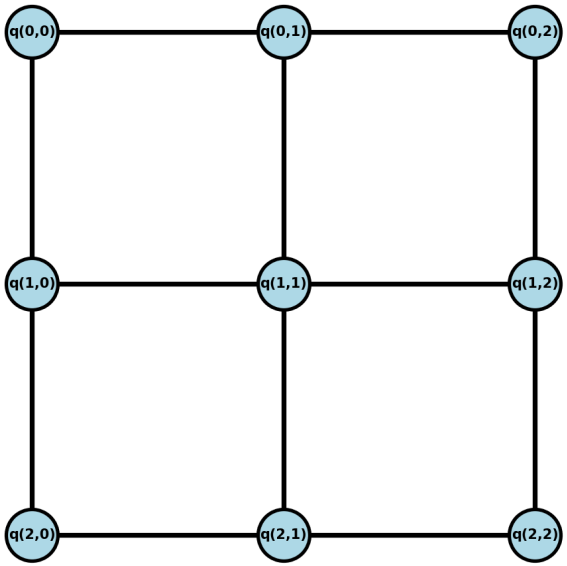
Variational Problem

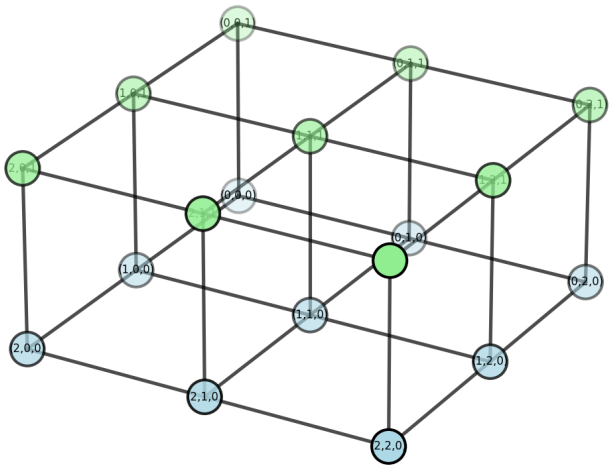
find the spin configuration on the lattice with the lowest expectation value of $H(Z)$, i.e. energy

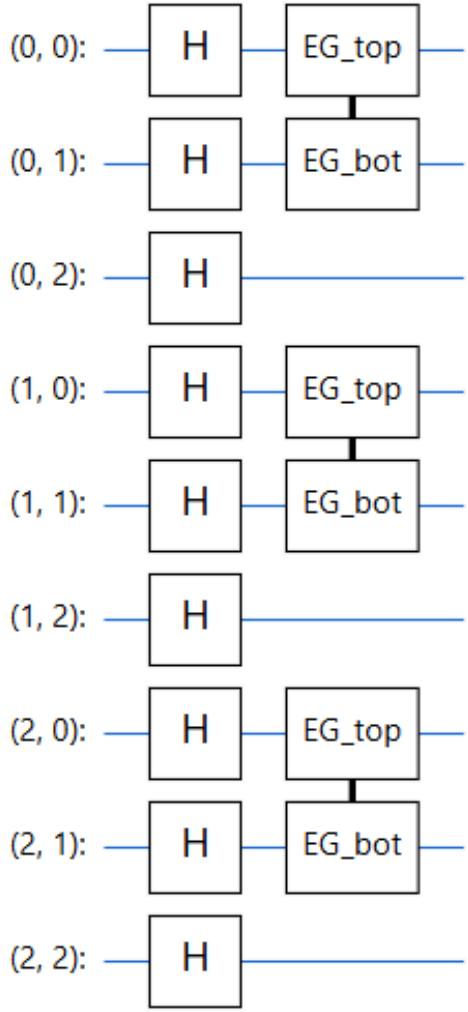
$$E = \langle \psi(\boldsymbol{\alpha}) | \mathcal{H} | \psi(\boldsymbol{\alpha}) \rangle$$

varying the set of parameters α.

Lattice Geometry and Coupling Scheme



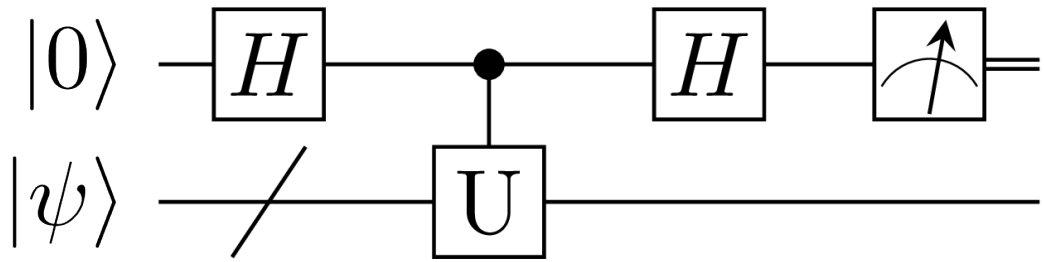




Evaluation of an average $\langle\psi(\boldsymbol{\alpha})|U|\psi(\boldsymbol{\alpha})\rangle$

First variant of measurement: Hadamard Test

$$\mathrm{H}=\frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix},\quad |0\rangle=\begin{bmatrix}1\\0\end{bmatrix},\quad |1\rangle=\begin{bmatrix}0\\1\end{bmatrix}$$



Measurements of the ancilla qubit $|0\rangle$ allow to evaluate the average $\langle\psi|U|\psi\rangle$.

$$P(1)=\frac{1}{2}\langle\psi(\boldsymbol{\alpha})|(\mathbb{I}-Re\,U)|\psi(\boldsymbol{\alpha})\rangle,\qquad P(0)=1-P(1)=\frac{1}{2}\langle\psi(\boldsymbol{\alpha})|(\mathbb{I}+Re\,U)|\psi(\boldsymbol{\alpha})\rangle$$

$$Re\,\langle\psi(\boldsymbol{\alpha})|U|\psi(\boldsymbol{\alpha})\rangle=1-2P(1),\qquad 2Re\,U=U+U^{\dagger}$$

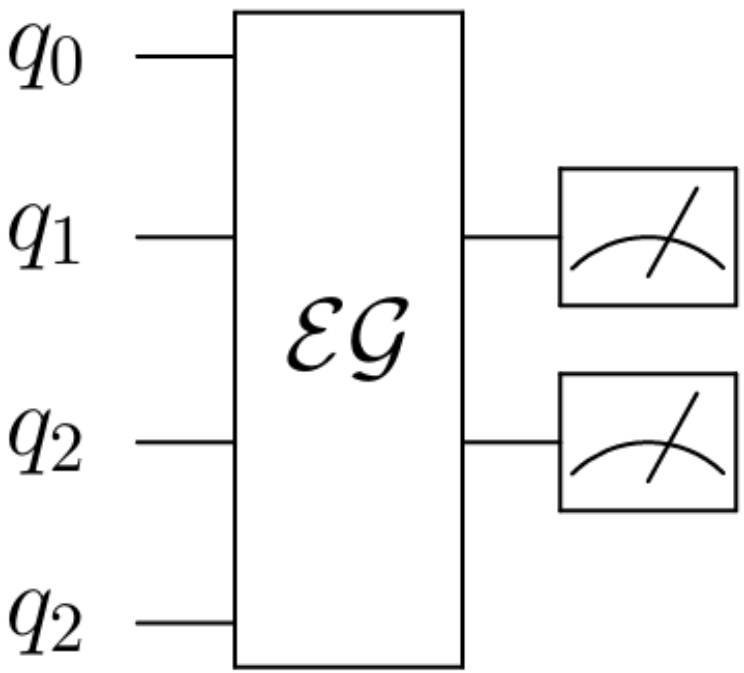
Evaluation an average $\langle\psi(\boldsymbol{\alpha})|U|\psi(\boldsymbol{\alpha})\rangle$ needs measuring only the ancilla qubit.

Evaluation of an average $\langle\psi(\boldsymbol{\alpha})|U|\psi(\boldsymbol{\alpha})\rangle$

Second variant of measurement: without ancilla qubit
Measurement of the Z-Z term

$$E_{Z_1Z_2} = -J\langle\bar{\psi}_1\bar{\psi}_2|(Z_1 \otimes Z_2)|\psi_1\psi_2\rangle$$

$$E_{Z_1Z_2} = -J\Big[P(p_1(0),p_2(0))+P(p_1(1),p_2(1))-\big(P(p_1(0),p_2(1))+P(p_1(1),p_2(0))\big)\Big]$$



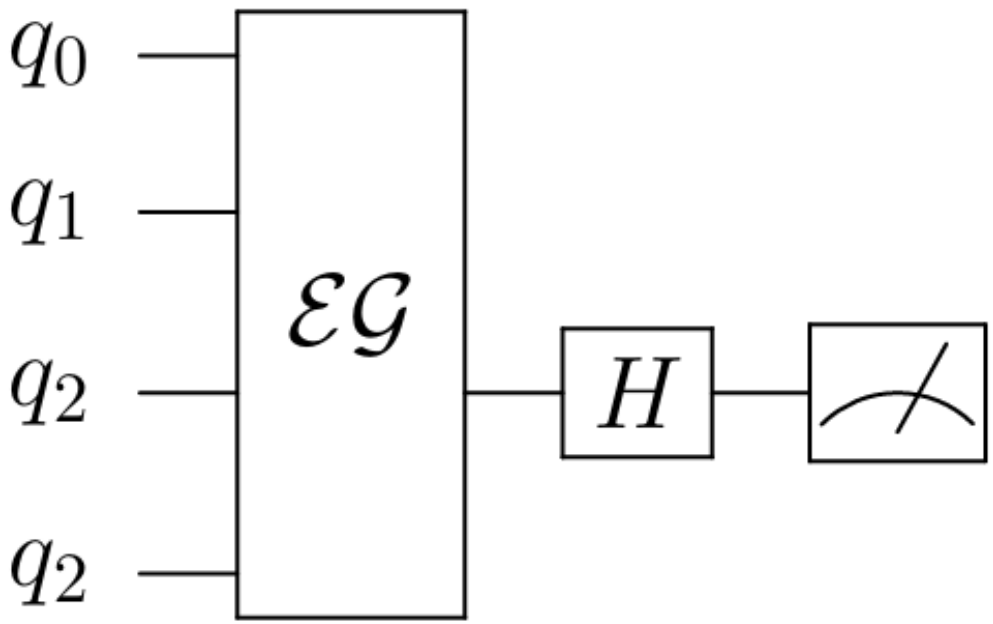
Evaluation of an average $\langle\psi(\boldsymbol{\alpha})|U|\psi(\boldsymbol{\alpha})\rangle$

Second variant of measurement: without ancilla qubit
Measurement of the X term

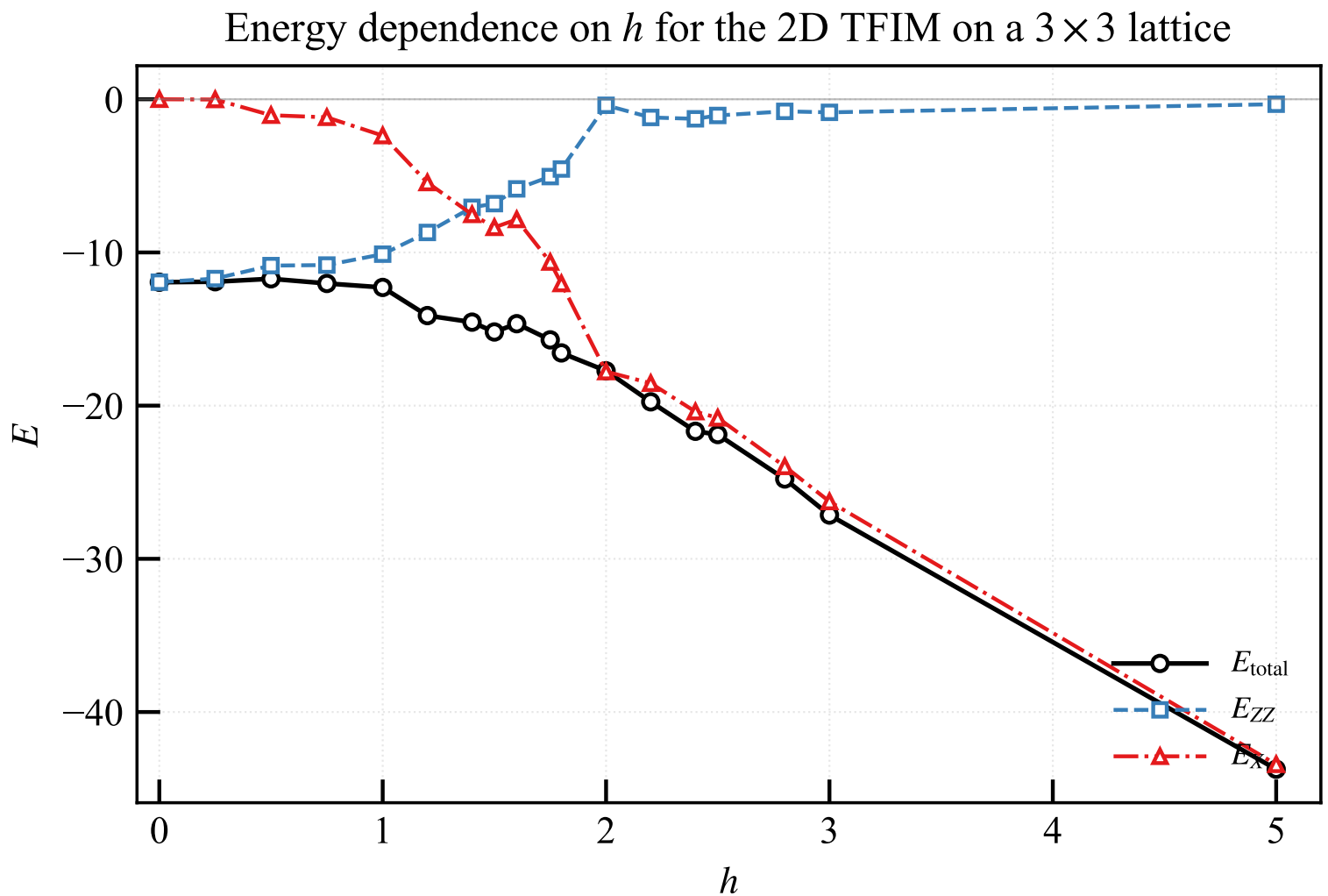
$$X = HZH \qquad |\psi'\rangle = H|\psi\rangle$$

$$\langle\psi'|Z|\psi'\rangle = \langle\psi|X|\psi\rangle$$

$$E_{X_2} = -\langle\psi|X_2|\psi\rangle = -(P'_2(0) - P'_2(1))$$

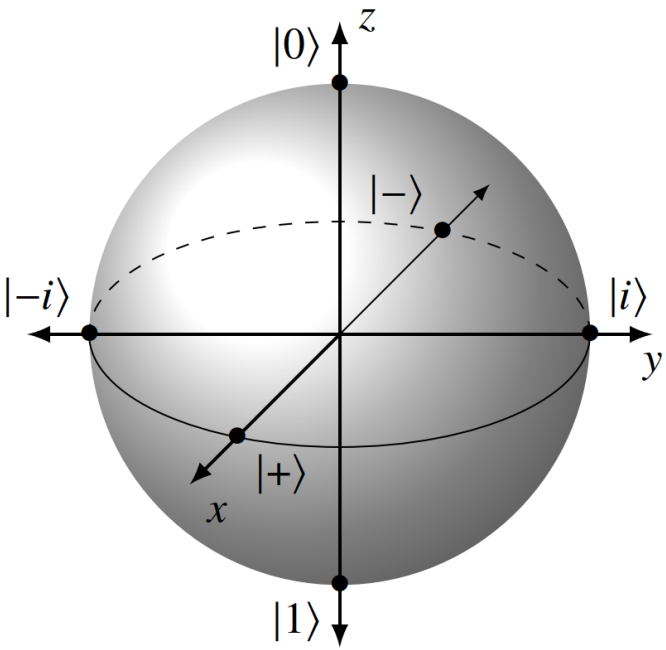


Phase Transition: Ordered to Disordered Phase



Ground States on the Bloch Sphere

$$E = \langle \mathcal{H} \rangle = -\frac{1}{N}\left(\sum_{i}^{N-1} \langle Z_i Z_{i+1} \rangle + h \sum_{i}^{N} \langle X_i \rangle \right)$$



Ordered Phase ($h \ll 1$)

TFIM ground states:
qubit states $|0\rangle$ and $|1\rangle$ (z-axis)

$$\langle 0|Z|0\rangle = +1, \qquad \langle 1|Z|1\rangle = -1,$$

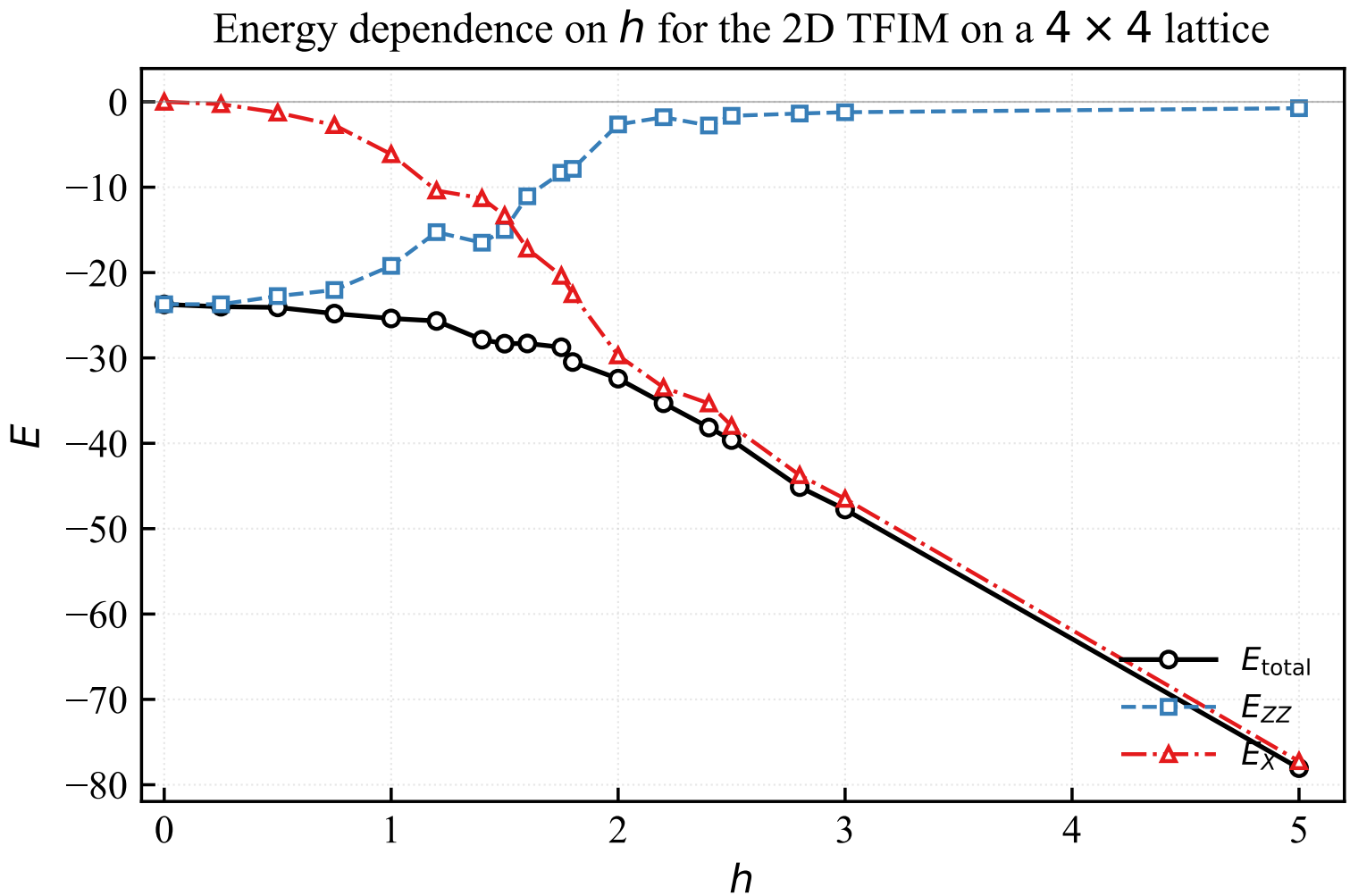
$$\langle 0|X|0\rangle = \langle 1|X|1\rangle = 0$$

Disordered Phase ($h \gg 1$)

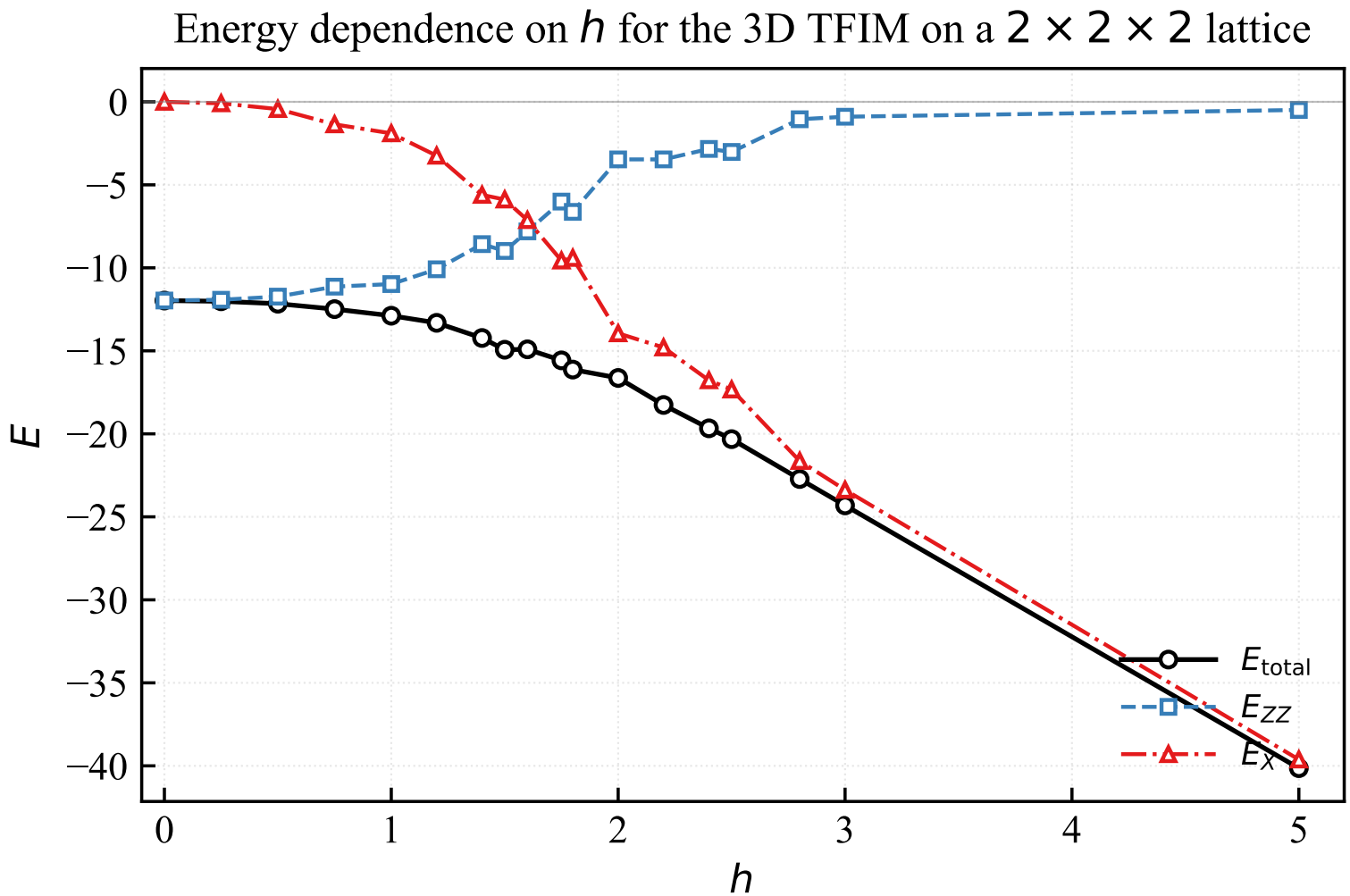
TFIM ground state:
qubit state $|+\rangle = H|0\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$ (x-axis, positive direction)

$$\langle +|X|+\rangle = 1, \qquad \langle +|Z|+\rangle = 0$$

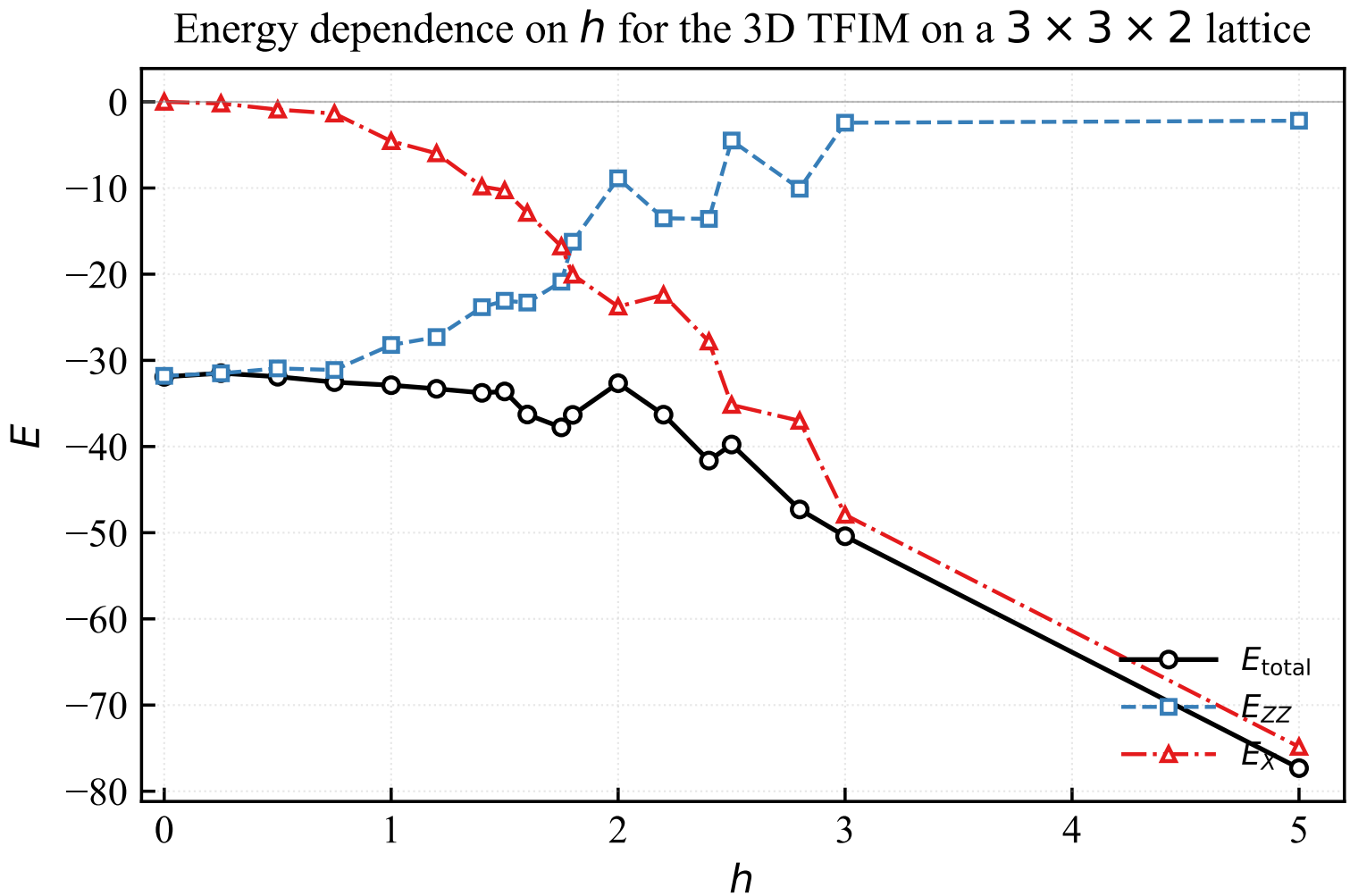
Results for 4×4 Lattice



Results for 2×2×2 Lattice



Results for 3×3×2 Lattice



Summary

Key Contributions

VQA Implementation Successfully implemented on the HybriLIT Quantum Testbed using `Cirq + qsim`

Expectation Value Evaluation Both Hadamard Test and ancilla-free measurement schemes were compared

Transverse Field Ising Model Ground state energies were calculated for 3×3, 4×4, $2\times2\times2$, and $3\times3\times2$ lattices

Phase Transition Quantum phase transition observed at $h/J \approx 1$ in agreement with theoretical predictions

Acknowledgments

Thank you for your attention!

Questions and comments are welcome.

We thank the HybriLIT team for consultations and their help in the setup and organization of computations with the HybriLIT and Govorun resources.