III. Neutron stars and nuclear equation of state

E.E. Kolomeitsev

BLTP, JINR, Dubna

To describe neutron stars and their evolution:
we have to construct
a bridge betweem micro and macro worlds

20 orders in distance: from ~10^{-13} cm to ~ 10^{3} km

15 orders in density: from mean solar density ~ g/cm^3 to ~10^{15} g/cm^3

40 orders in time: from 10^{-23} s to ~10^{10} $years$

60 orders in mass/energy: from m_π~10^{-25} g to 10^{33} g

Strong,
electromagnetic,
weak and
gravitational interactions are involved

Units & Numbers

speed of light: $c = 3 \times 10^{10} \, \frac{\mathrm{cm}}{\mathrm{s}}$

gravitational constant: $G = 6.7 \times 10^{-8} \, \frac{\mathrm{cm}^3}{\mathrm{g\,s}^2}$

Planck constant: $\hbar c = 197.327 \, \mathrm{MeV\,fm}$

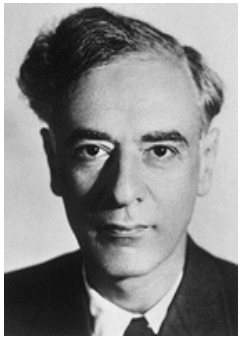
electrodynamic fine structure constant: $\alpha = \frac{e^2}{\hbar c} = \frac{1}{137}$ $\qquad e^2 = 1.44 \, \mathrm{MeV\,fm}$

gravitational fine structure constant: $\alpha_G = \frac{G\, m_n^2}{\hbar c} \simeq 10^{-38}$

nuclear matter density: $\varepsilon_0 \approx m_N \, \rho_0 \; = \; 940 \, \mathrm{MeV} \; \times 0.17 \, \mathrm{fm}^{-3} = 2.8 \times 10^{14} \, \frac{\mathrm{g}}{\mathrm{cm}^3}$

solar mass: $M_\odot = 2.0 \times 10^{33} \, \mathrm{g}$

Courageous theorists



Lev Landau (Phys. Z. Sowjetunion, 1, 285, 1932) speculated that one could compose a stellar object out of neutral particles held by gravity



Discovery of neutron 1932 by James Chadwick

Neutron would be a good candidate to build up
"unheimliche Sterne" (weird stars)
Landau @ seminar in NBI, 1932





Baade and Zwicky @ Stanford Meeting, 15-16 Dec. 1933

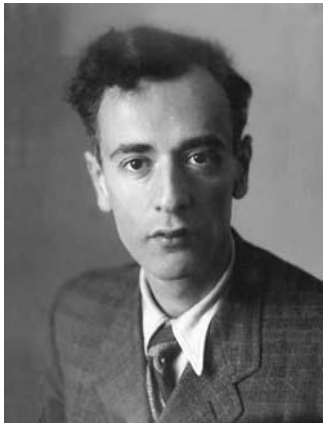
*"... With all reserve we advance the view that **supernovae** represent the transitions from ordinary stars into **neutron stars**, which in their final stages consist of extremely closely packed neutrons..."*

Courageous theorists



Friedrich Hund 1936
role of inverse β-decays in the stability of neutron stars

Landau, Nature 1938, “Origin of stellar energy”







Tolman, Oppenheimer & Volkoff 1939
calculated properties of neutron stars:
(only neutrons) R~10 km and M~1 M_{sol}

Courageous sinologist in Leiden. 1942



J.J.L. Duyvendak

"Amice, I have succeeded in finding another place where your Nova is mentioned. There exists an extensive work, of which a facsimile edition was published only a few years ago (and which could not have been known to earlier researchers), treating the institutions of the Sung dynasty, which includes the year 1054. The name is Sung Hui Yao. In vol. 54 of this work,..."

letter to Jan Oort, July 30, 1940

On this, Oort added in pencil,

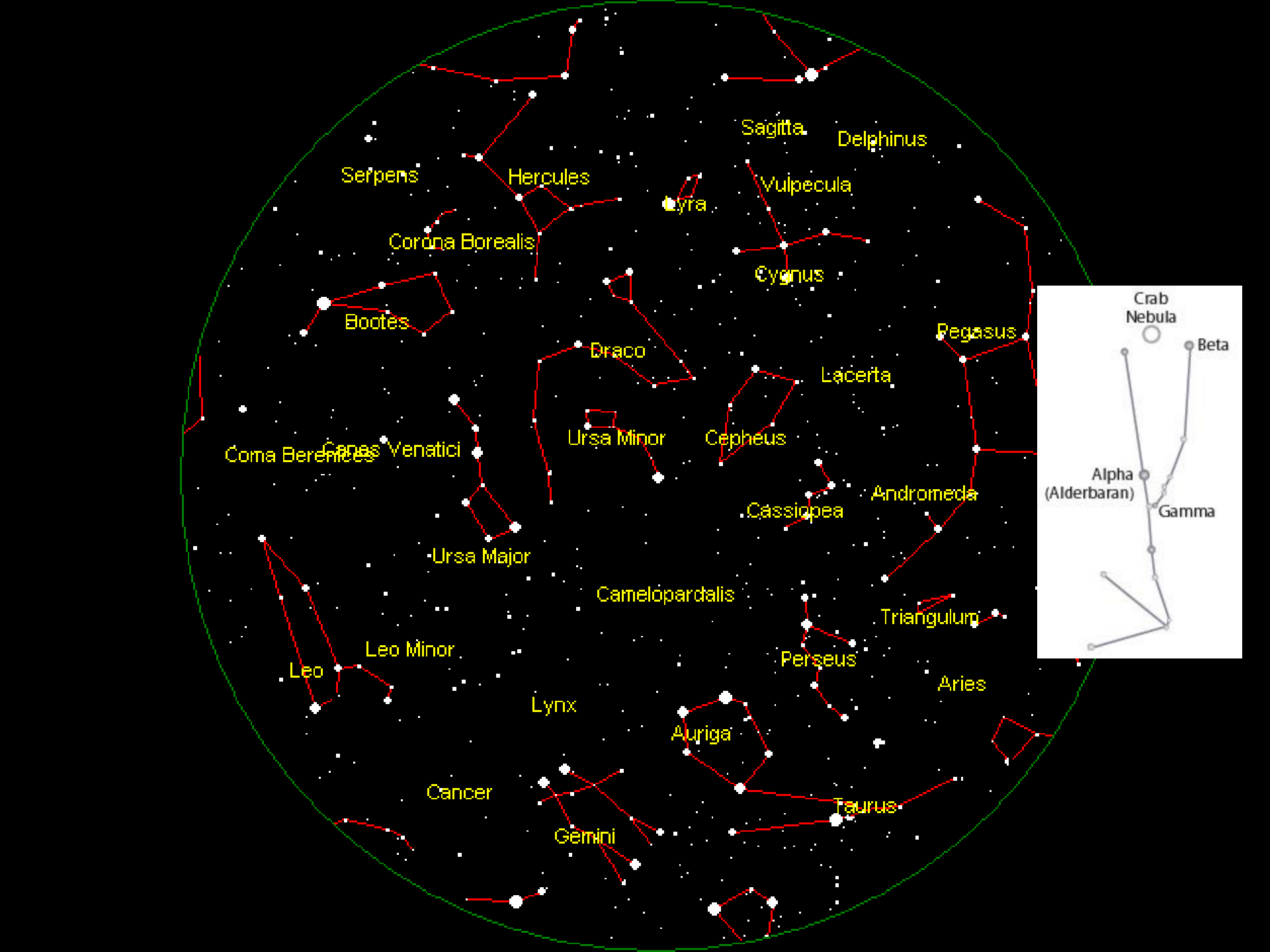
"Must write an airmail letter about this to Mayall and Baade, as soon as I am back in Leiden."

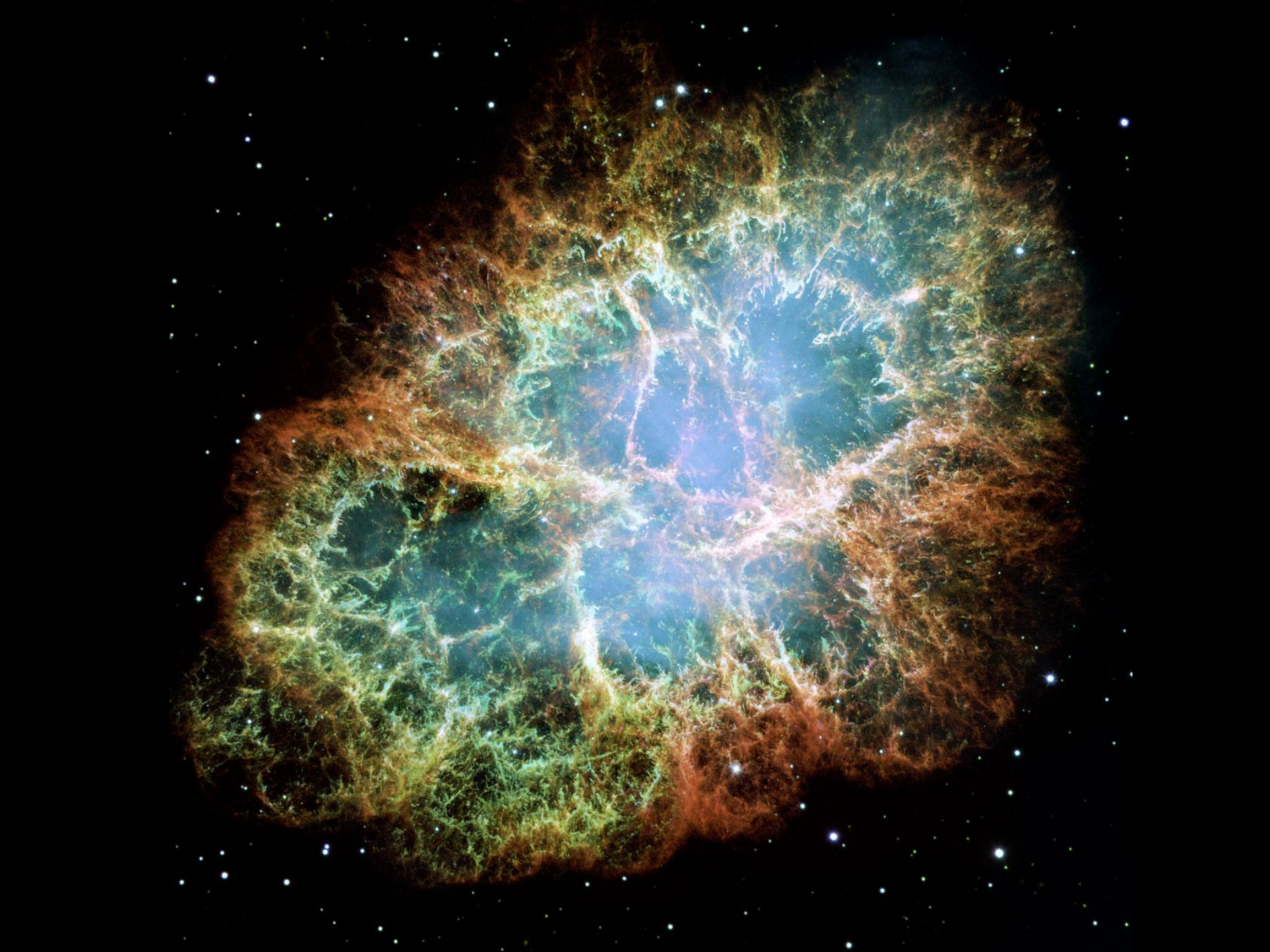


Jan Oort

The original text (quoted by Biot from Ma Tuan-lin's *Wen-hsien T'ung-k'ao*) is found in the *Sung-shih* ("History of the Sung Dynasty") by T'o-t'o (1313 to 1355).[2] It runs:

In the 1st year of the period *Chih-ho* [1054], the 5th moon, the day *chi-ch'ou* [July 4] [a guest-star] appeared approximately several inches south-east of *T'ien-kuan* [ζ Tauri]. After more than a year[3] it gradually became invisible.


Sagitta
Delphinus
Serpens
Hercules
Vulpecula
Lyra
Corona Borealis
Cygnus
Bootes
Pegasus
Draco
Lacerta
Ursa Minor
Cepheus
Coma Berenices
Canes Venatici
Andromeda
Cassiopea
Ursa Major
Camelopardalis
Triangulum
Leo Minor
Perseus
Leo
Aries
Lynx
Auriga
Cancer
Taurus
Gemini
Crab
Nebula
Beta
Alpha
(Alderbaran)
Gamma



Baade, Astrophysical J. 98, 188 (1942)

probed deeper into the nebula, observing that a prominent star near the nebula's center might be related to its origin.

Crab Nebula can be a result of an explosion happened about 1000 years ago

Six years later, scientists discovered that the Crab was emitting among the strongest radio waves of any celestial object.

Baade noticed in 1954 that the Crab possessed powerful magnetic fields.

1963 a high-altitude rocket detected X-rays from the nebula.

On the verge of discovery…



… somewhere in UK



Jocelyn Bell

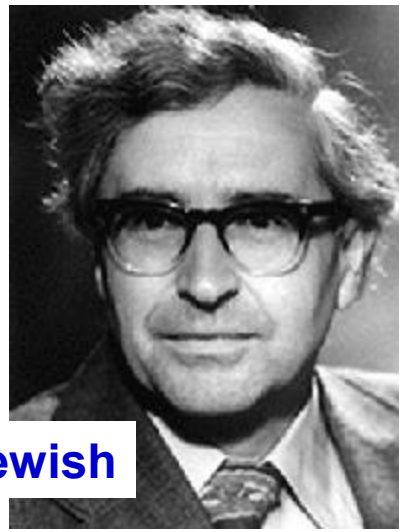


1967

: The Cambridge interplanetary scintillation telescope ("the 4-acre array")

While studying distant galaxies, Jocelyn Bell noticed small pulses of radiation when the telescope was looking at a particular position in the sky.

For a short time she and A. Hewish thought the pulses might be coming from an extra-terrestrial civilizations.



Anthony Hewish

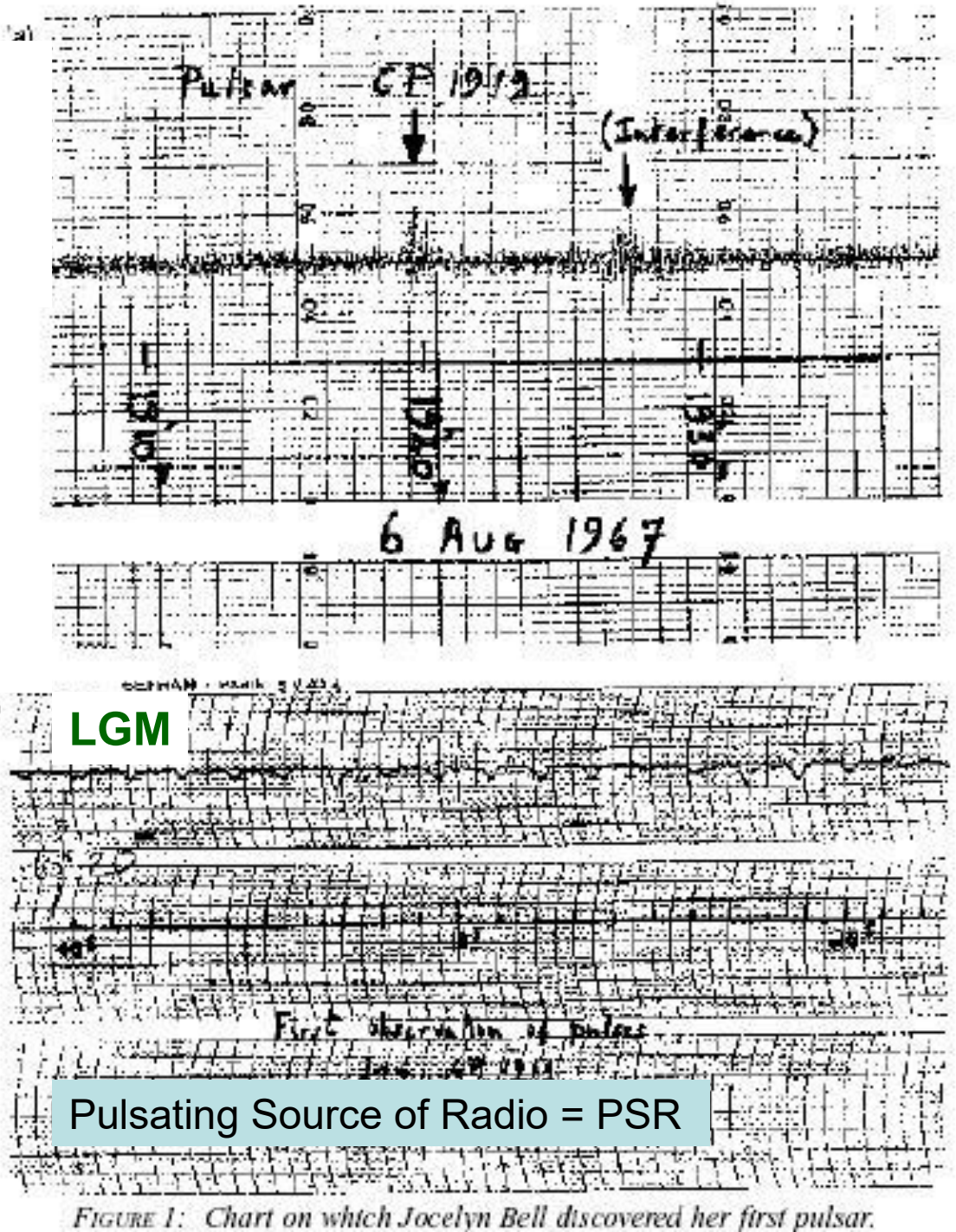


FIGURE 1: Chart on which Jocelyn Bell discovered her first pulsar.

Lighthouse model

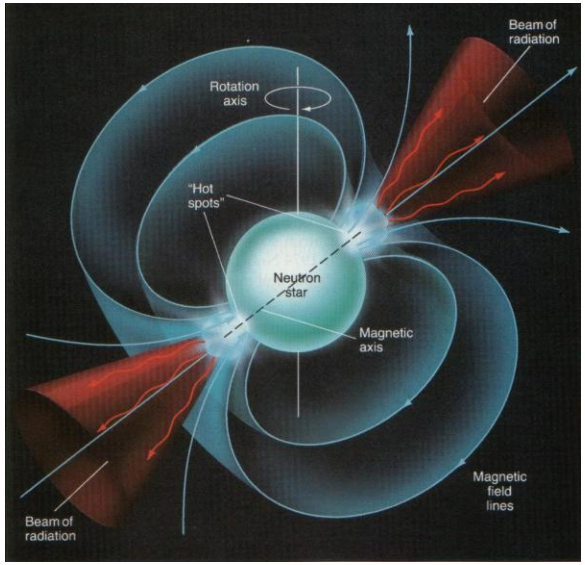
Energy Emission from a Neutron Star

It seems more rewarding therefore to look for some mechanisms by which the neutron star can release either its magnetic or its rotational energy or both. In this communication I would like to outline the principal features of a possible model of this kind.

F. PACINI

Center for Radiophysics and Space Research,
Cornell University,
New York.

[Nature, 216 (1967) 567]



Rotating Neutron Stars as the Origin of the Pulsating Radio Sources

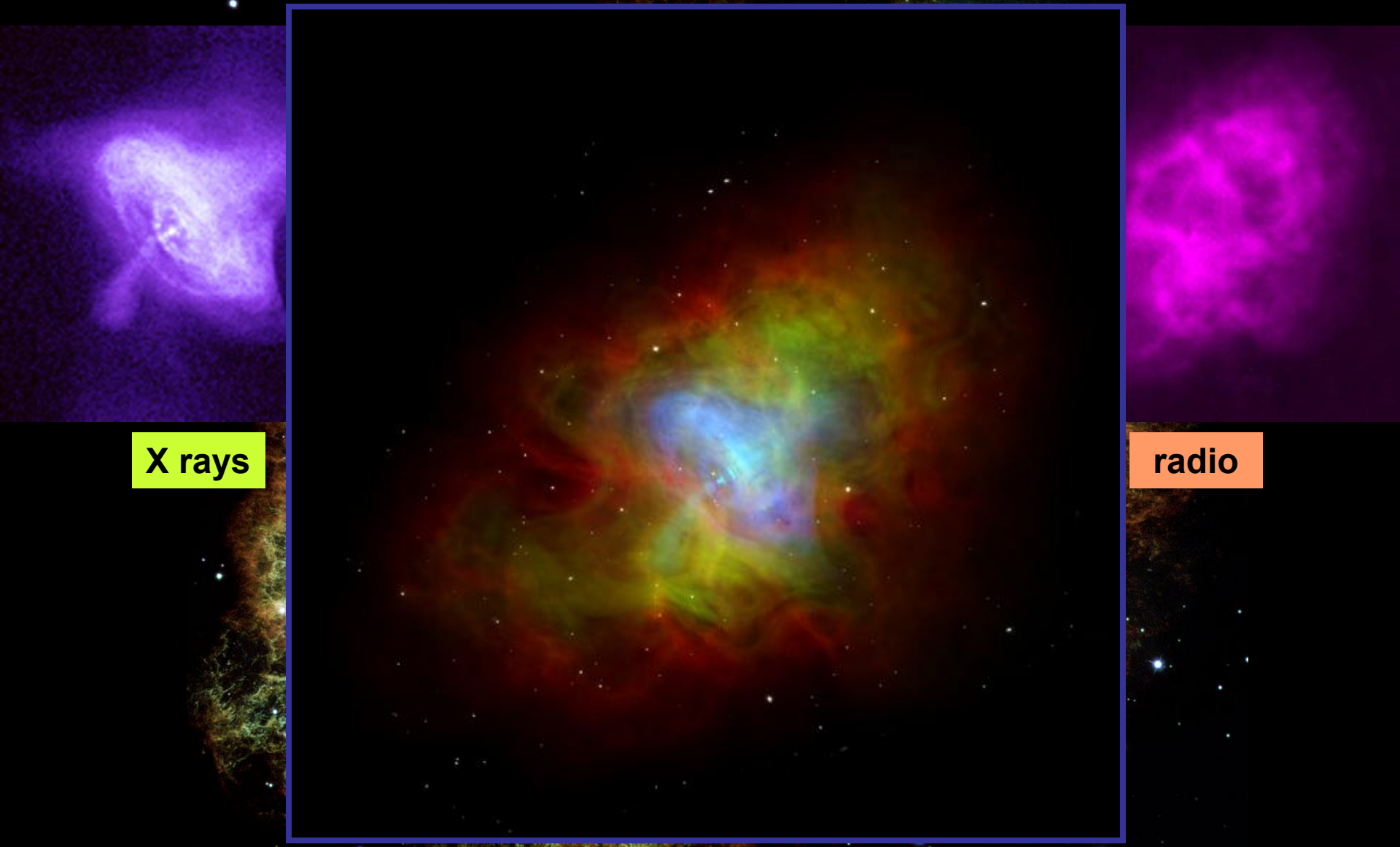
[Nature, 218 (1968) 731]

by

T. GOLD

Center for Radiophysics and Space Research,
Cornell University,
Ithaca, New York

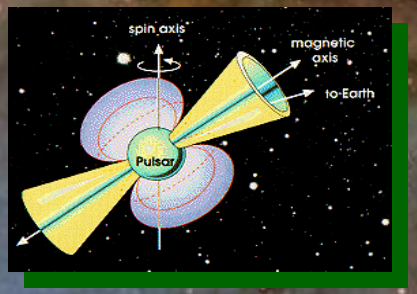
The constancy of frequency in the recently discovered pulsed radio sources can be accounted for by the rotation of a neutron star. Because of the strong magnetic fields and high rotation speeds, relativistic velocities will be set up in any plasma in the surrounding magnetosphere, leading to radiation in the pattern of a rotating beacon.



Crab's central pulsar was discovered in 1968 by radio astronomers.
The pulsar was then identified as a source of periodic optical and X-ray radiation.

Neutron Star Zoo

>2000 neutron stars in isolated rotation-powered pulsars
~ 30 millisecond pulsars



>100 neutron stars in accretion-powered X-ray binaries
~ 50 x-ray pulsar
intense X-ray bursters (thermonuclear flashes)



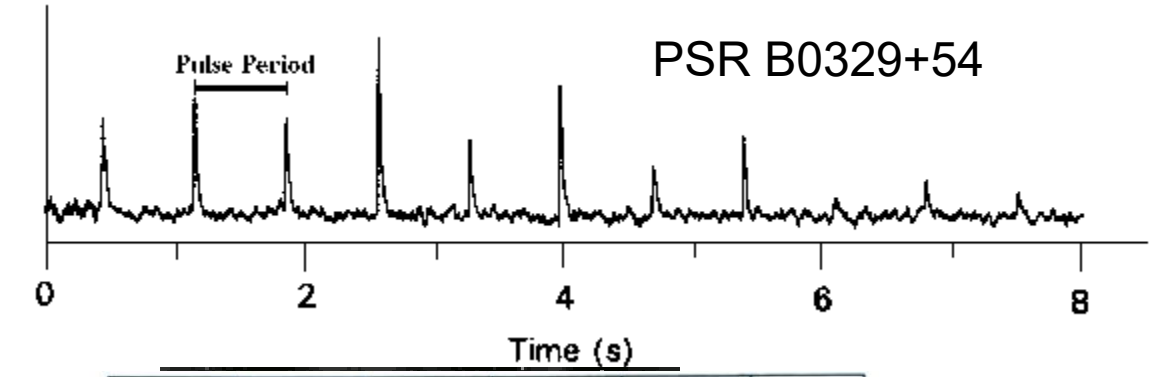
short gamma-ray bursts
neutron star -- neutron star,
neutron star -- black-hole mergers

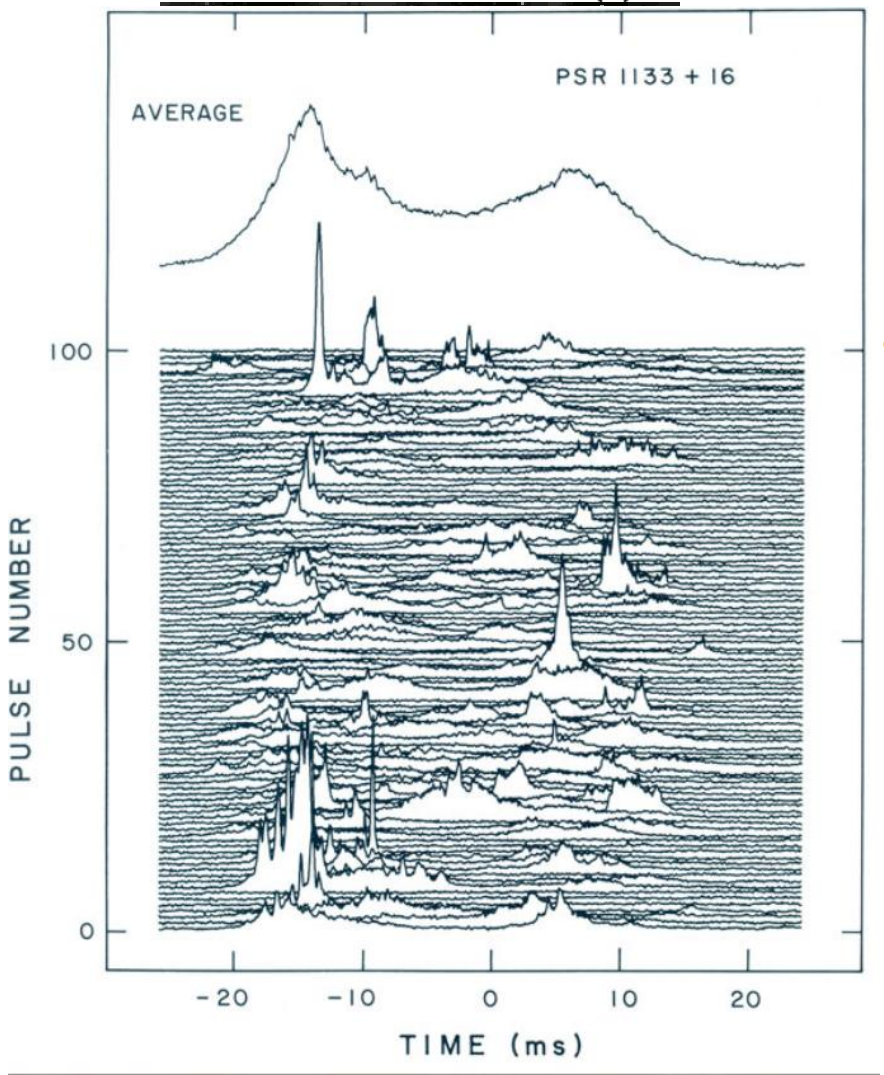


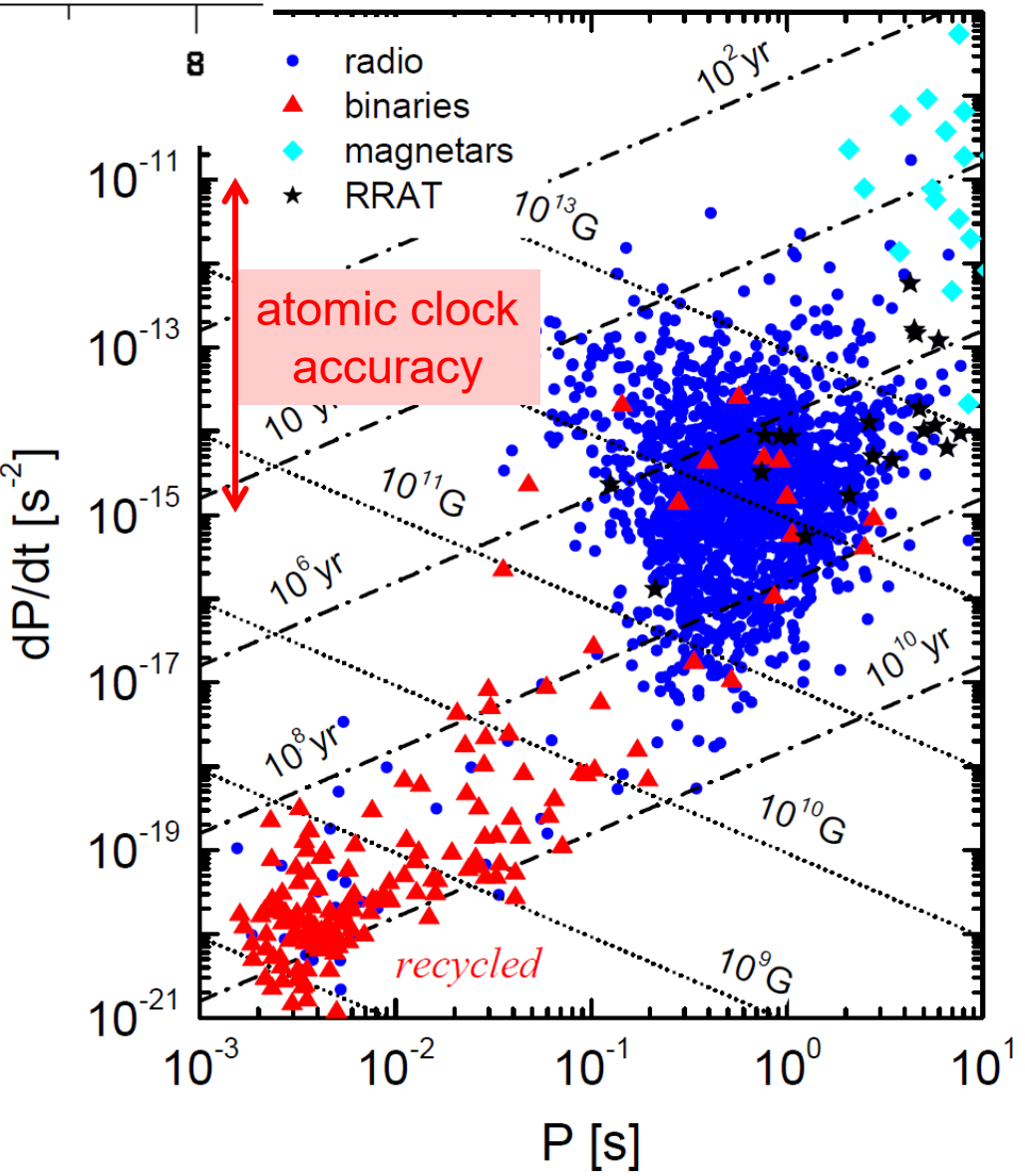
soft gamma-ray repeaters – magnetars
(super-strong magnetic fields)



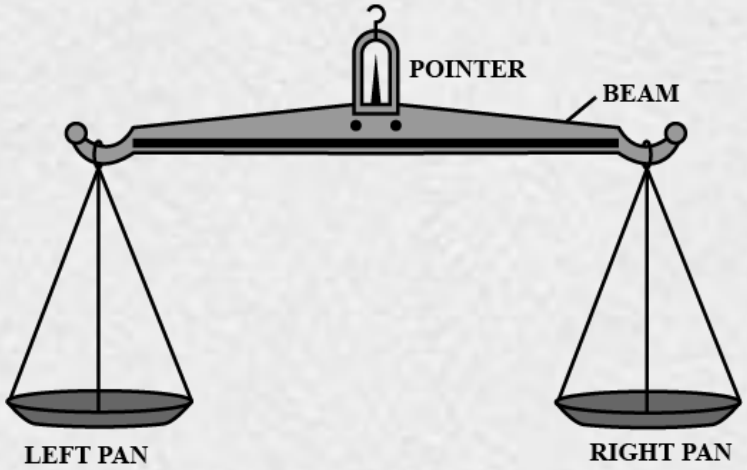
Pulsar timing







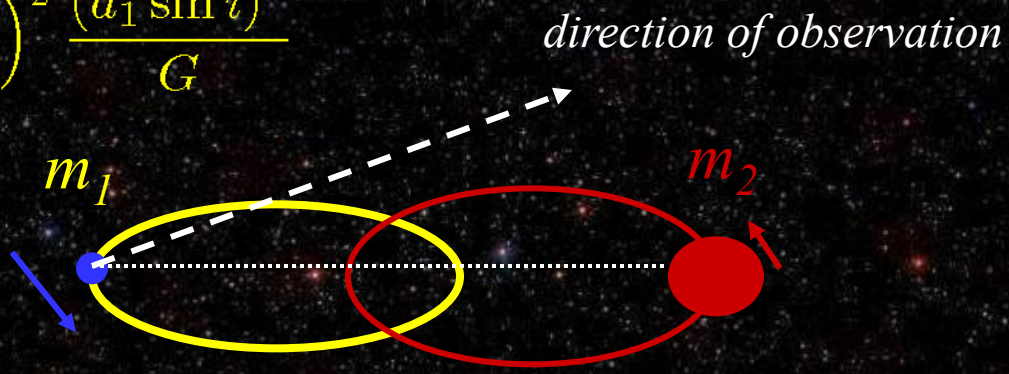
Neutron star masses



Measuring pulsar mass

Pulsar mass can be measured only in binary systems

$$\frac{(M_1 \sin i)^3}{(M_1 + M_2)^2} = \left(\frac{2\pi}{P_b}\right)^2 \frac{(a_1 \sin i)^3}{G}$$



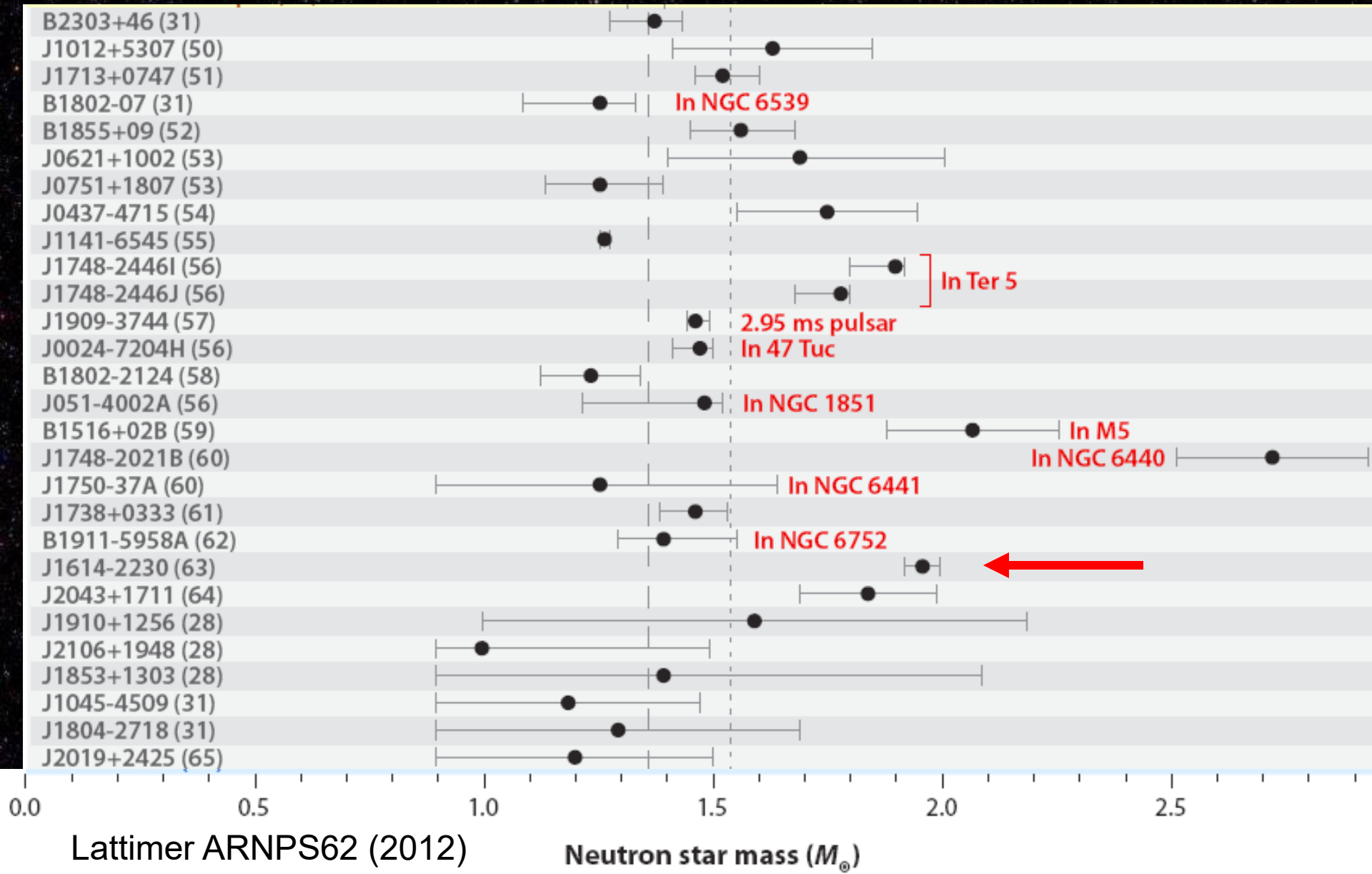
Newton gravity ⟶ 5 Keplerian orbital parameters:
orbital period, semi-major axis length,excentricity, …

Do not determine individual masses of stars and the orbital inclination.

Einstein gravity ⟶ 5 potentially measurable post-Keplerian parameters:
orbit precession, Shapiro delay, gravitational redshift, ….

Measurement of any 2 post-Keplerian parameters allows to determine the mass of each star.

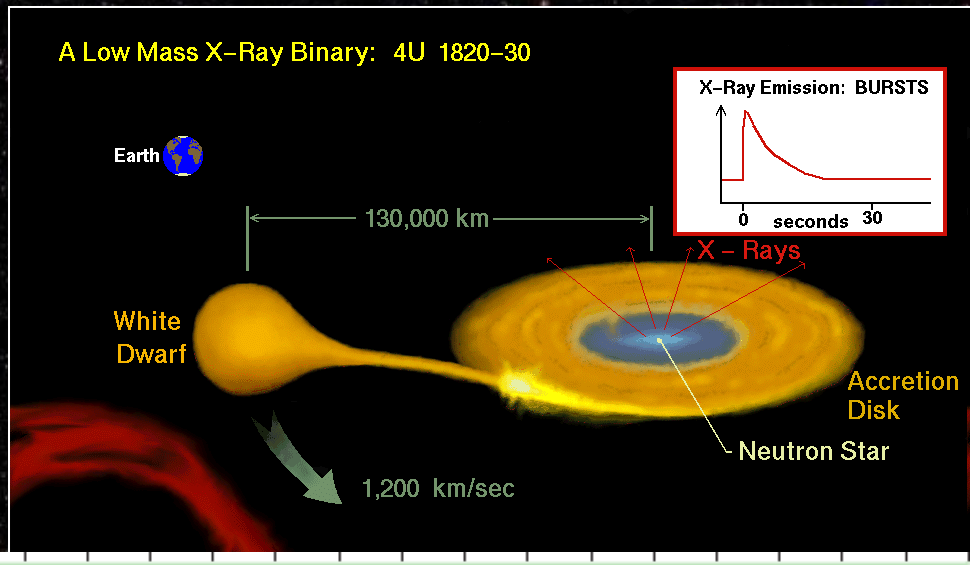
White dwarf -- neutron star binaries



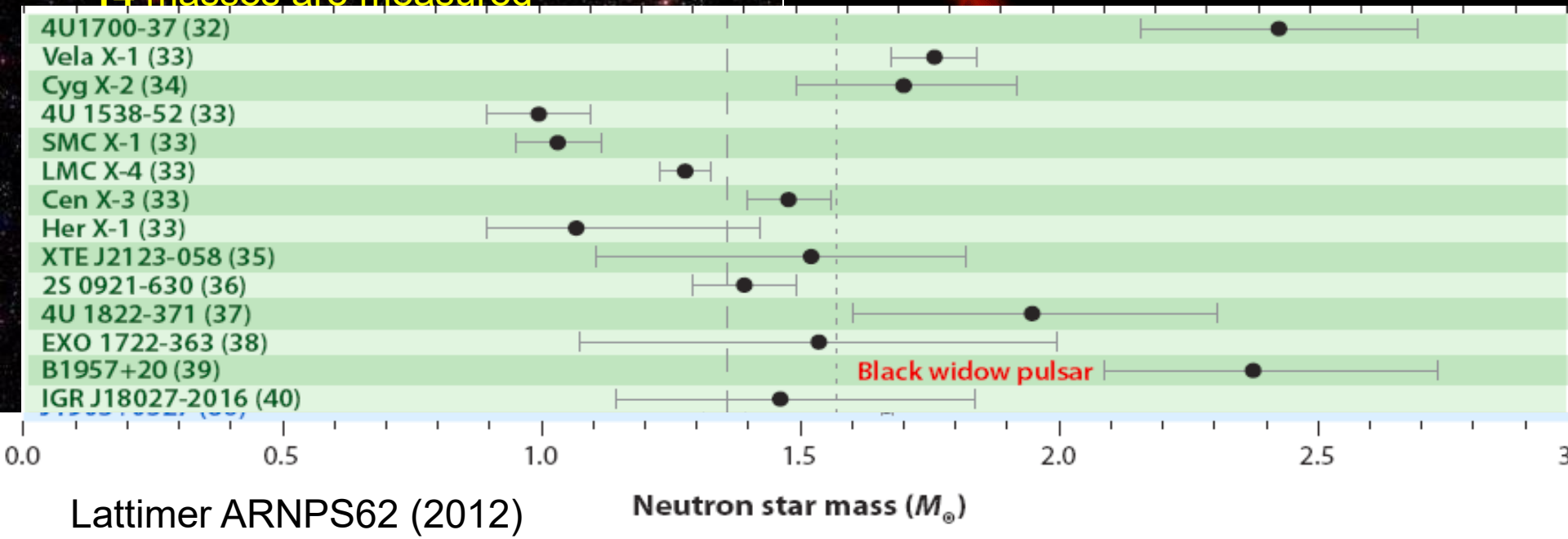
Lattimer ARNPS62 (2012)

Measuring pulsar mass

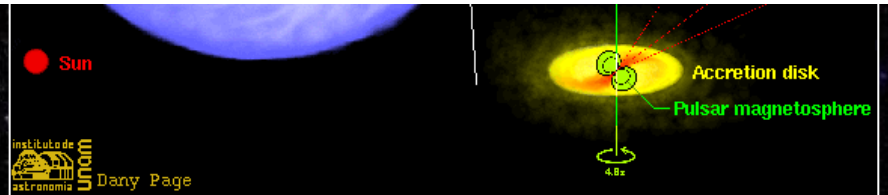
X-ray binaries



14 masses are measured



Lattimer ARNPS62 (2012)

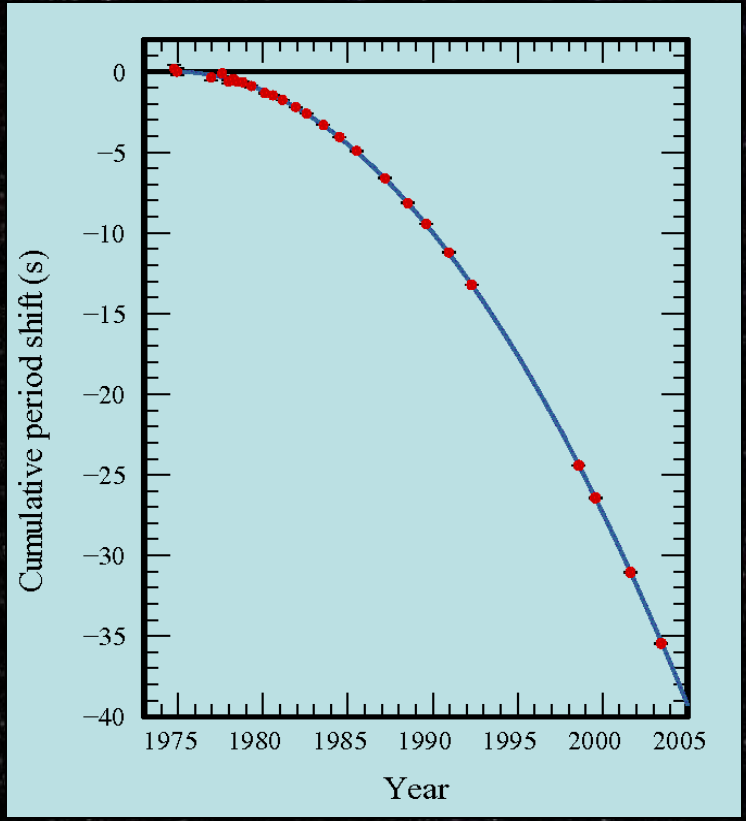


Measuring pulsar mass

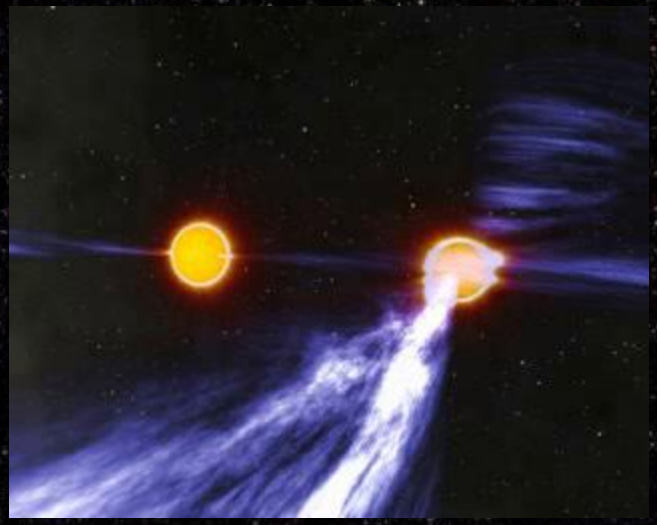
Double neutron star binaries

1974 PSR B1913+16 Hulse-Taylor pulsar

First precise test of Einstein gravitation theory



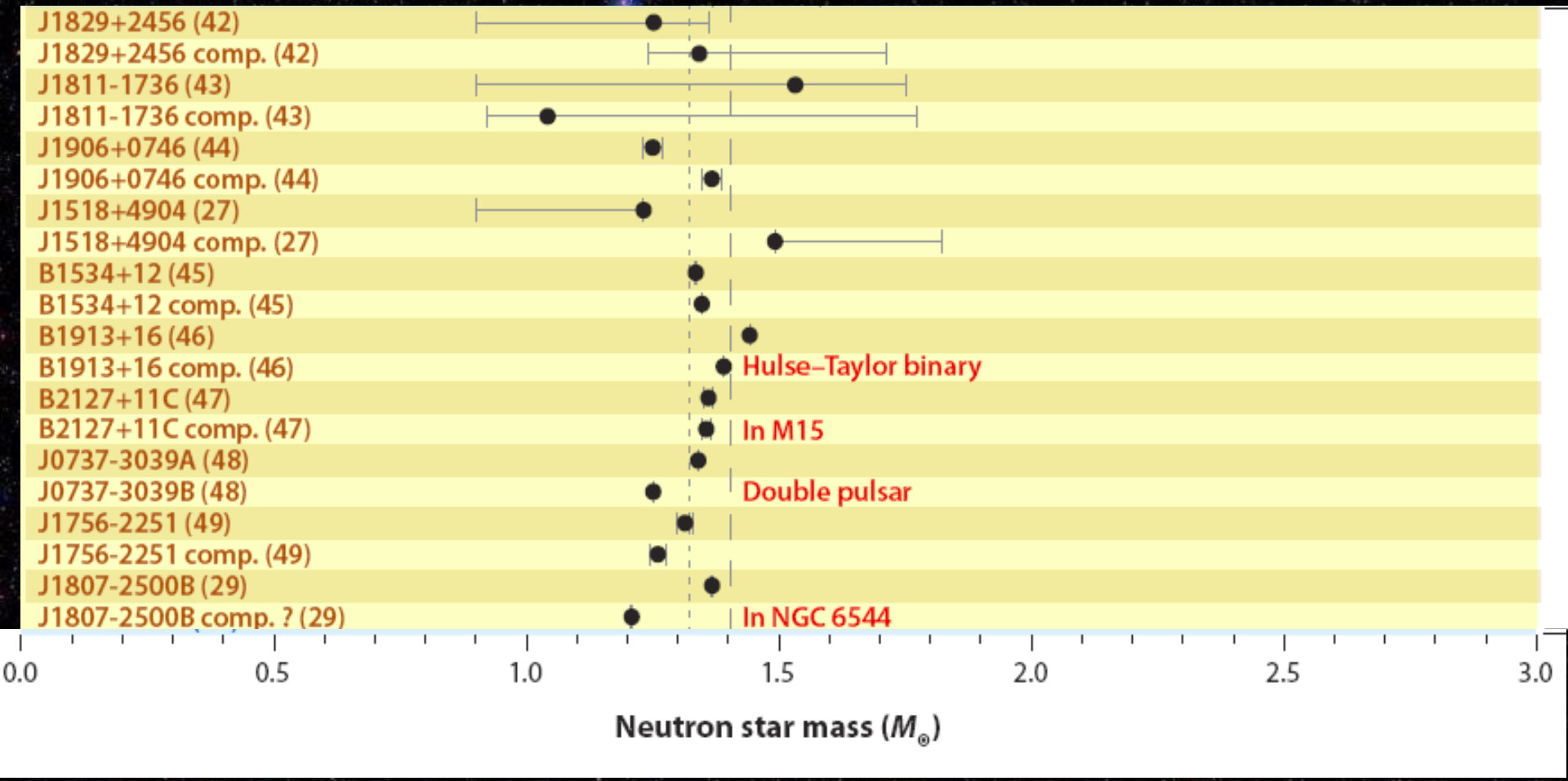
2003 J0737-3039 first double pulsar



Pulsar A: $P^{(A)}$=22.7 ms, $M^{(A)}$=1.338 M_{sol}
Pulsar B: $P^{(B)}$=2.77 ms, $M^{(A)}$=1.249±0.001 M_{sol}
Orbiting period 2.5 hours

[Nature 426, 531 (2003), Science 303, 1153 (2004)]

Double neutron star binaries



Most massive neutron stars



Pulsar J1614-2230

$$M = (1.97 \pm 0.04)\ M_{sol}$$

Highest well-known mass of NS

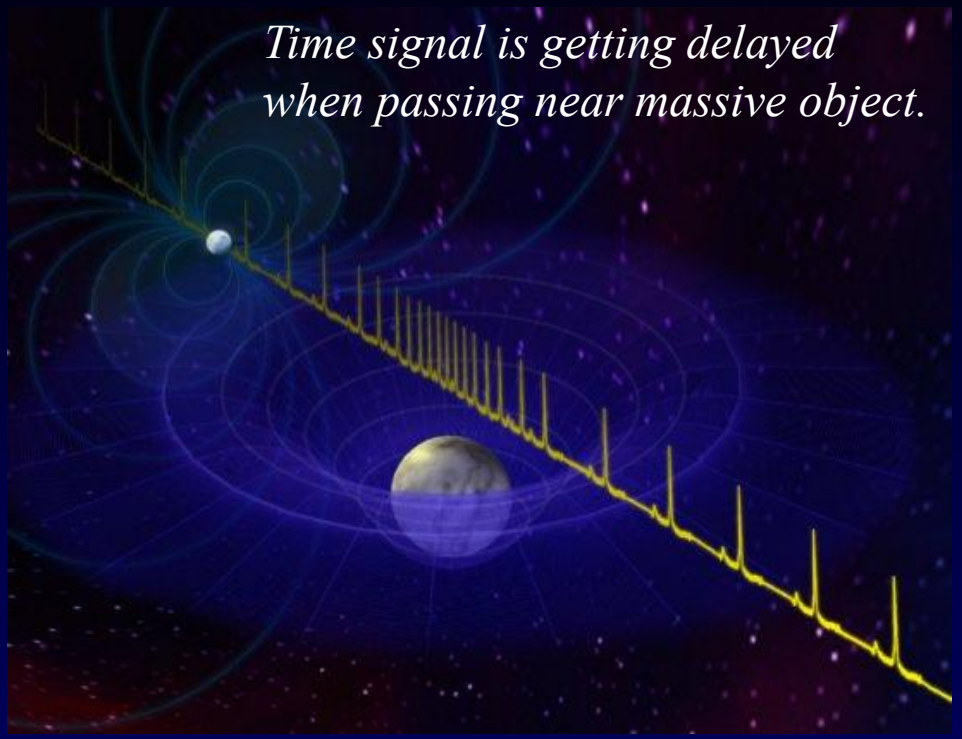
P.Demorest et al., Nature 467, 1081 (2010)

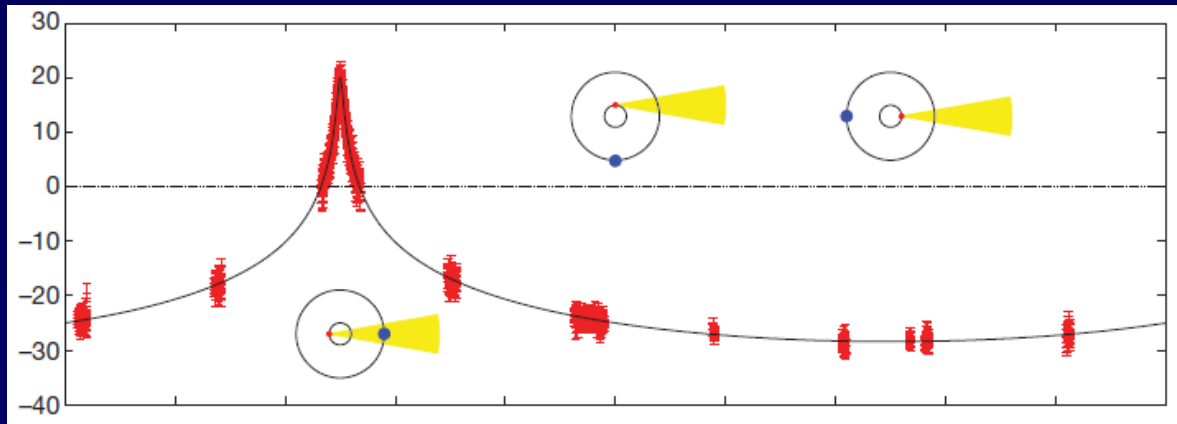
$$M = 2.14^{+0.20}_{-0.18}\ M_{sol} \quad (95\%)$$

H.T. Cromartie et al., Nat. Astr. (2019)

there are heavier, but far less precisely measured candidates)

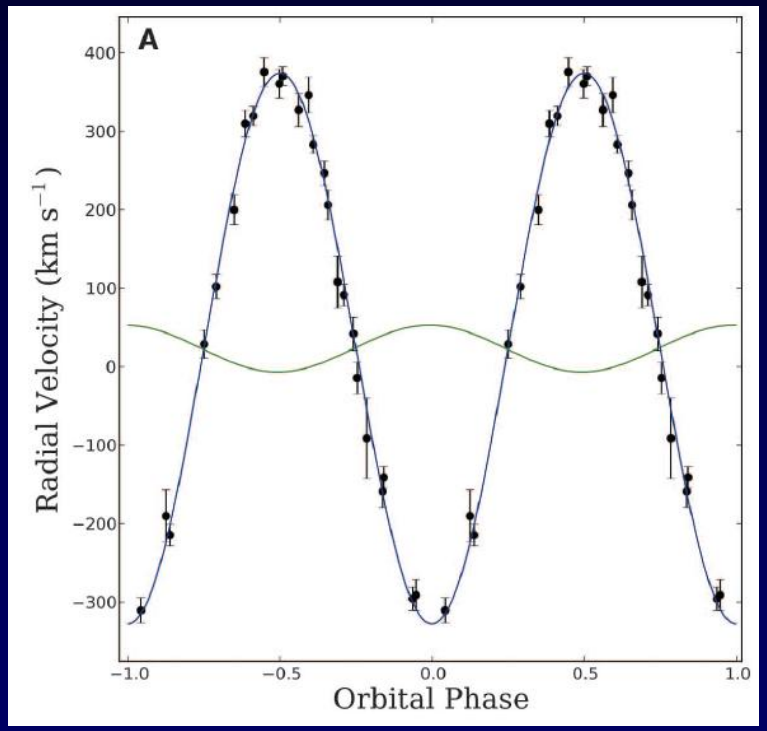
Measured Shapiro delay with high precision





Pulsar J0348+0432

$$M = (2.01 \pm 0.04)\ M_{sol}$$



Antonadis et al, Science 340, 1233232

pulsar-white dwarf binary

Measured phase-resolved spectra of the optical counterpart. hydrogen Balmer lines

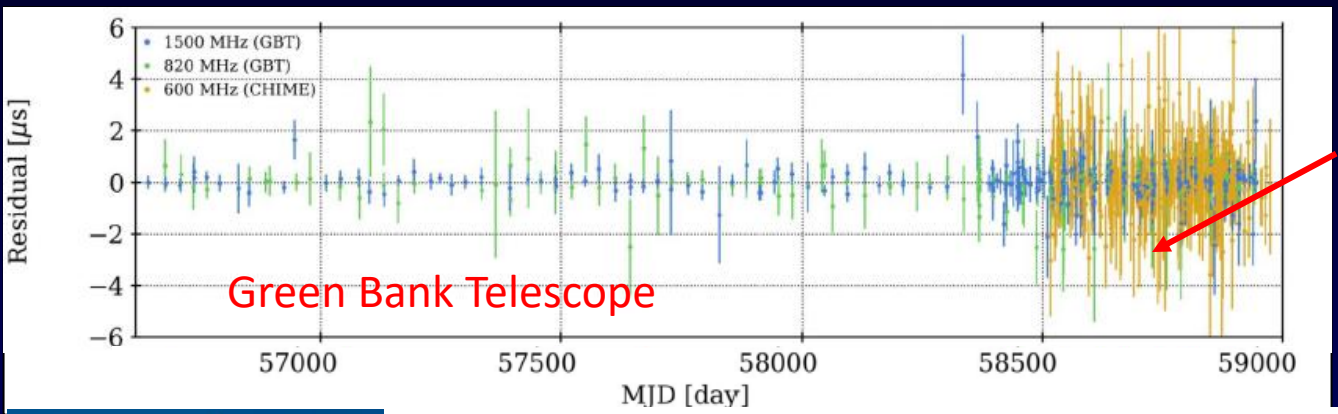
pulsar-white dwarf binary

Pulsar J0740+6620

$$\mathrm{M} = (2.08 \pm 0.07)\ \mathrm{M}_{\mathrm{sol}}$$

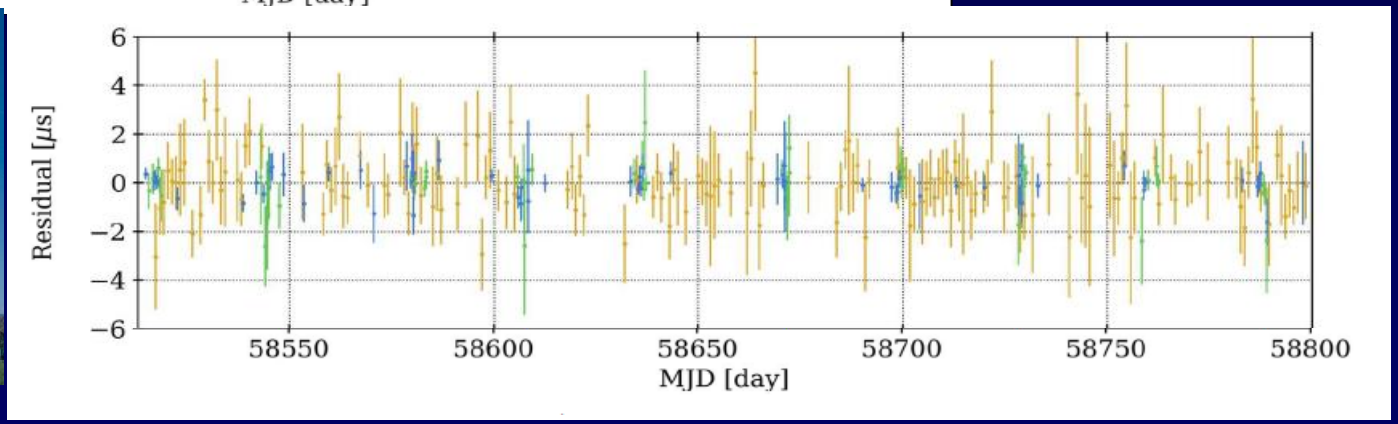
Shapiro delay





Canadian Hydrogen Intensity Mapping Experiment (CHIME) telescope





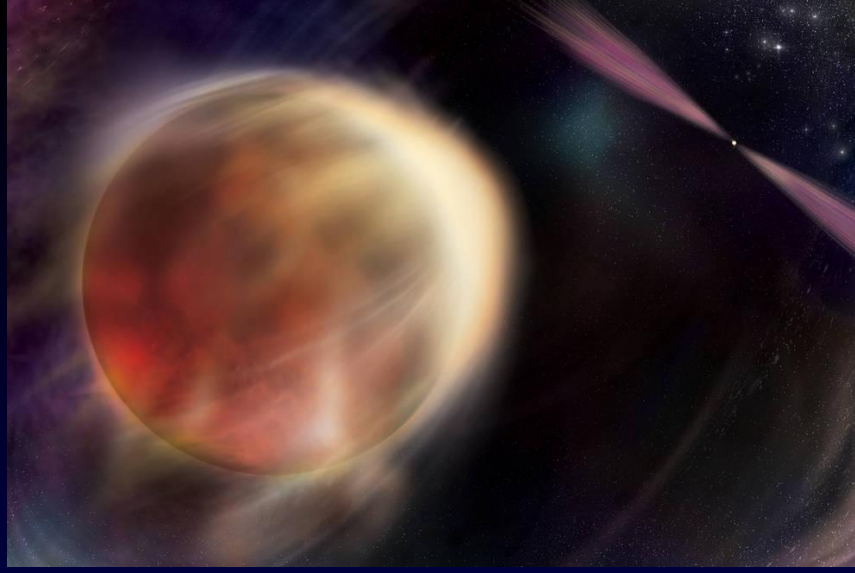
Fonseca et al, Astrophys. J. Lett., 915 (2021) L12

Pulsar J0952−0607

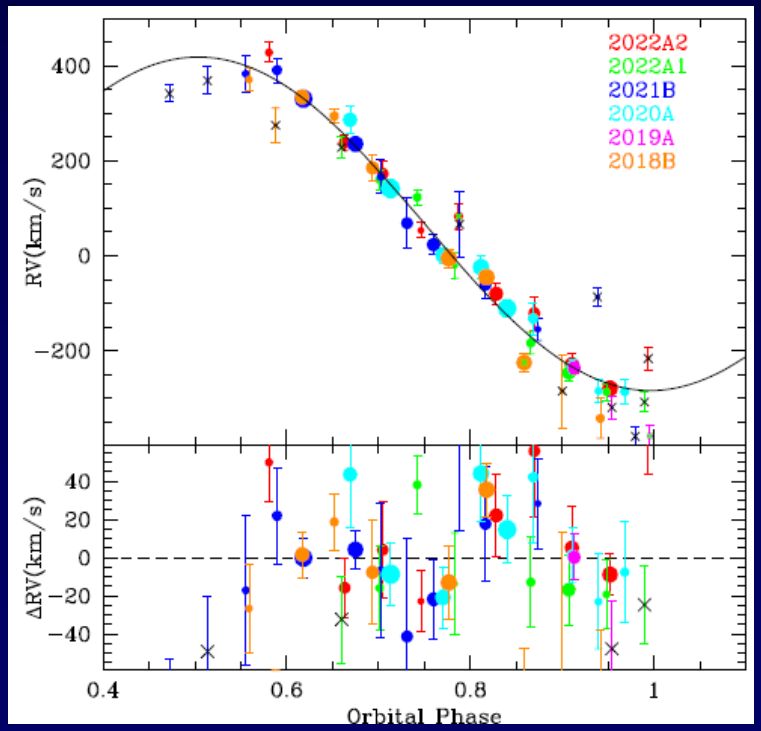
$$\mathrm{M} = (2.35 \pm 0.17)\ \mathrm{M_{sol}}$$

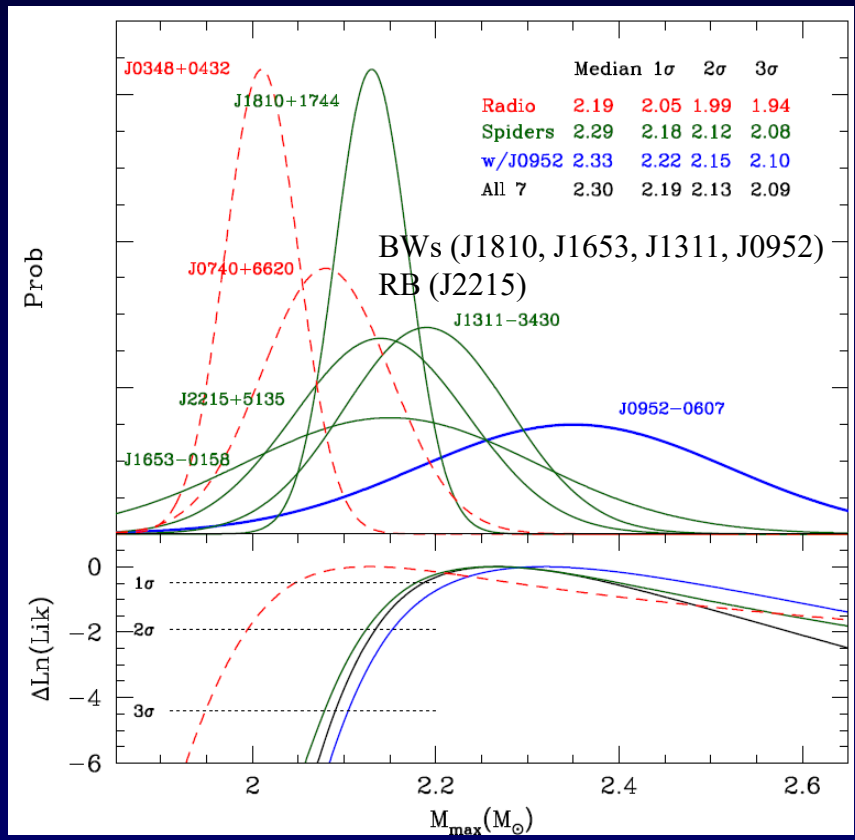
"Black widow" pulsar

the companion is being destroyed by the strong powerful outflows, or winds, of high-energy particles caused by the neutron star.

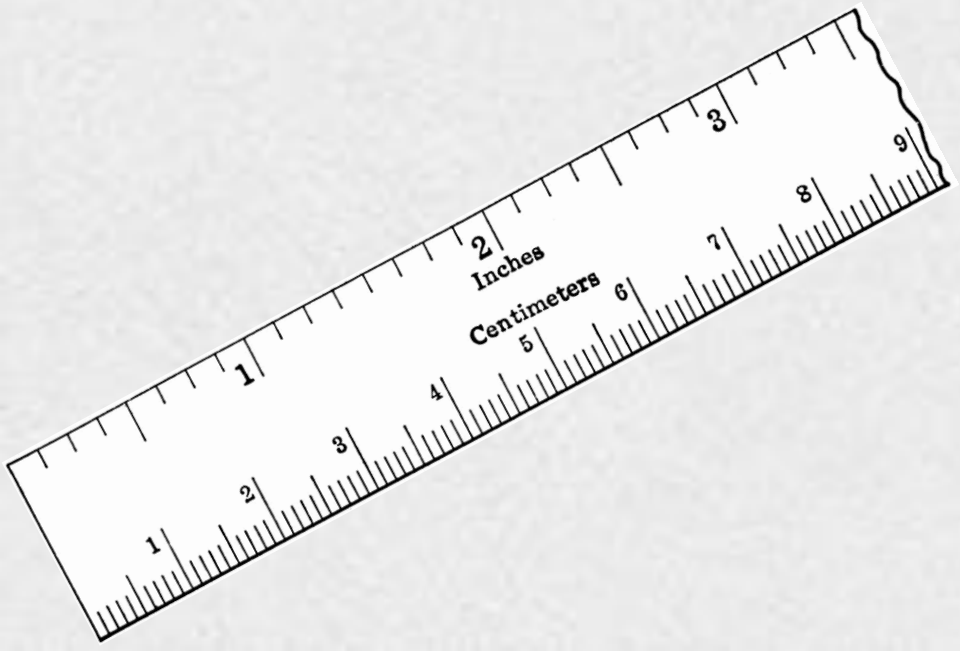


Radial velocity from the companion spectrum

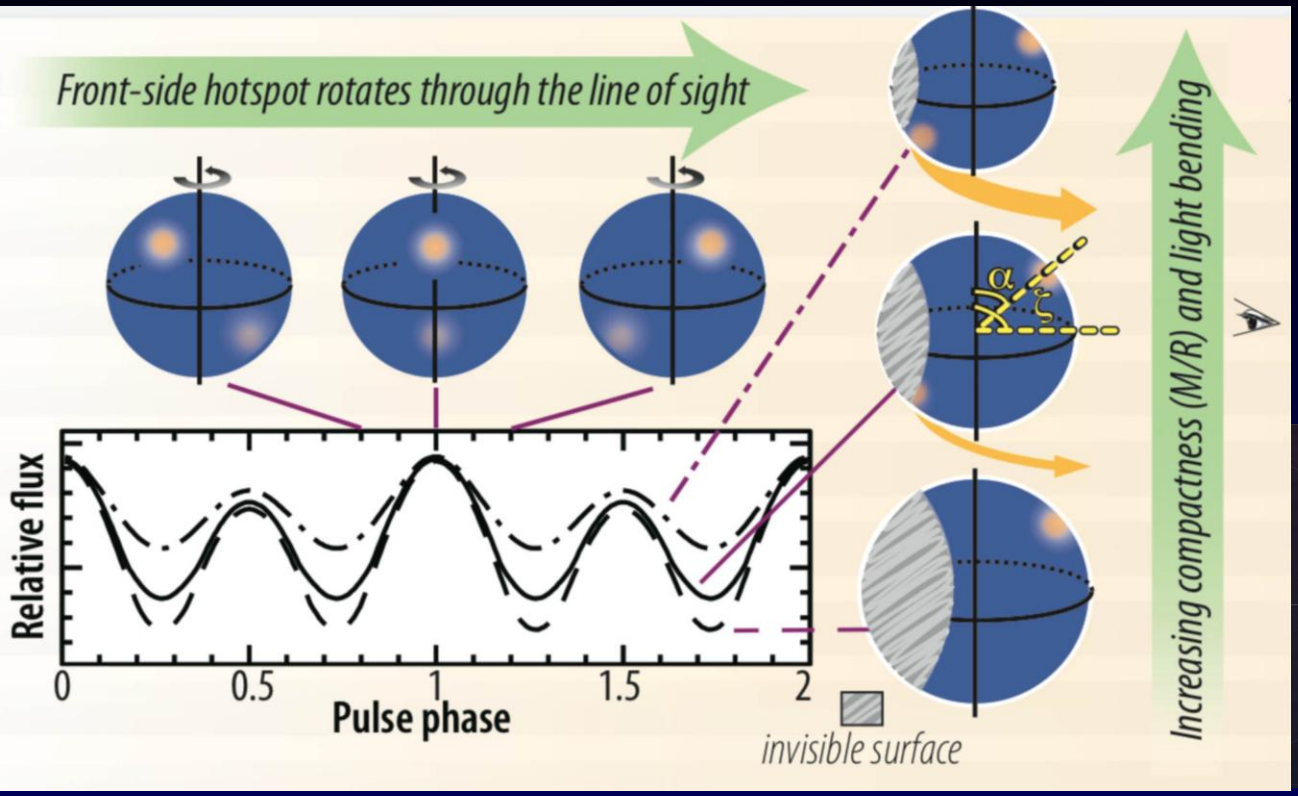




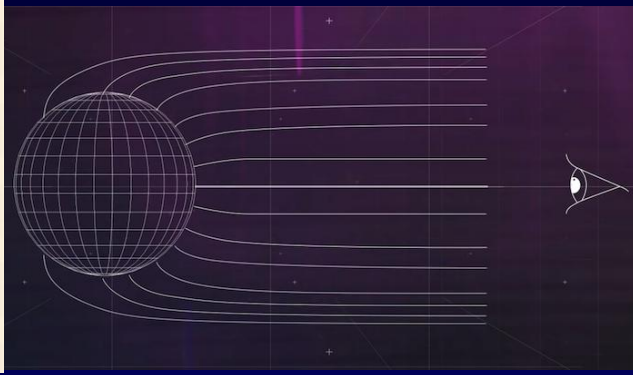
Neutron star radius



The Neutron Star Interior Composition Explorer Mission (NICER)



Lightcurve modeling constrains the compactness (GM/Rc^2) and viewing geometry of a non-accreting millisecond pulsar

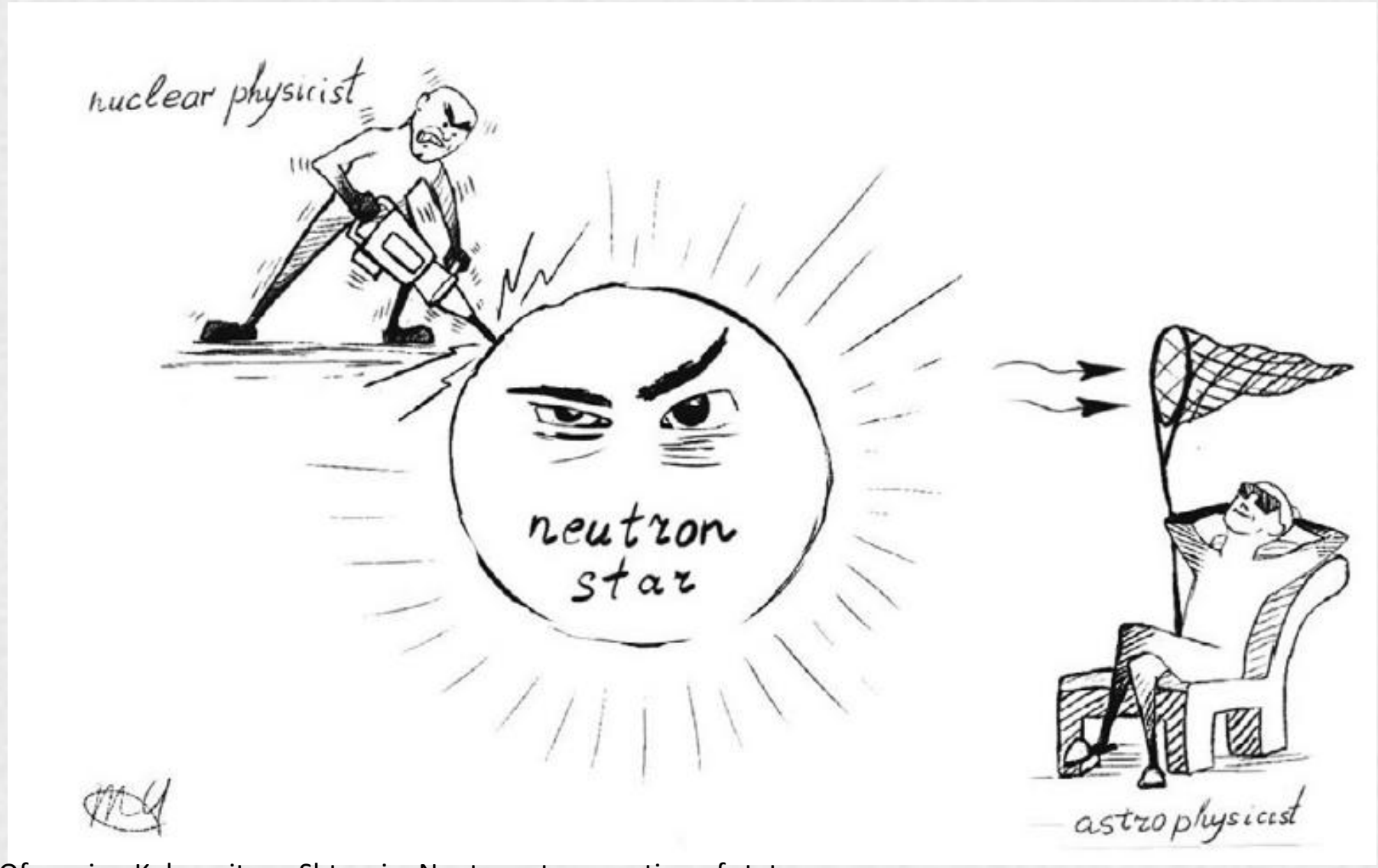


PSR J0740+6620

Thomas E. Riley *et al* 2021 *ApJL* **918** L27 $R = 12.39^{+1.30}_{-0.98}\,\mathrm{km}$ and $M = 2.072^{+0.067}_{-0.066}\,M_{\mathrm{sol}}$

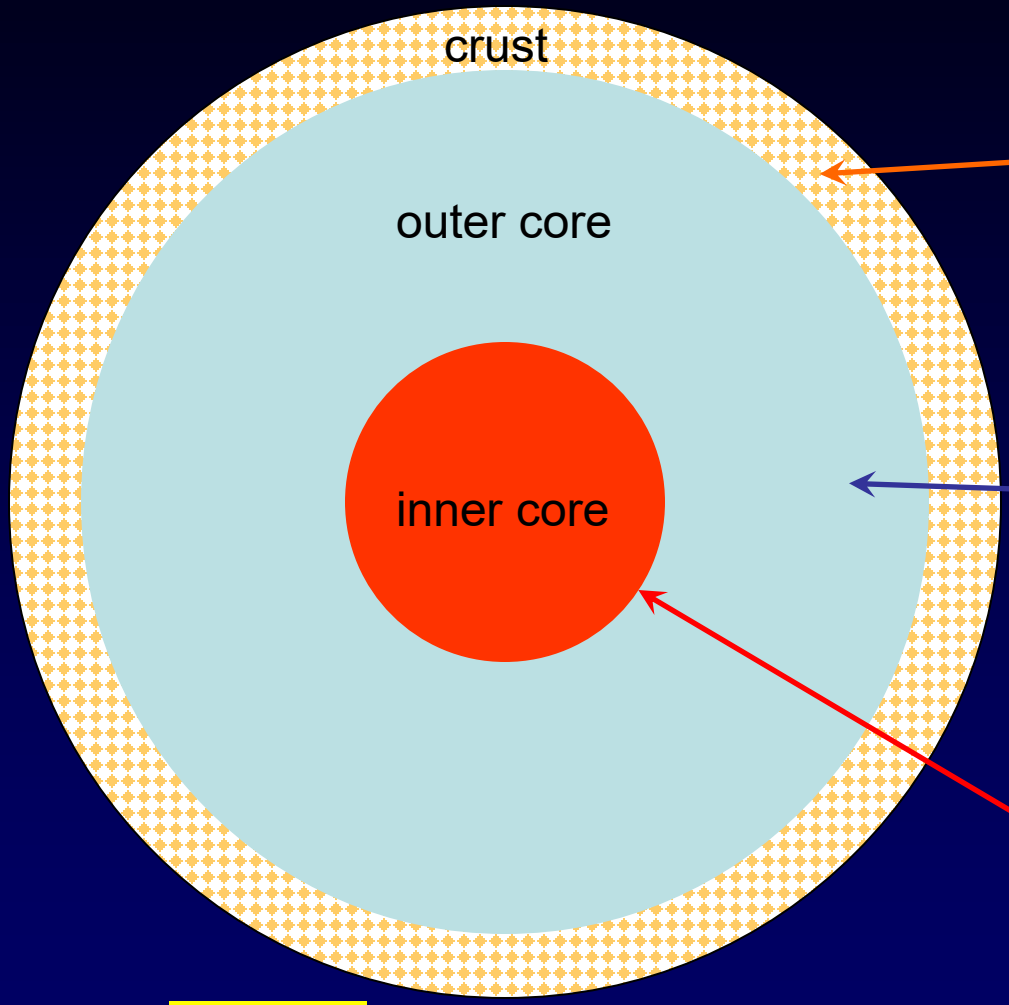
M. C. Miller *et al* 2021 *ApJL* **918** L28 $R = 13.7^{+2.6}_{-1.5}\,\mathrm{km}$ and $M = 2.08 \pm 0.07\,M_{\mathrm{sol}}$

The two strategies of studying physics of neutron star matter. Artist's impression



Ofengeim, Kolomeitsev, Shternin, Neutron star equation of state.
In Encyclopedia of Astrophysics; 2026; Vol. 3, pp.160–204

Cross section of a neutron star

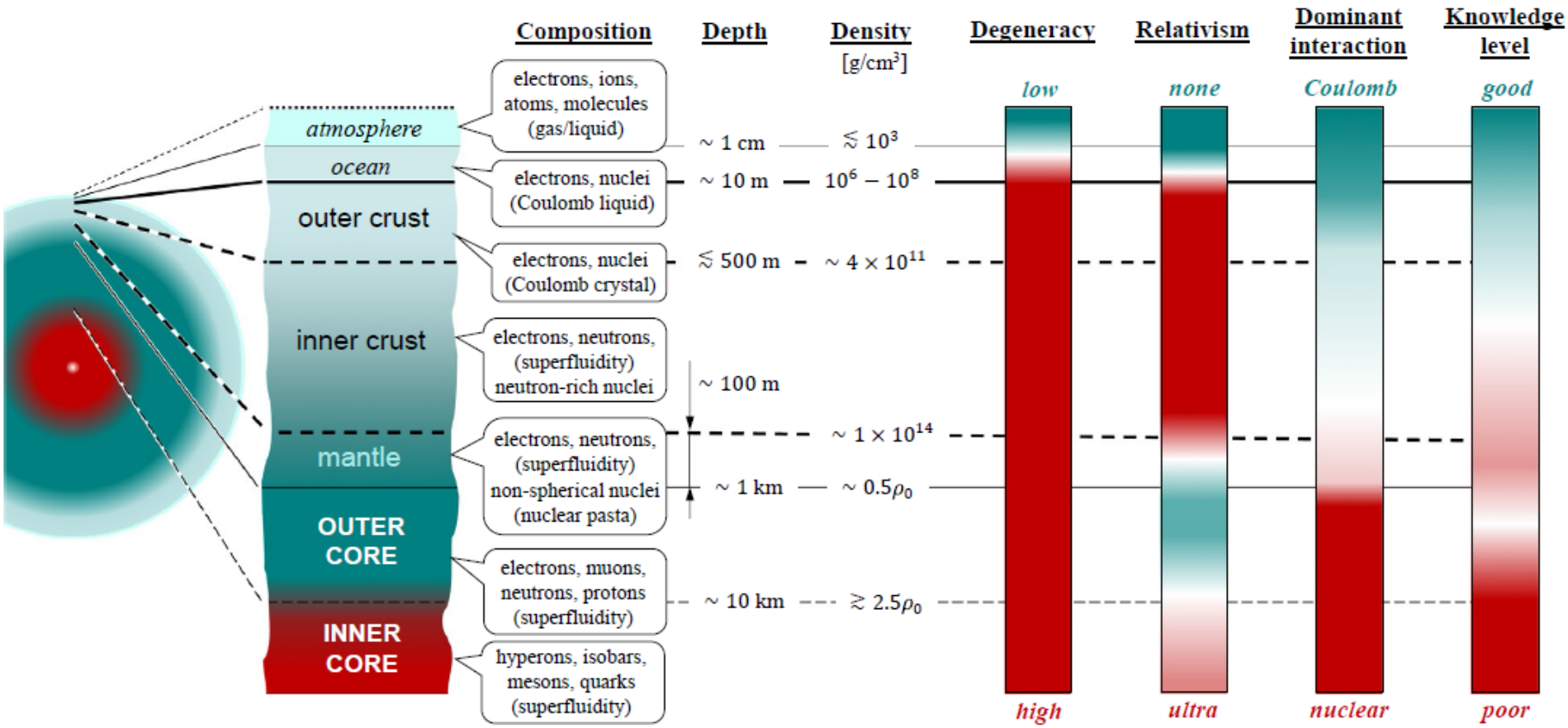


Nuclei and electrons
Neutron-rich nuclei and electrons
Nuclei, electrons and neutrons

neutrons, protons, electrons, muons

Exotics:
hyperons,
meson-condensates,
quark matter

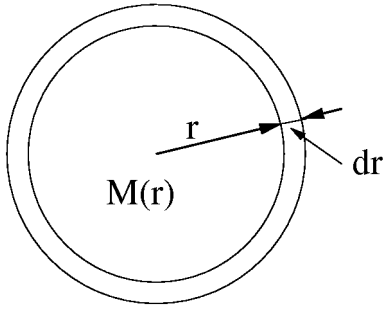
Graphic summary of our knowledge of NS structure and superdense matter properties



Ofengeim, Kolomeitsev, Shternin, Neutron star equation of state.
In Encyclopedia of Astrophysics; 2026; Vol. 3, pp.160–204

Tolman-Oppenheimer-Volkov equation

Equilibrium condition for a shell in a non-rotating neutron star



$$S_\Omega(r)\,dp = dF_G$$ Newton's Law

$$4\,\pi\,r^2\,(p(r) - p(r+\mathrm{d}r)) = -4\,\pi\,r^2\,dp = G\,\frac{M(r)\,dM}{r^2}$$

$$dM = 4\,\pi\,r^2\,\varepsilon(p)\,dr$$

INPUT: equation of state (EoS)

$$\varepsilon = \varepsilon(p) \quad \text{or} \quad \begin{cases} p = p(n) \\ \varepsilon = \varepsilon(n) \end{cases}$$

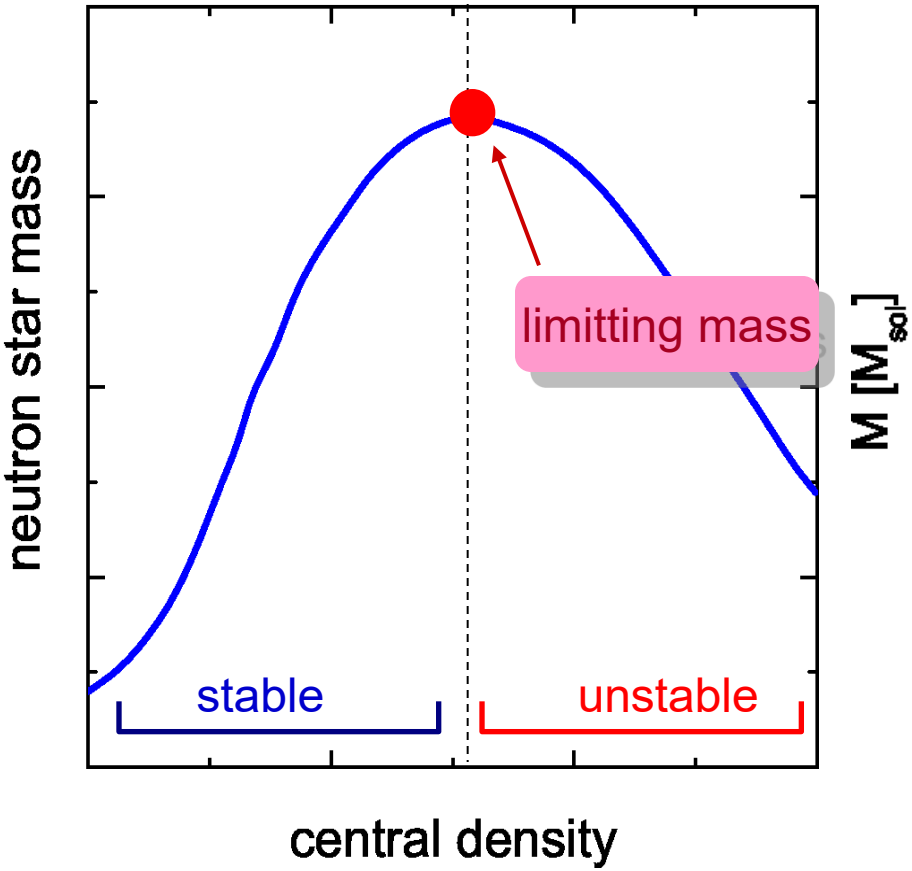
boundary conditions: $\varepsilon(r=0) = \varepsilon_\mathrm{c}\,, \quad M(r=0) = 0\,, P(r=R) = 0$

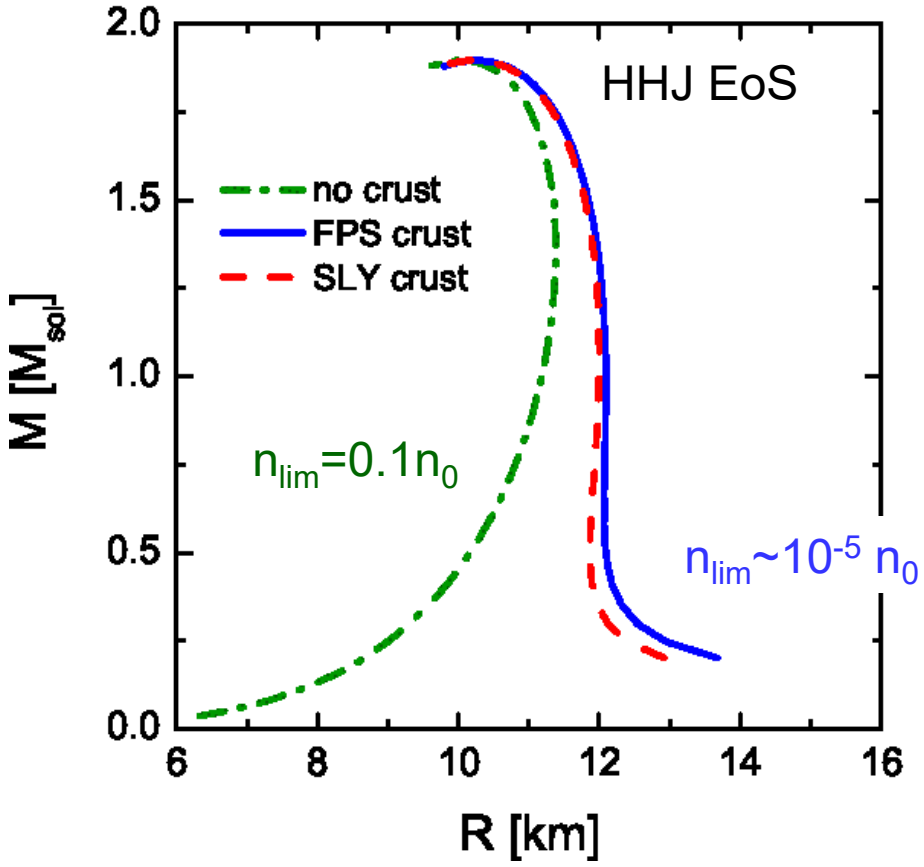
OUTPUT: neutron star density profile, radius R and mass M

relativistic corrections

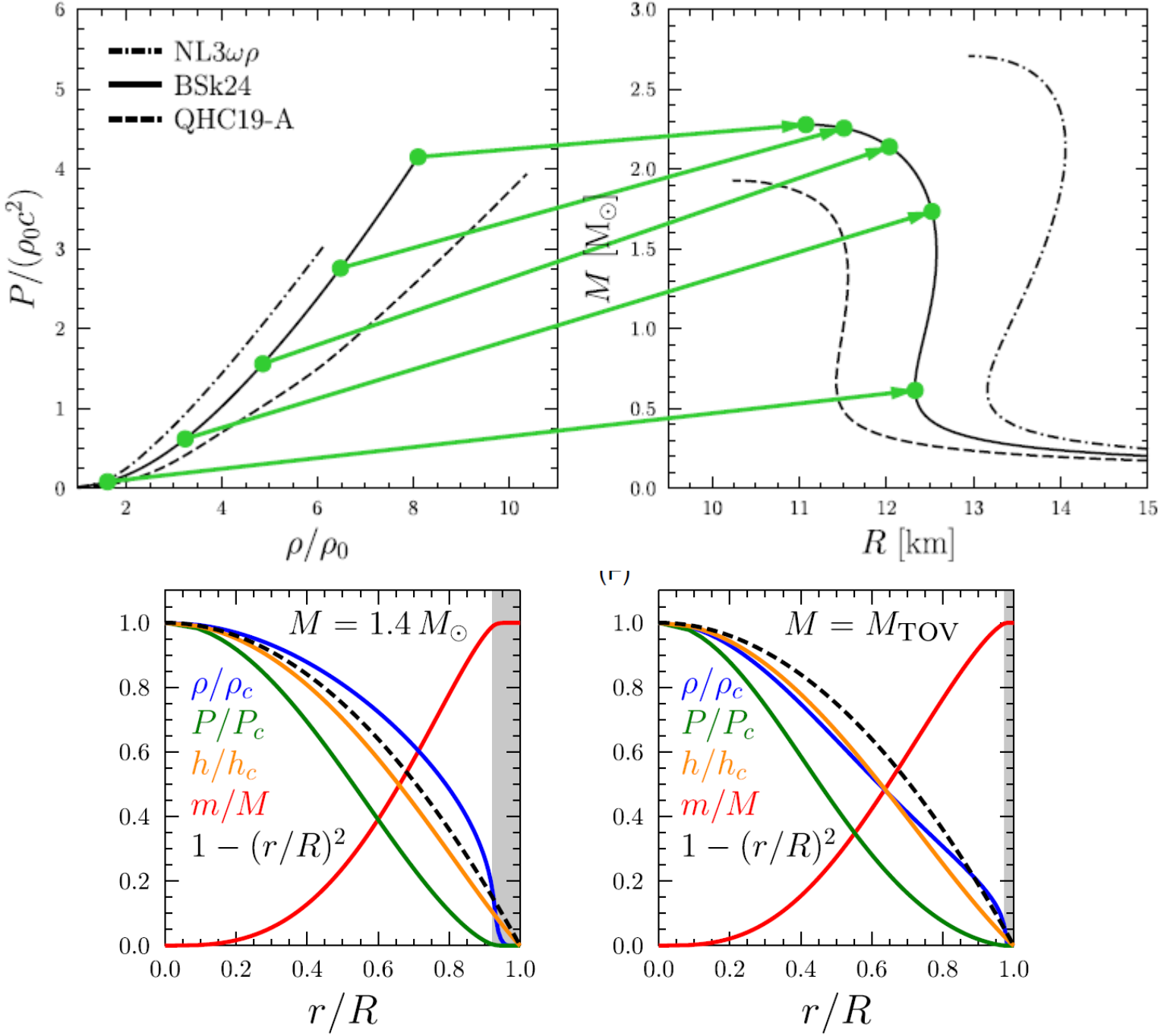
$$\frac{dp}{dr} = -\frac{G\,\rho\,M}{r^2}\left(1+\frac{p}{\rho\,c^2}\right)\left(1+\frac{4\pi P r^3}{Mc^2}\right)\left(1-\frac{2GM}{c^2 r}\right)^{-1}$$

Neutron star configuration





uncertainty in R~10^3 m

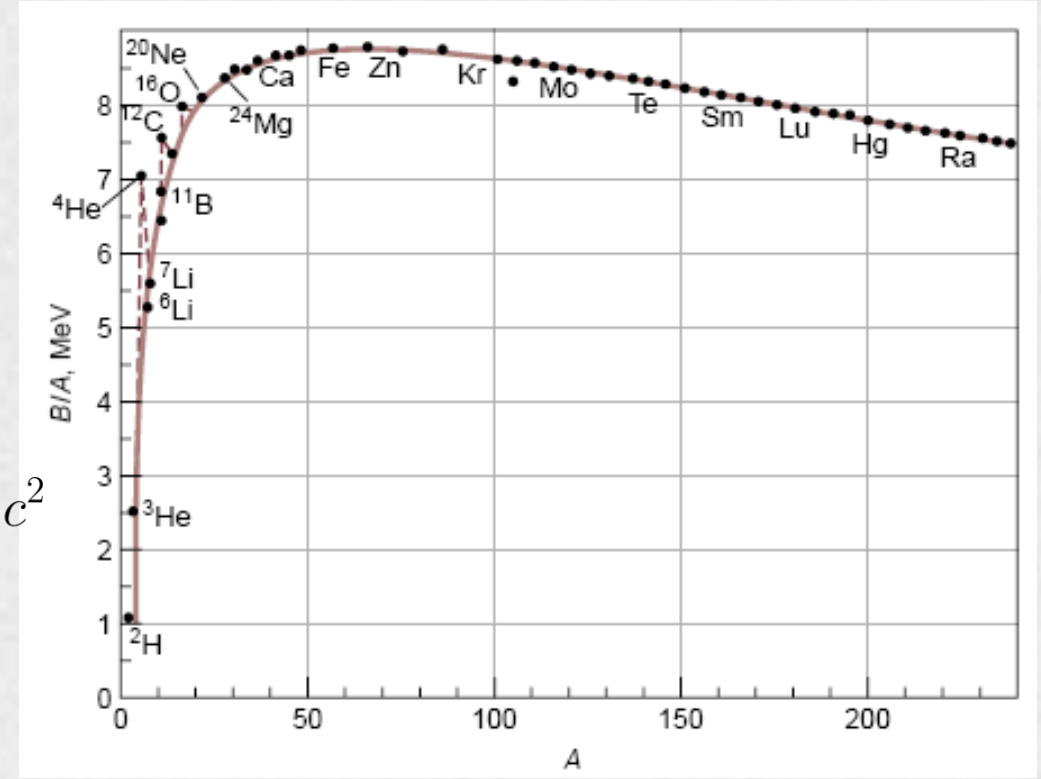

NL3ωρ
BSk24
QHC19-A
$P/(\rho_0 c^2)$
ρ/ρ_0
M [$M_\odot$]
R [km]
$M = 1.4\,M_\odot$
$M = M_{\rm TOV}$
ρ/ρ_c
P/P_c
h/h_c
m/M
$1-(r/R)^2$
r/R

Construction of the nuclear EoS

Nuclear equation of state

1932 Liquid-drop model of a nucleus:

Weizsäcker's
semiempirical mass formula



$$M(A,Z)\,c^2 = (A-Z)\,m_n\,c^2 + Z\,m_p\,c^2 - B(A,Z)$$

$$R = r_0\,A^{1/3}$$

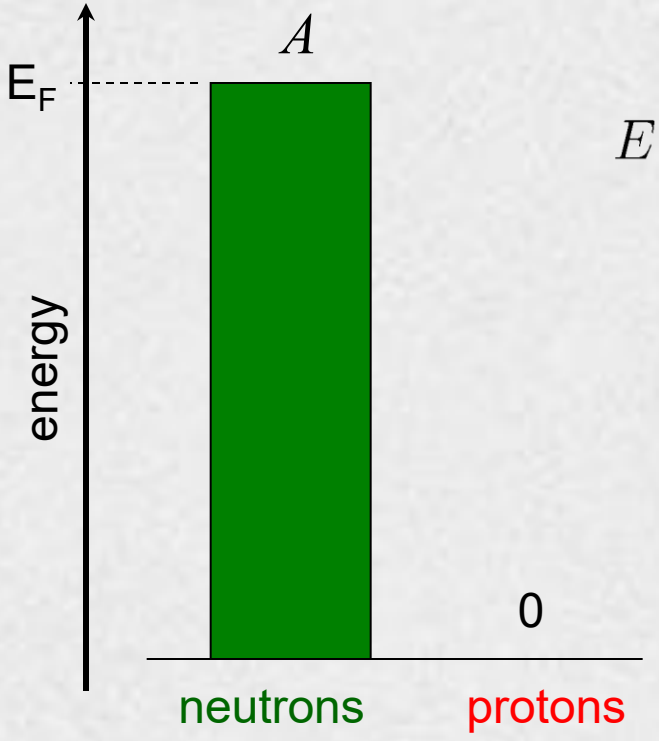
$$B(A,Z) = a_{\mathrm{Vol}}\,A - a_{\mathrm{Surf}}\,A^{2/3} - a_{\mathrm{Sym}}\,\frac{(A-2\,Z)^2}{A} - a_{\mathrm{Coul}}\,\frac{Z\,(Z-1)}{A^{1/3}} + \delta(A,Z)$$

volume e. | surface e. | symmetry e. | Coulomb e. | pairing e.

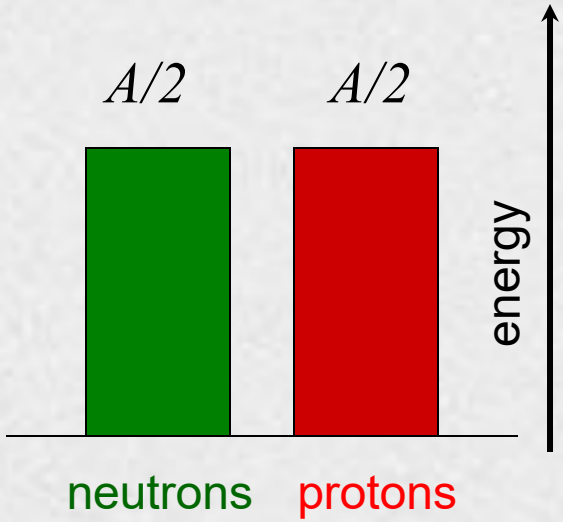
Pauli exclusion principle: nuclear symmetry energy

Neglect electric charge of protons: isospin symmetry.

We want to distribute A nucleons



$$E(A, Z = 0) > E(A, Z = A/2)$$



Two Fermi seas are better than one Fermi sea!

Two basic physical principles determine the structure of compact stellar objects:

Electroneutrality and **Pauli exclusion principle**.

Macroscopic object held by gravity must be electrically neutral:

Consider a sphere of a radius *R*

with the uniform charge density n_Q and the baryon density *n*

Coulomb energy:

$$E_{\rm C} = e^2\, 2\,\pi\, R^2\, n_Q \;= 1.5\times 10^{36}\ {\rm MeV}\ \left(\frac{n_Q}{n_0}\right)\ \left(\frac{R}{1{\rm km}}\right)^2$$

(n_0=0.16 fm^{-3})

Gravitational energy:

$$E_{\rm G} \;=\; -G\, 2\,\pi\, R^2\, n\, m_N = -0.7\ {\rm MeV}\times\left(\frac{n}{n_0}\right)\ \left(\frac{R}{1{\rm km}}\right)^2$$

$$E_{\rm C} + E_{\rm G} = 0 \qquad \Longrightarrow \qquad \frac{n_Q}{n} \sim 10^{-36}$$

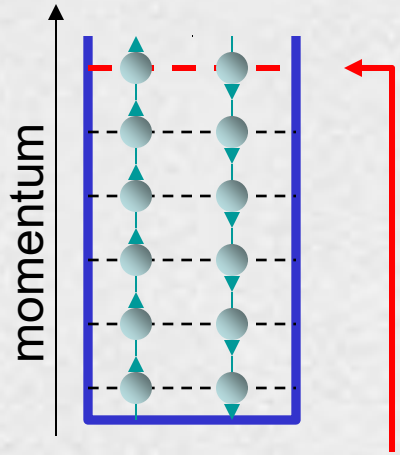
Quantum mechanics of a neutron star



Pauli exclusion principle

Two identical particles with uneven spin cannot occupy simultaneously the same quantum states, i.e. have the same quantum numbers

spin ½ particles in a box



$$n = 2\int_0^{p_\mathrm{F}} \frac{d^3p}{(2\pi)^3} = \frac{p_\mathrm{F}^3}{3\,\pi^2}$$

$$\epsilon = 2\int_0^{p_\mathrm{F}} \frac{d^3p}{(2\pi)^3}\, e(p) \approx n\,m + \frac{3}{5}\,n\,\frac{p_F^2}{2\,m}$$

$$P = 2\int_0^{p_\mathrm{F}} \frac{d^3p}{(2\pi)^3}\,\frac{p^2}{3\,e(p)} \approx \frac{2}{5}\,n\,\frac{p_F^2}{2\,m}$$

Fermi momentum p_F
Fermi energy E_F=e(p_F)

Fermi pressure supports a neutron star against a gravitational collapse

Infinite nuclear matter

1) make A, V, big by keeping $n_p = \frac{Z}{V},\ n_n = \frac{A-Z}{V} = n - n_p$ fixed

2) switch off electromagnetic interaction

3) $m_n = m_p = m_N$

Energy density of the infinite nuclear matter as function of the proton and neutron densities:

$$\lim_{A\to\infty} \frac{M(A,Z)c^2}{A} = E(n_p, n_n) = m_N - \lim_{A\to\infty} \frac{B(A,Z)c^2}{A}$$

Binding energy per nucleon: $\varepsilon(n,x) = E(n,x)/n - m_N$

where, $n = n_p + n_n$ total density, $x = n_p/n$ proton fraction

chemical potentials:

$$\mu_n = \frac{\partial E(n_p,n_n)}{\partial n_n} = \frac{\partial E(n,x)}{\partial n} - \frac{x}{n}\frac{\partial E(n,x)}{\partial x}$$

$$\mu_p = \frac{\partial E(n_p,n_n)}{\partial n_p} = \frac{\partial E(n,x)}{\partial n} + \frac{1-x}{n}\frac{\partial E(n,x)}{\partial x}$$

Pressure: $P = \mu_n n_n + \mu_p n_p - E = n\frac{\partial E}{\partial n} - E$ T=0

Infinite nuclear matter. Symmetry energy

$$\varepsilon(n,x) = \varepsilon_0(n) + \varepsilon_S(n)\,(1-2\,x)^2 + \ldots$$

ISM energy: $\varepsilon_0(n)$ Symmetry energy: $\varepsilon_S(n)$

Two definitions of the symmetry energy: (1) $\varepsilon_S(n) = \frac{1}{8}\frac{\partial^2\varepsilon(n,x)}{\partial x^2}\Big|_{x=1/2}$ *local (x~1/2)*

(2) $\varepsilon_S(n) = \varepsilon(n,x=0) - \varepsilon(n,x=1/2)$

NS applications

If the derivative $\frac{\partial^4\varepsilon(n,x)}{\partial x^4}$ is very small, then both definitions are equivalent

Equation of state of nuclear matter

The energy per nucleon of the nuclear matter $E(n_p, n_n) = \varepsilon_0(n) + \varepsilon_S(n)\dfrac{(n_p - n_n)^2}{n^2}$

n_p – proton number density
n_n – neutron number density

$n = n_p + n_n$

- **nuclear matter parameters**

$$\varepsilon_0(n) = \varepsilon_0 + 0 + \frac{K}{18}\frac{(n-n_0)^2}{n_0^2} + \frac{Q}{162}\frac{(n-n_0)^3}{n_0^3} + O\left(\frac{(n-n_0)^4}{n_0^4}\right)$$

symmetry energy

$$\varepsilon_S(n) = J + \frac{L}{3}\frac{n-n_0}{n_0} + \frac{K_{\rm sym}}{18}\frac{(n-n_0)^2}{n_0^2} + \frac{Q_{\rm sym}}{162}\frac{(n-n_0)^3}{n_0^3} + O\left(\frac{(n-n_0)^4}{n_0^4}\right)$$

S_0

saturation density n_0 and binding energy ε_0 $\quad n_0 \simeq 0.16 \pm 0.015\ \mathrm{fm}^{-3}$ $\quad \varepsilon_0 \simeq -15.6 \pm 0.6\,\mathrm{MeV}$

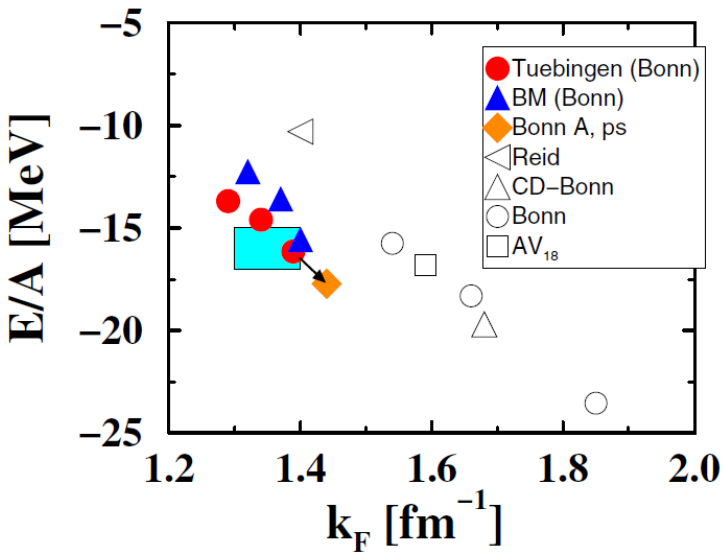
- *Correlations among parameters*

  n_0 vs ε_0 –Coester line problem: role of TNF, relativistic effects, chiral forces

- *Stiffness of EoS*

  frequently characterized by the compressibility modulus K

  $K = 240 \pm 20\,\mathrm{MeV}$



Giant Monopole Resonance (GMR)

- *Correlations among parameters L-J* $\varepsilon_S[n] = J + \frac{L}{3}\frac{n-n_0}{n_0} + \frac{K_{\rm sym}}{18}\frac{(n-n_0)^2}{n_0^2} + \ldots$

Masses: UNEDF0 Skyrme DF+BHF
[Kortelainen *et al.*, PRC **82**, 024313 (2010)]

Isobaric analog states+isovector skin:
[Danielewicz et al. NPA 958, 147 (2017)]

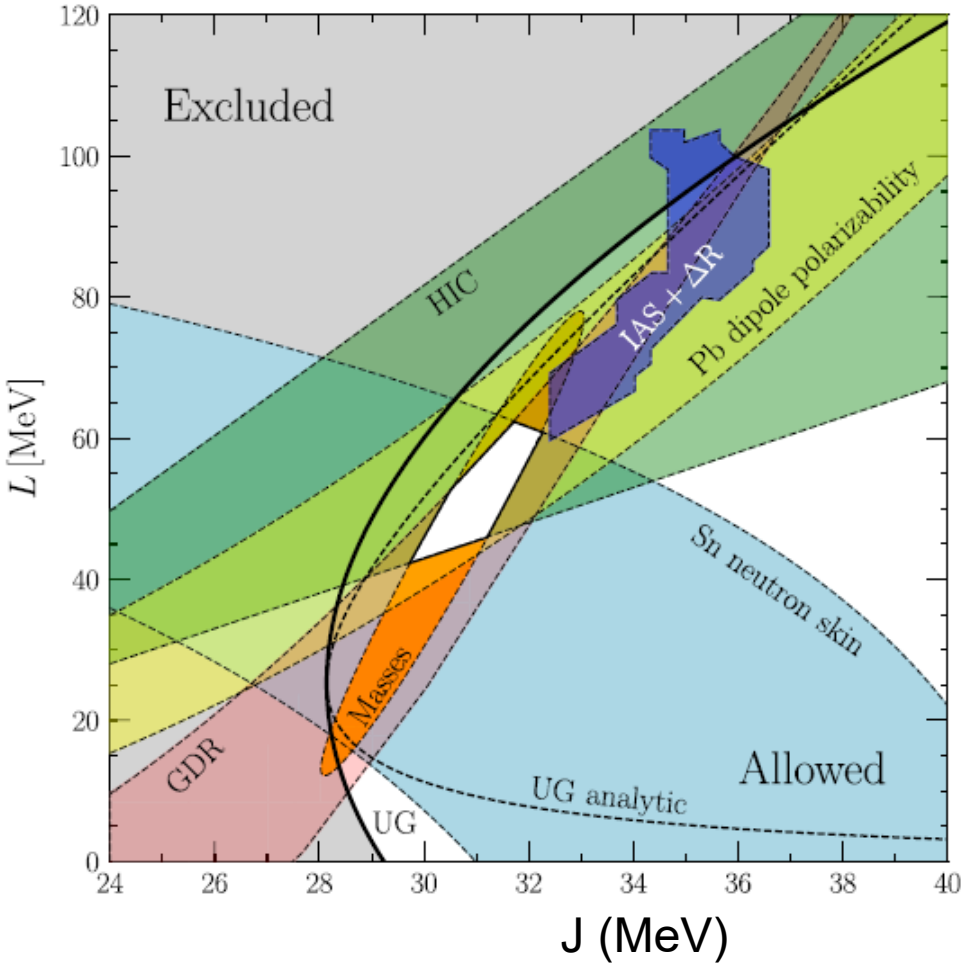
Pb dipole polarizability:
[Roca-Maza *et al.*, PRC **88**, 024316 (2013)]

Sn neutron skin:
[Chen et al., PRC **82**, 024321 (2010)]

GDR:
[Trippa et al., PRC **77**, 061304 (2008)]

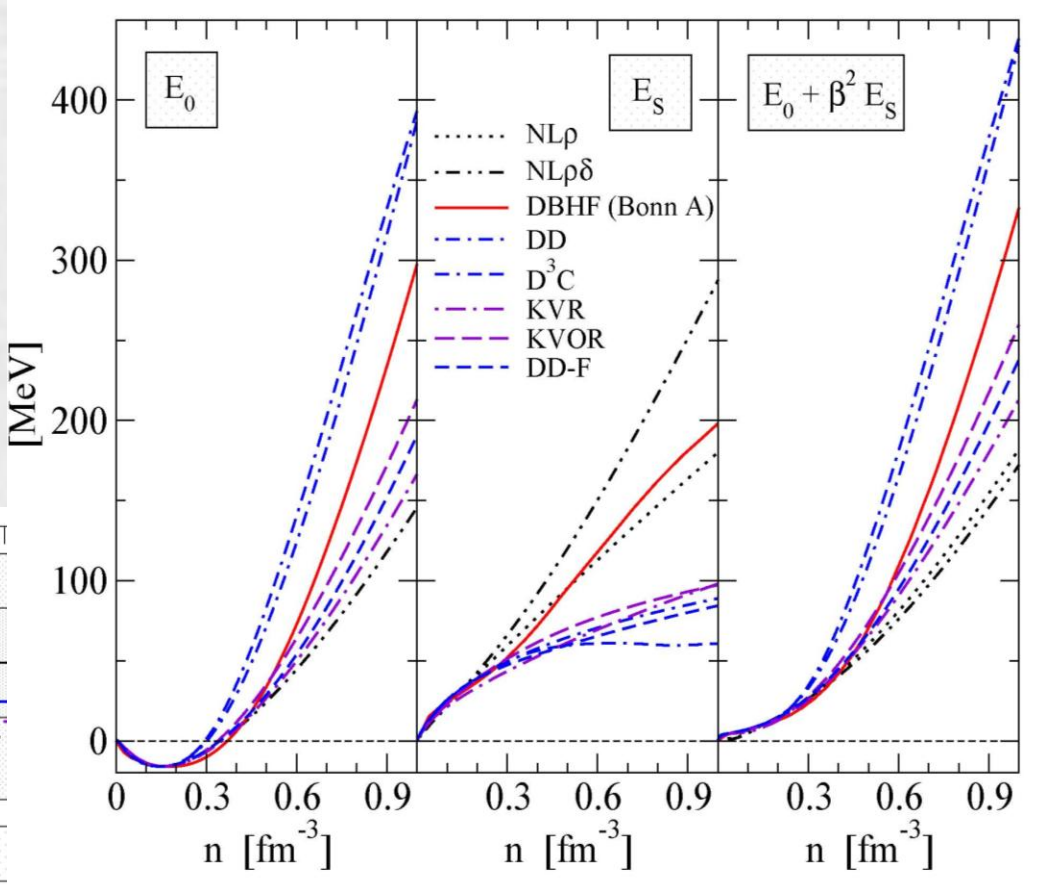
Isospin defusion in HIC
[Tsang et al., PRL 102, 122701 (2009)]

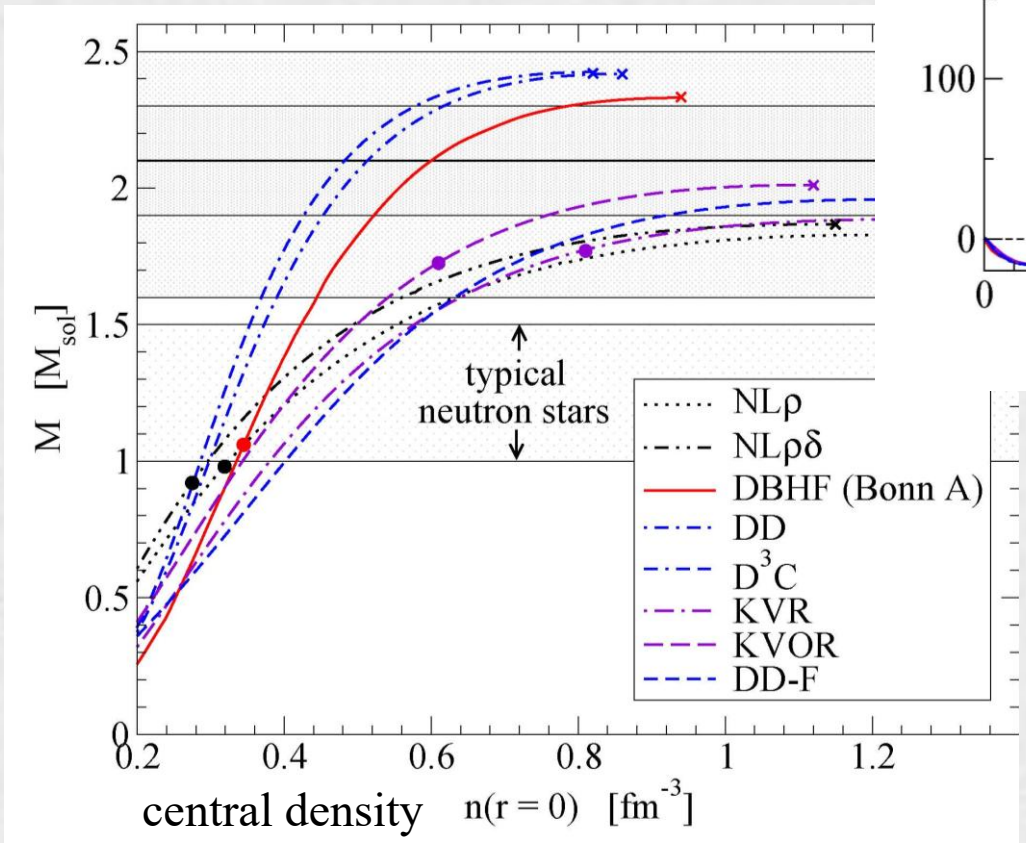
Lattimer Lim plot



Behind all calculation are particular models for NN interactions and many-body techniques

Selection of Nuclear Equations of State





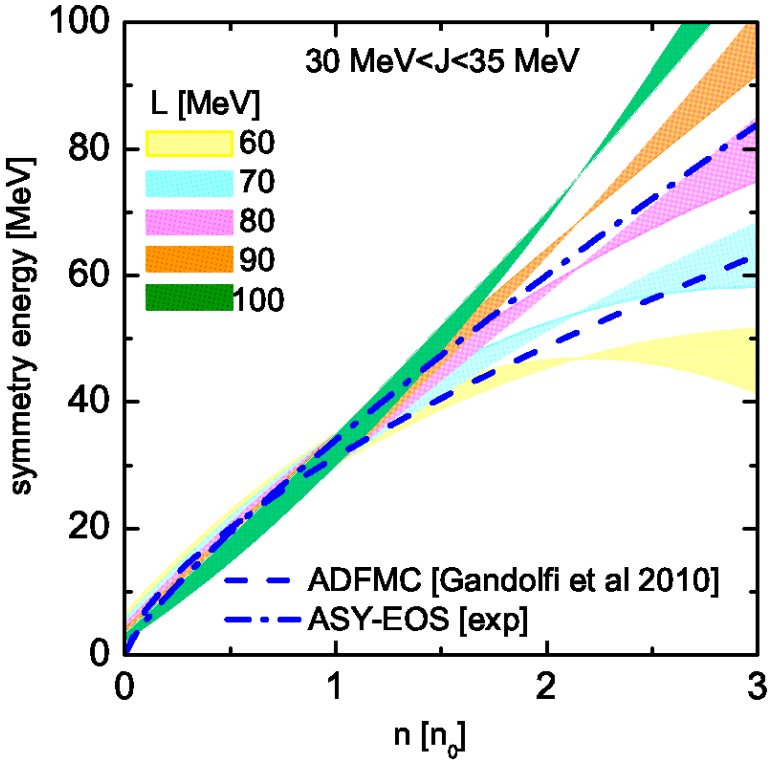
Nucleon part of EoS should be [sufficiently stiff]

If we assume some model for the density dependence of the symmetry energy

$$\varepsilon_S(n) = C_k\,(n/n_0)^{2/3} + C_1\,n/n_0 + C_2\,(n/n_0)^{\gamma}$$

$$J = C_1 + C_2 + C_k \qquad 3L = C_1 + 2C_k + 3\gamma C_2 \qquad K = -2C_k + 9C_2(\gamma - 1)\gamma$$

Eliminate C_1 and C_2



$$L = \left(\frac{2}{3\,\gamma} - 1\right) C_k + 3\,J + \frac{K_{\rm sym}}{3\,\gamma}$$

$$L = -19.5\ \text{MeV} + 3\,J + \frac{K_{\rm sym}}{5.50}$$

Analysis of 36 RMF models gives

[Dong, et al PRC85, 034308 (2012)]

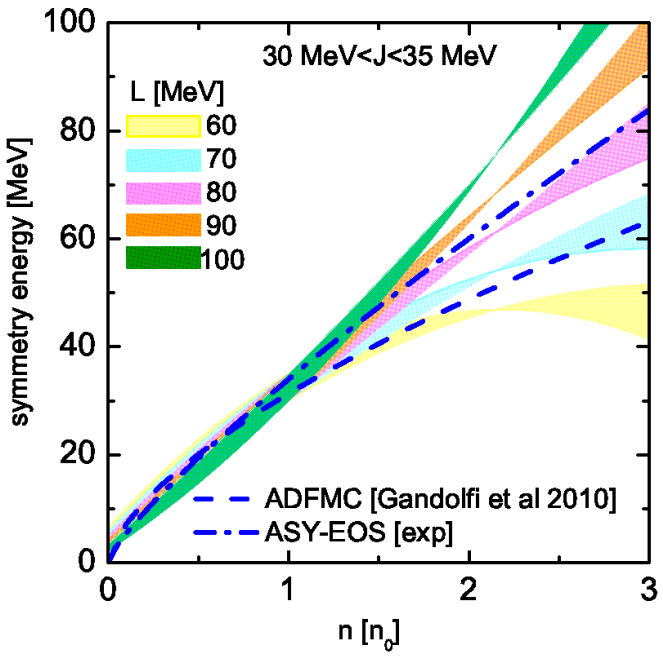
chemical potentials: $\mu_n = \frac{\partial E(n_p,n_n)}{\partial n_n} = \frac{\partial E(n,x)}{\partial n} - \frac{x}{n}\frac{\partial E(n,x)}{\partial x}$ $\mu_p = \frac{\partial E(n_p,n_n)}{\partial n_p} = \frac{\partial E(n,x)}{\partial n} + \frac{1-x}{n}\frac{\partial E(n,x)}{\partial x}$

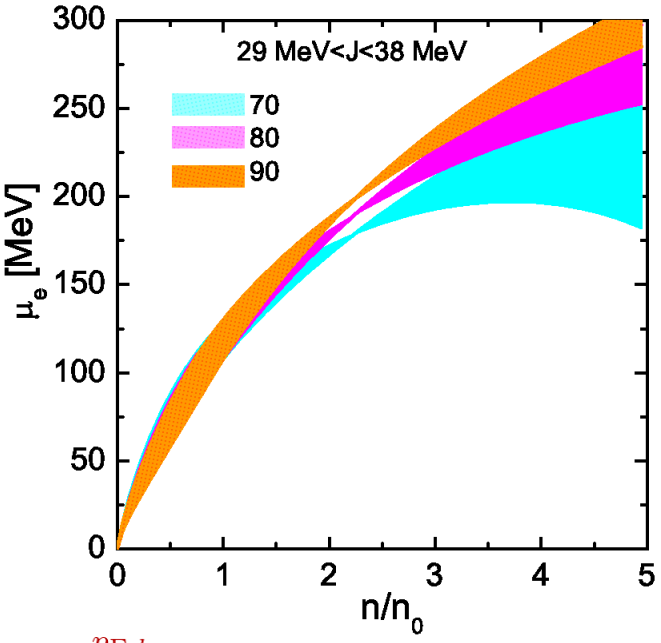
Condition of the beta equilibrium $n \leftrightarrow p + e^-$ $n \leftrightarrow p + \mu^-$ $\mu^- \leftrightarrow e^-$

$$\mu_n = \mu_p + \mu_e \qquad \mu_\mu = \mu_e$$

$$\mu_e = \mu_n - \mu_p = -\frac{1}{n}\frac{\partial E(n,x)}{\partial x} = 4\,\varepsilon_S(n)\,(1-2\,x)$$

equation for the proton concentration $n_e(\mu_e) + n_\mu(\mu_e) = \frac{(\mu_e^2 - m_e^2)^{3/2}}{3\pi^2} + \frac{(\mu_e^2 - m_\mu^2)^{3/2}}{3\pi^2} = n_p = x\,n$

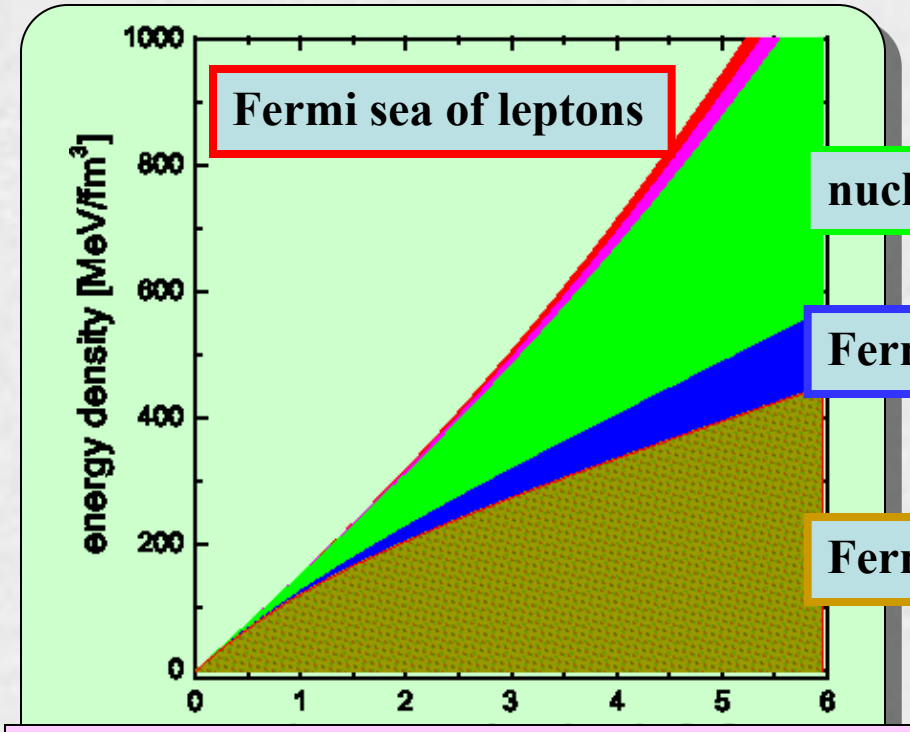


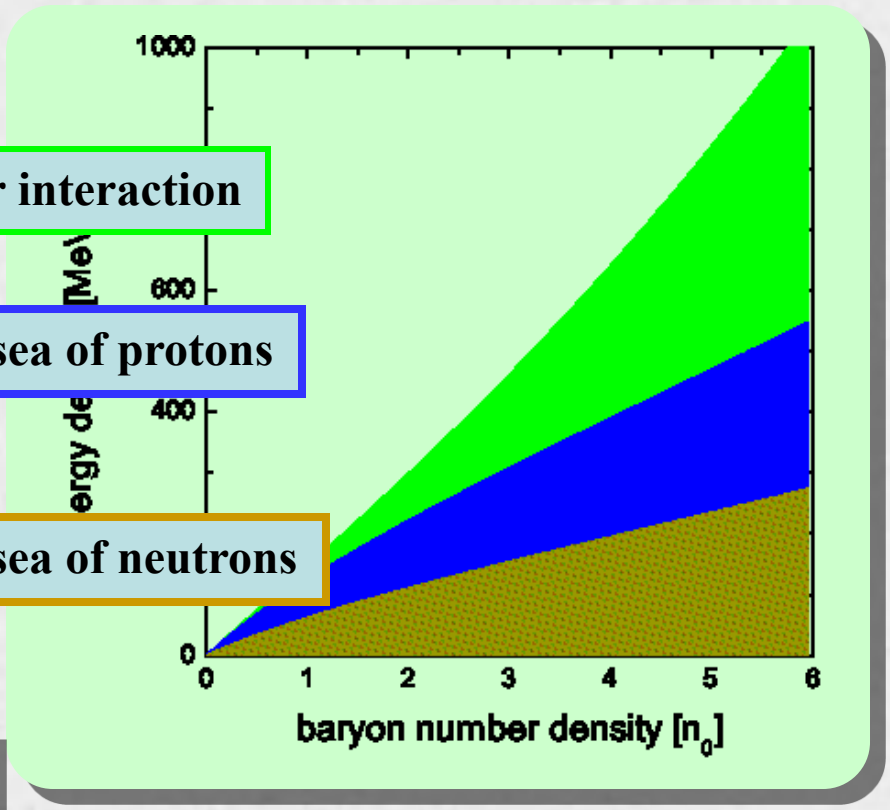


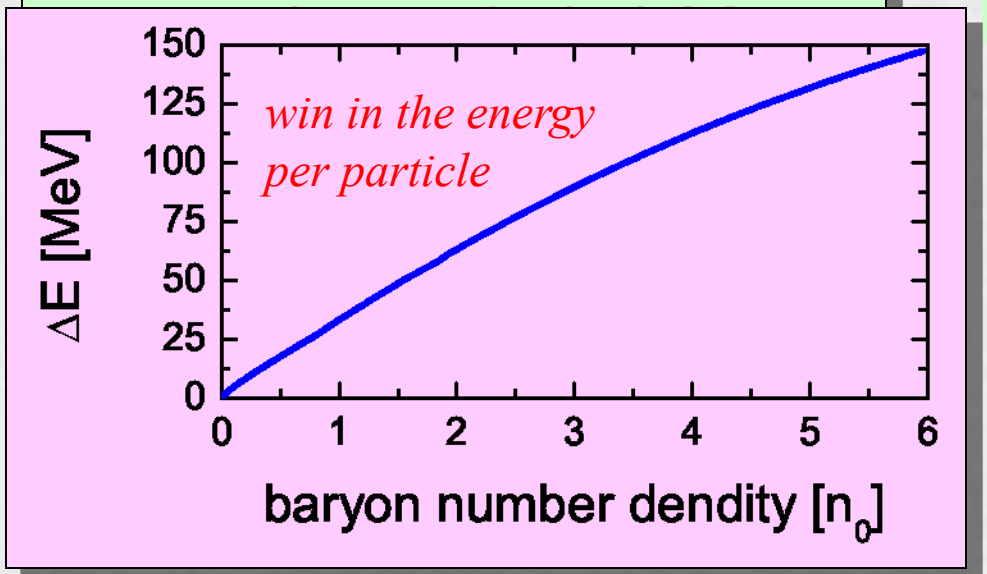
$$E_{\rm tot} = E_{\rm nucl} + E_{\rm lept} = E_{\rm nucl} + \sum_{l=e,\mu} \int_0^{p_{{\rm F},l}} \frac{2{\rm d}^3p}{(2\pi)^3}\sqrt{m_l^2 + p^2}$$

β equlibrium

isospin symmetric $N_n=N_p$

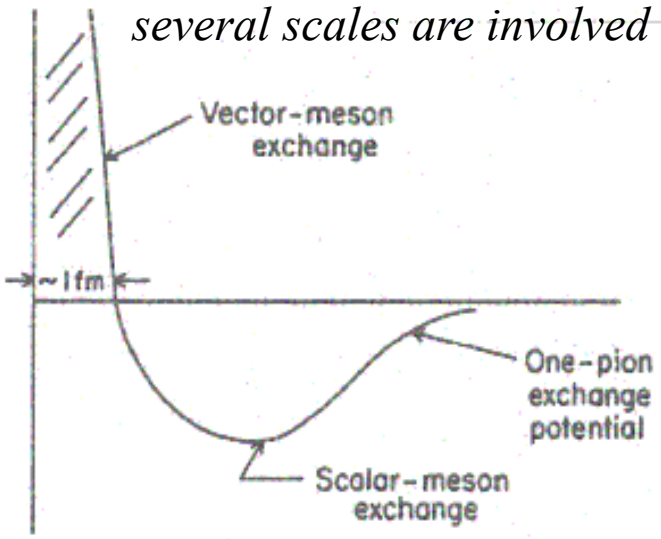


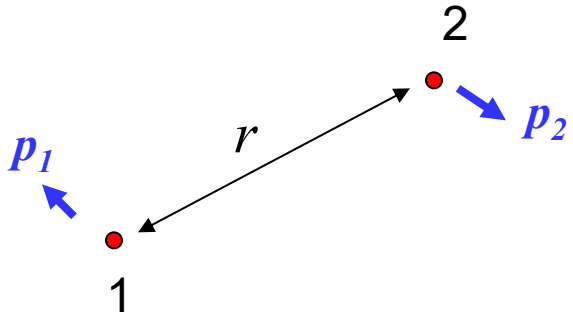




Compensate charge not by fermions but by light bosons

Nucleon-nucleon interaction





non-relativistic description

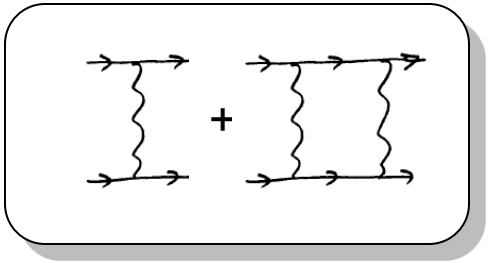
$$\hat{V}(r) = V_C(r)\,\hat{1} + V_{spin}(r)\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2 + V_{LS}(r)\,(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2)\cdot\boldsymbol{L} + V_{tensor}(r)\,\hat{S}_{12}(r)$$

vector mesons: $m_{\omega,\rho}$~800 MeV , **r~0.24 fm**

correlated 2π exchange: m~200-600 MeV **r~0.3—1fm**

1-pion exchange: m_{π}=140 MeV **r~1.4 fm**

relativistic description



Equilibrium density of an atomic nucleus n_0=0.16 fm^{-3}

inter-nuclear distance $(n_0)^{-1/3}$=**1.8 fm**

Relativistic mean-field models

nucleon-nucleon interaction — **vacuum:** one boson-exchange for NN-potential + Lippmann-Schwinger equations

a model

$$\mathcal{L} = \sum_N \overline{\Psi_N}\left[i\left(\hat{\partial} + i\,g_{\omega N}\hat{\omega} + i\,g_{\rho N}\boldsymbol{\tau}\,\hat{\boldsymbol{\rho}}\right)\right] - \left(m - g_{\sigma N}\,\sigma\right)\Psi_N$$
$$+ \underbrace{\frac{1}{2}\left(\partial_\mu\sigma\,\partial^\mu\sigma - m_\sigma^2\,\sigma^2\right) - U(\sigma)}_{\text{scalar}} \underbrace{- \frac{1}{4}\,\omega_{\mu\nu}\,\omega^{\mu\nu} + \frac{1}{2}\,m_\omega\,\omega_\mu\,\omega^\nu}_{\text{vector}} \underbrace{- \frac{1}{4}\,\boldsymbol{\rho}_{\mu\nu}\,\boldsymbol{\rho}^{\mu\nu} + \frac{1}{2}\,\boldsymbol{\rho}_\mu\,\boldsymbol{\rho}^\mu}_{\text{iso-vector}}$$

$$\omega_{\mu\nu} = \partial_\mu\omega_\nu - \partial_\nu\omega_\mu \qquad \boldsymbol{\rho}_{\mu\nu} = \partial_\mu\boldsymbol{\rho}_\nu - \partial_\nu\boldsymbol{\rho}_\mu$$

Euler-Lagrange equations for $q \equiv q(\vec{x},t) = \{\Psi, \sigma, \omega, \rho\}$ $\qquad \partial_\mu\left[\dfrac{\partial\mathcal{L}}{\partial(\partial_\mu q)}\right] - \dfrac{\partial\mathcal{L}}{\partial q} = 0$

$$\left[i\gamma_\mu(\partial^\mu + ig_{\omega N}\omega^\mu + ig_{\rho N}\boldsymbol{\tau}\boldsymbol{\rho}^\mu) - (m_N - g_{\sigma N}\sigma)\right]\Psi_N = 0$$

$$(\partial^2 + m_\sigma^2)\sigma + \frac{dU}{d\sigma} = g_{\sigma N}\sum_{N=p,n}\bar{\Psi}_N\Psi_N$$

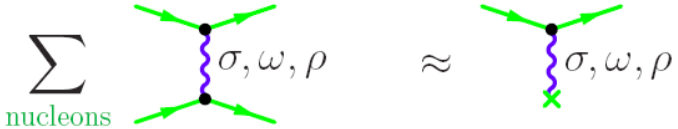
$$(\partial^2 + m_\omega^2)\omega_\mu = g_{\omega N}\sum_{N=p,n}\bar{\Psi}_N\gamma_\mu\Psi_N$$

$$(\partial^2 + m_\rho^2)\boldsymbol{\rho}_\mu = g_{\rho N}\sum_{N=p,n}\bar{\Psi}_N\boldsymbol{\tau}\gamma_\mu\Psi_N$$

nucleon sources for meson fields

$$\Psi_N = \begin{pmatrix}\Psi_p\\ \Psi_n\end{pmatrix}$$

medium: mean-field approximation



$\sigma(r,t) = \sigma$

$\omega_\mu(r,t) = \delta_{\mu,0}\,\omega_0$

$\rho^a_\mu(r,t) = \delta^{a,3}\,\delta_{\mu,0}\,\rho^{(3)}_0$

constant fields

$$\omega_0 = \frac{g_{\omega N}}{m_\omega^2} < \Psi^\dagger\Psi > \equiv \frac{g_{\omega N}}{m_\omega^2}\, n_B = \frac{g_{\omega N}}{m_\omega^2}\,(n_p + n_n)$$

(vector) density

$$\rho_0^{(3)} = \frac{g_{\rho N}}{m_\rho^2} < \Psi^\dagger \tau^{(3)} \Psi > \equiv \frac{g_{\rho N}}{m_\rho^2}\, n_{\rm iso} = \frac{g_{\rho N}}{m_\rho^2}\,(n_p - n_n)$$

$$m_\sigma^2\sigma_0 + \left.\frac{dU}{d\sigma}\right|_{\sigma_0} = g_{\sigma N} < \bar\Psi\Psi > \equiv g_{\sigma N} n_s = g_{\sigma N}(n_{s,p} + n_{s,n})$$

scalar density

$$\left[i\gamma_\mu\partial^\mu - \underbrace{(g_{\omega N}\omega_0 + (g_{\rho N}\rho_0^{(3)})\gamma^0}_{V} - \underbrace{(m_N - g_{\sigma N}\sigma_0)}_{m^*_N}\right]\Psi = 0$$

$m_N^* = m_N - g_{\sigma N}\,\sigma$

$$\begin{bmatrix} p_0 - V - m_N^* & -(\boldsymbol{p}\boldsymbol{\sigma}) \\ (\boldsymbol{p}\boldsymbol{\sigma}) & -p_0 + V - m_N^* \end{bmatrix} \Psi_N = 0 \quad \Rightarrow \quad -(p_0 - V)^2 + m_N^{*2} + (\boldsymbol{p}\boldsymbol{\sigma})^2 = 0$$

$$(p_0 - V)^2 = m_N^{*2} + \boldsymbol{p}^2$$

nucleon spectrum in MF approximation

$$\epsilon_N(p) = \sqrt{m_N^{*2} + p^2} + g_{\omega N}\,\omega_0 + g_{\rho N}\,I_N\,\rho_{03}$$

[Serot, Walecka]

pion dynamics falls out completely in this approx.

The energy-momentum tensor $T_{\mu\nu} = -g_{\mu\nu}\mathcal{L} + \sum_i \frac{\partial \mathcal{L}}{\partial(\partial q_i/\partial x_\mu)} \cdot \frac{\partial q_i}{\partial x^\nu}$

$E = \langle T_{00} \rangle \quad P = \frac{1}{3}\langle T_{ii} \rangle$

$$T_{\mu\nu} = \Big[-\frac{1}{2} m_\omega^2 \omega_0^2 - \frac{1}{2} m_\rho^2 \rho_0^{(3)2} + \frac{1}{2} m_\sigma^2 \sigma^2 + U(\sigma) \Big] g_{\mu\nu} + i \bar{\Psi}_N \gamma_\mu \partial_\nu \Psi_N$$

$$E = \frac{m_\sigma^2 \sigma^2}{2} + U(\sigma) - \frac{m_\omega^2 \omega_0^2}{2} - \frac{m_\rho^2 \rho_0^{(3)2}}{2} + \sum_N \int_0^{p_{\mathrm{F,N}}} \frac{2\mathrm{d}^3 p}{(2\pi)^3} \epsilon_N(p)$$

$$= \frac{m_\sigma^2 \sigma^2}{2} + U(\sigma) + \frac{m_\omega^2 \omega_0^2}{2} + \frac{m_\rho^2 \rho_0^{(3)2}}{2} + \sum_N \int_0^{p_{\mathrm{F,N}}} \frac{2\mathrm{d}^3 p}{(2\pi)^3} \sqrt{m_N^{*2} + p^2}$$

$$\epsilon_N(p) = \sqrt{m_N^{*2} + p^2} + g_{\omega N}\, \omega_0 + g_{\rho N}\, I_N\, \rho_{03}$$

Energy-density functional

$$E[n_p, n_n; \sigma] = \frac{m_\sigma^2\, \sigma^2}{2} + U(\sigma) + C_\omega^2 \frac{(n_n + n_p)^2}{2\, m_N^2} + C_\rho^2 \frac{(n_n - n_p)^2}{8\, m_N^2} + \sum_N \int_0^{p_{\mathrm{F},N}} \frac{dp\, p^2}{\pi^2} \sqrt{(m_N - g_{\sigma N}\, \sigma)^2 + p^2}$$

evaluated for σ–field following from the equation $\frac{\delta E[n_p, n_n, \sigma]}{\delta \sigma} = 0$ ➡ $\sigma = \sigma(n_p, n_n)$

Paremeters $C_i^2 = \frac{g_{iN}^2\, m_N^2}{m_i^2}$ are adjusted to properties of nuclear matter at saturation

Dimensionless scalar field $f = \frac{g_{\sigma N}\sigma}{m_N} \qquad \frac{m_\sigma^2\, \sigma^2}{2} \to \frac{m_N^4 f^2}{2 C_\sigma^2}$

$$U(f) = m_N^4 (b f^3/3 + c f^4/4)$$

n_0	$\simeq$	0.16 ± 0.015 fm^{-3}
ε_0	$\simeq$	-15.6 ± 0.6 MeV
$m_N^*(n_0)$	$\simeq$	$(0.75 \pm 0.1)\, m_N$
K	$\simeq$	240 ± 40 MeV
J	$\simeq$	32 ± 4 MeV

If we add gradient terms this energy density functional can be used for a description of properties of atomic nuclei.

$$E(n,x) = (m_N + \varepsilon(n,x))n$$

ISM: $n_p = n_n = n/2$

condition for the minimum

$$\left.\frac{\partial E[n/2, n/2; f]}{\partial n}\right|_{n_0,f_0} = \frac{1}{n_0} E[n_0; f_0]$$

binding energy

$$\frac{1}{n_0} E[n_0; f_0] = m_N + \varepsilon_0$$

f_0 is the scalar field at the density $n_p = n_n = n_0/2$

condition for the compressibility modulus

$$K = 9\, n_0 \left[\left.\frac{\partial^2 E}{\partial n^2}\right|_{n_0,f0} - \left(\left.\frac{\partial^2 E}{\partial n\, \partial f}\right|_{n_0,f_0} \right)^2 \left[\left.\frac{\partial^2 E}{\partial f^2}\right|_{n_0,f_0} \right]^{-1} \right]$$

effective nucleon mass

$$1 - f_0 = \frac{m_N^*}{m_N}$$

symmetry energy

$$\varepsilon_{\rm S}(n) = \frac{n}{8}\frac{\partial^2}{\partial n_p^2} E[n - n_p, n_p]\Big|_{n_p = n/2} = \frac{C_\rho^2\, n}{8 m_N^2} + \frac{\pi^2\, n}{4\, p_{\rm FN} \sqrt{m_N^{*2} + p_{\rm FN}^2}}$$

ρ-meson repulsion Pauli exclusion principle

equation for the scalar field

$$\frac{m_N^4 f}{C_\sigma^2} + \frac{{\rm d}U}{{\rm d}f} = 2 \int_0^{p_{\rm FN}} \frac{2{\rm d}^3 p}{(2\pi)^3} \frac{m_N(1-f)}{\sqrt{m_N^2(1-f)^2 + p^2}}$$

$$p_{\rm FN} = (3\pi^2 n/2)^{1/3}$$

n_0	$\simeq$	0.16 ± 0.015 fm^{-3}
ε_0	$\simeq$	-15.6 ± 0.6 MeV
$m_N^*(n_0)$	$\simeq$	$(0.75 \pm 0.1)\, m_N$
K	$\simeq$	240 ± 40 MeV
J	$\simeq$	32 ± 4 MeV

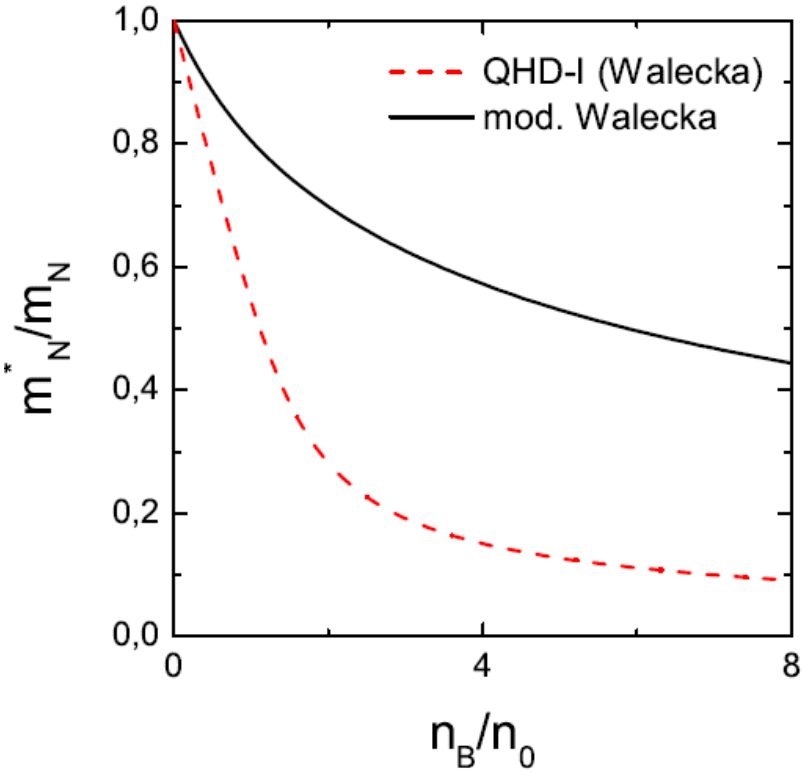
Input: $n_0 = 0.16\ \mathrm{fm}^{-3}, \quad \varepsilon_0 = -16\ \mathrm{MeV}, \quad J = 32\ \mathrm{MeV}, \quad m_N = 938\ \mathrm{MeV}$

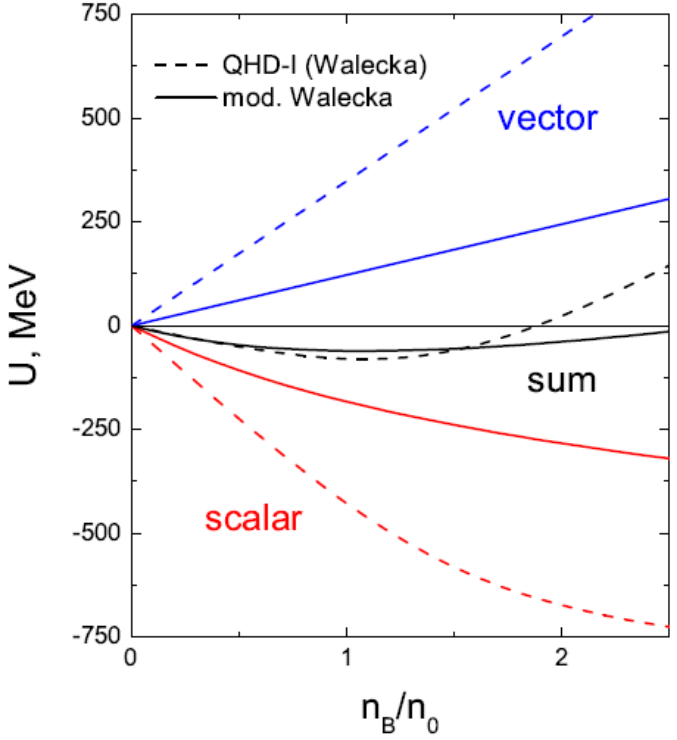
Walecka model $U(f) = 0 \quad \mathrm{W}: \quad C_\sigma^2 = 329.70\,, \; C_\omega^2 = 249.40\,, \quad C_\rho^2 = 68.09$

➡ $K \simeq 553\ \mathrm{MeV}, \quad m_N^*(n_0)/m_N \simeq 0.54$

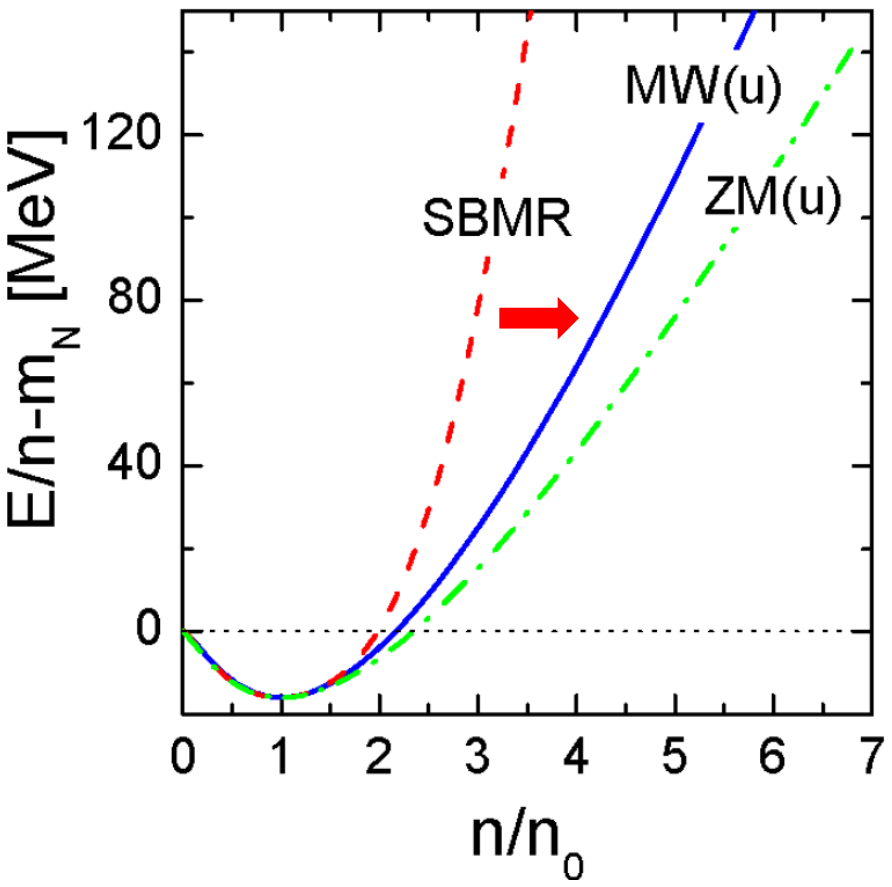
Modified Walecka model $U(f) = m_N^4(bf^3/3 + cf^4/4)$

Additional input: $K = 270\ \mathrm{MeV}, \quad m_N^*(n_0)/m_N = 0.8$

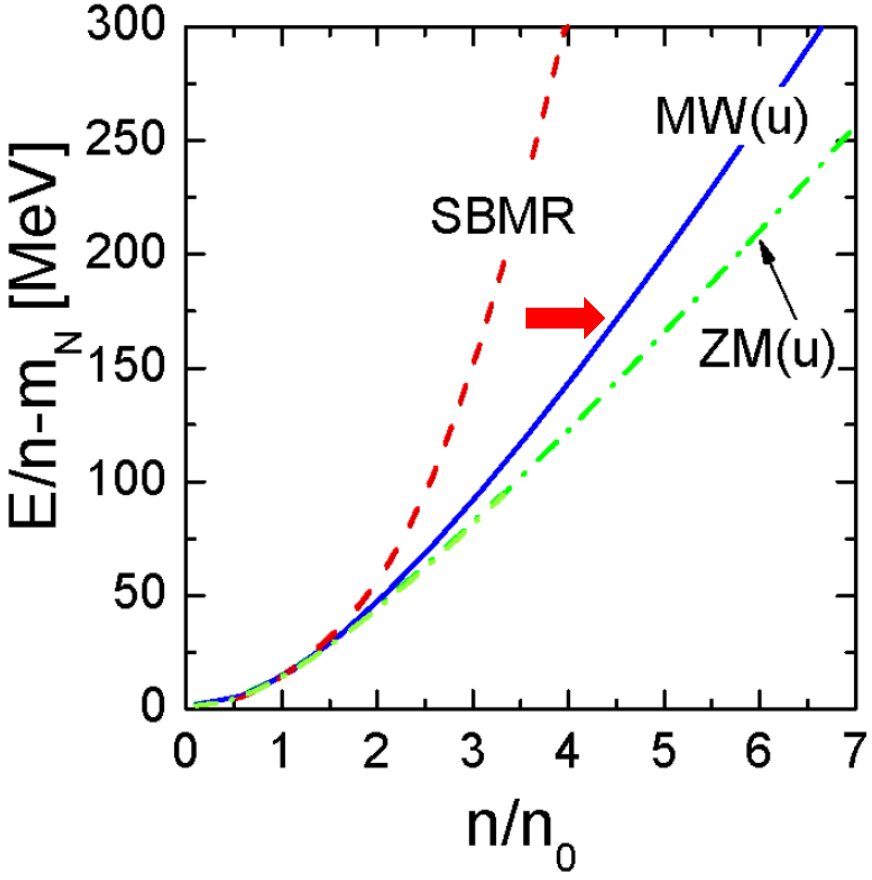




Isospin symmetric matter (ISM)



Pure neutron matter (PNM)



Compact stars and HICs II

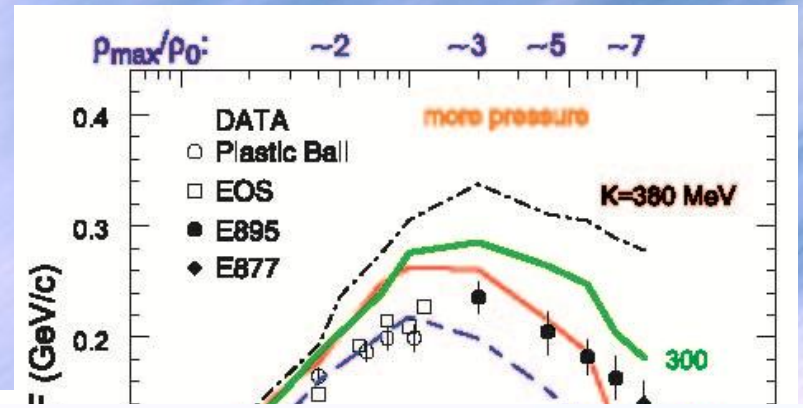
...per collisionis ad astra …

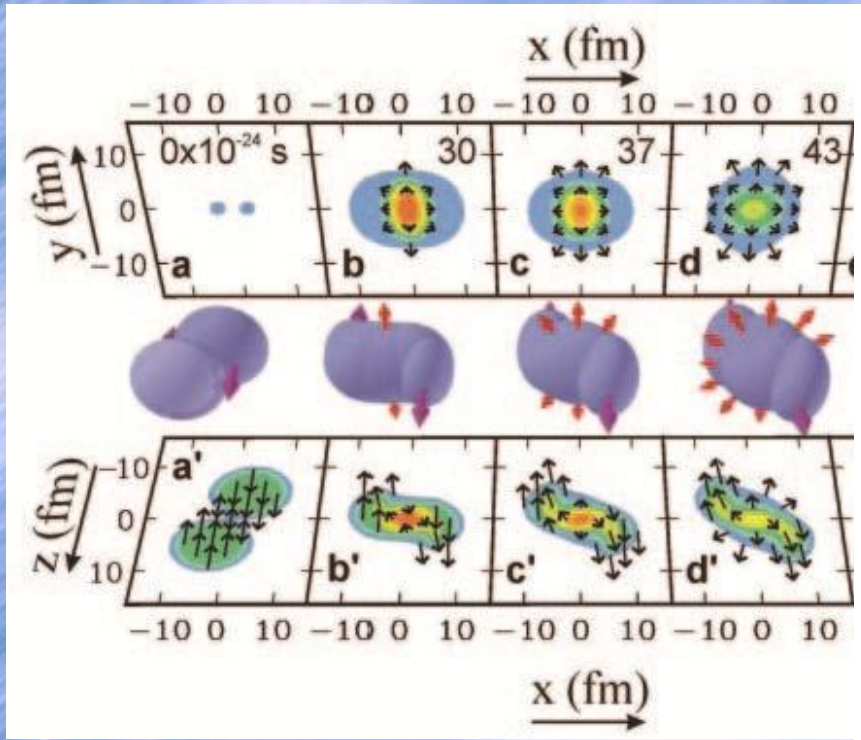
Constraints from heavy-ion collisions

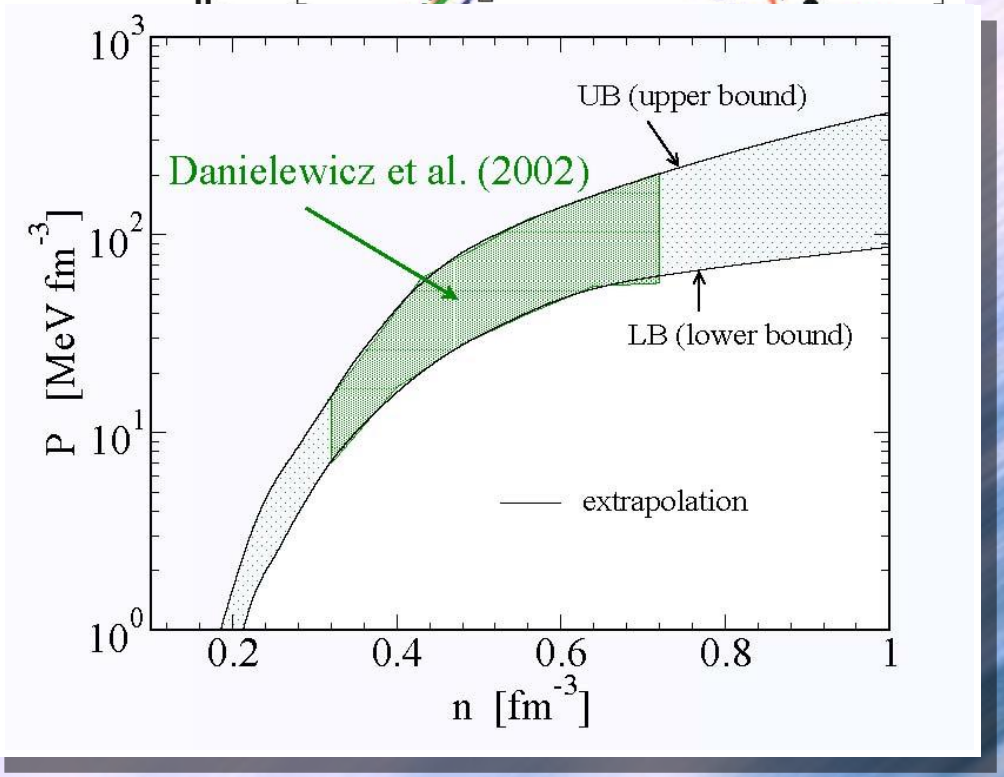
- Boltzmann kinetic equation
- Mean-field potential

$$U = (a\rho + b\rho^{\nu})/[1+(0.4\rho/\rho_0)^{\nu-1}] + \delta U_p$$

fitted to directed & elliptic flow

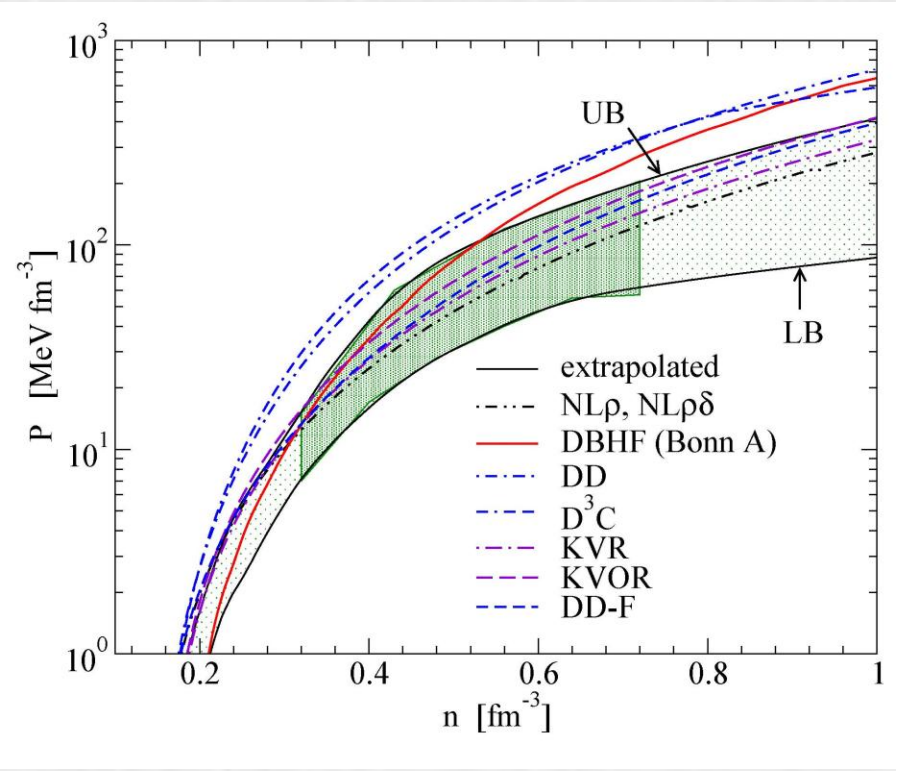




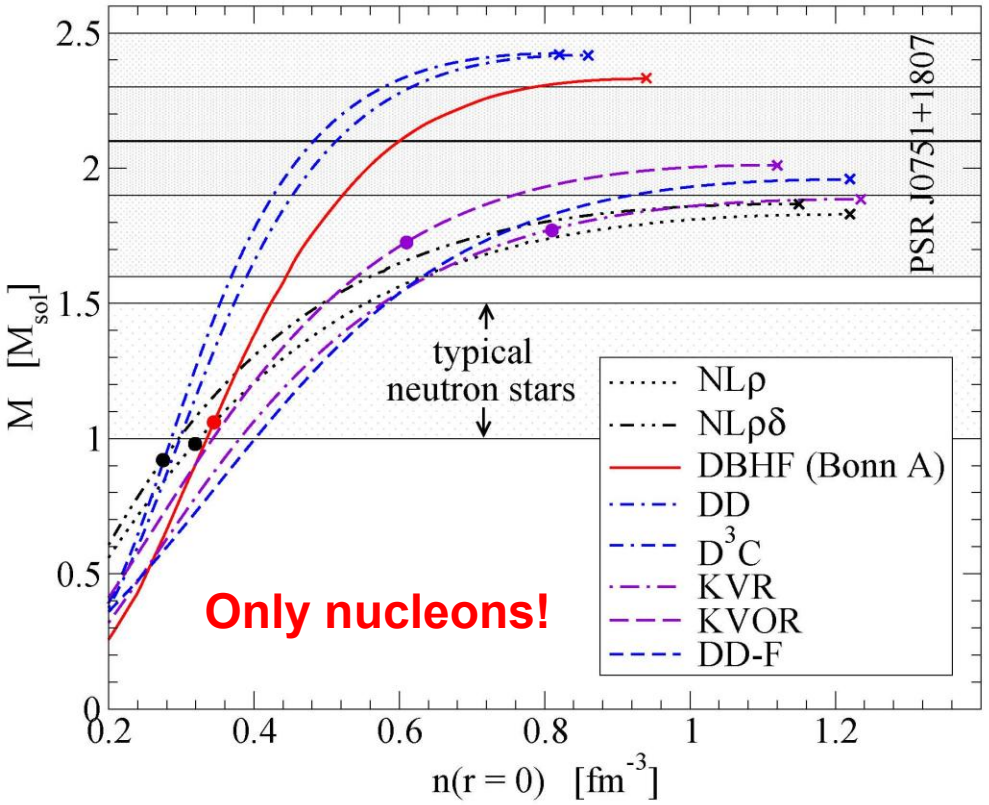


[Danielewicz, Lacey, Lynch, Science 298, 1592 (2002)]

✔ constraints from heavy-ion collisions



✔ maximum mass constraints

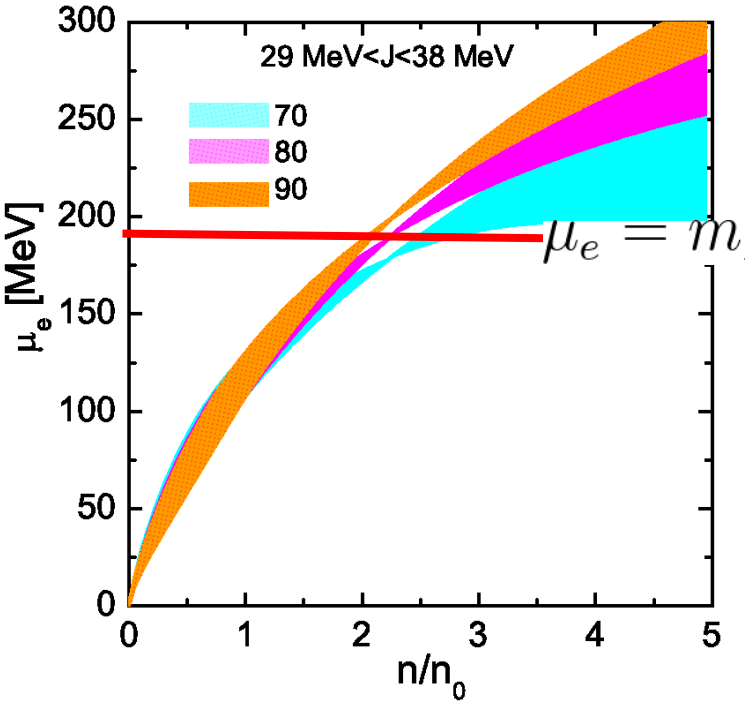


Puzzle

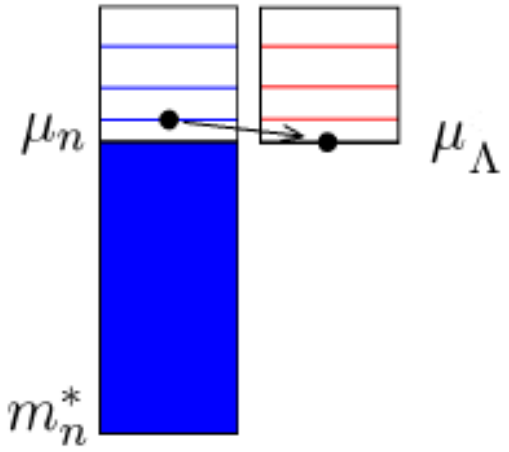
Nucleon part of EoS should be sufficiently stiff.
Pressure in isospin symmetric matter should be no to high. (particle flow in HIC)

[Klaehn et al., PRC 74 (2006) 035802.]

Hyperonization of nuclear matter



$$\mu_e = m_\Lambda - m_N$$

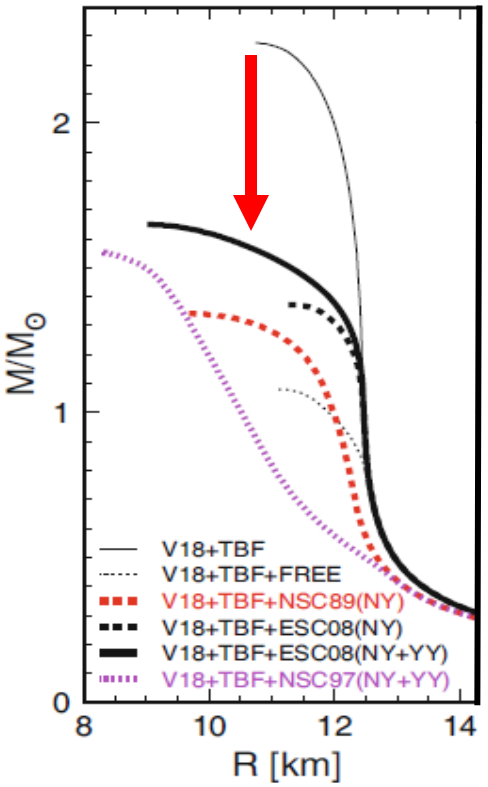


$$e^- + p \longrightarrow \Lambda + \nu_e$$

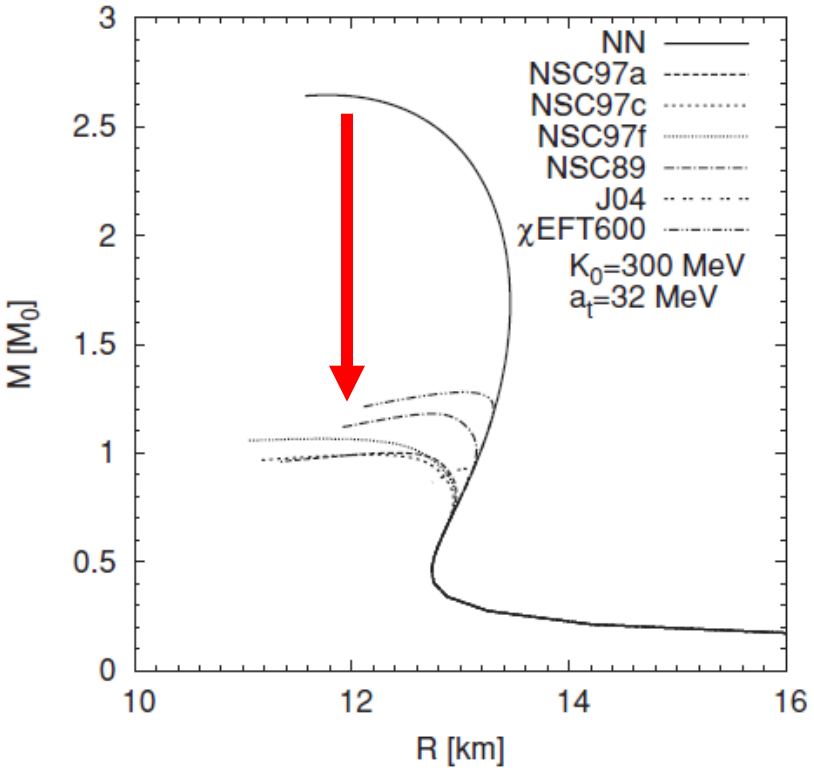
$$n \longrightarrow \Lambda + \nu + \bar{\nu}$$

“Hyperon puzzle”

If we allow for a population of new Fermi seas (hyperon, Δ baryons, …)
EoS will be softer and the NS will be smaller



[Rijken, Schulze, EPJA52 (2016) 21]



[Dapo, Schaefer, Wambach PRC 81 (2010) 035803]

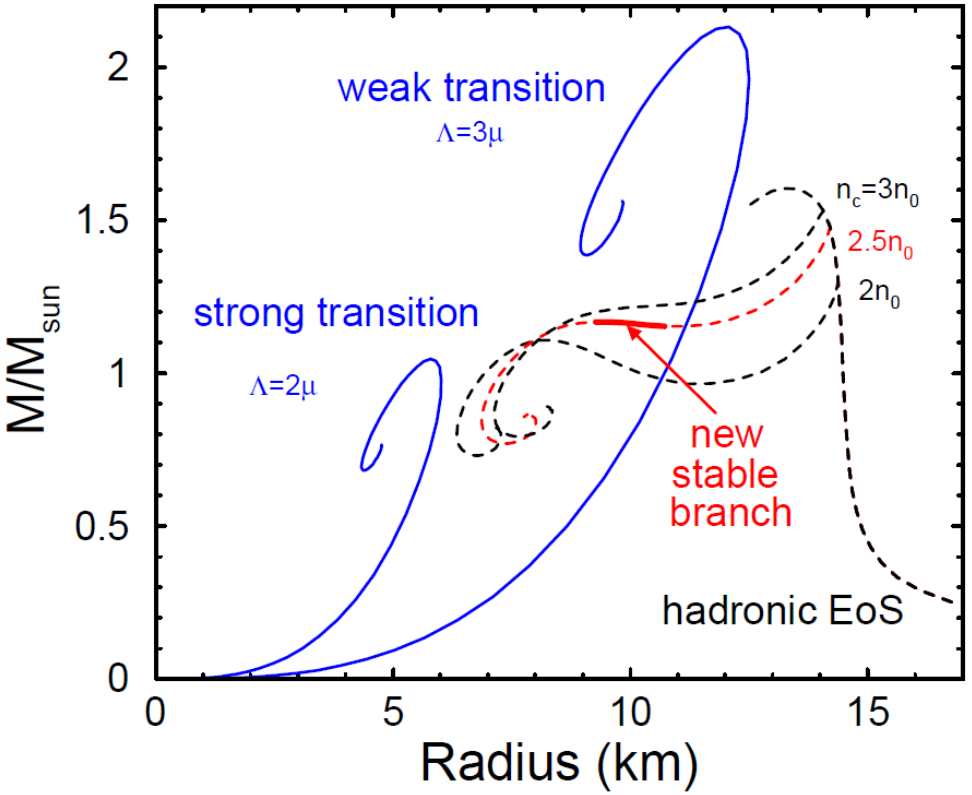
Simple solutions: **-- make nuclear EoS as stiff as possible** [flow constraint]
-- suppress hyperon population (increase repulsion/reduce attraction)

against phenomenology of YN,NN,YY interaction in vacuum
+hypernuclear physics

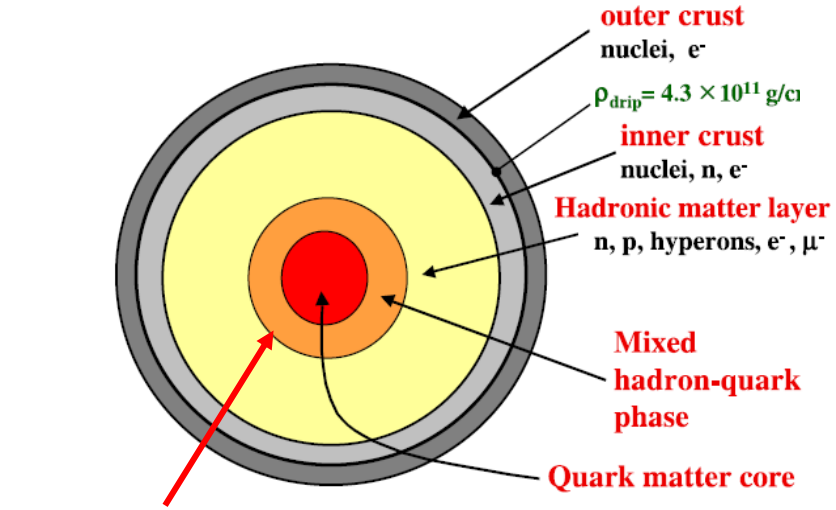
Quark Stars

Perturbative QCD EoS

[Fraga, Pisarski, Schaffner-Bielich, NPA 702,217]

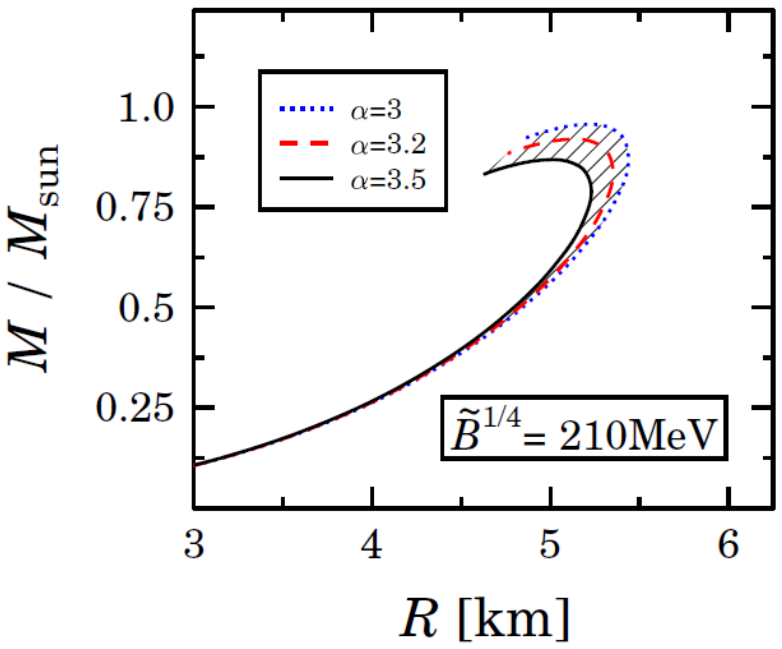


Λ renormalization subtraction point



[Bombaci 0809.4228]

no constraints from Lattice QCD!



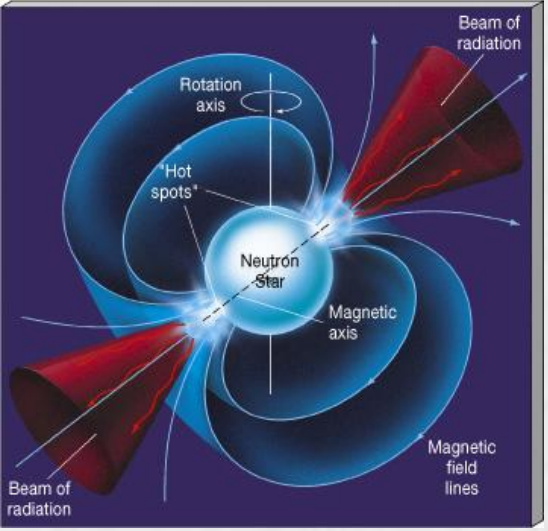
Quark stars are
very small (R<6 km)
and very light (M<1 Msol)

Matching between the realistic hadronic EoS and the quark EoS fitted to the lattice date is not always possible

[Ivanov et al. PRC72 (2005) 025804]

No hybrid stars are possible

Neutron star cooling



Pulsar age

Pulsar rotation period/frequency changes with time:

$$\dot{\Omega} = -\frac{B^2 R^6}{6 c^3 I} \sin^2 \alpha \, \Omega^3 - G\frac{32\varepsilon^2}{5c^5} I \, \Omega^5$$

e.m. wave emission

grav. wave emission

α *angle between rotation a and magnetic axes*

ε *neutron star eccentricity*

$$\dot{\Omega} = \varkappa \Omega^n$$

$$P = 2\pi/\Omega$$

$$(n-1)\frac{\dot{P}(t)}{P(t)}t = 1 - \left(\frac{P_0}{P(t)}\right)^{n-1}$$

initial period

current period

$$P_0/P \ll 1$$

spin-down age:

$$\tau_{\rm sd} = \frac{1}{n-1}\frac{P}{\dot{P}}$$

$$n = \frac{\ddot{\Omega}\,\Omega}{\dot{\Omega}^2}$$

braking index:

kinematic age:

1) age of the associated SNR

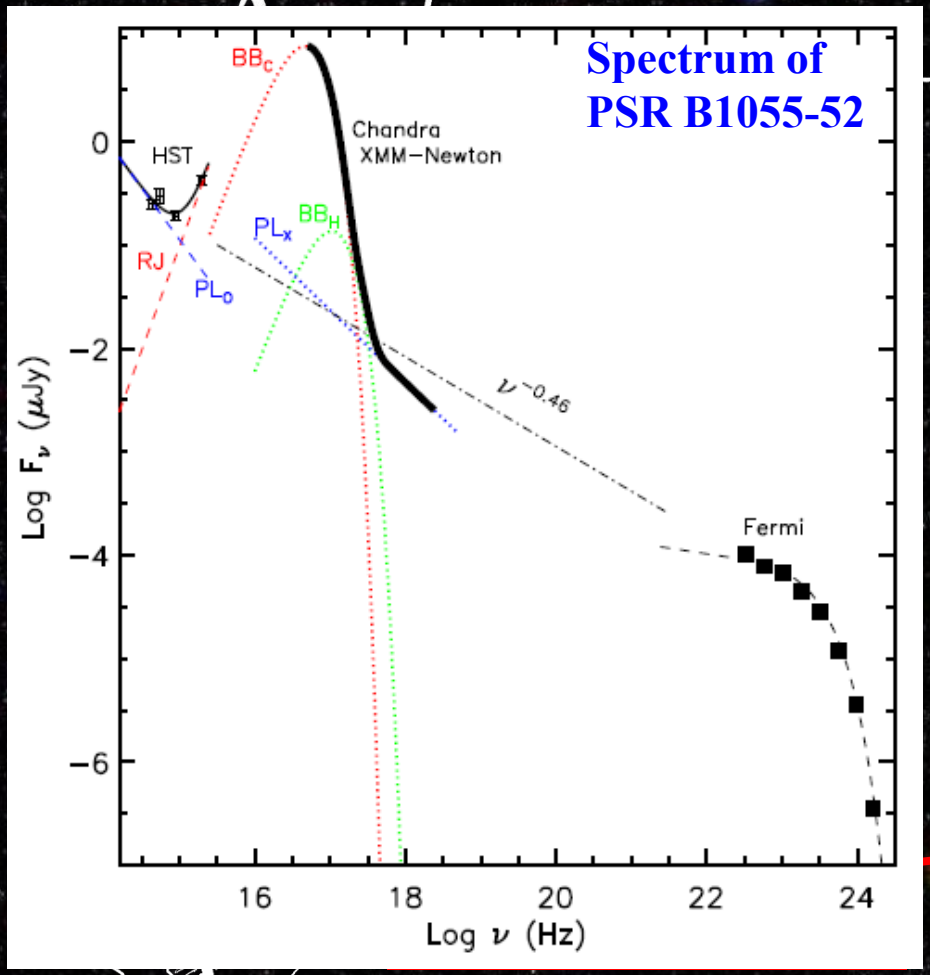
2) pulsar speed and position w.r. to the geometric center of the associated SNR

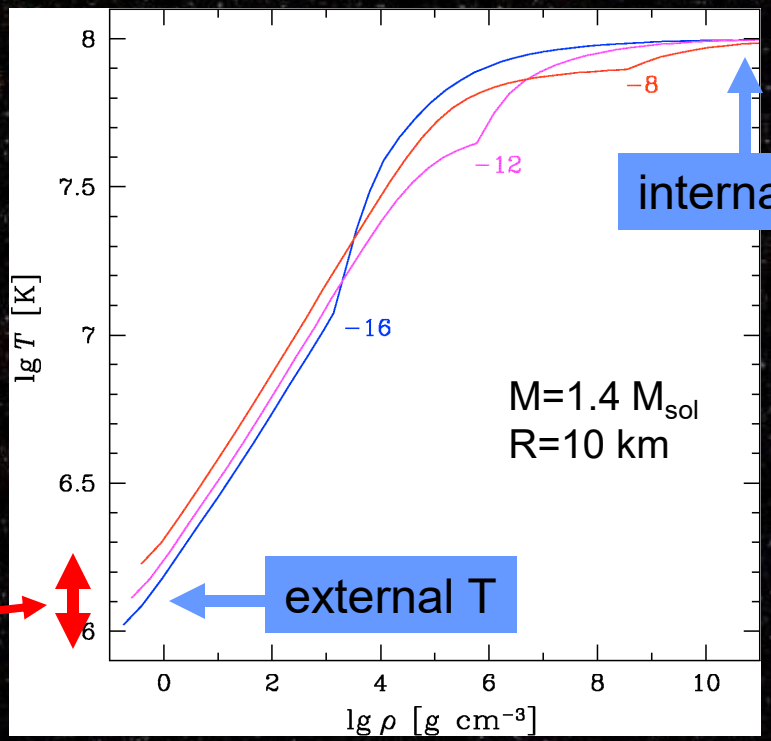
3) historical events

Crab : 1054 AD
Cassiopeia A: 1680 AD
Tycho's SN: 1572 AD

Measuring pulsar temperature

- radiation spectrum and source geometry
- internal vs. external temperature
- heat transfer in envelop





envelop

debris of supernova explosion;
accreted "nuclear trash"

[Yakovlev et al., A&A 417, 169]

Neutron star cooling data

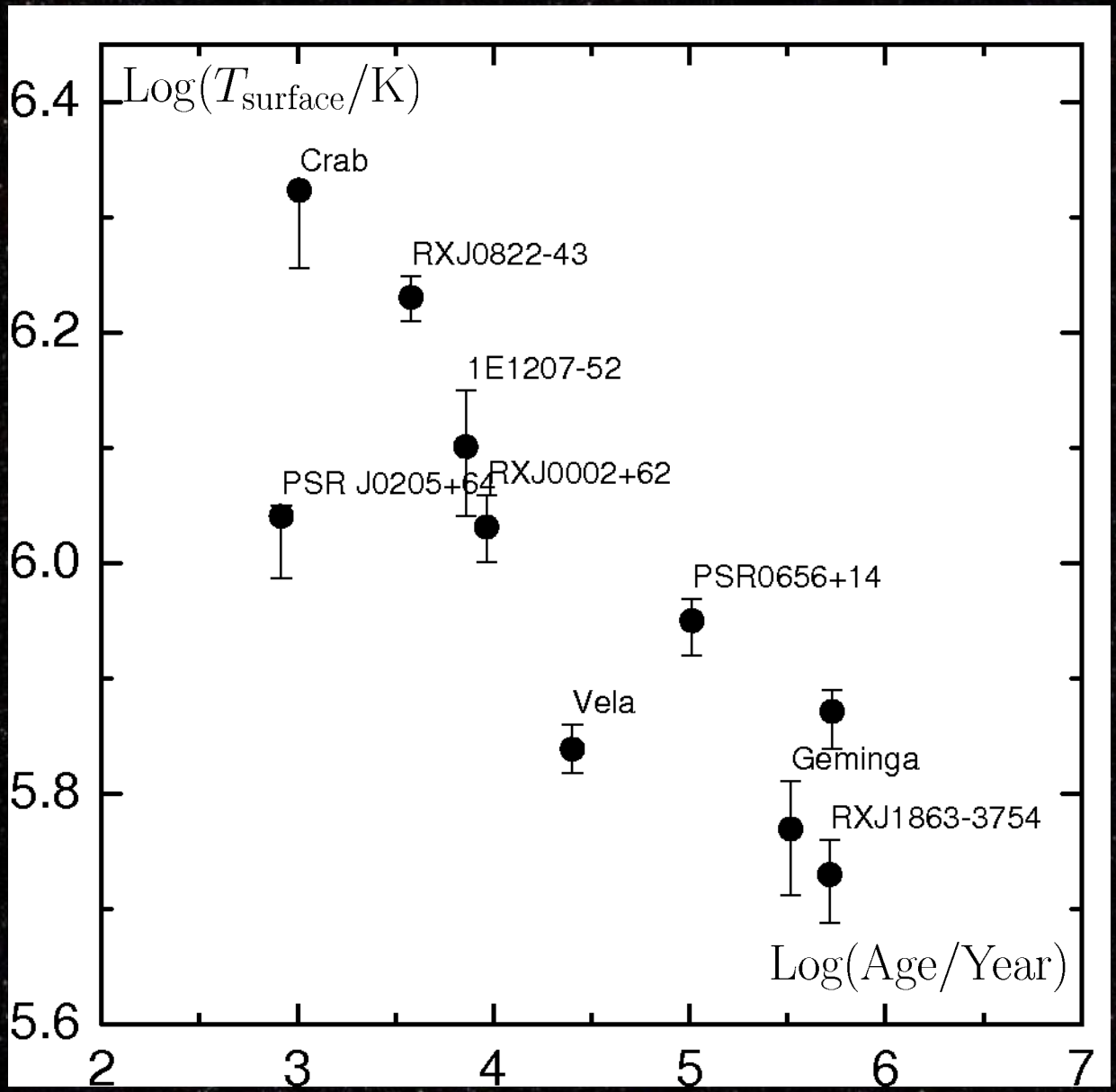
Given:

- **EoS**
- **Cooling scenario**
  [neutrino production]

Mass of NS

↓

Cooling curve



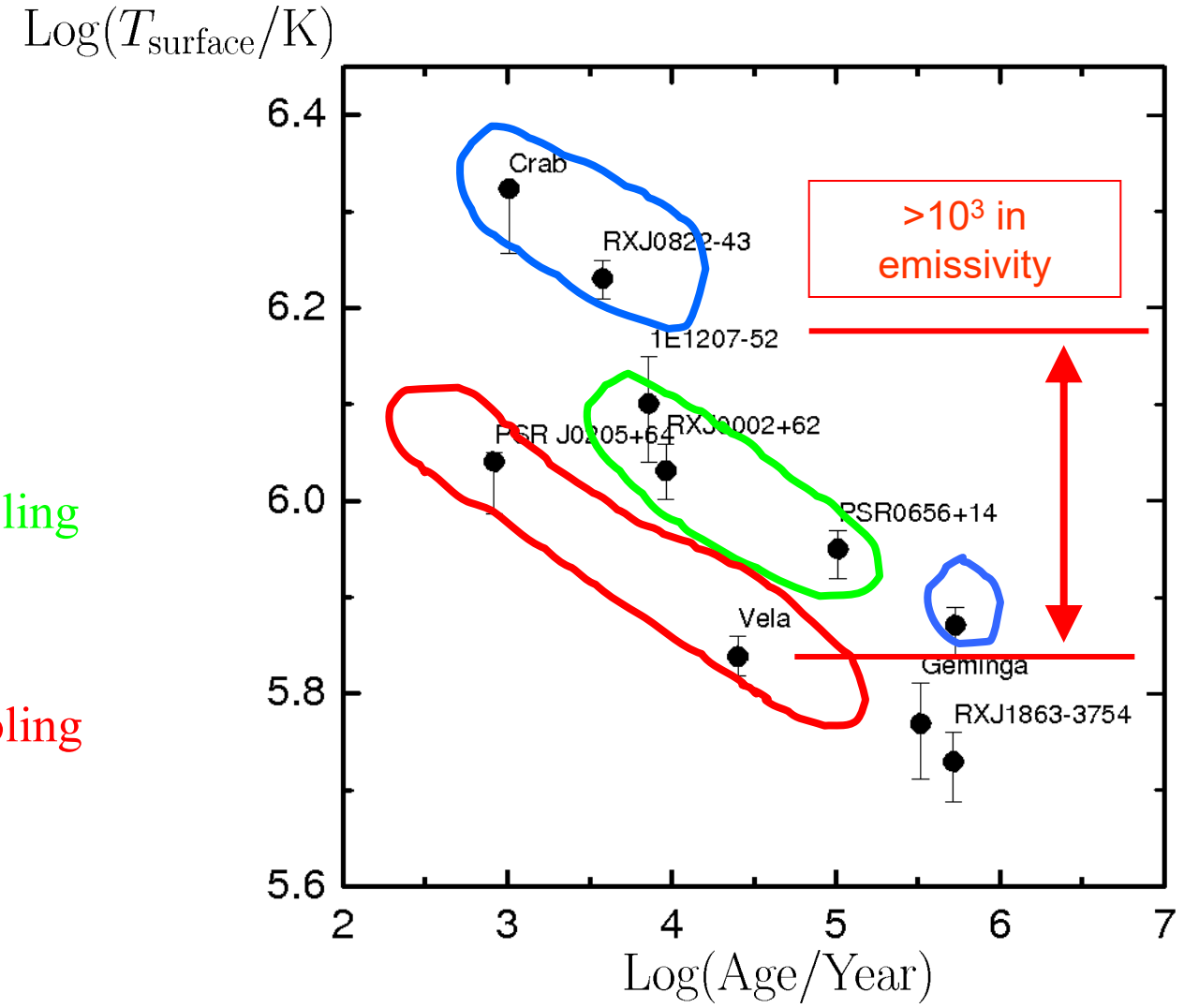
Neutron Star Cooling Data

3 groups:

slow cooling

intermediate cooling

rapid cooling



All groups can be described within one cooling scenario if there is a strong dependence of neutrino luminocity on the NS mass ⟷ strong density-dependence of emissivity

[Voskresensky, Senatorov 1986]

Neutrino emission reactions

$T < T_{\rm opac} \sim 10^{-1} \div 10^{0}$ MeV neutron star is transparent for neutrino

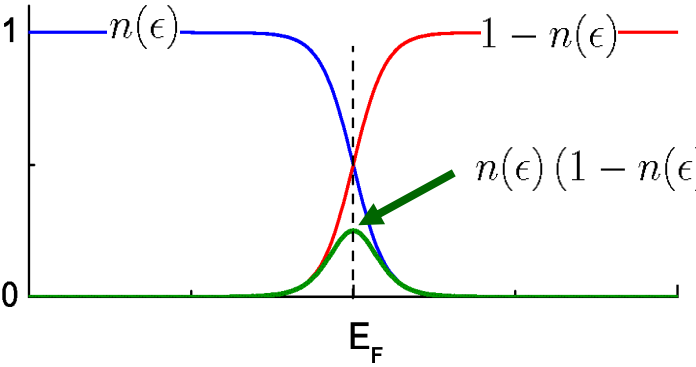
$$C_V \frac{\mathrm{d}T}{\mathrm{d}t} = -L$$

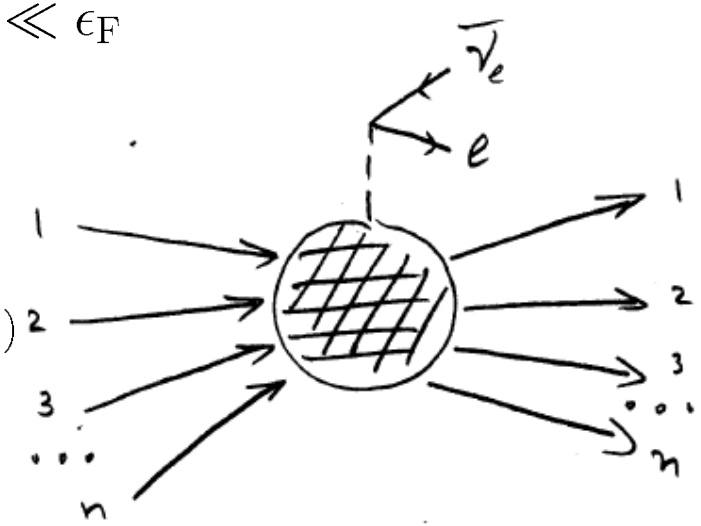
C_V – *heat capacity, L - luminosity*

$$L = \int \mathrm{d}V \sum_{\text{reaction r}} \epsilon_\nu^{(r)}$$

emissivity

✔ *n*-nucleon reaction $T \ll \epsilon_{\rm F}$



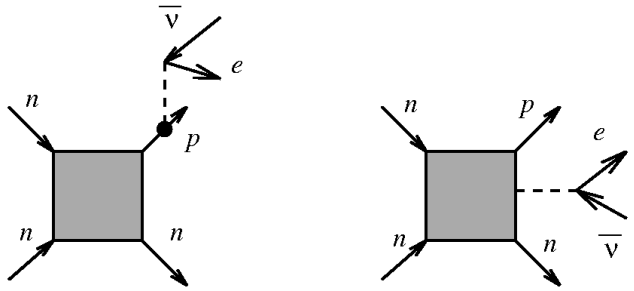


$\epsilon_\nu \sim T^{2n+4}$

each leg on a Fermi surface / T

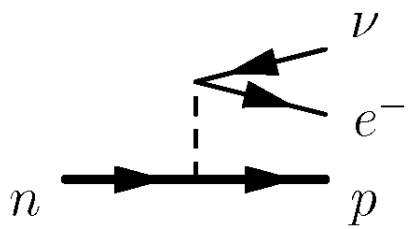
neutrino phase space ´ neutrino energy $\omega_{\bar\nu} \times \delta(\omega_{\bar\nu} - \dots)\,\omega_{\bar\nu}^2\, d\omega_{\bar\nu} \sim T^3$

standard: modified Urca



$$L = 10^{22} \left(\frac{T}{10^9\,\mathrm{K}}\right)^8 \left(\frac{n_e}{n_0}\right)^{\frac{1}{3}} \quad \frac{\mathrm{erg}}{\mathrm{cm}^3\,\mathrm{s}}$$

exotic: direct Urca



$$\text{allowed if } |p_{\mathrm{F},n} - p_{\mathrm{F},p}| < p_{\mathrm{F},e}$$

$$L = 10^{27} \times \left(\frac{T}{10^9\,\mathrm{K}}\right)^6 \left(\frac{n_e}{n_0}\right)^{\frac{1}{3}} \quad \frac{\mathrm{erg}}{\mathrm{cm}^3\,\mathrm{s}}$$

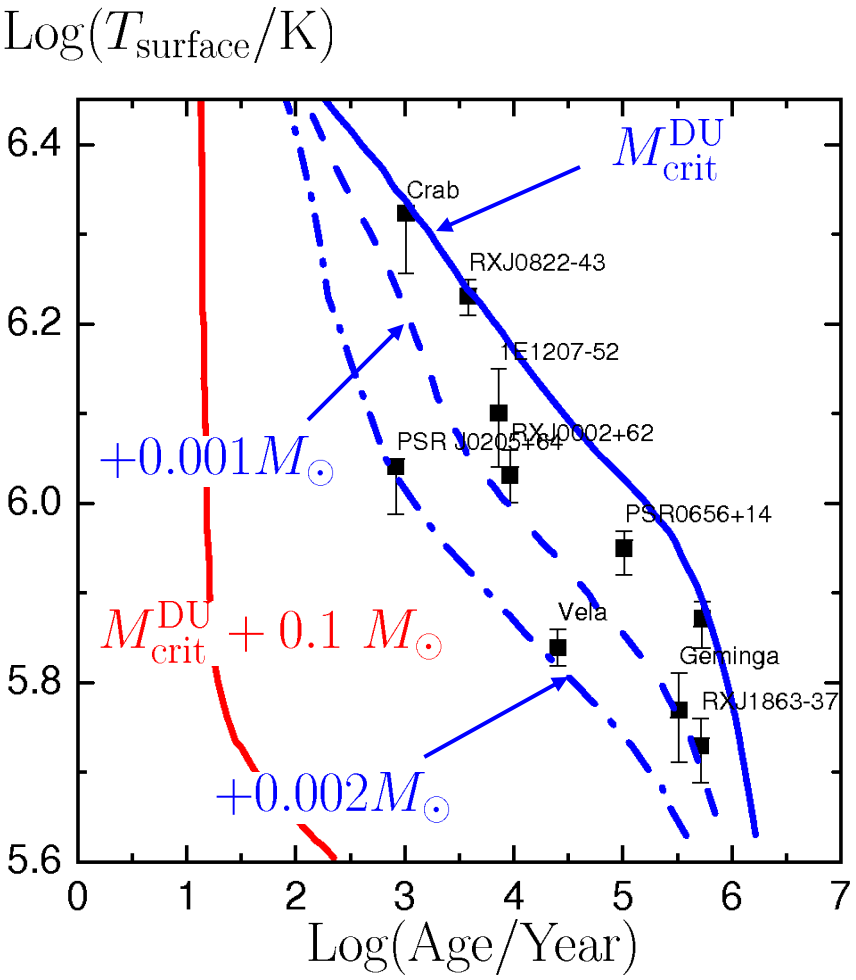
starts at some critical density, i.e. in stars with $M > M^{DU}_{crit}$

Neutron Star Cooling Scenario

standard scenario (MU+pairing)

only "slow" cooling can be described

Neutron stars with $M > M^{\rm DU}_{\rm crit}$ will be too cold



[Blaschke, Grigorian, Voskresensky A&A 424 (2004) 979]

DU process schould be „exotics"

(if DU starts it is dificult to stop it)

$M^{\rm DU}_{\rm crit} \gtrsim 1.3\ M_\odot \qquad n^{\rm DU}_{\rm crit} \gtrsim 4\, n_0$

EoS should produce a large DU threshold in NS matter !

[EEK, Voskresensky NPA759 (2005) 373]

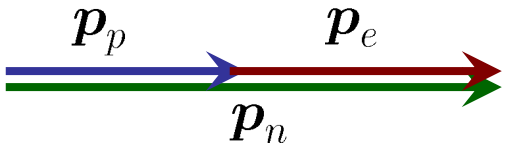
Direct Urca Constraint

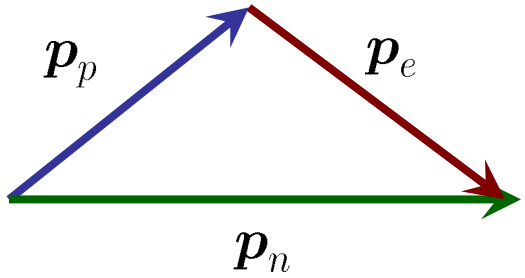
$$n \to p + e^- + \tilde{\nu}_e$$

triangle inequality

$$p_{\mathrm{F},n} \le p_{\mathrm{F},n} + p_{\mathrm{F},e}$$

critical condition





charge neutrality: $n_p = n_e + n_\mu$ $\qquad p_{\mathrm{F},p}^3 = p_{\mathrm{F},e}^3 + p_{\mathrm{F},\mu}^3$

total density $n = n_p + n_n$ $\to$ $p_{\mathrm{F},n}^3 + p_{\mathrm{F},p}^3 = 3\pi^2 n$

$$(p_{\mathrm{F},p} + p_{\mathrm{F},e})^3 + p_{\mathrm{F},p}^3 = 3\pi^2 n \quad \to \quad \left(1 + \frac{p_{\mathrm{F},e}}{p_{\mathrm{F},p}}\right)^3 + 1 = \frac{3\pi^2 n}{p_{\mathrm{F},p}} = \frac{n}{n_p} = \frac{1}{x_p} \qquad \frac{p_{\mathrm{F},e}^3}{p_{\mathrm{F},p}^3} = \frac{x_e}{x_p} = x_{e/p}$$

$$\to \quad x_p^{(\mathrm{DU})}(x_{e/p}) = \frac{1}{1 + \left(1 + x_{e/p}\right)^{1/3}}$$

no muons: $x_{e/p} = 1$ $\qquad x_p^{(\mathrm{DU})} = 11.1\%$

relativistic limit ($n_e = n_\mu$): $x_{e/p} = 1/2$ $\qquad x_p^{(\mathrm{DU})} = 14.8\%$

the value of $x_{e/p}$ depends on the symmetry energy of the nuclear matter

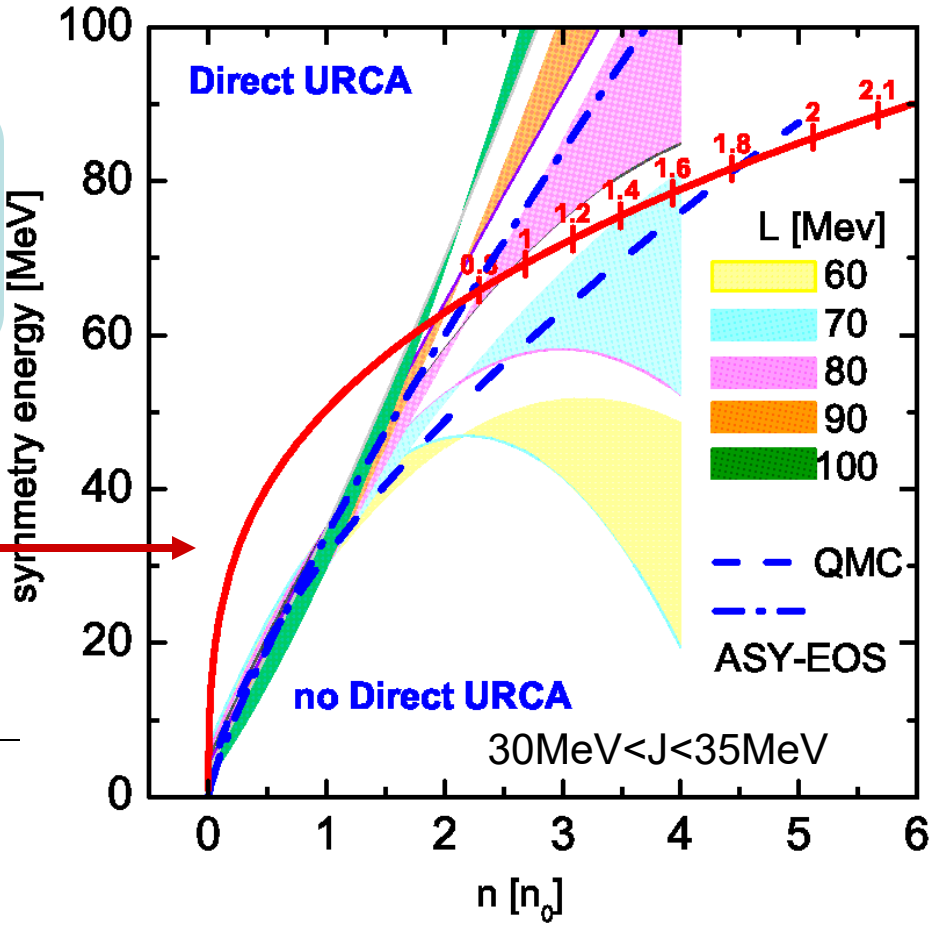
The universal "DU curve"

$$\mu_e = 4\,E_S(n)\,(1-2\,x_{\rm DU}(\mu_e))$$

$$\frac{(\mu_e^2-m_e^2)^{3/2}}{3\,\pi^2}+\frac{(\mu_e^2-m_\mu^2)^{3/2}}{3\,\pi^2}=x_{\rm DU}(\mu_e)\,n$$

"DU curve" for the symmetry energy

$$E_S^{\rm DU}(n)$$



Low density expansion

$$E_S(n)=J+\frac{L}{3}\frac{n-n_0}{n_0}+\frac{K_{\rm sym}}{18}\frac{(n-n_0)^2}{n_0^2}+\ldots$$

$$L=-19.5\ {\rm MeV}+3\,J+\frac{K_{\rm sym}}{5.50}$$

[Dong, et al PRC85, 034308 (2012)]

Pionization of neutron star matter

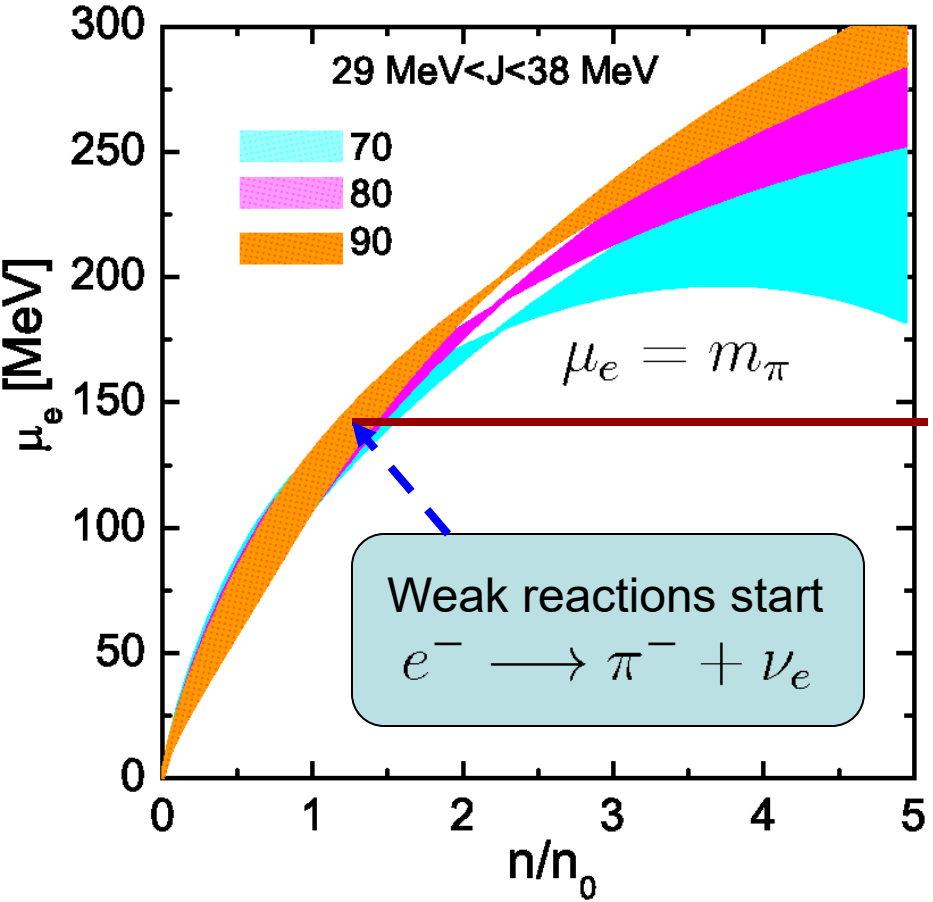
Simple model for NS matter

n+p+e+μ matter

Proton concentration $x = n_p/n$ follows from equations

$$\mu_e = \mu_n - \mu_p = \frac{\partial E}{\partial n_n} - \frac{\partial E}{\partial n_n} = 4\,E_S(n)\,(1-2\,x)$$

$$\frac{(\mu_e^2 - m_e^2)^{3/2}}{3\,\pi^2} + \frac{(\mu_e^2 - m_\mu^2)^{3/2}}{3\,\pi^2} = x\,n$$



Lightest negatively charged bosons: π^-

$$m_\pi = 140 \text{ MeV}$$

$\sqrt{m^2 + k^2}$ → minimum at k=0

Pionization of neutron star matter

Chiral symmetry for pion-nucleon interaction

isospin even and odd amplitudes $T^{(\pm)} = \frac{1}{2}\left[T(\pi^- p) \pm T(\pi^- n)\right]$

Chiral symmetry: for forward scattering amplitude: $T^{(\pm)}(\omega, q^2)$

on mass-shell $q^2 = \omega^2 - \mathbf{k}^2 = m_\pi^2$

$$\lim_{\omega\to 0} \omega^{-1} T^{(-)}(\omega, m_\pi^2) = \frac{1}{2 f_\pi^2}$$ Weinberg Tomazawa theorem

$$T^{(-)}(\omega) \approx \frac{\omega}{2 f_\pi^2}$$

- **polarization operator**

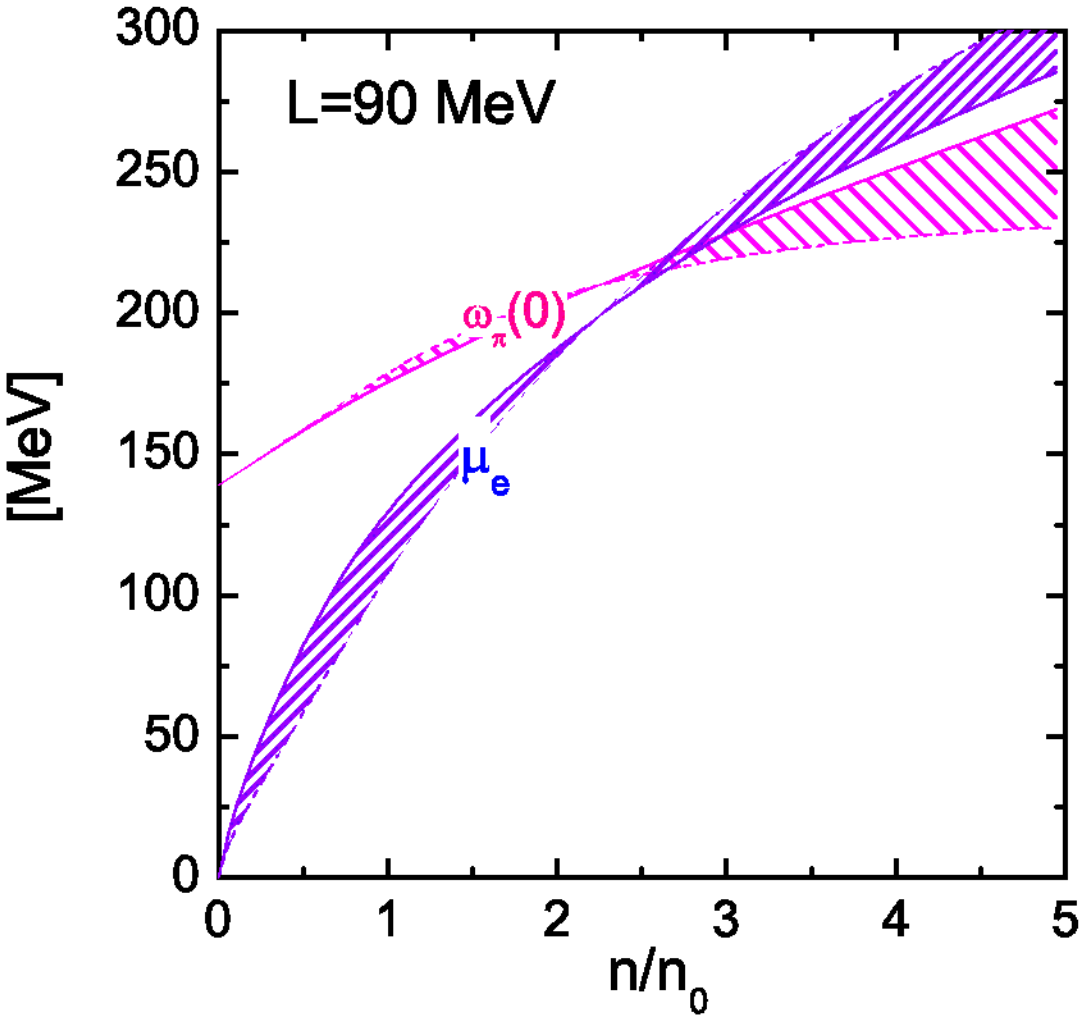
$$\Pi_S(\omega) = -T^{(-)}(\omega)\,(\rho_p - \rho_n)$$

repulsive in neutron reach matter

- **spectrum** $D^{-1}(\omega, k=0) = \omega^2 - m_\pi^2 - \Pi_S(\omega) = 0$

29 MeV<J<38 MeV

chiral shield against pionization



Detailed analysis of the possibility of the s-wave pion condensation in Onishi, Jido et al, PRC 80, 038202 (2009)

Stability of Vacuum and Limiting Fields

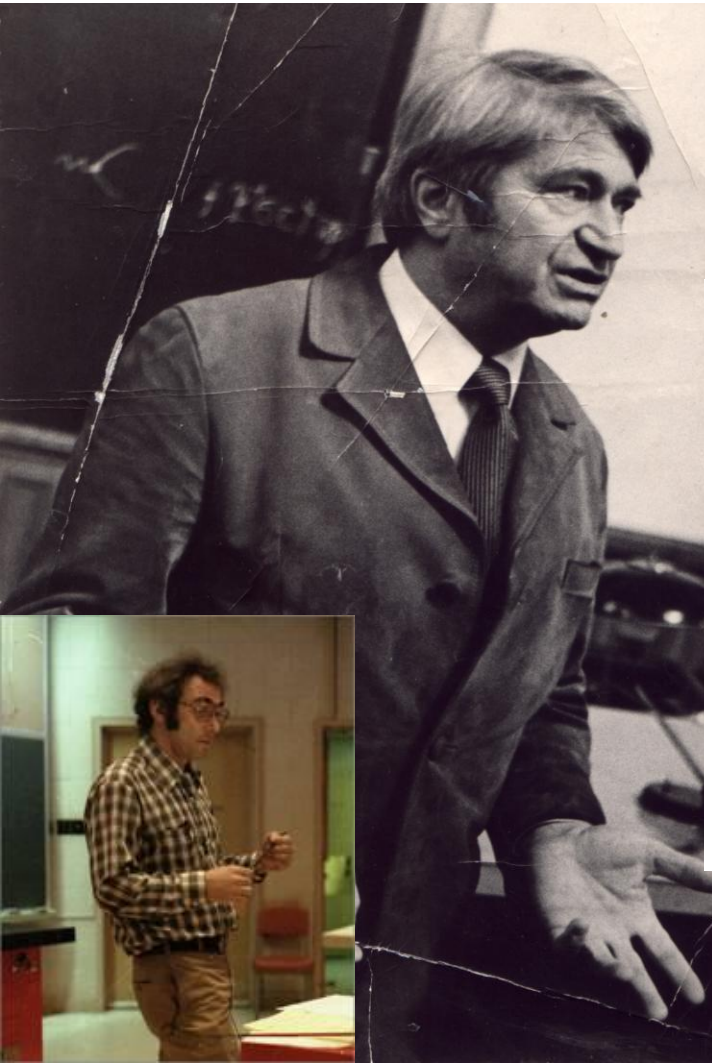
A. B. MIGDAL

L. D. Landau Institute of Theoretical Physics, USSR Academy of Sciences

Submitted June 21, 1971

Zh. Eksp. Teor. Fiz. **61**, 2209–2224 (December, 1972)





Condensed π^- Phase in Neutron-Star Matter*

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(Received 29 March 1972)

π^- Condensate in Dense Nuclear Matter*

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(Received 17 April 1972)

Pion Condensation in Nuclear and Neutron Star Matter*

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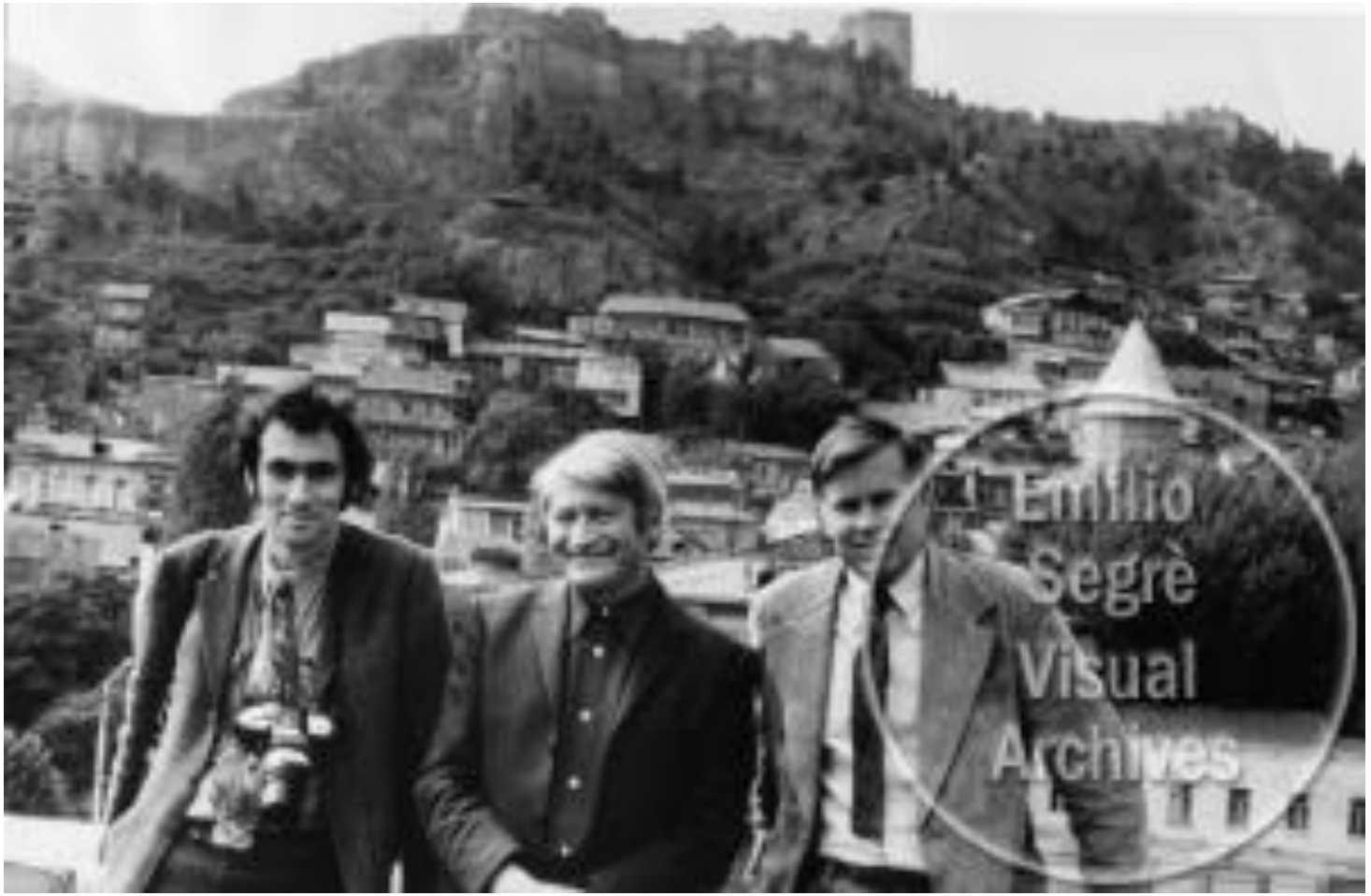
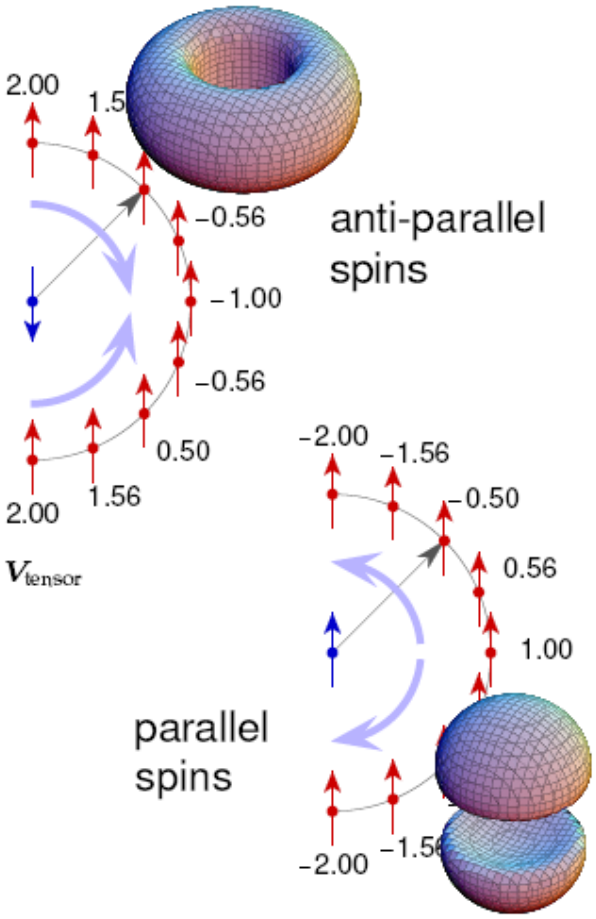
(Received 13 April 1973)



Baym Migdal Scalapino

1974 Tbilisi

Tensor forces in NN interaction



$$V_{\text{tensor}} = V_t(r) \left(3 \frac{(\boldsymbol{\sigma}_1 \boldsymbol{r})(\boldsymbol{\sigma}_2 \boldsymbol{r})}{r^2} - \boldsymbol{\sigma}_1 \boldsymbol{\sigma}_2 \right)$$

Resummed, enhanced pion exchange

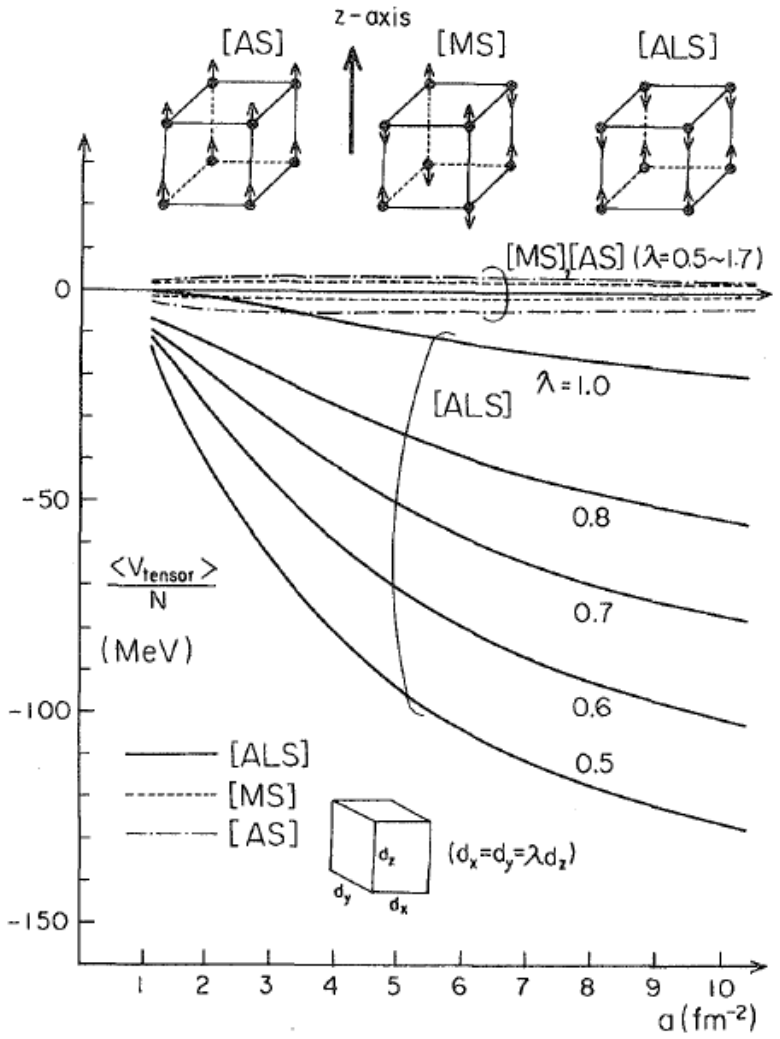
nucleon arrangement

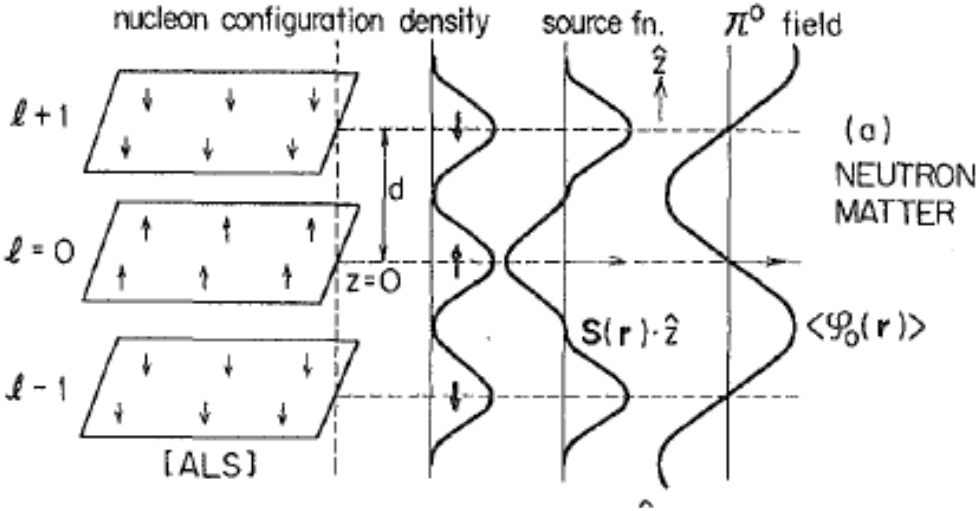
Alternating-layer-spin configurations

Ryozo Tamagaki

Tatsuyuki Takatsuka

1972'-73'



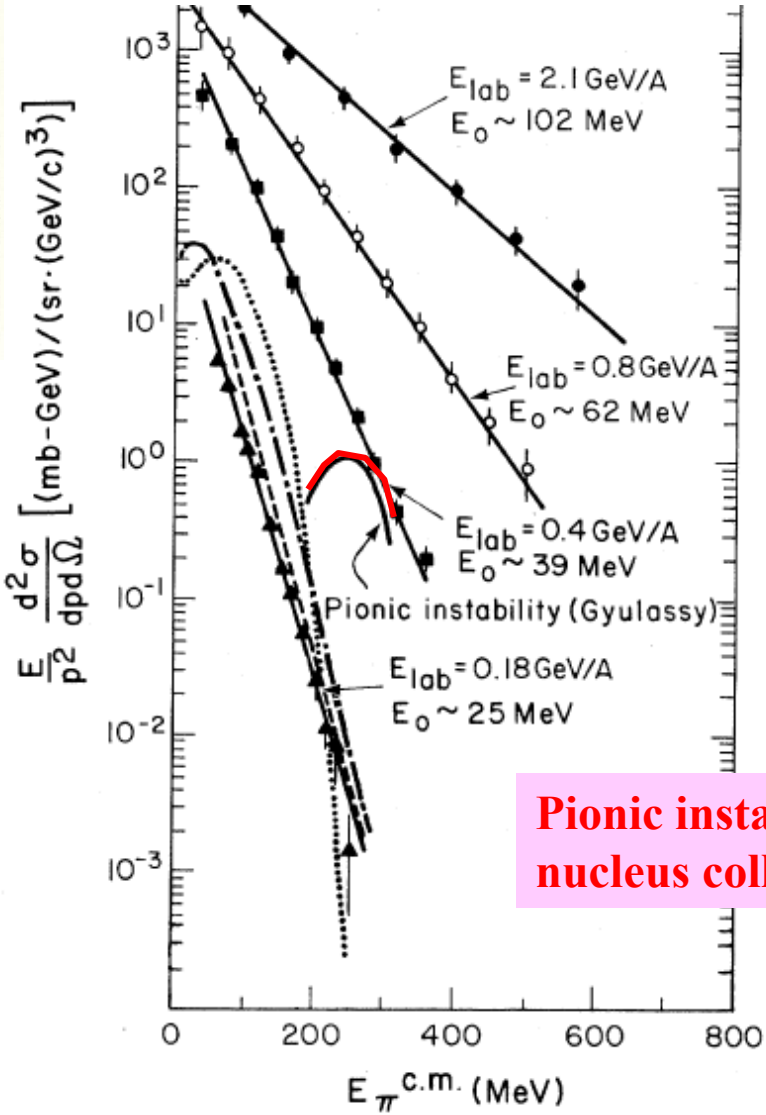


π^0 condensate



Pion condensation

A.B. Migdal, and D. Scalapino, R. Sawyer



Pionic instability in nucleus collisions

Conclusion

NSs and HICs are the only sources of the information about properties of the strongly interacting matter under extreme condition.

They provide test for our theories and models in dynamical systems.

Problems in discussion:

Relativistic equation of state?
Inclusion of new particles (hyperons, Deltas)?
Meson in medium?