**Mathematical Modelling and Computational Physics (MMCP) 2026,
22.-26. June 2026, Dubna, Russia**

Reaction-diffusion processes: Non-perturbative renormalization group treatment of advection effects

Georgii Kalagov, Matej Kecer

Bogoliubov Laboratory of Theoretical Physics, JINR, 141980 Dubna, Moscow Region, Russia

Outline

- Problem definition

- Methods

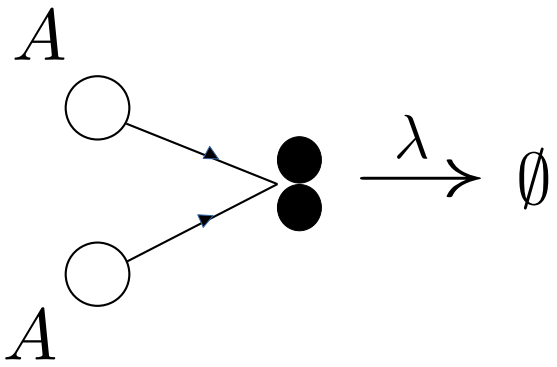
- Results

Outline

What do we study?

Introduction

Particle diffuse and can react on mutual contact



example: $A + A \rightarrow \emptyset$

Irreversible reactions

Non-equlibrium phenomenon

Tauber 2014; Krapivsky et. al 2010; Ovchinnikov & Timashev 1989;

Introduction

Branching and annihilating random walks (BARW) with odd number of offspring

$$A \to 2A, \qquad \text{branching process with rate } \sigma,$$
$$2A \to \emptyset, \qquad \text{pair annihilation with rate } \lambda,$$



Cardy&Täuber 2004

Introduction

- Active to absorbing phase transition

$a(t) \neq 0, \ t \to \infty$

Active state

$a(t) = 0, \ t \to \infty$

Absorbing state

Continuous phase transition
Emergent universality
Power law behavior
Critical exponents

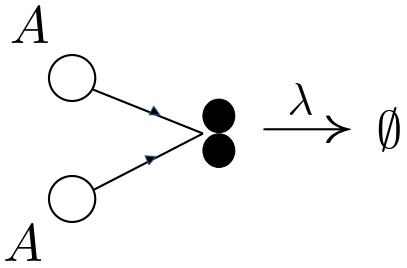
e.g. Hinrichsen 2008

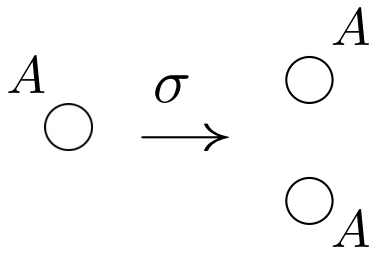
Introduction

- Mean field treatment

$$\frac{da(t)}{dt} = -2\lambda a(t)^2 + \sigma a(t)$$

- transition at $\sigma = 0$, i.e. NO Transition
- system always in Active state





Introduction

- simulations in 1D and 2D: Grassberger et al 1984, Takayasu et al 1992, Kim et al 1994, Jenssen 1993

Phase transition found

Introduction

Diffusion

$d \leq 2$

- self recurrence property
- large particle density fluctuations

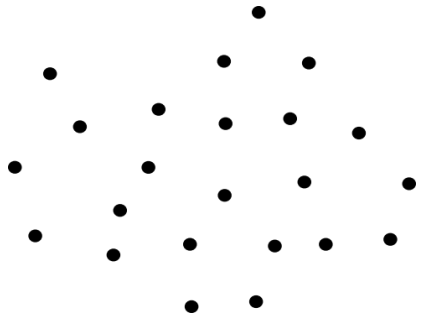
$d > 2$

- No density fluctuations
- system well mixed

See e.g. Krapivsky et al 2010

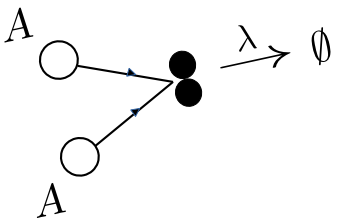
Introduction

$$d \leq 2$$

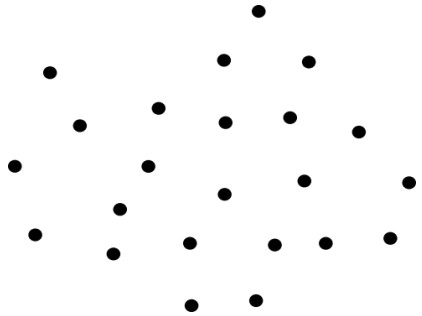


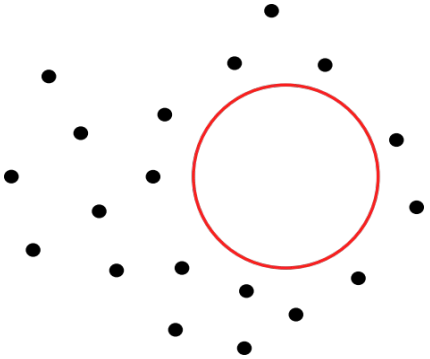
Introduction

$d \leq 2$



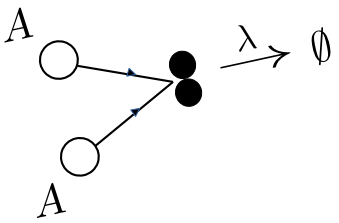
Particle destruction

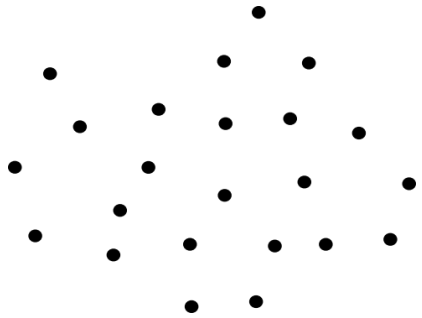




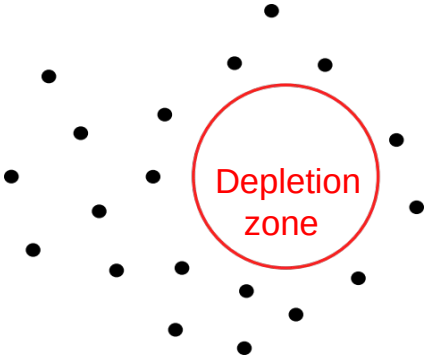
Introduction

$d \leq 2$



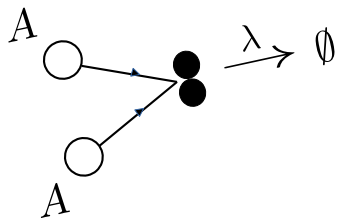


Particle destruction

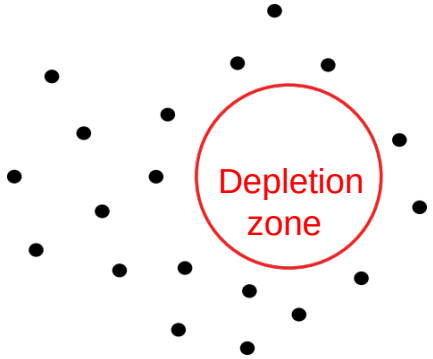


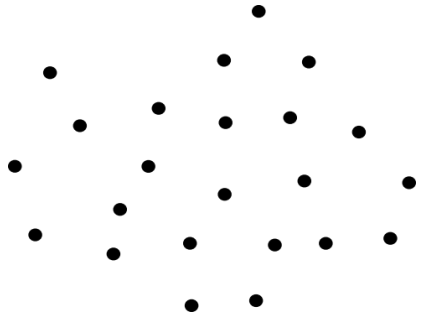
Introduction

$d \leq 2$

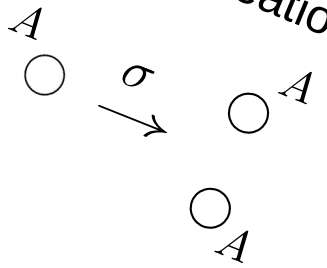


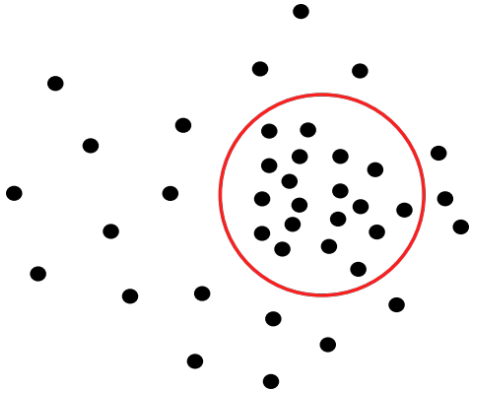
Particle destruction





Particle creation





Introduction

Diffusion

$d \leq 2$

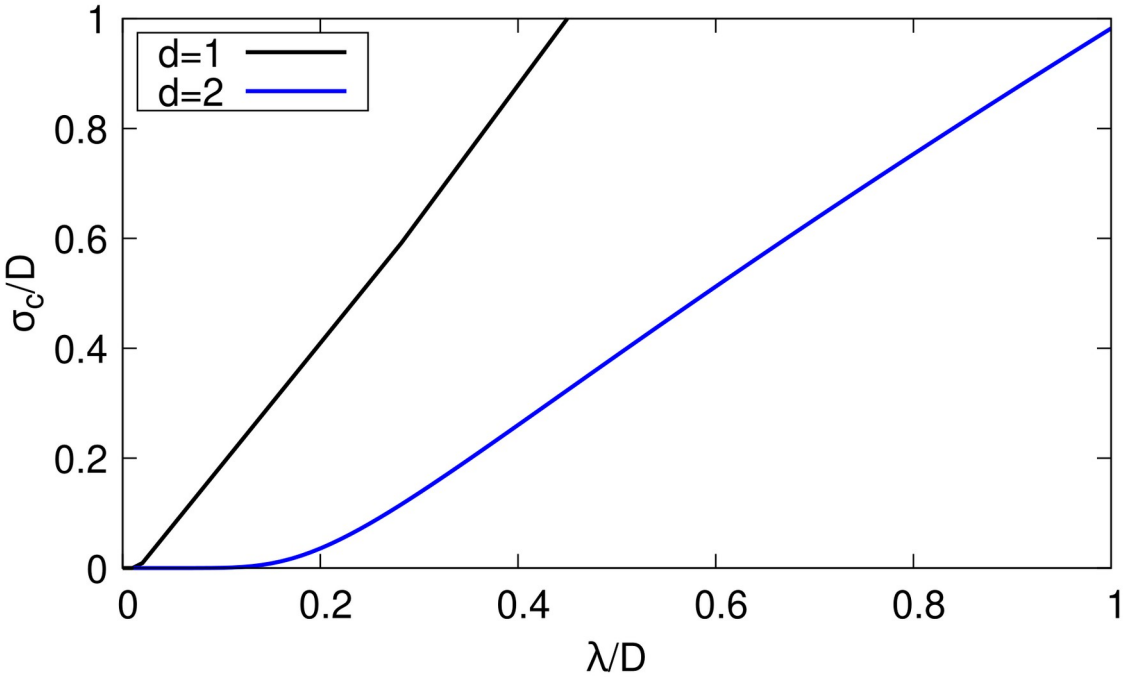
- self recurrence property
- large particle density fluctuations
- Mean-field treatment breaks down

$d > 2$

- No density fluctuations
- system well mixed

See e.g. Krapivsky et al 2010

Introduction



Introduction

3D BARW

- Mean-field holds?

Nonperturbative Renormalization-Group Study of Reaction-Diffusion Processes

Léonie Canet,[1] Bertrand Delamotte,[1] Olivier Deloubrière,[2] and Nicolas Wschebor[3]

[1]*Laboratoire de Physique Théorique et Hautes Energies, Universités Paris VI Pierre et Marie Curie, Paris VII Denis Diderot, 2 place Jussieu, 75251 Paris Cedex 05, France*

[2]*Department of Physics, Virginia Polytechnic Institute and State University, Blacksburg, Virginia 24061-0435, USA*

[3]*Instituto de Fisica, Facultad de Ingenieria, J.H.y Reissig 565, 11000 Montevideo, Uruguay*

(Received 22 September 2003; published 13 May 2004)

We generalize nonperturbative renormalization group methods to nonequilibrium critical phenomena. Within this formalism, reaction-diffusion processes are described by a scale-dependent effective action, the flow of which is derived. We investigate branching and annihilating random walks with an odd number of offspring. Along with recovering their universal physics (described by the directed percolation universality class), we determine their phase diagrams and predict that a transition occurs even in three dimensions, contrarily to what perturbation theory suggests.

DOI: 10.1103/PhysRevLett.92.195703 PACS numbers: 64.60.Ak, 05.10.Cc, 11.10.Hi, 64.60.Ht

Introduction

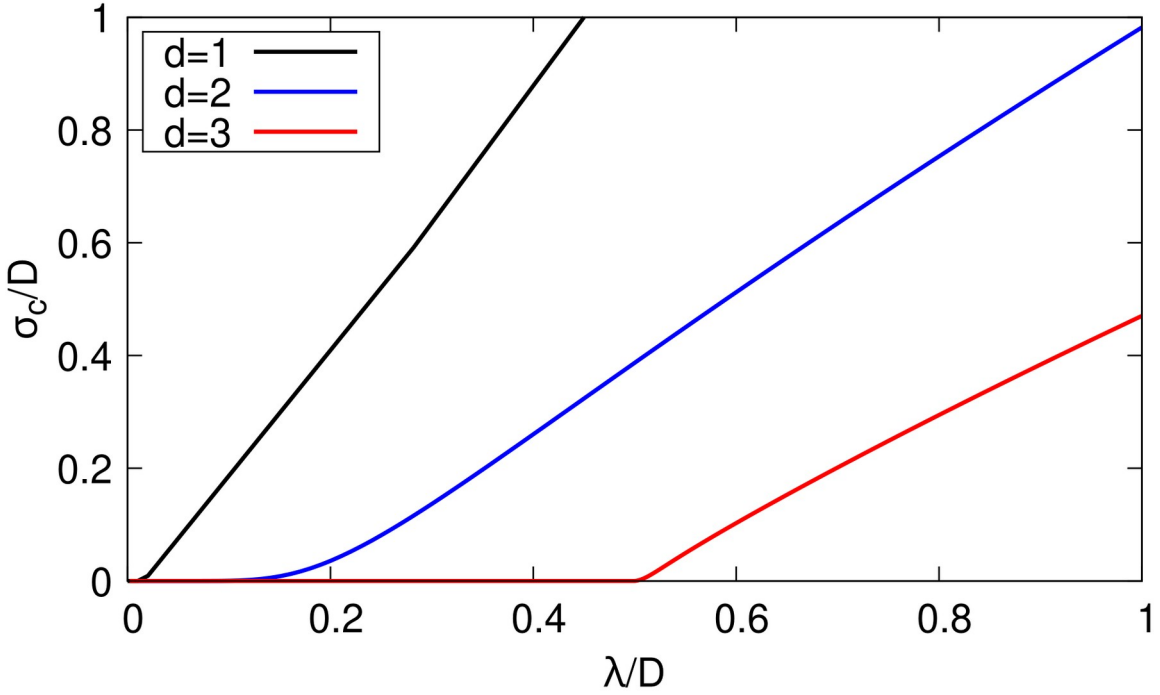
- Self-destruction mechanism

- consider $D << 1$

- particles annihilating with their own offspring

Canet et al 2004

Introduction



Advection effects

Why advection?

Idealized setting → Considering dynamics of the surrounding medium

Velocity-fluctuation-induced anomalous kinetics of the $A+A\rightarrow\varnothing$ reaction

M. Hnatich
Institute for Experimental Physics, SAS, Košice, Slovakia

J. Honkonen
Theory Division, Department of Physics, University of Helsinki, Helsinki, Finland

(Received 22 November 1999)

The effect of a random velocity field on the kinetics of the single-species annihilation reaction $A+A\rightarrow\varnothing$ is analyzed near two dimensions with the aid of the perturbative renormalization group. The previously found asymptotic behavior induced by density fluctuations only in the diffusion-limited reaction is shown to be unstable to any velocity fluctuations (including thermal fluctuations near equilibrium) in spatial dimensions $d\leqslant d_c=2$. Four different stable long-time asymptotic regimes induced by the combined effect of velocity and density fluctuations are identified and the corresponding decay rates calculated in the leading order.

PACS number(s): 05.70.Ln, 47.27.Qb, 82.20.Mj

Velocity-fluctuation-induced anomalous kinetics of the $A+A\rightarrow\varnothing$ reaction

M. Hnatich

Slovakia

IOP PUBLISHING JOURNAL OF PHYSICS A: MATHEMATICAL AND THEORETICAL

J. Phys. A: Math. Theor. **42** (2009) 135001 (19pp) doi:10.1088/1751-8113/42/13/135001

nki, Helsinki, Finland

s annihilation reaction $A+A\rightarrow\varnothing$ is
:ation group. The previously found
on-limited reaction is shown to be
equilibrium) in spatial dimensions
the combined effect of velocity and
lated in the leading order.

Effects of turbulent mixing on the nonequilibrium critical behaviour

N V Antonov, V I Iglovikov and A S Kapustin

Department of Theoretical Physics, St. Petersburg University, Uljanovskaja 1, St. Petersburg, Petrodvorez 198504, Russia

E-mail: nikolai.antonov@pobox.spbu.ru

Received 31 July 2008, in final form 31 January 2009
Published 6 March 2009
Online at stacks.iop.org/JPhysA/42/135001

Abstract
We study the effects of turbulent mixing on the critical behaviour of a nonequilibrium system near its second-order phase transition between the absorbing and fluctuating states. The model describes the spreading of an agent (e.g., infectious disease) in a reaction-diffusion system and belongs to

Velocity-fluctuation-induced anomalous kinetics of the $A+A \rightarrow \varnothing$ reaction

IOP PUBLISHING JOURN

J. Phys. A: Math. Theor. **42** (2009) 135001 (19pp)

Effects of turbulent mixing on critical behaviour

N V Antonov, V I Iglovikov and A

Department of Theoretical Physics, St. Peters
Petrodvorez 198504, Russia

E-mail: nikolai.antonov@pobox.spbu.ru

Received 31 July 2008, in final form
Published 6 March 2009
Online at stacks.iop.org/JPhysA/42/

Abstract

We study the effects of turbulent mixing on the critical behaviour of a nonequilibrium system near its second-order phase transition between the absorbing and fluctuating states. The model describes the spreading of an agent (e.g., infectious disease) in a reaction-diffusion system and belongs to

Theoretical and Mathematical Physics, **200**(3): 1335–1347 (2019)

PASSIVE ADVECTION IN A PERCOLATION PROCESS: TWO-LOOP APPROXIMATION

Š. Birnšteinová,* M. Hnatič,*† T. Lučivjanský,* L. Mižišin,† and V. Škultéty‡

We study an instructive model of the directed percolation process near its second-order phase transition between absorbing and active states. We first express the model as a Langevin equation and then rewrite it in a field theory formulation. Using the Feynman diagram technique and the perturbative renormalization group method, we then analyze the resulting response functional. The percolation process is assumed to occur in an external velocity field, which has an additional effect on the properties of spreading. We use the Kraichnan rapid-change ensemble to generate velocity fluctuations. We obtain the structure of the set of fixed points in the two-loop approximation.

Keywords: directed percolation, renormalization group, multiloop Feynman diagram

DOI: 10.1134/S0040577919090071

Advection effects

Why advection?

- new regimes

- different exponents

- changes in position of transition line

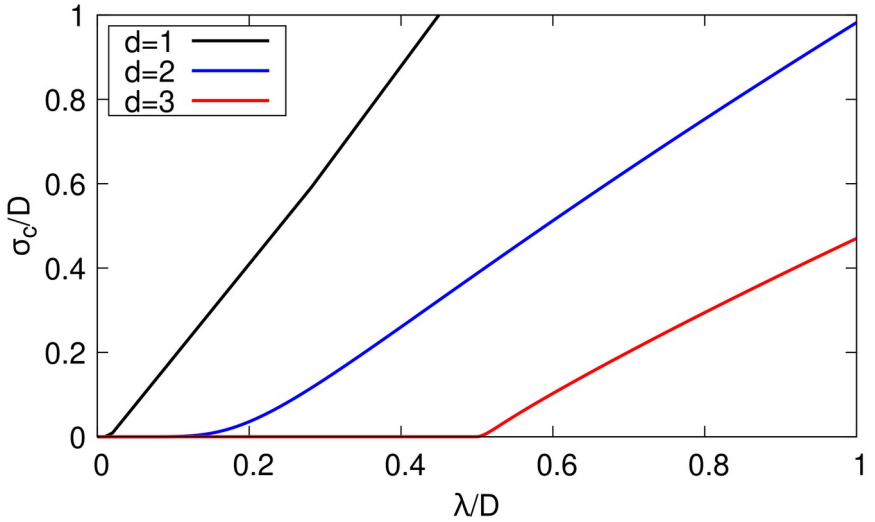
Advection effects

Why advection?

- new regimes

- different exponents

- changes in position of transition line



Introduction

What do we study?

Analyse the effects of diffusion-enhancement by means of advection by Kraichnan velocity field of BARW system

Methods

Non-Perturbative Renormalization Group (NPRG)

Main idea

Microscopic scale

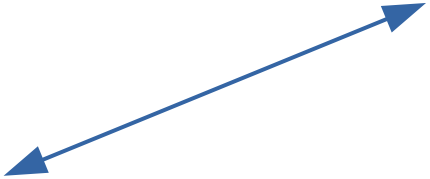
Γ_Λ

See e.g. Delamotte 2012

Main idea

Microscopic scale

Γ_Λ



$$\psi^\dagger[\partial_t - D\partial^2 + (\mu - \sigma)]\psi + \sqrt{2\lambda\sigma}(\psi^\dagger\psi^2 - \psi^{\dagger 2}\psi)$$
$$+ \lambda\psi^{\dagger 2}\psi^2 + \psi^\dagger v_j \partial_j \psi + v_i D_{ij} v_j/2$$

See e.g. Delamotte 2012

Main idea

Macroscopic scale

$$\Gamma = \Gamma_{k \to 0}$$

Microscopic scale

$$\Gamma_{\Lambda}$$

See e.g. Delamotte 2012

Main idea

Macroscopic scale

$$\Gamma = \Gamma_{k\to 0}$$



$$\Gamma_k$$

Microscopic scale

$$\Gamma_\Lambda$$

See e.g. Delamotte 2012

Our model

$$\Gamma_k = Z_k \psi^\dagger (\partial_t + v_a \partial_a)\psi - D_k \psi^\dagger \Delta \psi + \Delta S_k + U_k[\psi, \psi^\dagger] + v_i D_{ij} v_j / 2$$

Our model

$$\Gamma_k = Z_k \psi^\dagger (\partial_t + v_a \partial_a)\psi - D_k \psi^\dagger \Delta \psi + \Delta S_k + U_k[\psi, \psi^\dagger] + v_i D_{ij} v_j / 2$$

Our model

$$\Gamma_k = Z_k \psi^\dagger (\partial_t + v_a \partial_a)\psi - D_k \psi^\dagger \Delta \psi + \Delta S_k + U_k[\psi, \psi^\dagger] + v_i D_{ij} v_j / 2$$

Rapidity reversal:

$$\psi(t, \vec{x}) \to -\psi^\dagger(-t, \vec{x}), \quad \psi^\dagger(t, \vec{x}) \to -\psi(-t, \vec{x}), \quad \vec{v}(t, \vec{x}) \to -\vec{v}(-t, \vec{x}),$$

Our model

$$\Gamma_k = Z_k \psi^\dagger (\partial_t + v_a \partial_a)\psi - D_k \psi^\dagger \Delta \psi + \Delta S_k + U_k[\psi, \psi^\dagger] + v_i D_{ij} v_j / 2$$

Rapidity reversal:

$$\psi(t,\vec{x}) \to -\psi^\dagger(-t,\vec{x}), \quad \psi^\dagger(t,\vec{x}) \to -\psi(-t,\vec{x}), \quad \vec{v}(t,\vec{x}) \to -\vec{v}(-t,\vec{x}),$$

$$\sigma = \psi^\dagger \psi$$
$$\rho = \psi - \psi^\dagger$$



$$\begin{aligned} U_k = {} & g_{01}\sigma \\ & + g_{11}\rho\sigma \\ & + g_{02}\sigma^2 + g_{12}\rho^2\sigma \\ & + \ldots \end{aligned}$$

Our model

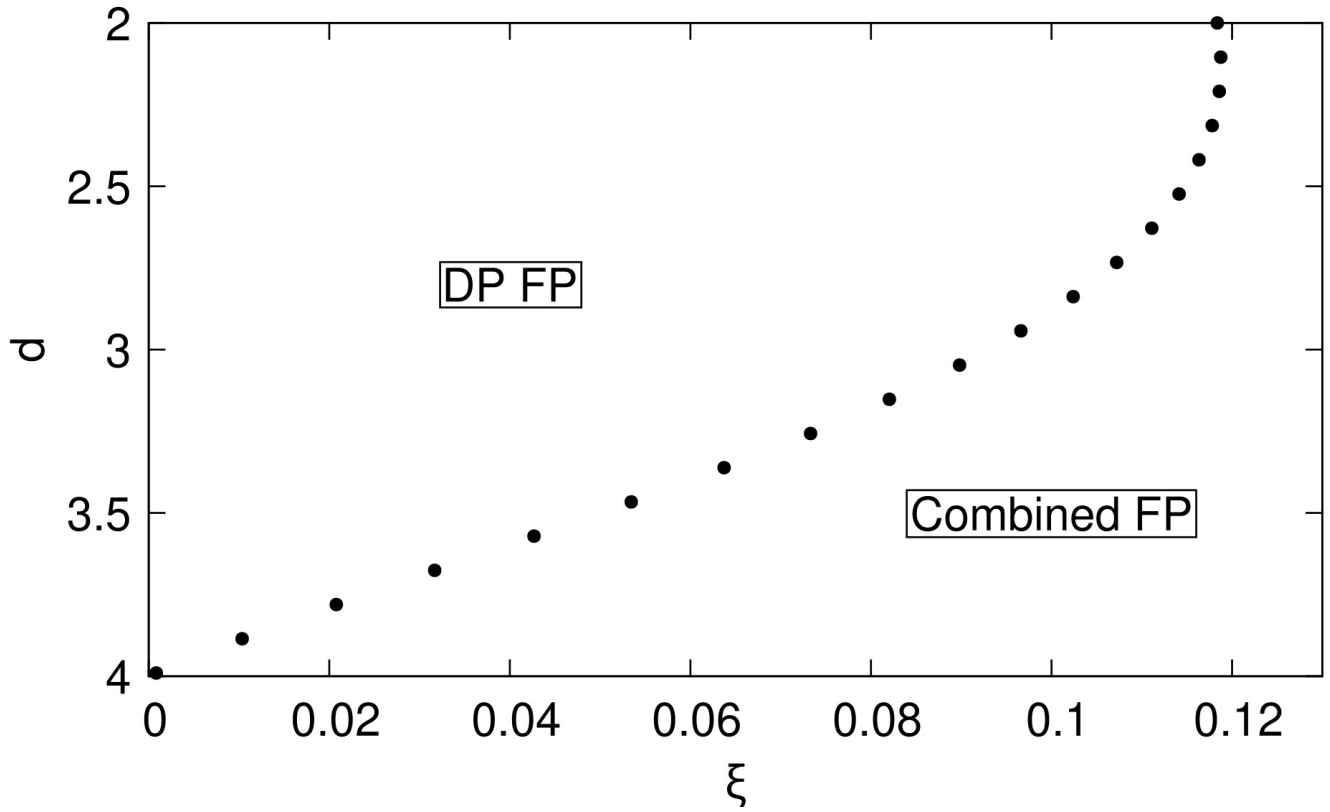
$$\Gamma_k = Z_k \psi^\dagger (\partial_t + v_a \partial_a)\psi - D_k \psi^\dagger \Delta \psi + \Delta S_k + U_k[\psi, \psi^\dagger] + v_i D_{ij} v_j / 2$$

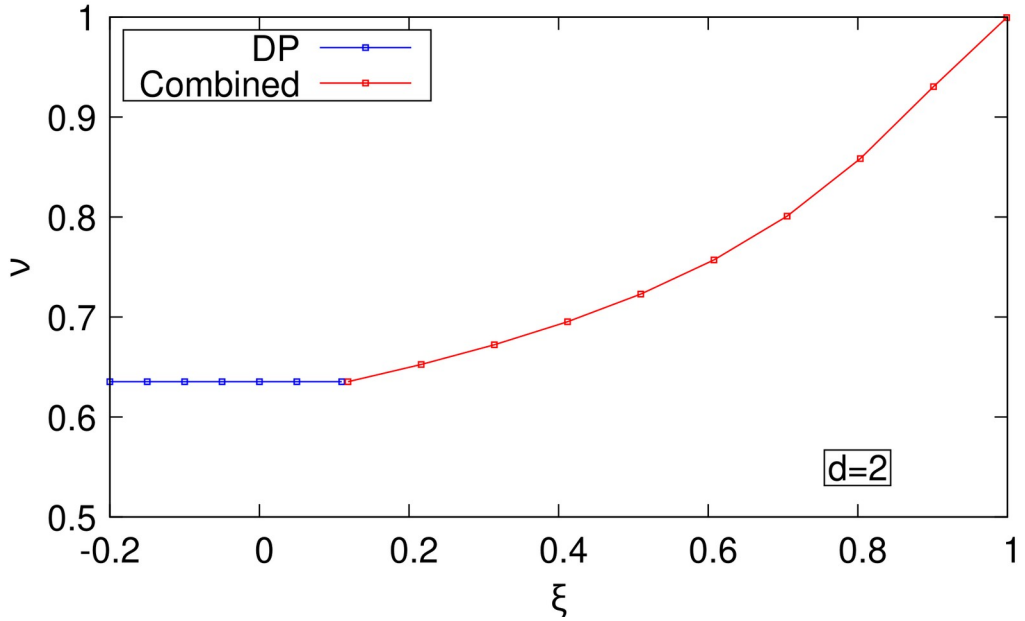
IR regulator

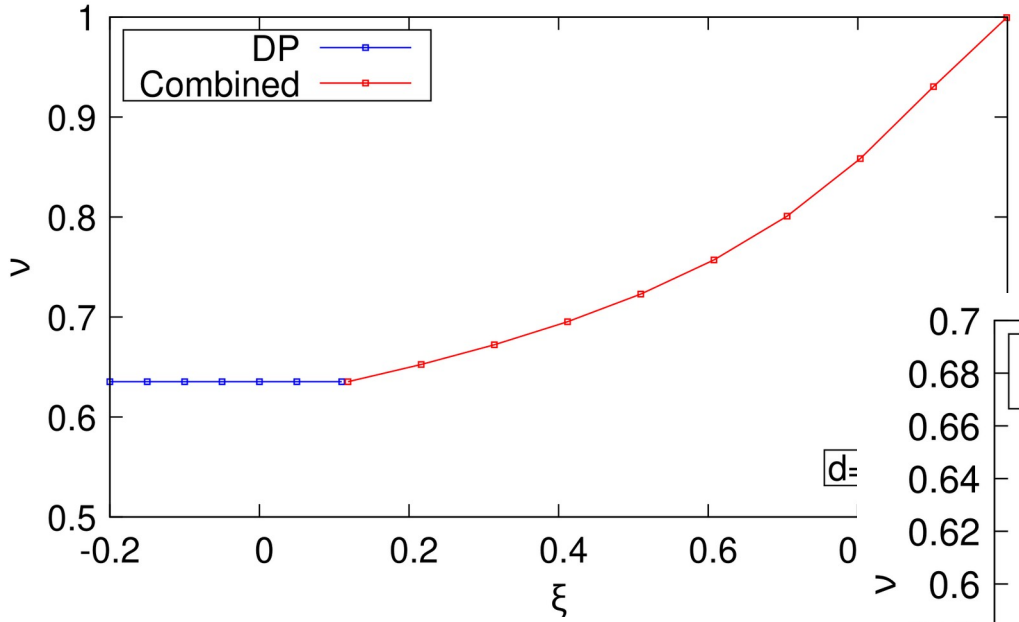
Advantage:

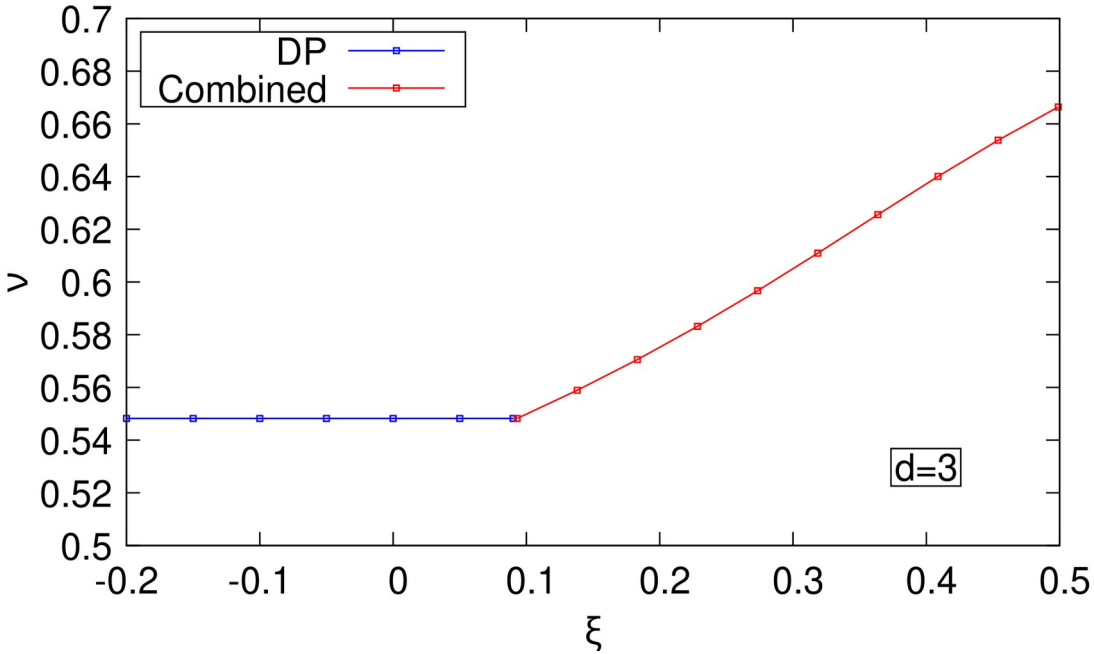
- straightforward use of numerical methods

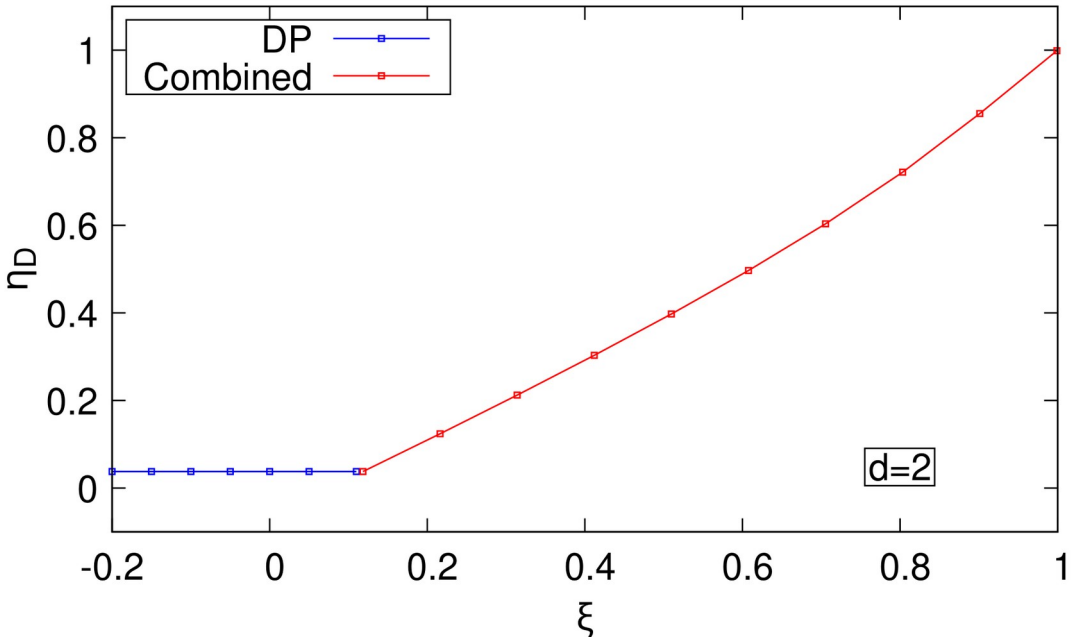
Results

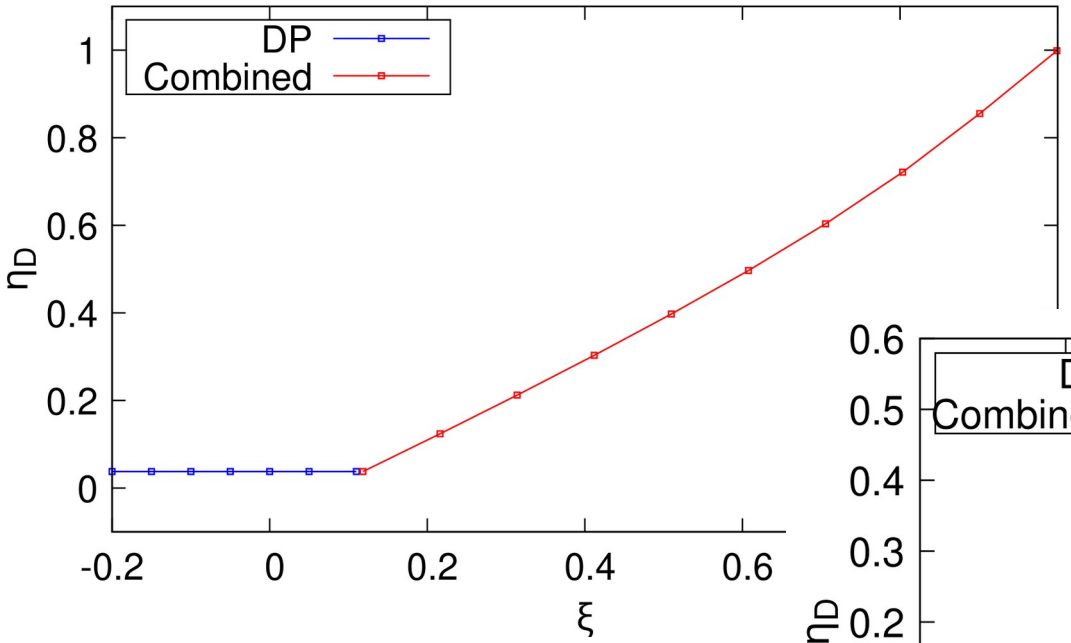

2
2.5
3
3.5
4
d
0
0.02
0.04
0.06
0.08
0.1
0.12
ξ
DP FP
Combined FP

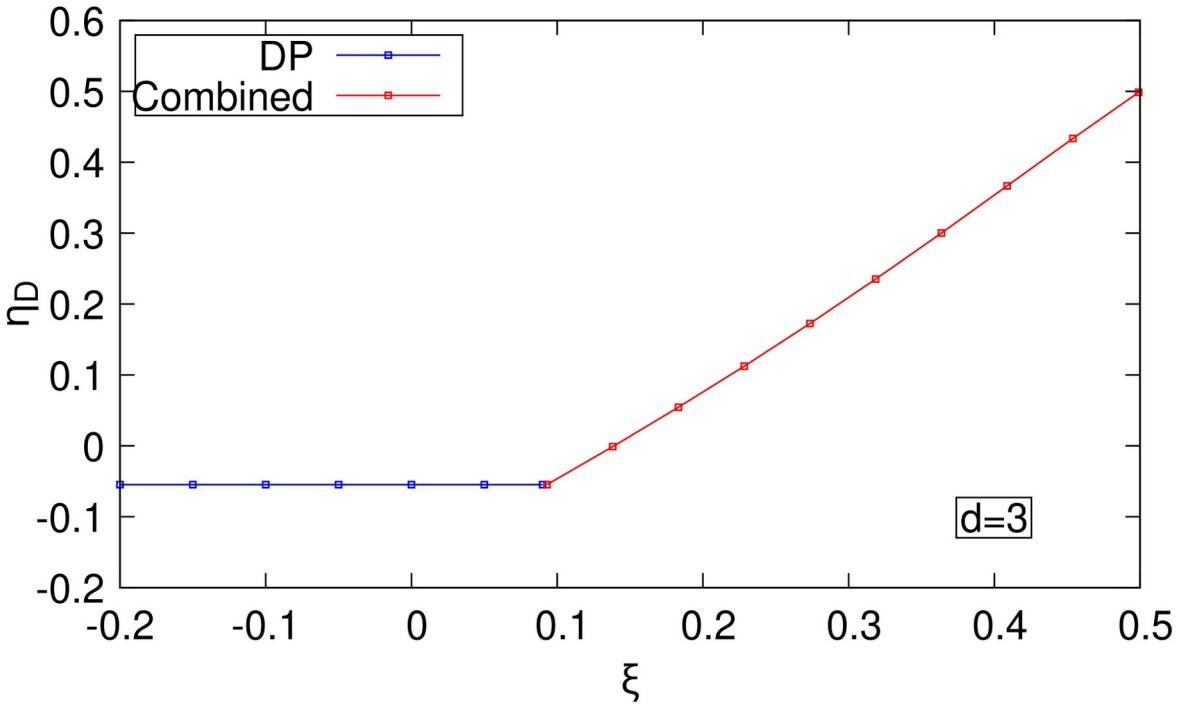

DP
Combined
d=2
ν
ξ
1
0.9
0.8
0.7
0.6
0.5
-0.2
0
0.2
0.4
0.6
0.8
1


DP
Combined
1
0.9
0.8
0.7
0.6
0.5
ν
-0.2
0
0.2
0.4
0.6
0
ξ
d=

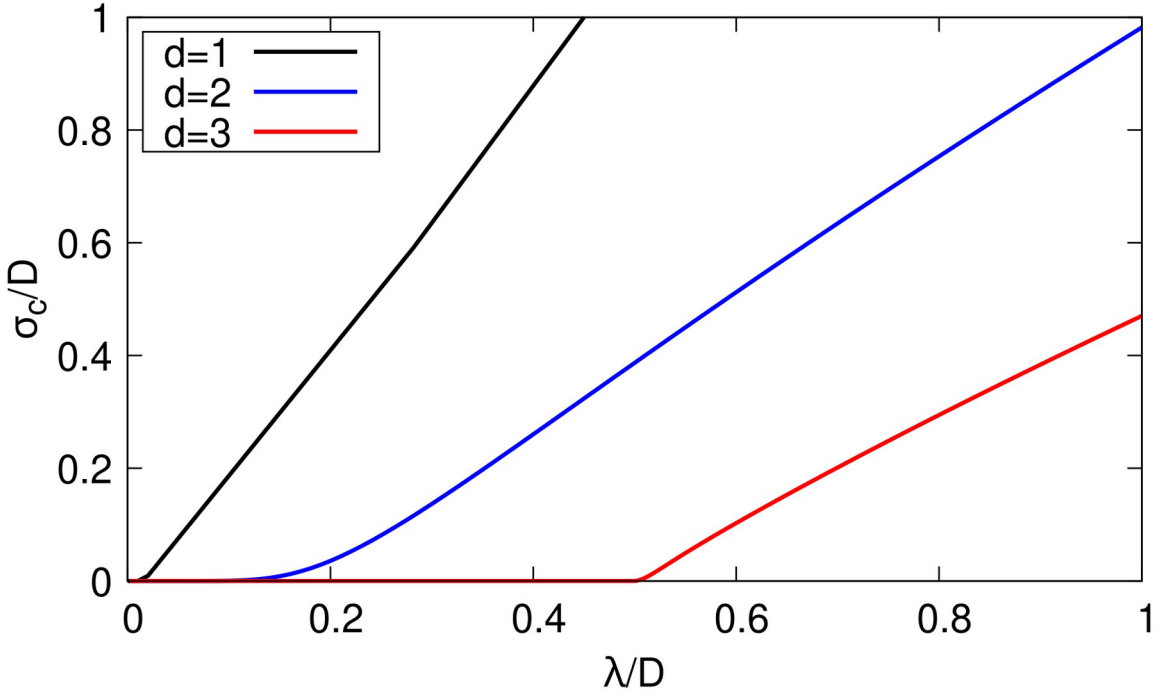

DP
Combined
0.7
0.68
0.66
0.64
0.62
0.6
0.58
0.56
0.54
0.52
0.5
ν
-0.2
-0.1
0
0.1
0.2
0.3
0.4
0.5
ξ
d=3

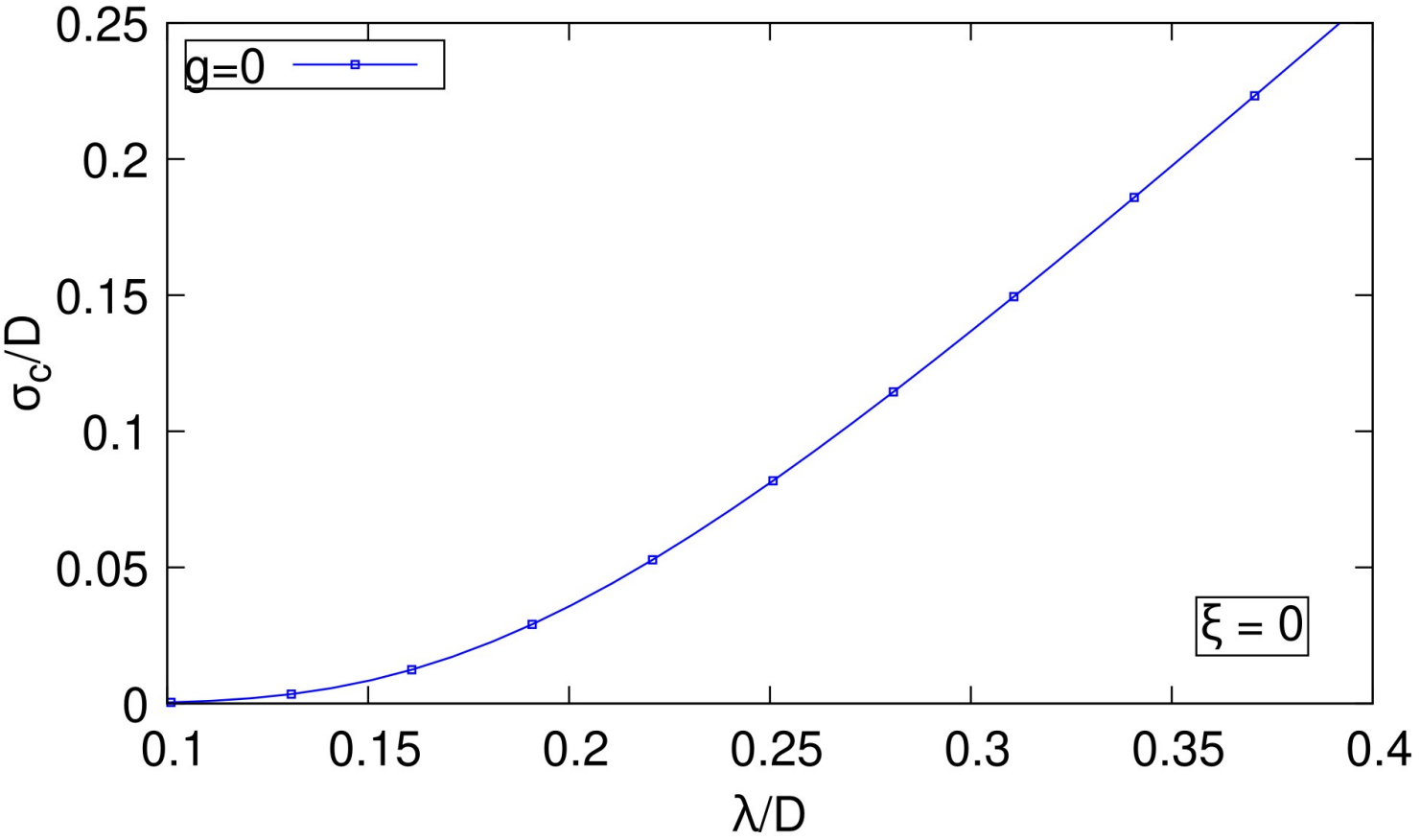

DP
Combined
1
0.8
0.6
0.4
0.2
0
η_D
-0.2
0
0.2
0.4
0.6
0.8
1
ξ
d=2

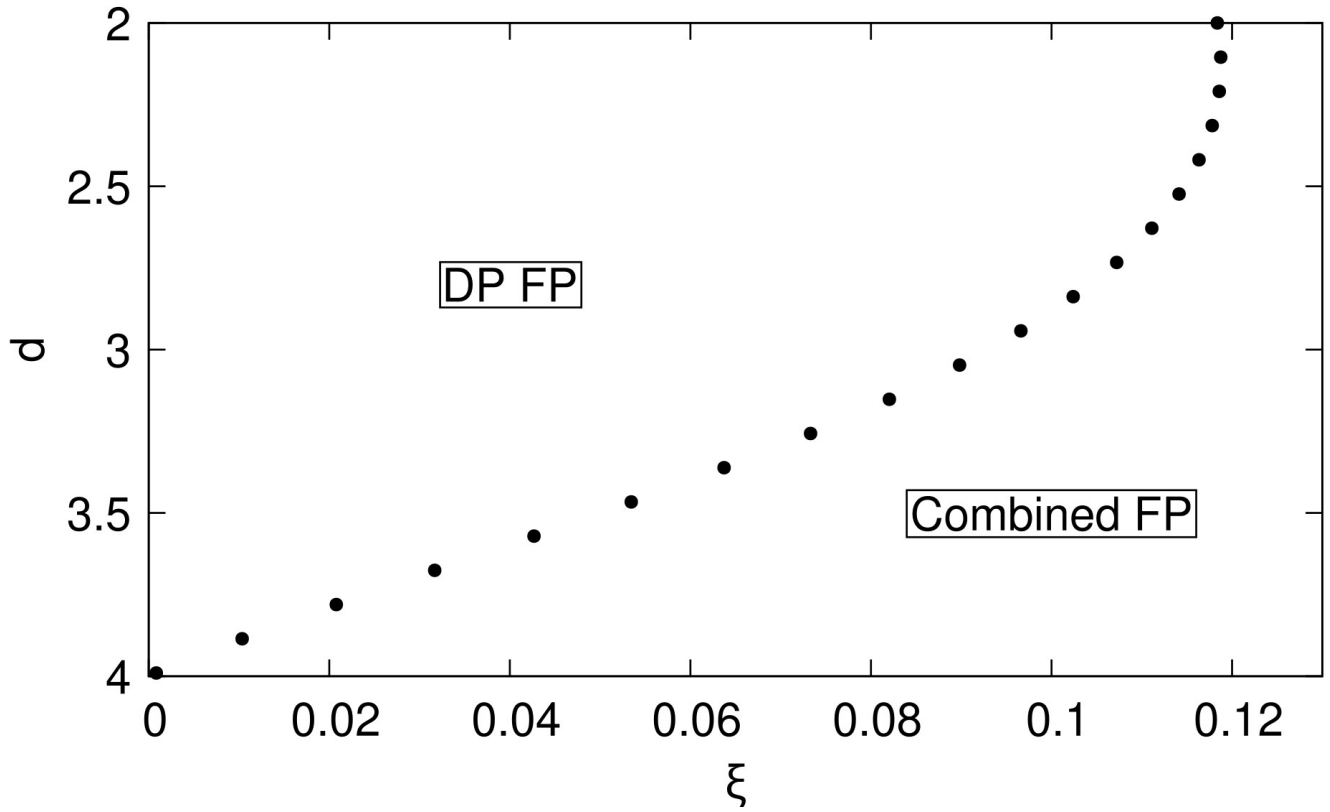

DP
Combined
1
0.8
0.6
0.4
0.2
0
η_D
-0.2
0
0.2
0.4
0.6
ξ

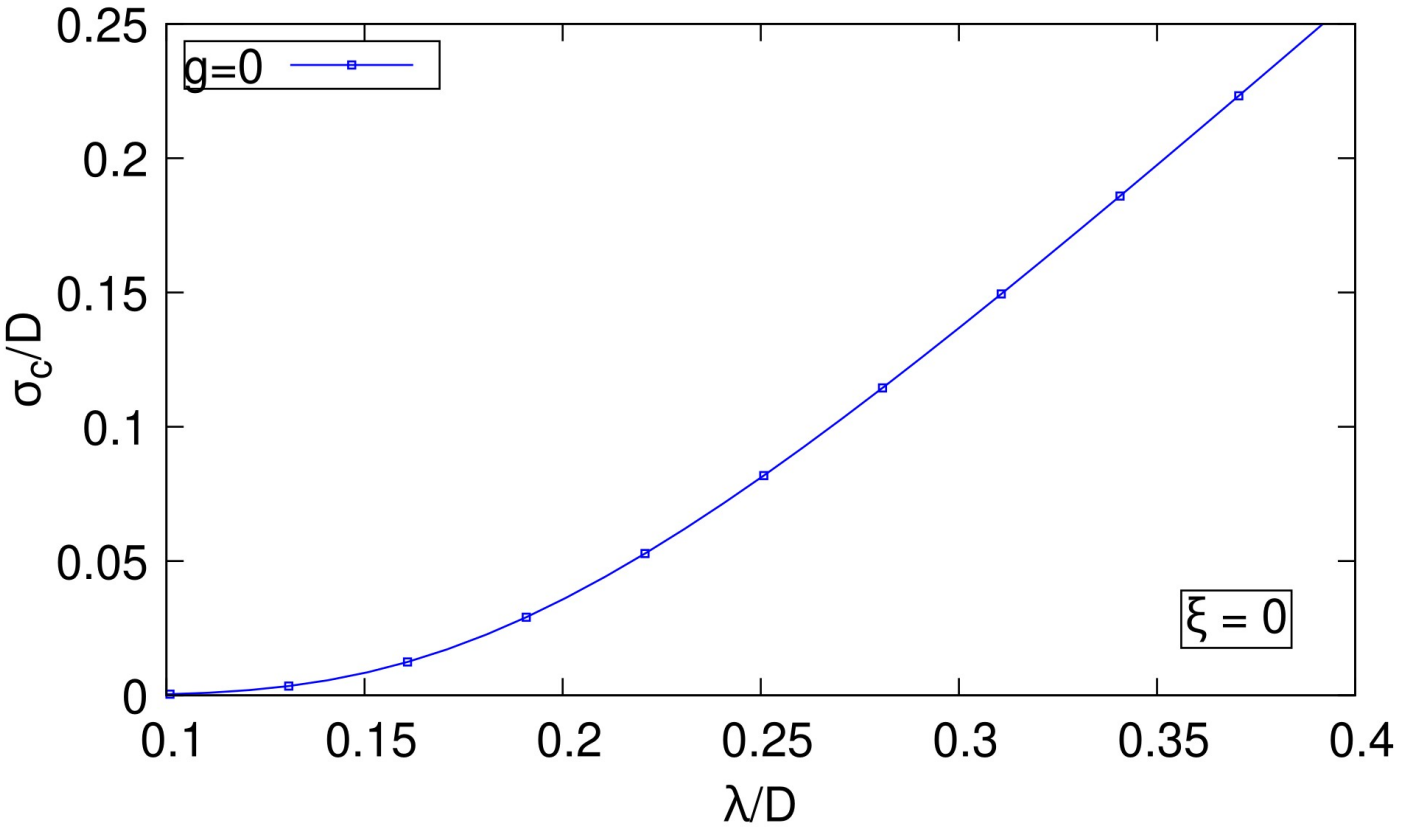

DP
Combined
0.6
0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.2
η_D
-0.2
-0.1
0
0.1
0.2
0.3
0.4
0.5
ξ
d=3

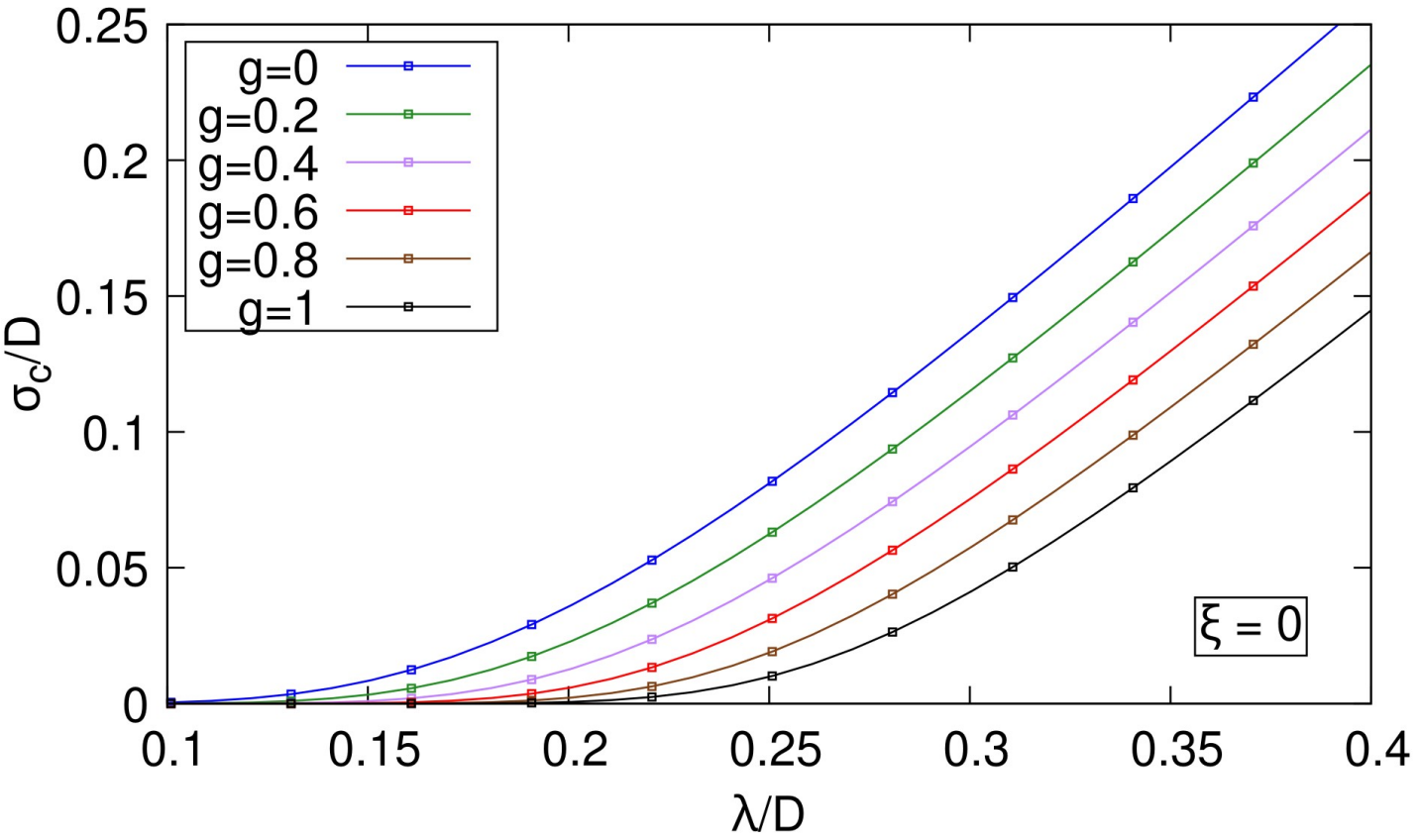
Phase diagram

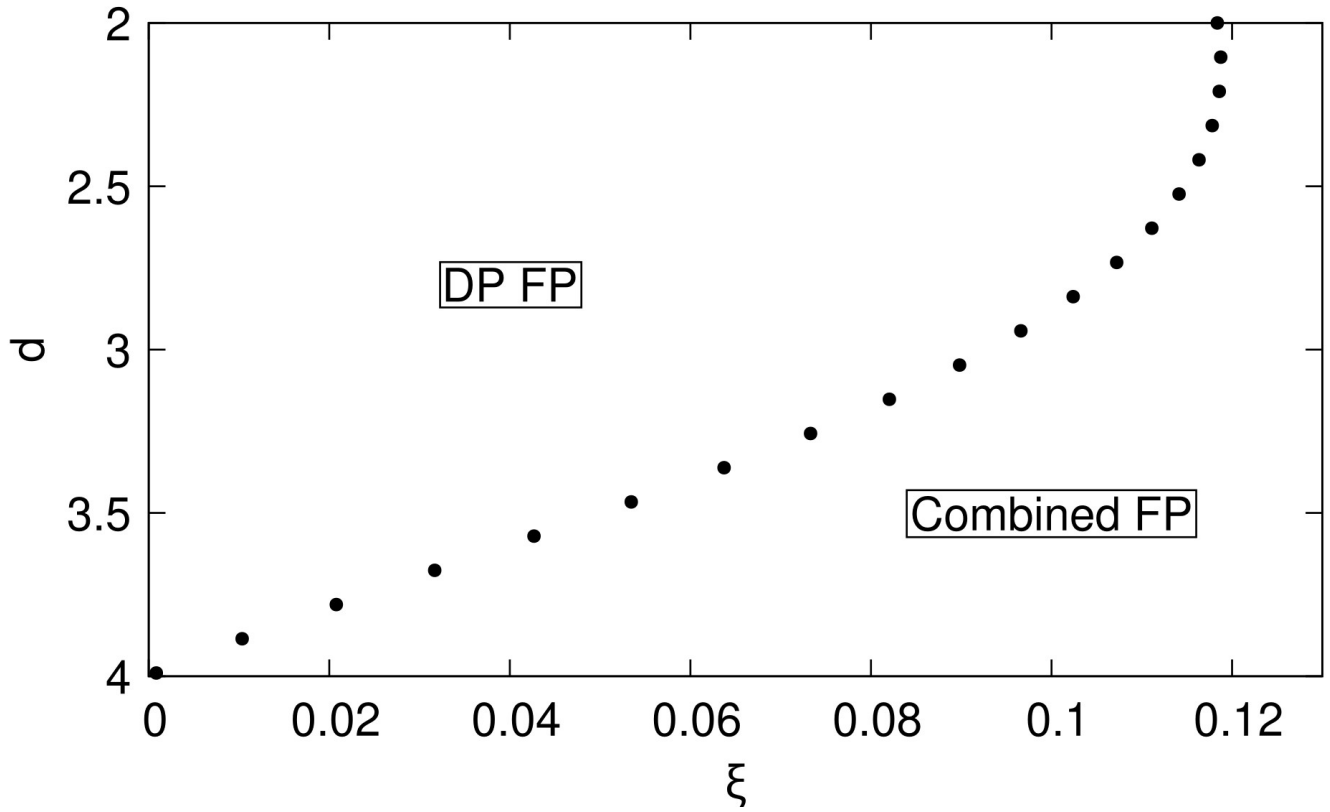

d=1
d=2
d=3
σc/D
λ/D
0
0.2
0.4
0.6
0.8
1

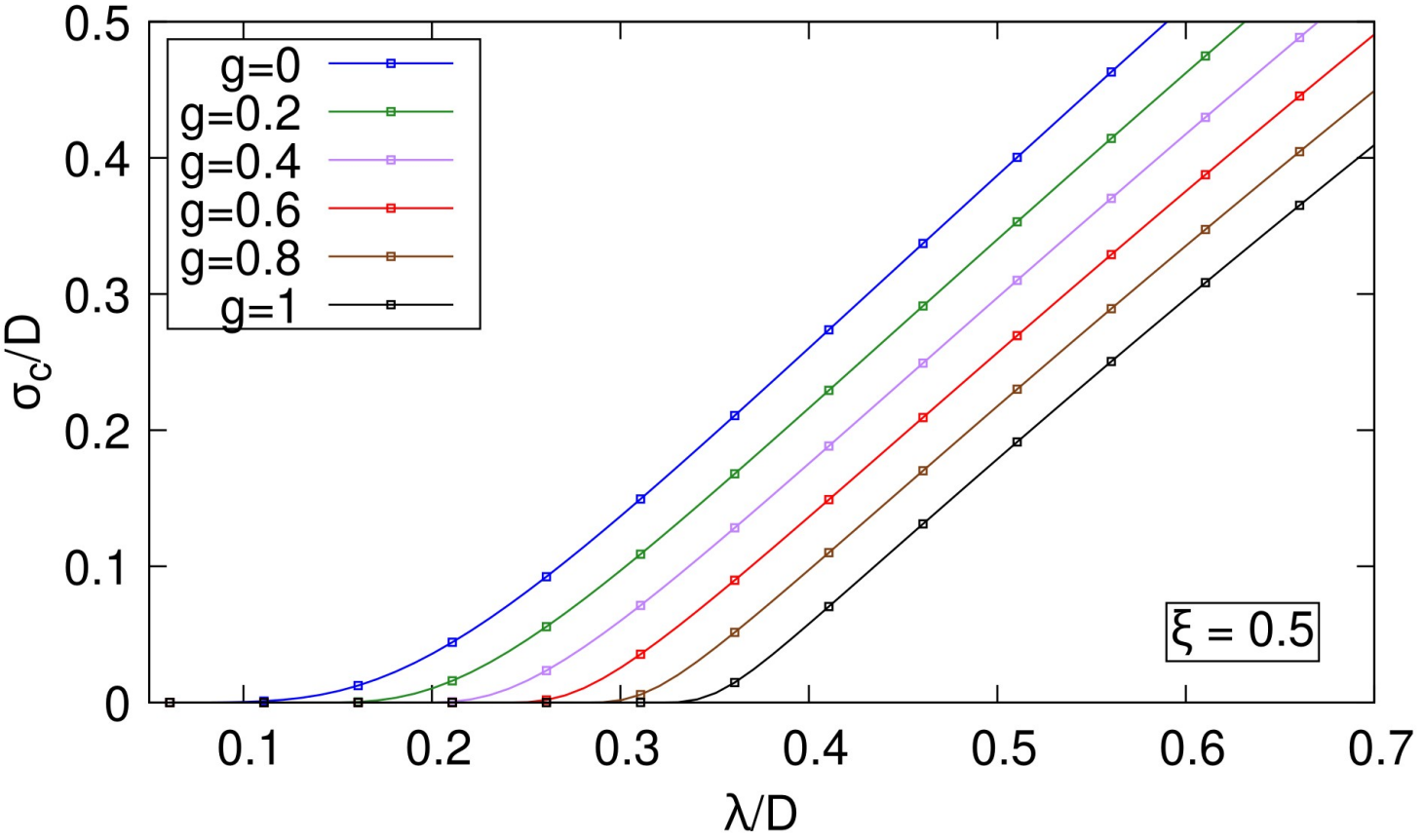

g=0
ξ = 0
σc/D
λ/D
0.25
0.2
0.15
0.1
0.05
0
0.1
0.15
0.2
0.25
0.3
0.35
0.4


2
2.5
3
3.5
4
d
0
0.02
0.04
0.06
0.08
0.1
0.12
ξ
DP FP
Combined FP


g=0
ξ = 0
σc/D
λ/D
0.25
0.2
0.15
0.1
0.05
0
0.1
0.15
0.2
0.25
0.3
0.35
0.4


0.25
0.2
0.15
0.1
0.05
0
0.1
0.15
0.2
0.25
0.3
0.35
0.4
σc/D
λ/D
g=0
g=0.2
g=0.4
g=0.6
g=0.8
g=1
ξ = 0


2
2.5
3
3.5
4
d
0
0.02
0.04
0.06
0.08
0.1
0.12
ξ
DP FP
Combined FP


g=0
g=0.2
g=0.4
g=0.6
g=0.8
g=1
ξ = 0.5
σc/D
λ/D
0.5
0.4
0.3
0.2
0.1
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7

References

- U. C. Täuber, Critical Dynamics: A Field Theory Approach To Equilibrium And Non-Equilibrium Scaling Behavior (Cambridge, 2014).
- P. L. Krapivsky, S. Redner and Ben–Naim A Kinetic View of Statistical Physics (Cambridge, 2010).
- A. A. Ovchinnikov and S. Timashev. Kinetics of Diffusion controlled chemical processes (Nova Science Pub Incorporated, 1989).
- J. L. Cardy, U. C. Täauber, Phys. Rev. Lett. 77, 4780 (2004)
- H. Hinrichsen: Non-Equilibrium Phase Transitions: Volume I: Absorbing Phase Transitions (Spinger, 2008)
- L. Canet, B. Delamotte, O. Deloubrière and N. Wschebor, Phys. Rev. Lett. 92, 195703 (2004)
- P. Grassberger, F. Krause, T. von der Twer, J. Phys. A: Math. And Gen. 17, L105 (1984)
- H. Takayasu, A. Y. Tretyakov, Phys. Rev. Lett. 68, 3060 (1992)
- B. Delamotte, An introduction to the nonperturbative renormalization group (Springer, 2012)
- M. H. Kim, H. Park, Phys. Rev. Lett. 73, 2579 (1994)
- I. Jensen, Phys. Rev. E 47, R1 (1993)