SUPERCOMPUTING WITHOUT SUPERCOMPUTERS

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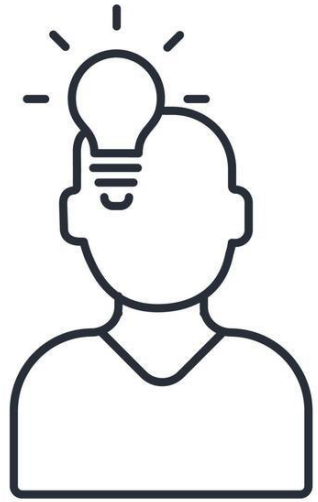
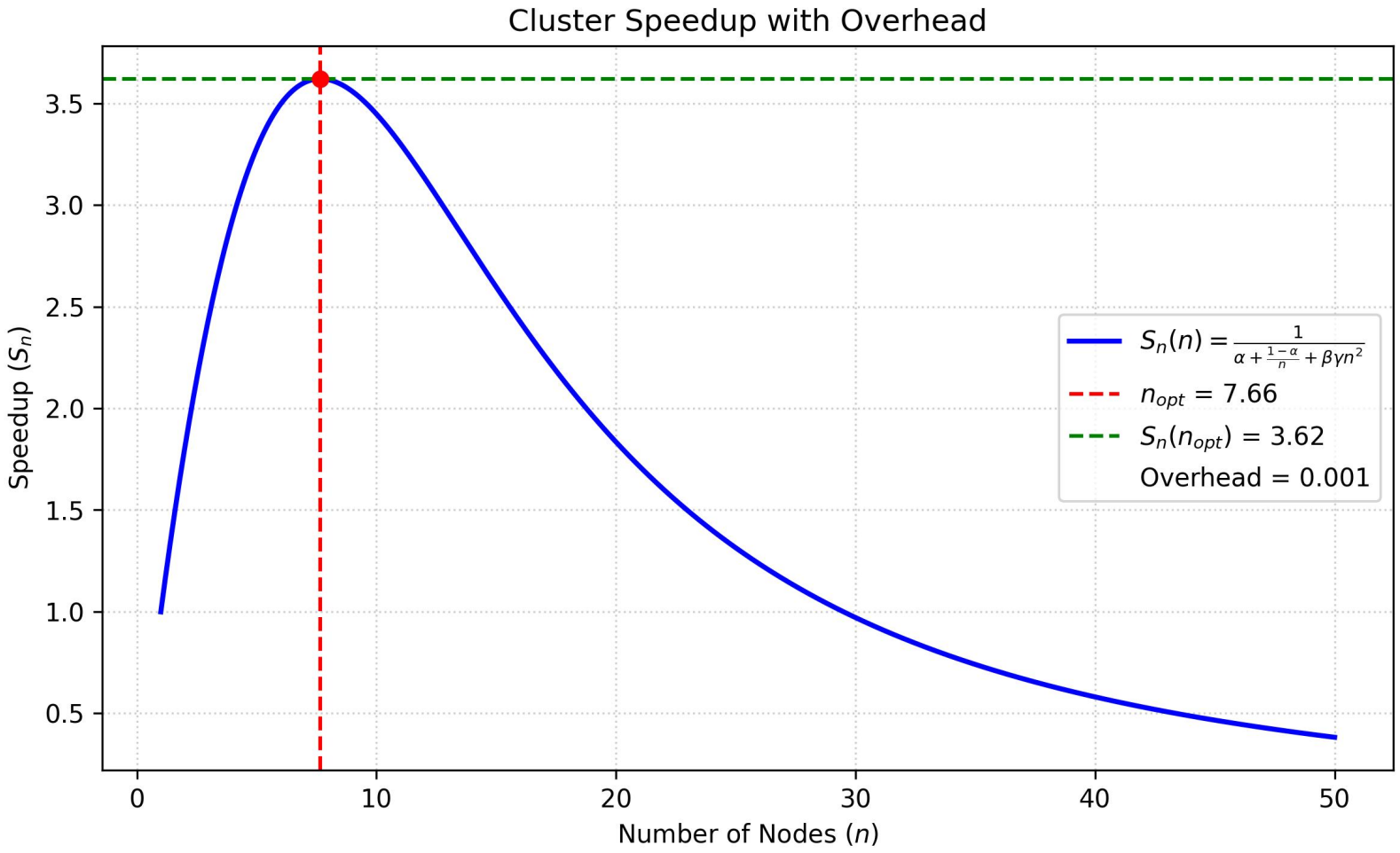
What we need to understand

1. The problems of classical architectures
2. Hopes with quantum systems
3. Why quantum computing anyway
4. What about underlying mathematics
5. How to improve it
6. Can we use quantum math in practice
7. Parallel algorithms
8. A way to qualitative analysis



Shape of cluster acceleration



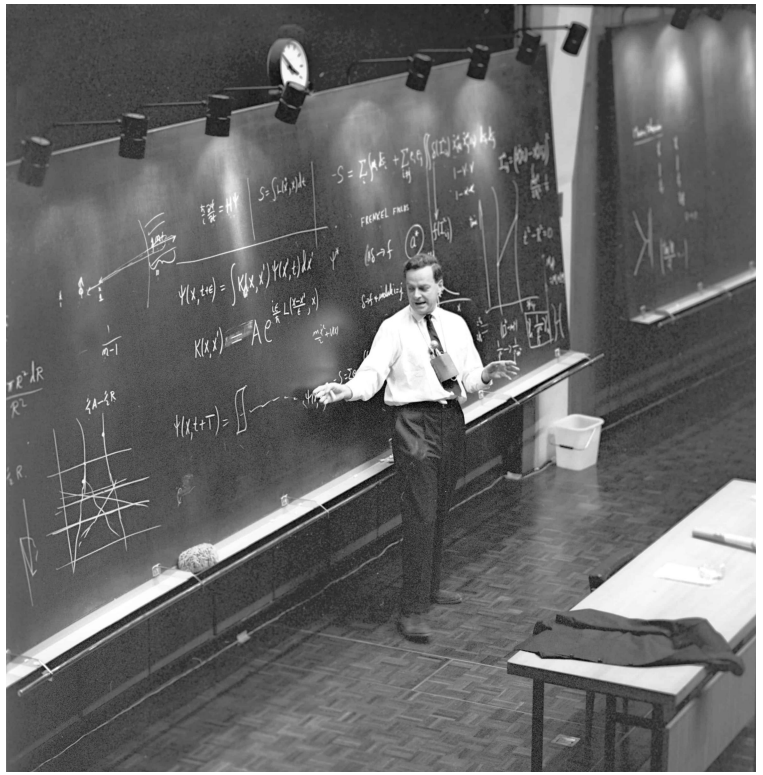
$$n_{opt} = \sqrt[3]{\frac{1-\alpha}{2\beta\gamma}}$$

$$S_n(n_{opt}) = \frac{1}{\alpha + \frac{3 \cdot (1-\alpha)}{2n_{opt}}}$$

Quantum supremacy

The concept of a qualitative quantum computational advantage, specifically for simulating quantum systems:

“Nature isn't classical, dammit, and if you want to make a simulation of nature, you'd better make it quantum mechanical, and by golly it's a wonderful problem, because it doesn't look so easy.” R. Feynman



Church–Turing thesis.

Any computational problem that can be solved by a classical computer can also be solved by a quantum computer.[2]

Conversely, any problem that can be solved by a quantum computer can also be solved by a classical computer, at least in principle given enough time

Problems:

- bits and qubits,
- need of measurement,
- stability of quantum system,
- quantum analog systems

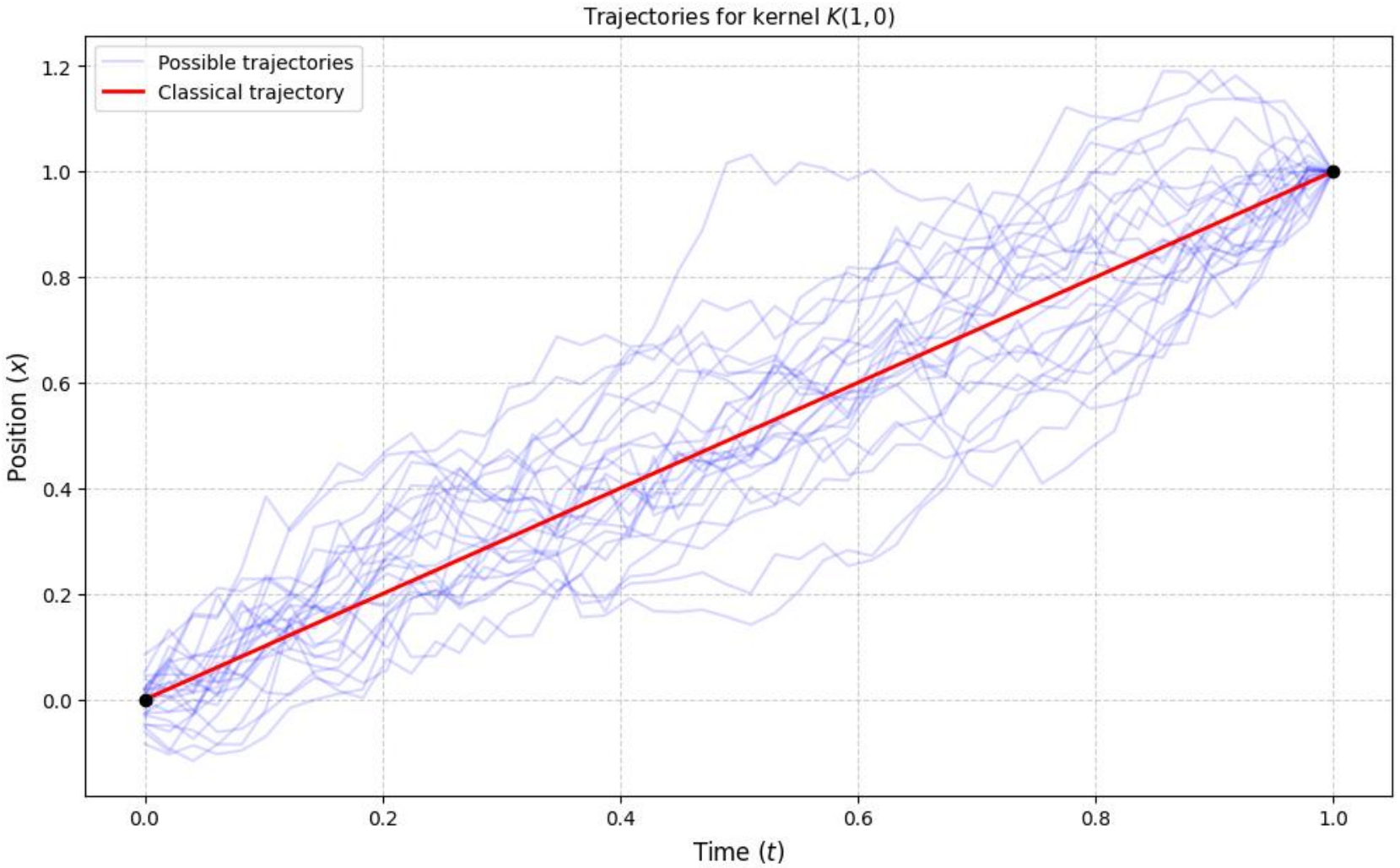
While we wait for quantum computer, let us turn to quantum computing

R. Feynman prophesay the need of new math for quantum computing and claimed path integral to be the tool

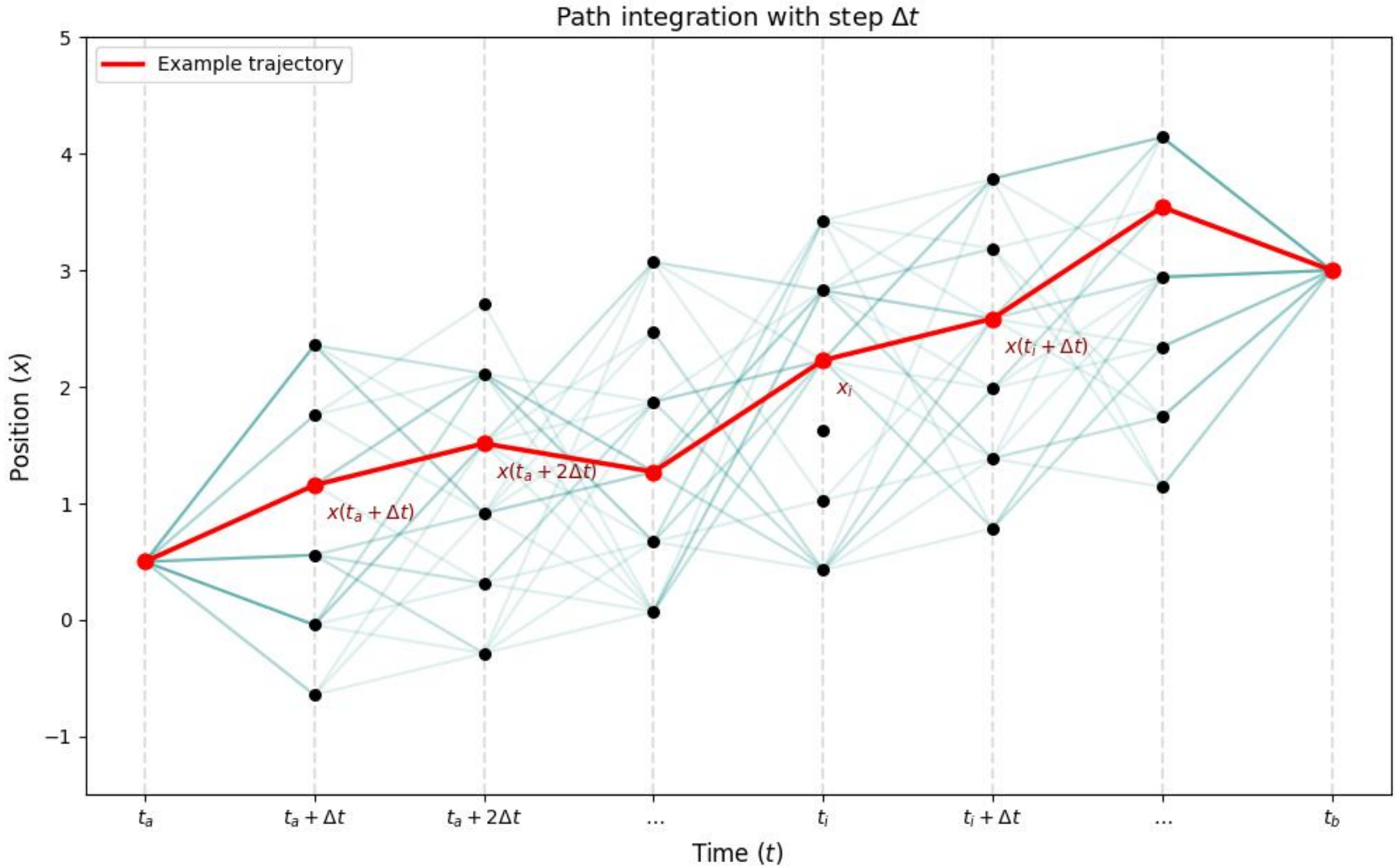
$$G(X,X') = \int DX \exp\{ i/h S_d(X,X') \}$$

$$DX = \Pi_t A dx, \qquad A=(2\pi i h\gamma)^{-1/2}$$

Path integral formulation



Finite approximation



The limit does’t exist

Feynman idea: to use

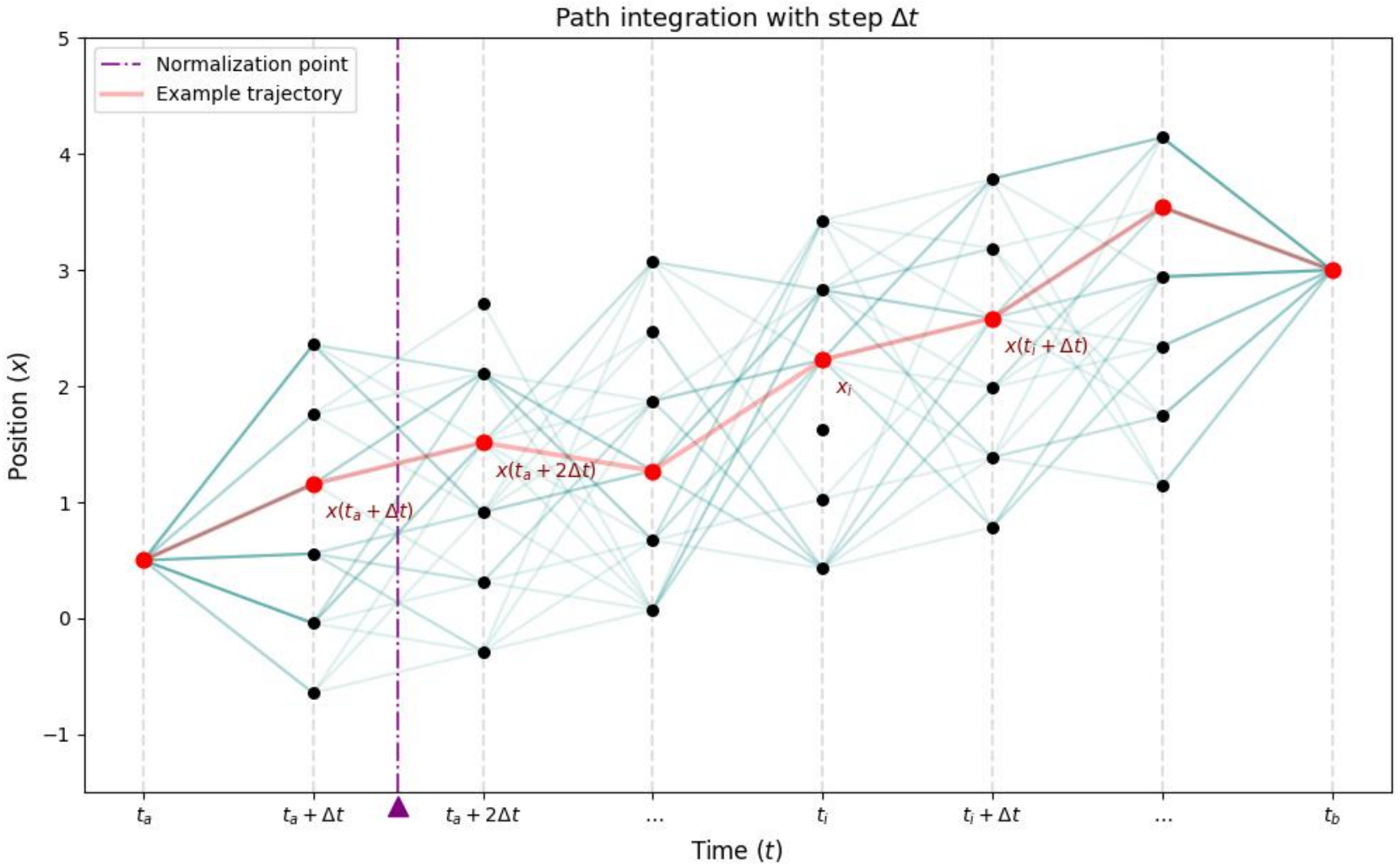
$(2\pi h/i\gamma)^{-1/2} = \int dp\ EXP(\ i/h\gamma[p - \{x' - x\}]^2)$

to get

$$\mathbf{G\ (X,P) = \int DPDX/(2\pi h)^s\ exp\{\ i/h\ \int XdP - i/h \int Hd\alpha + i\phi\ \}}$$

with $DPDX = \Pi_t\, dp\ dx$

This is not a solution even in finite dimensions



The problem

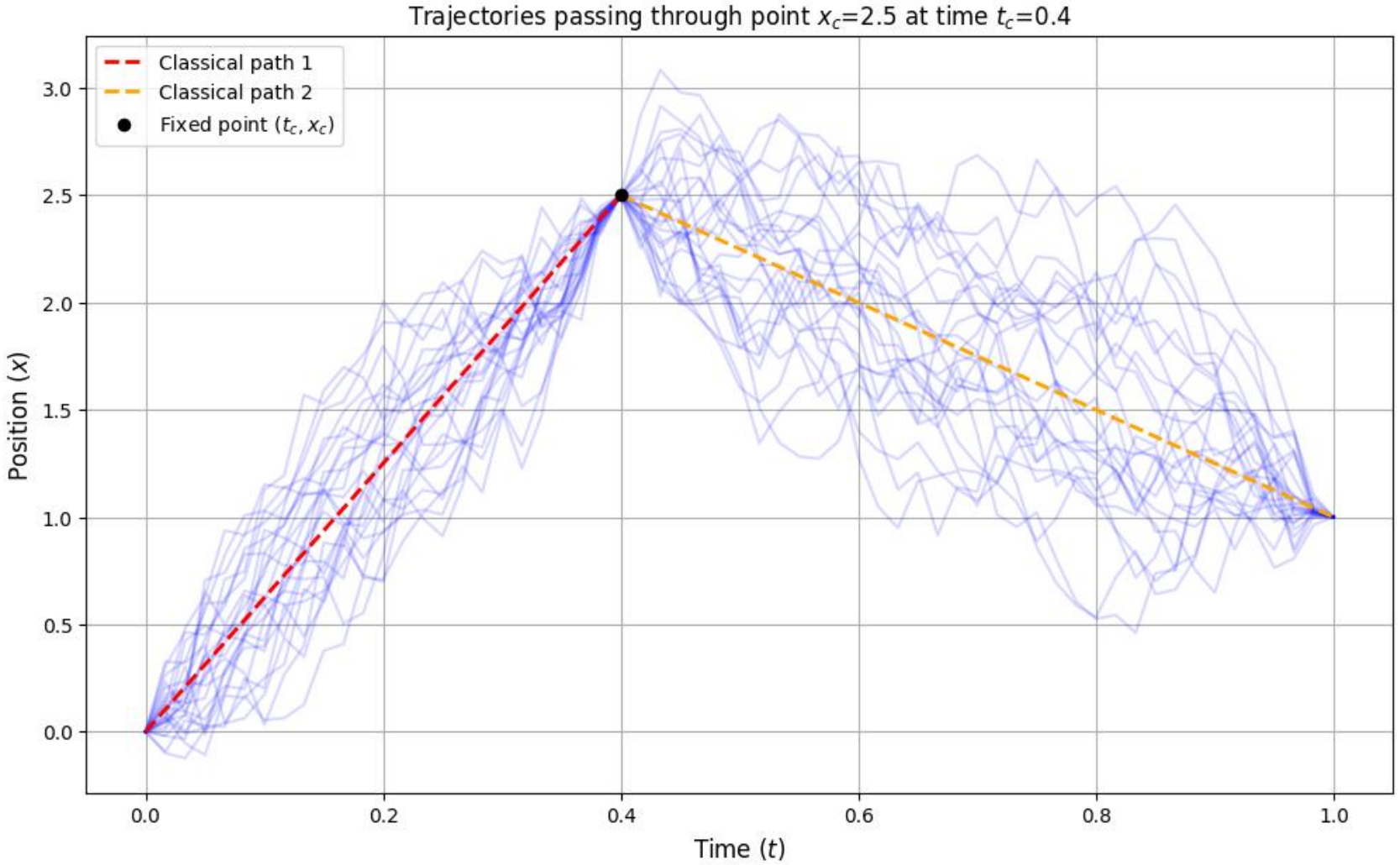
In finite presentation number of integrations over p and x is different

How to solve:

1. If ends are coordinates insert one more integral with delta function and make a shift of coordinate by t p (V. Popov, L. Faddeev)

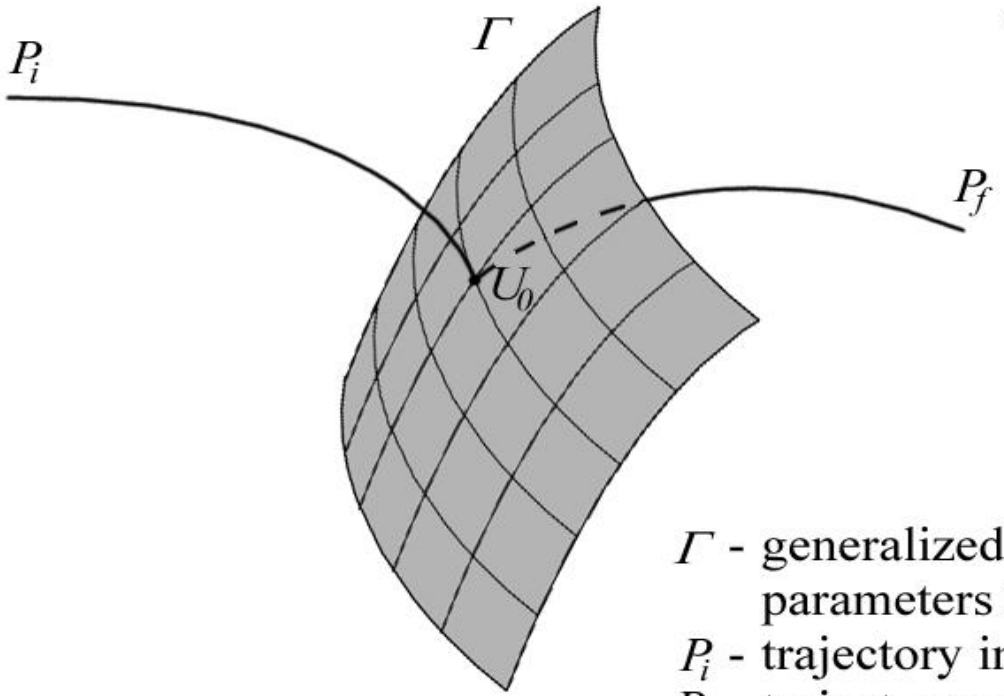
2. If ends are momentums take one internal integral, represent next momentum over previous and again make shift (A. Bogdanov et al.)

Pass through fixed point



The result is

$$A(i, f') = \langle G(i, t) G(t, f') \rangle$$



Γ - generalized impact parameters hyper-sphere
P_i - trajectory incoming branch
P_f - trajectory outgoing branch
U_0 - point in which Green function calculated

The importance of intermediate hypersurface

1. Possible change of coordinates
2. Taking care of quantum jumps
3. Separating different channels
4. Use of approximate methods

All hopes for speed - up are now in

$$G(P_*X) = \int DPDX \exp\{-i/h\, P_*X_* + i/hS|_* + i/hQ_*(Y_0 - Y_*)\}$$

There are three approaches, that show this expression to be mathematically correct without Feynman reasoning

As a result, we have functional delta function

$$G(P_*X) = \exp\{-i/h\ P_*X_* + i/hF_1|_* + i/hQ_*(Y_0 - Y_*)\}$$

This is correct for real vector fields

For complex fields we have to find four generators

Transition to field theory

- Postulate such an integral as a fundamental object of the theory
- Perform analytic continuation of the integration results

The final result is

$$A(P_i \rightarrow P_f) = \int dX_0 C\, \delta(X_0 P_0)\, \exp\{ i/h(P_f X_f - P_i X_i)$$

$$+ i/hF_1 |_* + i/hF_1 |_* + i/hQ_i(Y_0 - Y_i) + i/hQ_f(Y_f - Y_0) \}$$

$$F_1: \quad H(X, \partial F_1/\partial x) = E$$

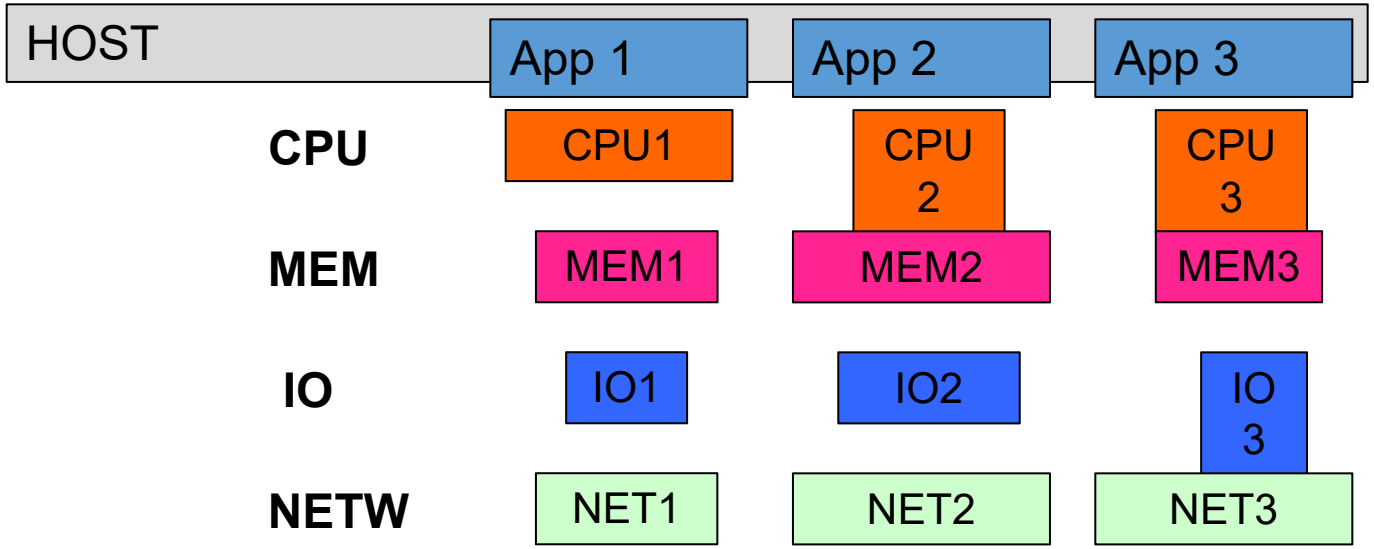
–

Look at this from the point of view of HPC

1. You have an integral with up to four PDE of the first order
2. Nothing can be more parallel
3. PDE of the first order is equivalent to the system of the ODE
4. Algorithm is well suited for hybrid system

Why virtual clusters?

- Precise control on allocated resources (CPU, memory, etc)
- Applications get exactly what they need (or what they request): one app needs fast disk IO and not much CPU, another one – fast network and fast CPU with no disk IO, etc
- Capacity of unclaimed resources available for other applications on a limited set of hardware



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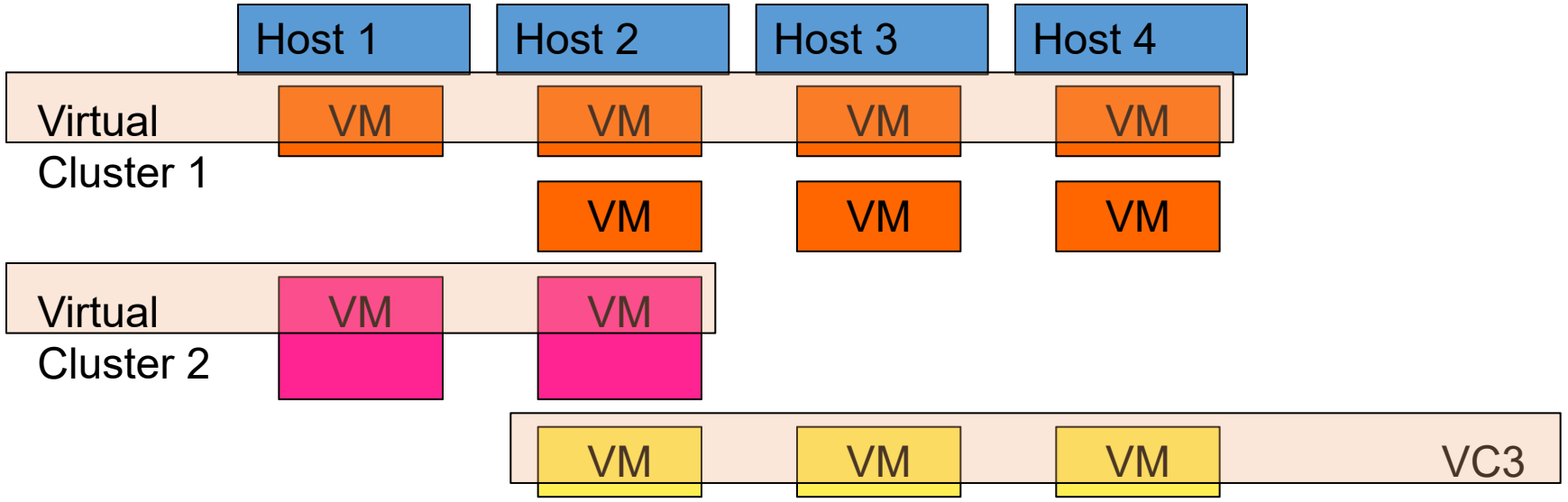
Virtual clusters

- Collection of virtual machines working together to solve a computational problem
- Can be configured by advanced users; they know exactly what they want (CPU, memory, IO, network)
- Can be flexibly adjusted to the needs of an application

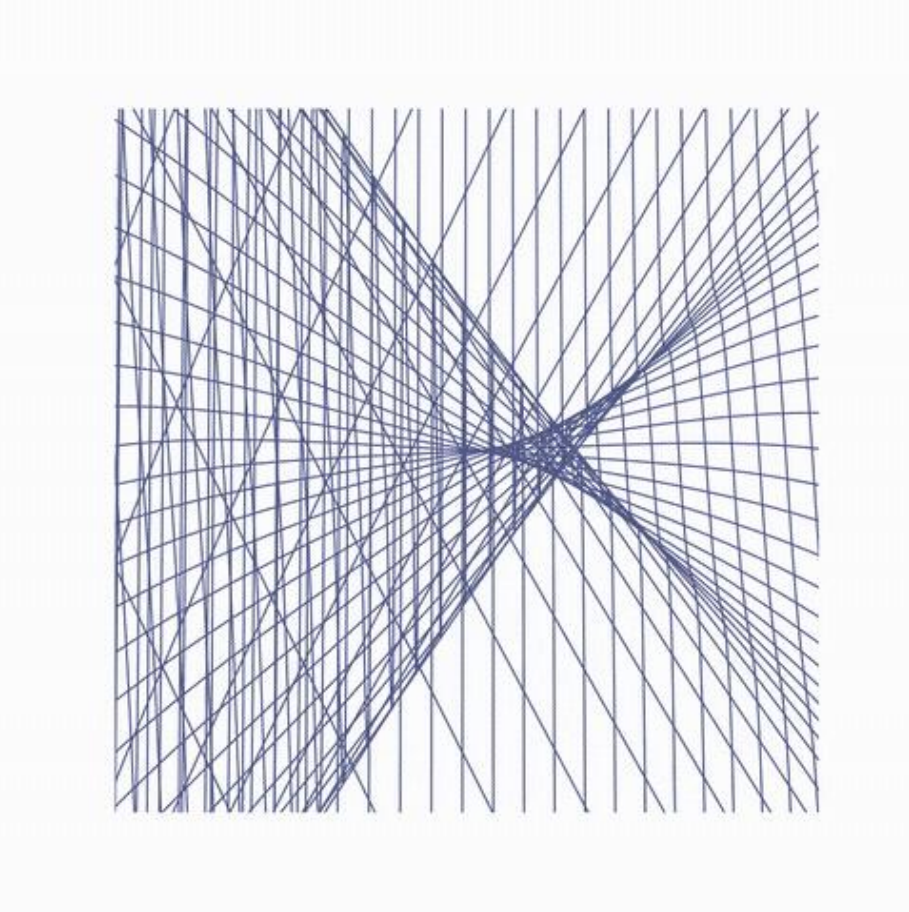


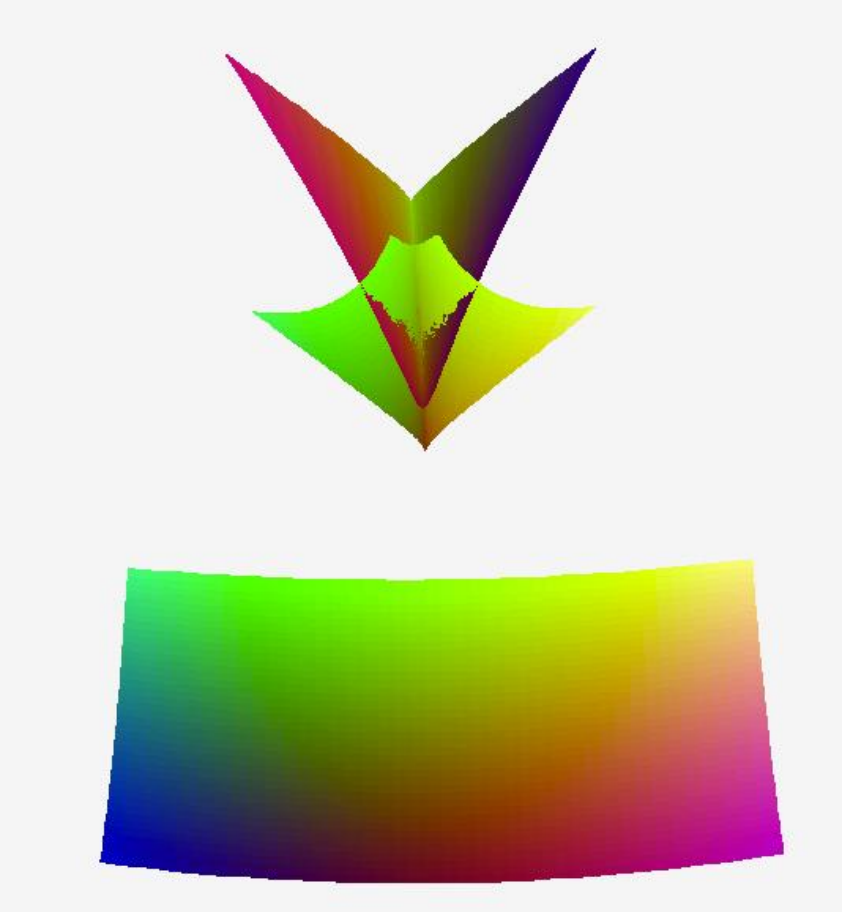
A. Bogdanov et al. Virtual Supercomputer as optimization tool

Before computations to make preliminary analysis

1. To use catastrophe theory
2. To choose one of the Toma elementary catastrophe
3. To make representation of the integral via one of functions, connected to elementary catastrophes

See couple of examples





Does it like a qualitative theory of PDE

If it is not, I will be surprised

To move forward, we have

1. To build functional representation for other PDE.
2. To find proper procedures for reduction

Thank you for attention!

1. It was my personal opinion
2. I can very well agree to other opinions
3. Only discussion will generate ideal approach