Giant resonances in nuclei

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Lecture at International Summer School
"Nuclear Dynamics: from Stars to Colliders",
Dubna, JINR, June 29 – July 10, 2026

Content of the talk:

Lecture of R.V. Jolos

- Basics:
  - mean field, quantum shells, single-particle potentials
  - deformation
  - resonance widths, Landau damping
  - sum rules
  - density functional theory (DFT) → *Lectures of A.P. Severyukhin, E.V. Mardyban*

- Variety of giant resonances (GR) in nuclei:
  - basic properties, main research directions, open problems
  - Giant dipole resonance E1(T=1),
  - Giant monopole resonance E0(T=0)
  - exotic E1(T=0) GR: pygmy, toroidal and compression
  - magnetic M1 (orbital scissors and spin-flip) GR

Mean field,
quantum shells,
single-particle potentials

Mean field

- Complicated task:
  - many particles,
  - interaction between particles
  - in nuclei interaction is not well determined.

Microscopic Hamiltonian:

$$H\Psi = \{\sum_{i=1}^{A}(-\frac{\hbar^2}{2m}\Delta_i) + \sum_{i<j}^{A} v(i,j)\}\ \Psi(\vec{r}_1,....,\vec{r}_A)$$

$$\Downarrow$$

$$h_0(\vec{r})\varphi\ (\vec{r}) = \varepsilon\ \varphi\ (\vec{r})$$

- Conception of mean field: to replace the action of other particles by action of some mean field.

- Then the complicated many-body task with interaction is simplified to the task of motion of one particle in the mean field.

$$H = H_0 + V_{res}, \quad H_0\Psi_0 = E\Psi_0,$$

$$H_0 = \sum_{j=1}^{A} h_0(\vec{r}_j), \quad h_0(\vec{r}_j) = -\frac{\hbar^2}{2m}\Delta_j + V(\vec{r}_j),$$

⇐ **Mean-field Hamiltonian**

$$V_{res} = H - H_0 = \sum_{i<j}^{A} V_{res}(i,j)$$

⇐ **Residual two-body interaction**

Mean-field potential can be found by two ways :

- self-consistent (Hartree-Fock method),
- phenomenological (not self-consistent): simple form with fitted parameters.

Phenomenological single-particle potentials:

$$V(r) = \frac{1}{2} m\omega_0^2 r^2 - \frac{4}{3}\sqrt{\frac{\pi}{5}} m\omega_0^2 r^2 \delta_{20} Y_{20}(\theta,\varphi) + C\vec{l}\cdot\vec{s} + Dl^2$$ **- Nilsson potential**

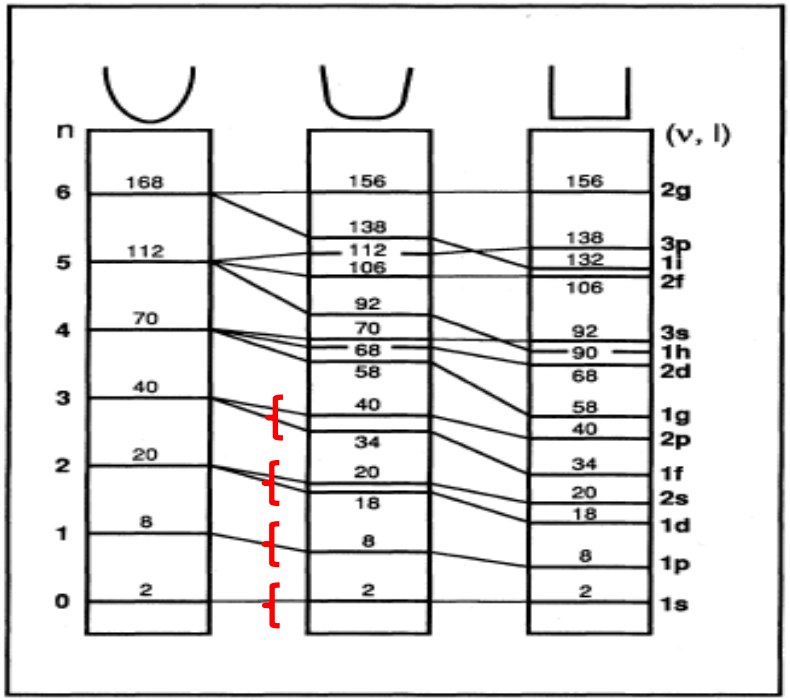
Unrealistic behavior on the nuclear boundary

$$V(r) = \frac{V_0}{1+e^{\frac{r-R}{\alpha}}} + C\vec{l}\cdot\vec{s}$$ **- Woods-Saxon potential**

- **Realistic diffuseness at the nuclear boundary**
- **Deformation is introduced through the radius expansion**

$$R = R_0(\delta_0 + \sum_{\lambda\mu} \delta_{\lambda\mu} Y_{\lambda\mu}(\Omega)) \Rightarrow R_0(\delta_0 + \delta_{20} Y_{20}(\Omega))$$

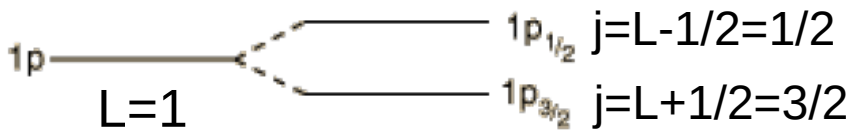
HO Nilsson square



Spin-orbit interaction:

$$(\vec{l}\cdot\vec{s})$$

Spin-orbit splitting:



Quantum shell = well separated group of levels in s-p spectrum

Quantum shells and magic numbers

nlj – **level notation,**
n – **node number,**
l – **orbital moment,**

$j = l \pm 1/2$

s, p, d, f, g, ...

l = 0, 1, 2, 3, 4, …

Magic numbers:
2, 8, 20, 28, 50, 82, 126

Nuclei with magic numbers

$^{A}_{Z}A_{N}$: $^{16}_{8}O_{8}$, $^{48}_{20}Ca_{28}$, $^{208}_{82}Pb_{126}$

are most stable and have spherical shape

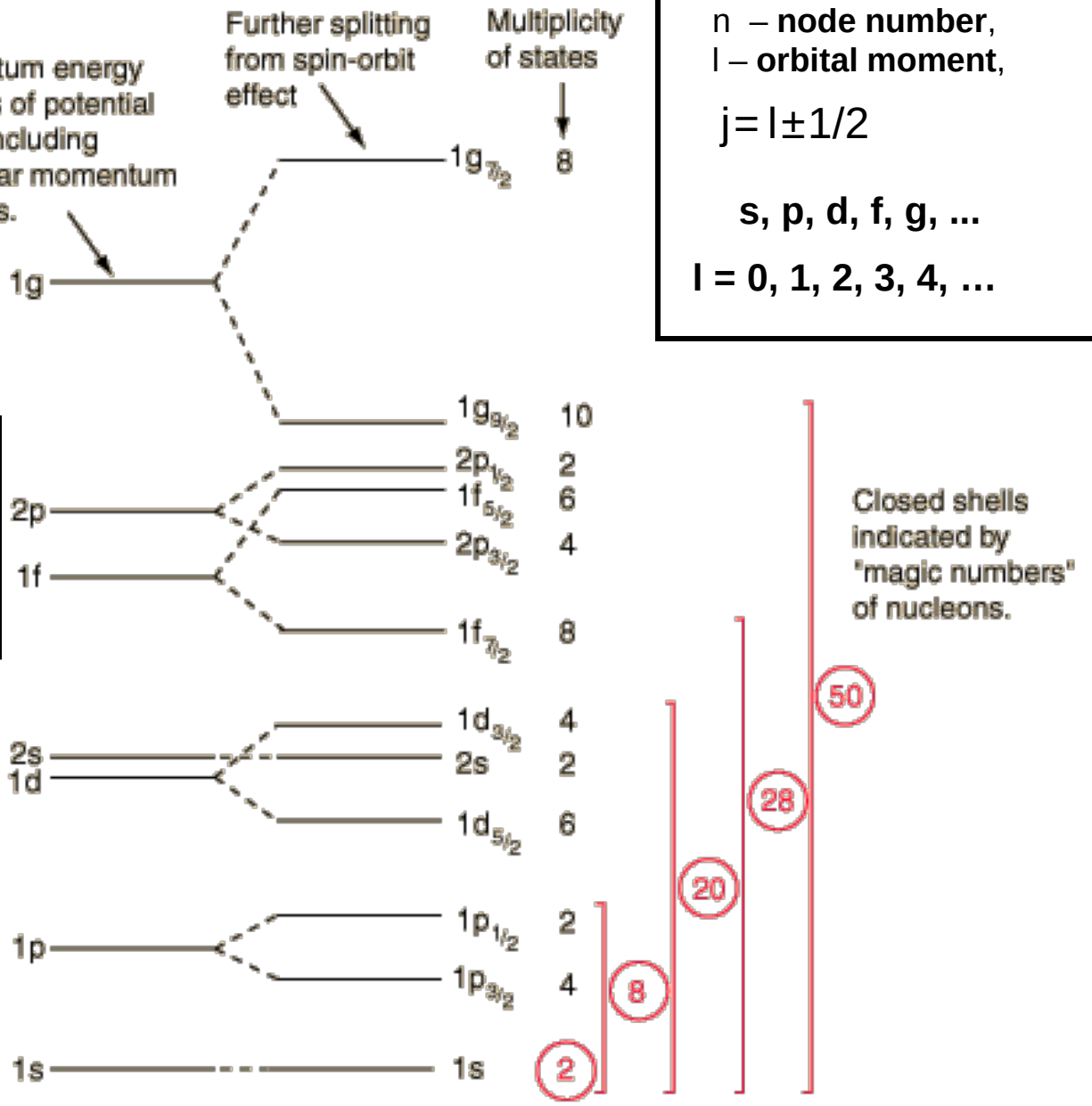
N – principle quantum shell number

$\pi = (-1)^{l}$ **- parity**

N=2, π = +

N=1,π = --

N=0, π = +



splitting of levels with $j = l \pm 1/2$

Quantum shells: influence on nuclear structure and dynamics

- giant resonances as $\Delta N = 1,2,3,...$ transitions

- deformation absent for closed shells and exists in nuclei with open shells

- pairing vanishes for closed shells and exists in nuclei with open shells

- fission: shell corrections (V.M. Strutinsky)

$$E_{tot} = E_{LDM} + \delta E$$

$$\delta E = 2\left(\sum_{n=1}^{N/2} E_n - < \sum_{n=1}^{N/2} E_n > \right)$$

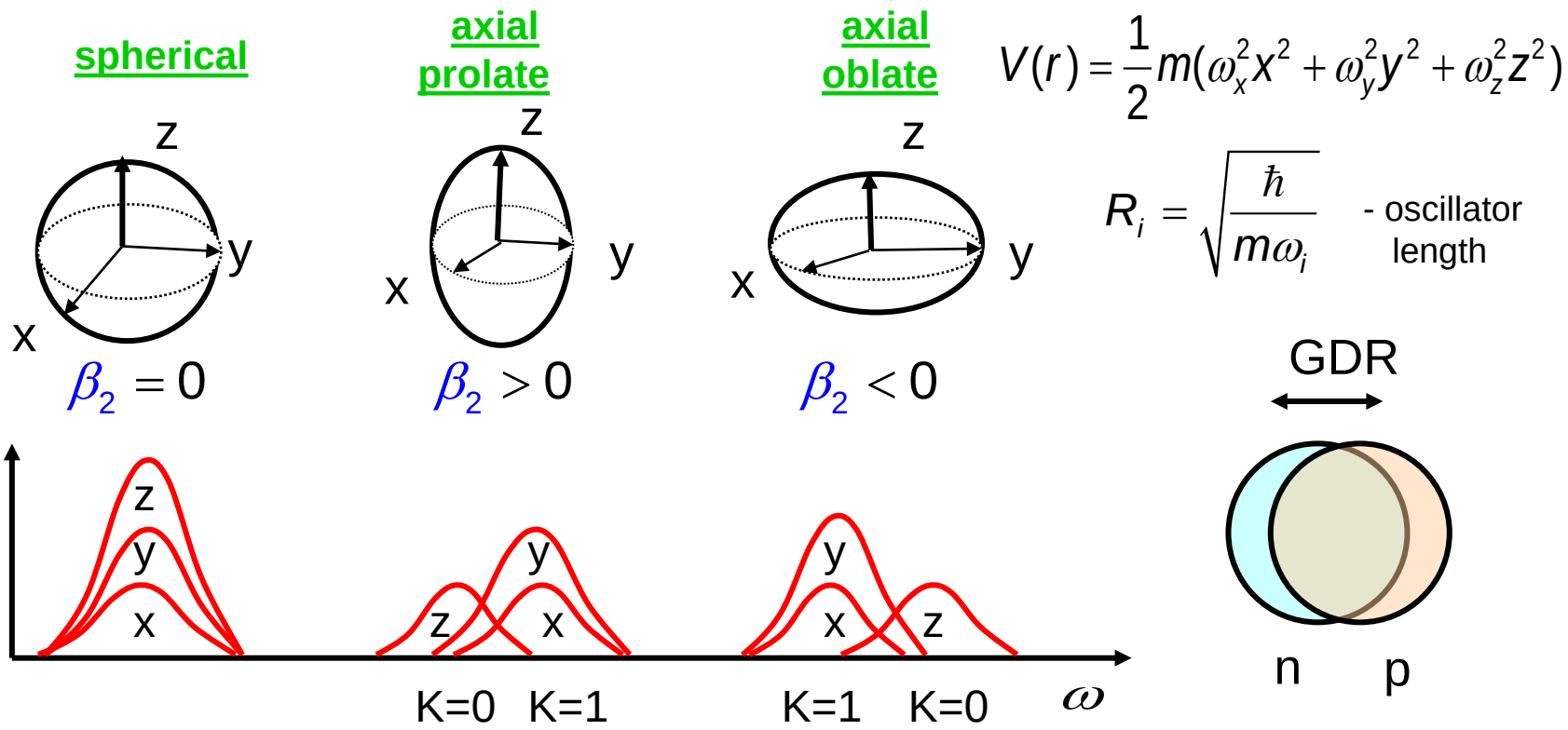
- superheavy nuclei: island of stability (island of superheavy nuclei exists only due to shell corrections)

Deformation of atomic nuclei

Fine structure of GDR: effect of deformation splitting

Nuclei with fully occupied valence shell are spherical (magic and semi-magic nuclei).
Nuclei with partly occupied valence shell can be deformed.
The nuclear shape with a minimum of the system energy is actually realized.

$$R = R_0(\beta_0 + \beta_2 Y_{20}(\theta))$$

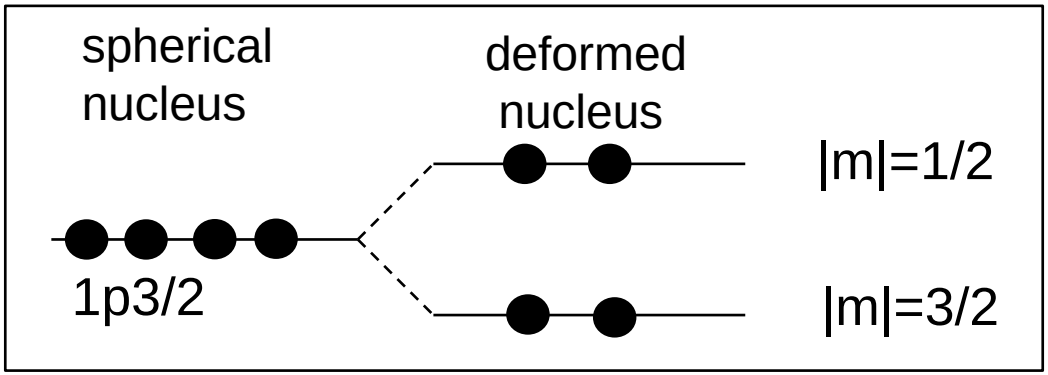


Axial quadrupole deformation causes splitting of GDR into K=0 and K=1 branches

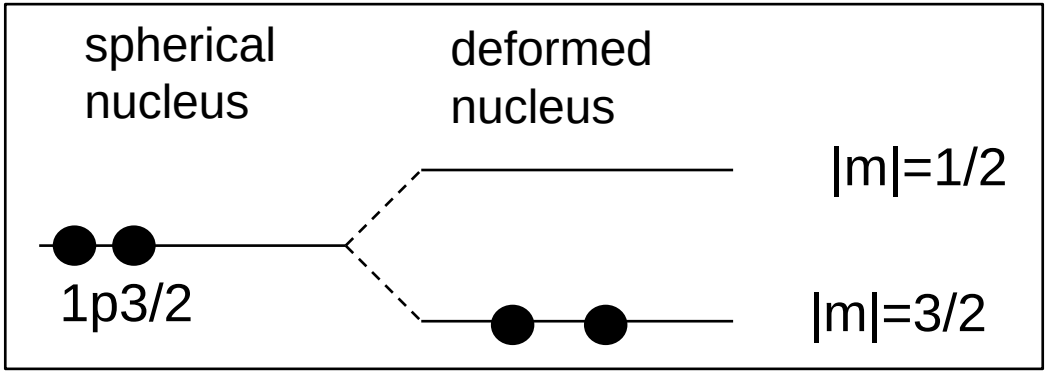
Decrease of the system energy due to deformation

Deformation decreases the system symmetry and thus the degeneration of quantum levels. For the axial quadrupole deformation,
we get splitting to levels with a given absolute value of quantum number m
E.g., the level **nlj = 1p3/2** with degeneration M=2j+1=4 is split into 4 levels.

j=3/2, m=-3/2, -1/2, 1/2, 3/2



Magic number of nucleons:
deformation splitting does not lead to the energy gain



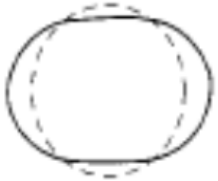
Non-magic number of nucleons:
deformation splitting does lead to the energy gain. Thus the system becomes deformed.

Different kinds of deformation

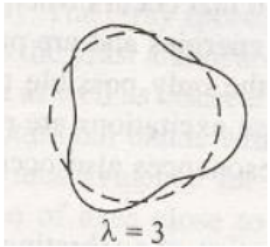
Generic nuclear shapes can be described by a development of spherical harmonics

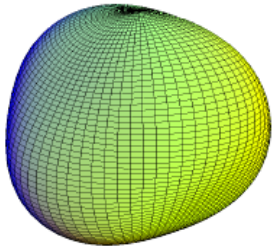
$$R(\vartheta,\phi) = R_0\left[1+\sum_{\lambda}\sum_{\mu=-\lambda}^{+\lambda} a_{\lambda\mu}\, Y_{\lambda\mu}(\vartheta,\phi)\right]$$

quadrupole $a_{20} = \beta_2 \cos\gamma$ $\quad a_{22} = a_{2-2} = \frac{1}{\sqrt{2}}\beta_2 \sin\gamma$ $\quad$ $\mathbf{a_{\lambda\mu}}$:deformation parameters

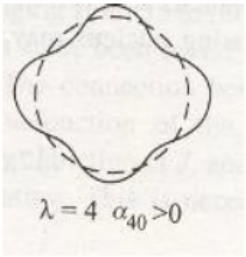


octupole

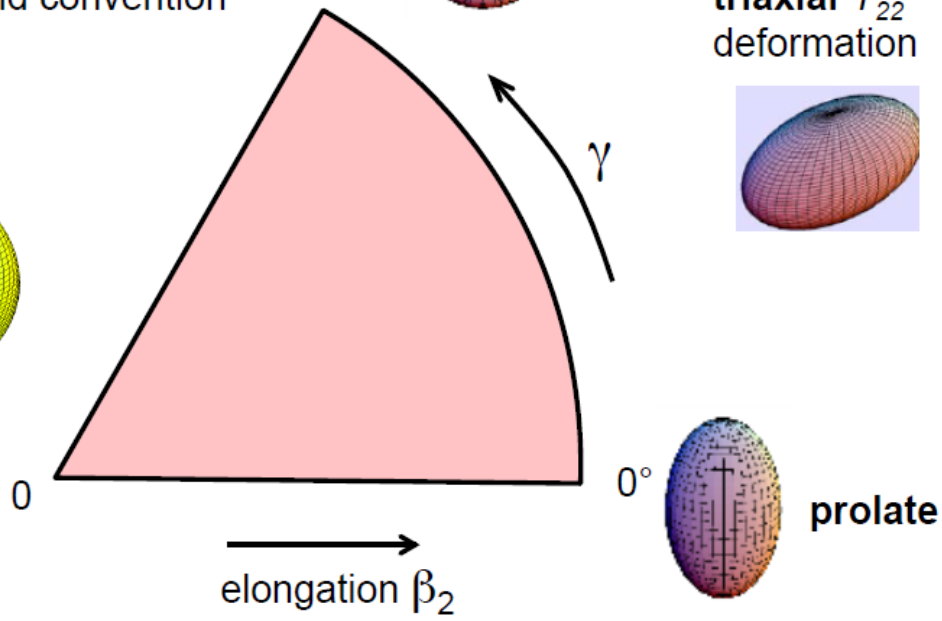




hexadecapole



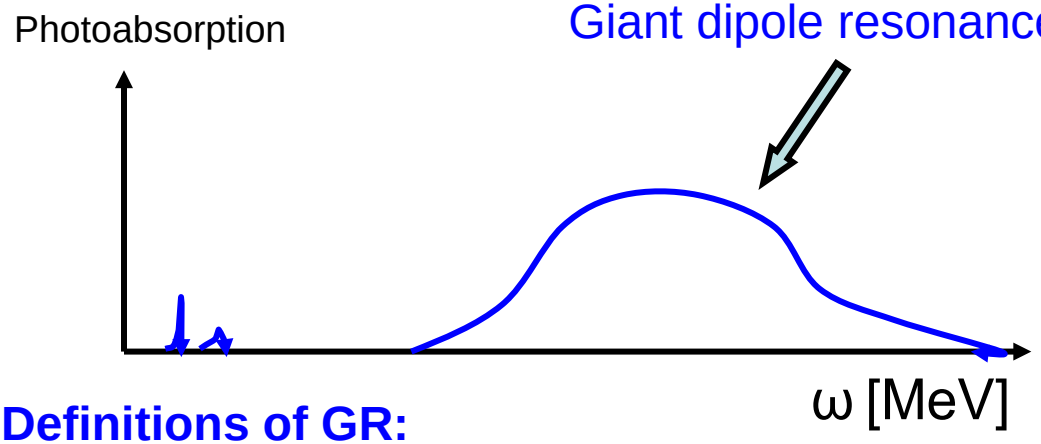
Lund convention

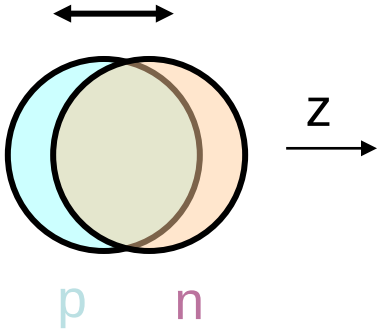


Giant resonances:

- definition
- variety
- test for DFT methods

Giant resonances: definition





E1 GDR (IV)

$$F^{E10}(\vec{r}) = \sum_{i=1}^{Z} e r_i \, Y_{10}(\Omega_i) \propto z$$

Definitions of GR:

**** Collective excitation with a dominant contribution to cross-section of some reaction.**

****Collective excitation exhausting a large part of the proper sum rule.**

$$m_1^{E\lambda} = \sum_j \omega_j \, | \langle j | F^{E\lambda} | 0 \rangle |^2 \Rightarrow \quad \approx \quad \omega_{GDR} \, | \langle GDR | F^{E\lambda} | 0 \rangle |^2$$

response to external field F

Sum rules look complicated but in many cases they are reduced to very simple expressions.

GDR $(I = 1, \pi = -1)$: $m_1^{E1} = \dfrac{\hbar^2}{2m} \dfrac{NZ}{A}$ - model independent!

Soviet physicist Migdal has predicted GDR using the sum rule estimation.

Sum rule:

$$[A,B] = AB - BA, \quad \{A,B\} = AB + BA$$

$$F^{E\lambda\mu}(\vec{r}) = \sum_{i=1}^{Z} e r_i^{\lambda} Y_{\lambda\mu}(\Omega_i)$$

$$H = T + V(\vec{r}), \; T = \frac{\hbar^2}{2m}\nabla^2$$

$$m_1^{E\lambda} = \sum_j \omega_j \left|\left\langle j|F^{E\lambda}|0\right\rangle\right|^2 = \frac{1}{2}\left\langle 0|[F^{E\lambda},[F^{E\lambda},H]]|0\right\rangle$$

$$= \frac{1}{2}\left\langle 0|[F^{E\lambda},[F^{E\lambda},T]]|0\right\rangle$$

If V and F are only coordinate dependent, then
[F,H] = [F,T] + [F,V] = [F,T]

$$= \frac{e\hbar^2}{8\pi m}\lambda(\lambda+1)Z\underbrace{\left\langle 0|r^{2\lambda-2}|0\right\rangle}_{\text{= 1 для } \lambda = 1} \longleftarrow$$

model-independent!

$$m_1 = \tfrac{1}{2}\langle 0|[F,[F,H]]|0\rangle$$
$$= \tfrac{1}{2}\langle 0|(F[F,H]-[F,H]F)|0\rangle$$
$$= \tfrac{1}{2}\langle 0|(2FHF - FFH - HFF)|0\rangle$$

$$I = \sum_i |i\rangle\langle i| \;\text{ - Unity operator}$$

$$I|k\rangle = \sum_i |i\rangle\langle i|k\rangle = \sum_i |i\rangle\delta_{ik} = |k\rangle$$

$$m_1 = \tfrac{1}{2}\sum_i (2\langle 0|FH|i\rangle\langle i|F|0\rangle - \langle 0|F|i\rangle\langle i|FH|0\rangle - \langle 0|HF|i\rangle\langle i|F|0\rangle)$$
$$= \tfrac{1}{2}\sum_i (2E_i|\langle i|F|0\rangle|^2 - 2E_0|\langle i|F|0\rangle|^2)$$
$$= \sum_i \omega_i |\langle i|F|0\rangle|^2$$

$$H|0\rangle = E_0|0\rangle$$
$$H|i\rangle = E_i|i\rangle$$
$$\langle i|F|j\rangle = \langle j|F^{\dagger}|i\rangle^*$$
$$\langle i|F|0\rangle\langle i|F|0\rangle^* = |\langle i|F|0\rangle|^2$$
$$\omega_i = E_i - E_0$$

There are SR with a different energy multiplier:

E. Lipparini ans S. Stringari,
Phys. Rep. 175, 104 (1989).

$$m_p^{E\lambda} = \sum_j \omega_j^p \, | \langle j | F^{E\lambda} | 0 \rangle |^2$$

SR can be expressed through commutators and anticommutators

$$m_0 = \tfrac{1}{2} \langle 0 | \{F,F\} | 0 \rangle - \langle 0 | F | 0 \rangle^2,$$

$$m_1 = \tfrac{1}{2} \langle 0 | [F,[F,H]] | 0 \rangle, \quad \Longleftarrow \text{ only commutators}$$

$$m_2 = \tfrac{1}{2} \langle 0 | \{[F,H],[H,F]\} | 0 \rangle,$$

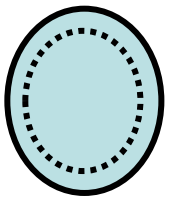
$$m_3 = \tfrac{1}{2} \langle 0 | [[F,H],[H,[H,F]]] | 0 \rangle, \quad \ldots \Longleftarrow \text{ only commutators}$$

SR with p= -1, 1, 3 are often used for estimation:

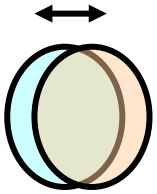
- if the calculation basis is large enough (m_1)
- GR energy

$$\omega_p = \sqrt{\frac{m_p}{m_{p-2}}} = \sqrt{\frac{\omega_{GR}^p \, | \langle GR | F | 0 \rangle |^2}{\omega_{GR}^{p-2} \, | \langle GR | F | 0 \rangle |^2}},$$

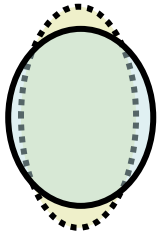
Variety of GR



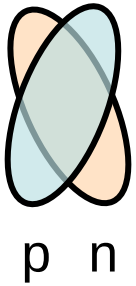
E0(T=0,1)
GMR



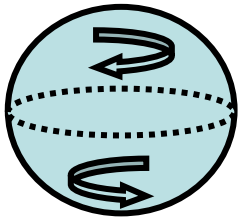
E1(T=1)
GDR



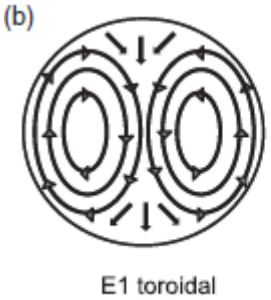
E2(T=0,1)
GQR



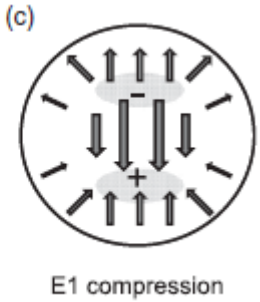
M1(T=1)
scissors



M2(T=0,1)
twist



E1(T=0,1)
toroidal
vortical



E1(T=0,1)
compression

isoscalar (IS,T=0): protons and neutrons oscillate in phase
isovector (IV,T=1): protons and neutrons oscillate in opposite phase

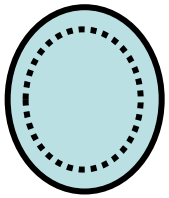
MeV :	$80\,A^{-1/3}$	$81\,A^{-1/3}$	$64\,A^{-1/3}$	2-3 MeV	$44\,A^{-1/3}$	$60\,A^{-1/3}$	$134\,A^{-1/3}$
E [MeV]:	13	13	10	2.5	7	10	22

^{238}U

For each GR there is the optimal reaction where it can be well observed and discriminated (photoabsorption, (p,p'), (α,α'), (e,e'), NRF, …):

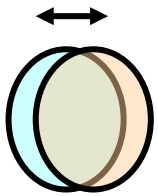
- IS (α,α') or IV (p,p')
- determination of multipolarity by angular distributions
- determination of E/M using specific reactions and measuring anysotropy

GR provide a useful information on particular features of finite nuclei and nuclear matter



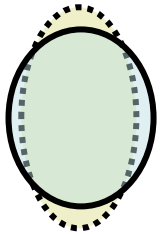
E0(T=0,1)
GMR

nuclear incomp. K_∞



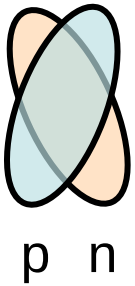
E1(T=1)
GDR

m_1^*



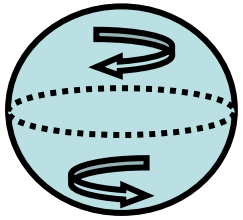
E2(T=0,1)
GQR

m_0^*



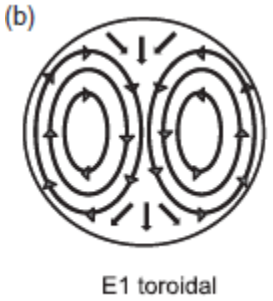
M1(T=1)
scissors

orbital magnetism



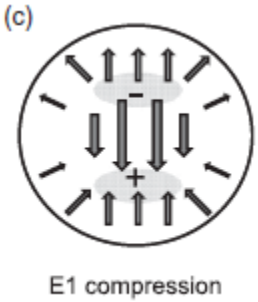
M2(T=0,1)
twist

orbital magnetism



E1(T=0,1)
toroidal

electric vortical motion



E1(T=0,1)
compression

nuclear incomp. K_∞

- spin-flip M1 GR(T=1), …..

- charge-exchange GR: Gamow-Teller GR, …

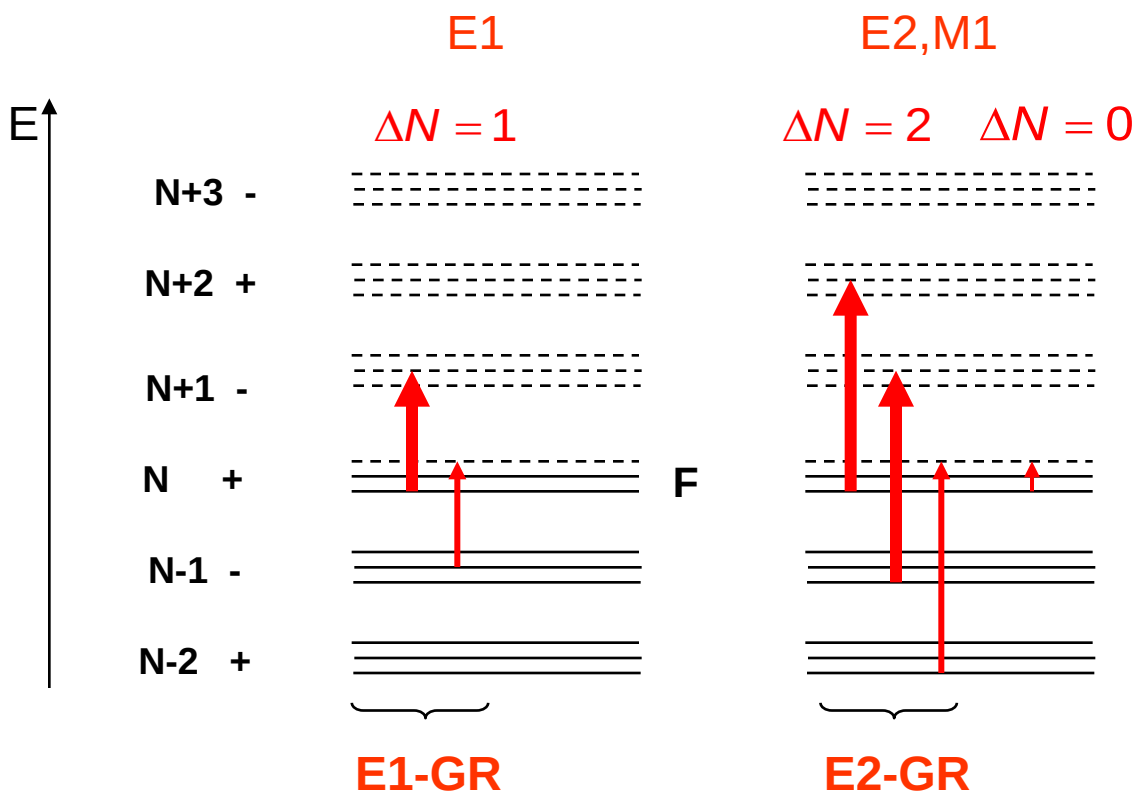
⟵ omitted in the present talk

Giant resonances and quantum shells

Parity:

$\tau = (-1)^{\lambda}$ - natural parity $E\lambda$

$\tau = (-1)^{\lambda+1}$ - unnatural parity $M\lambda$



$E_{GDR} \approx 81 A^{-1/3}$ MeV

$E(\Delta N = 1) \approx 41 A^{-1/3}$ MeV

(~ 7 MeV for ^{208}Pb)

$H = H_0 + V_{res}$

$E(E1-GR) \approx 7\ MeV$

$E(E2-GR) \approx 14\ MeV$

Giant resonances are collective transitions between quantum shells

The origin and main properties of GR are basically known.

Why then the study of GR is still actual?

- To determine nuclear matter characteristics:
  nuclear incompressibility, nucleon effective masses, …

- Fine structure of GR:
  deformation-induced splitting, E/M coupling, configuration structures in light nuclei
  decay mechanisms ⟶ wavelet analysis

- **Very important test for DFT self-consistent models. GR are often used for the fit of parameters of modern DFT models**

GR are the robust test for modern self-consistent mean field approaches (Skyrme, Gogni, relativistic, …) based on

density functional theory (DFT)

Lectures of A.P. Severjukhin
E.V. Marduban

These approaches:

- are **main theoretical tools (work horses)** to describe GR and other nuclear excitations

- are **self-consistent**: both mean field and residual interaction are obtained from the same initial functional (no additional free parameters), high predictive power.

- pretend to be **universal**: description of **astrophysical problems**, **nuclear matter** (symmetric, neutron), and **finite nuclei** of a different shape (spherical, deformed) through the full periodic table,

- pretend to describe both **static nuclear properties** and **nuclear dynamics**

- **parameters** of the initial functional are fitted to describe both finite nuclei (statics, **dynamics**) and nuclear matter

GR provide the robust check of the accuracy of the self-consistent models.

Skyrme functional with spin-isospin channel

$$\mathcal{H}_{\rm Sk} = \frac{b_0}{2}\rho^2 - \frac{b'_0}{2}\sum_q \rho_q^2 + b_1(\rho\tau - \mathbf{j}^2) - b'_1\sum_q\left(\rho_q\tau_q - \mathbf{j}_q^2\right)$$
$$- \frac{b_2}{2}\rho\Delta\rho + \frac{b'_2}{2}\sum_q \rho_q\Delta\rho_q + \frac{b_3}{3}\rho^{\alpha+2} - \frac{b'_3}{3}\rho^\alpha\sum_q\rho_q^2$$

spin-independent IS-IV channel (gs properties, EL excitations)

$$- b_4(\rho\nabla\mathbf{J} + (\nabla\times\mathbf{j})\cdot\mathbf{s}) - b'_4\sum_q(\rho_q\nabla\mathbf{J}_q + (\nabla\times\mathbf{j}_q)\cdot\mathbf{s}_q)$$

spin-orbit IS-IV channel (gs properties, EL-ML excitations)

$$+\frac{\tilde{b}_0}{2}\mathbf{s}^2 - \frac{\tilde{b}'_0}{2}\sum_q\mathbf{s}_q^2 - \frac{\tilde{b}_2}{2}\mathbf{s}\cdot\Delta\mathbf{s} + \frac{\tilde{b}'_2}{2}\sum_q\mathbf{s}_q\cdot\Delta\mathbf{s}_q + \frac{\tilde{b}_3}{3}\rho^\alpha\mathbf{s}^2$$
$$- \frac{\tilde{b}'_3}{3}\rho^\alpha\sum_q\mathbf{s}_q^2 + \gamma_{\rm T}\left(\underbrace{\tilde{b}_1(\mathbf{s}\cdot\mathbf{T} - \mathbf{J}^2) + \tilde{b}'_1\sum_q\left(\mathbf{s}_q\cdot\mathbf{T}_q - \mathbf{J}_q^2\right)}_{\text{tensor contribution}}\right)$$

spin-dependent IS-IV channel (ML excitations)

Spin-independent and spin-orbit channels can be fitted by gs properties of even-even nuclei. So these channels are rather well fitted.

Instead, spin-dependent channels cannot be fitted by gs properties of even-even nuclei. So this channel is still rather vague.

The impact of tensor contribution is also not yet well understood.

In modern DFT, GR are used in fit of the parameters.

Функционал Скирма

Y.M. Engel et al, NPA 249, 215 (1975).
J. Dobaczewski and J. Dudek,PRC, 52 1827 (1995).

$$E = \int \varepsilon(\vec{r})\, d\vec{r}, \qquad \varepsilon = t + \varepsilon_{Sk} + \varepsilon_{Coul}$$

$$\varepsilon_{Sk}(\rho,\tau,\vec{\Im},\vec{j},\vec{s},\vec{T}) = \frac{b_0}{2}\rho^2 - \frac{b_0'}{2}\sum_q \rho_q^2 + \frac{b_3}{2}\rho^{\alpha+2} - \frac{b_3'}{2}\sum_q \rho_q^{\alpha+2}$$

$$+b_1(\rho\tau - \vec{j}^2) - b_1'\sum_q(\rho_q\tau_q - \vec{j}_q^2) - \frac{b_2}{2}\rho\Delta\rho + \frac{b_2'}{2}\sum_q \rho_q\Delta\rho_q$$

$$-b_4(\rho\vec{\nabla}\cdot\vec{\Im} + \vec{s}\cdot(\vec{\nabla}\times\vec{j})) + b_4'\sum_q(\rho_q\vec{\nabla}\cdot\vec{\Im}_q + \vec{s}_q\cdot(\vec{\nabla}\times\vec{j}_q)) + \ldots$$

q=p,n

Galilean invariance

(+) **плотность нуклонов.** $\rho(\vec{r}) = \sum_j |\phi_j(\vec{r})|^2$

(+) **плот. кин. энергии** $\tau(\vec{r}) = \sum_j |\vec{\nabla}\phi_j(\vec{r})|^2$

(+) **спин-орбит. плот.** $\vec{\Im}(\vec{r}) = -i\sum_j \phi_j^+(\vec{r})\vec{\nabla}\times\vec{\sigma}\phi_j(\vec{r})$

T-even: $\hat{T}\,\rho\,\hat{T}^{-1} = \rho$

Contribute both to the ground state and excitations.

(-) **плот. тока** $\vec{j}(\vec{r}) = -\frac{i}{2}\sum_j(\phi_j^+(\vec{r})\vec{\nabla}\phi_j(\vec{r}) - \phi_j(\vec{r})\vec{\nabla}\phi_j^+(\vec{r}))$

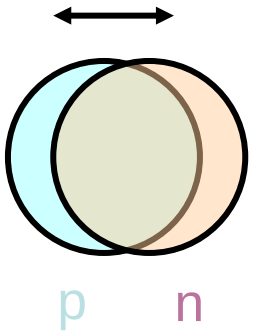
(-) спиновая плот. $\vec{s}(\vec{r}) = -i\sum_j \phi_j^+(\vec{r})\,\vec{\sigma}\,\phi_j(\vec{r})$

(-) **плот. спин. кинет. энергии** $\vec{T}(\vec{r}) = \sum_j \vec{\nabla}\phi_j(\vec{r})\vec{\sigma}\cdot\vec{\nabla}\phi_j(\vec{r})$

T-odd: $\hat{T}\,\vec{j}\,\hat{T}^{-1} = -\vec{j}$

Contribute only to excitations.

Giant dipole E1 resonance

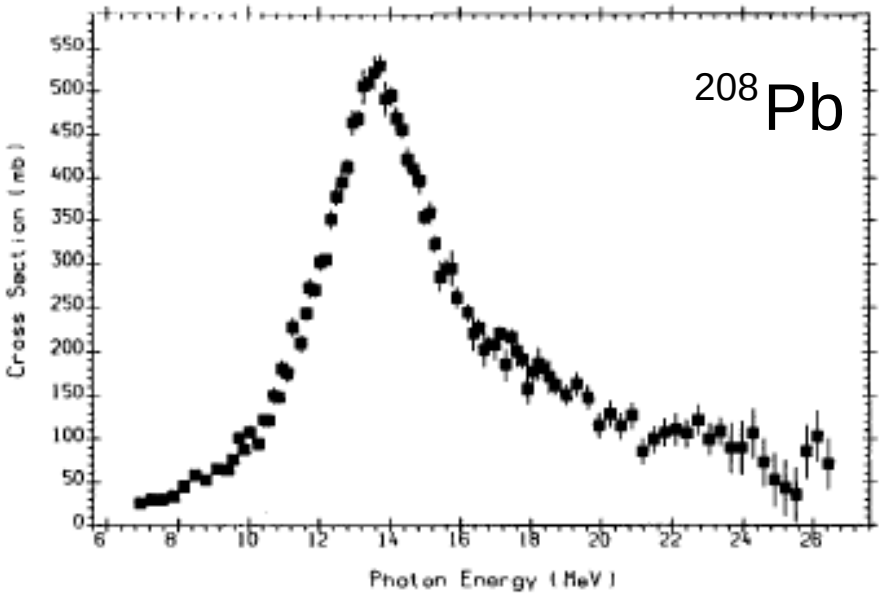


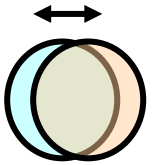
E1 GDR (IV)

E1(T=1) - GDR

A. V. Varlamov, V. V. Varlamov, D. S. Rudenko, and M. E. Stepanov, Atlas of Giant Dipole Resonances, INDC(NDS)-394,1999

Experimental photoabsorption cross-section





Goldhaber-Teller model, (1948)

It is convenient to describe GDR by the energy-weighted strength function

$$S(E1,\omega)=\sum_i \eta(\omega-\omega_i)\omega_i \left|\left\langle i\left|\hat{F}_{E1}\right|0\right\rangle\right|^2$$

with the Lorentz averaging weight

$$\eta(\omega-\omega_\nu)=\frac{1}{2\pi}\frac{\Delta}{[(\omega-\omega_\nu)^2+\frac{\Delta^2}{4}]},\quad \Delta=1-2\,\text{MeV}$$

Empirical estimation for GDR energy:

$$E_{GDR}=81A^{-1/3}\ \text{MeV}$$

Transition operator

$$\hat{F}_{E1}=\frac{N}{A}\sum_{k=1}^{Z}(rY_{1K})_k-\frac{Z}{A}\sum_{k=1}^{N}(rY_{1K})_k$$

includes effective charges to remove spurious admixtures (motion of the whole nucleus) in the transition rates.

We should keep the **translation invariance** = conservation of the total nuclear moment P:

$$[H,P]=0$$

The violation happens if Hamiltonian depends o the center-of mass coordinate R

$$[H(R),P]\neq 0$$

Extraction of spurious admixture is a general problem of the theory

E1 states: violation of translation invariance = conservation of the total nuclear moment)

$$[H,P]=0$$

E2(K=0) states: pairing-induced violation of the conservation of the particle number

$$[H,N]=0$$

E2(K=1) states: violation of rotational invariance = conservation of the total angular momentum

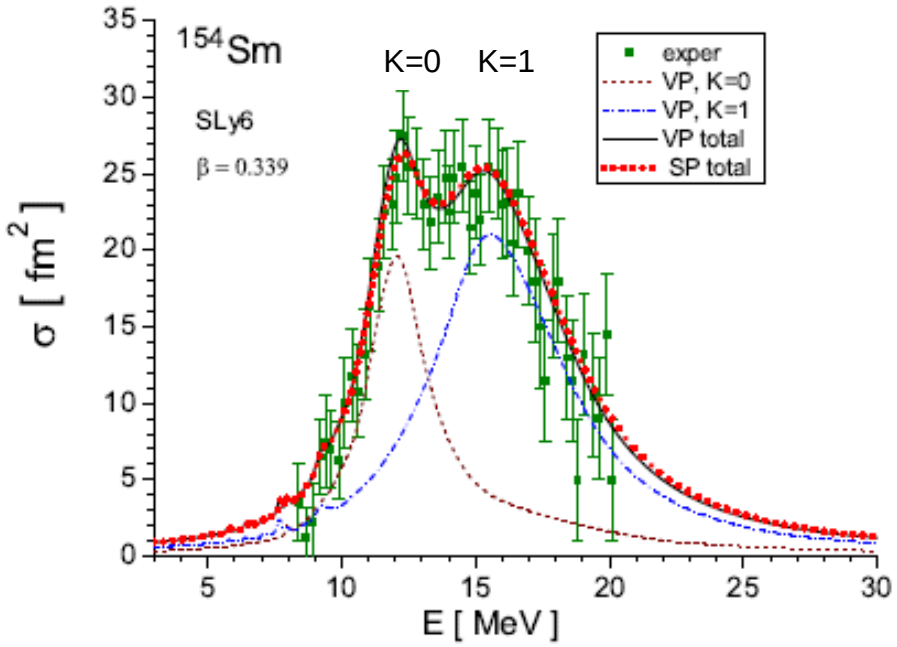
$$[H,J]=0$$

There were derived **general** methods for extraction of various spurious admixtures from nuclear excitations.
These methods reproduce the familiar effective charges etc.

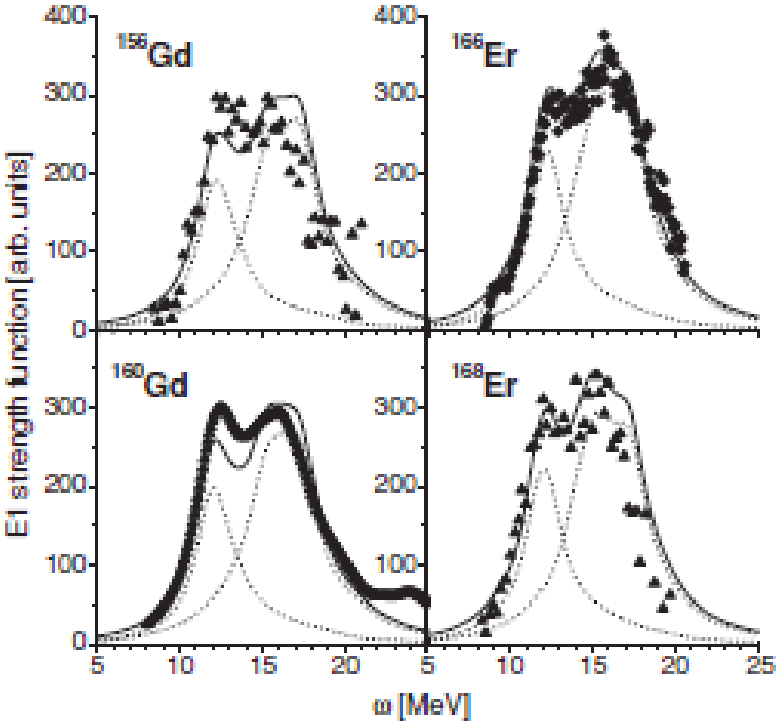
1) J. Kvasil, A. Repko, and V.O.N. Eur. Phys. J. A 55, 213 ((2019);
2) A. Repko, J. Kvasil, and V.O.N., Phys. Rev. C 99, 044307 (2019).

Fine structure of GDR: examples of deformation-induced GDR splitting

Exp: G.M. Gurevich et al,
Nucl. Phys. A351, 257 (1981).



A. Repko, J. Kvasil, V.O.N. and P.-G. Reinhard,
Eur. Phys. J. A, 53, 221 (2017).



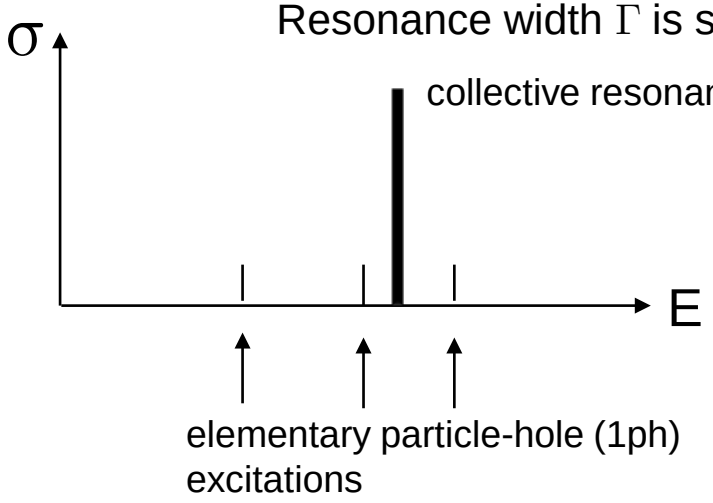
W. Kleinig, V.O.N., J. Kvasil, P.-G. Reinhard,
P. Vesely, PRC, 78, 044313 (2008)

- Prolate nuclei exhibit different relative altitudes of K=0 and K=1 branches.
- The GDR fine structure depends on the Landau damping of the branches.
- K=1 branch lies at a higher energy and so is more damped.
- In deformed nuclei, GDR width is almost fully provided by Landau damping.
- In spherical nuclei, the coupling with complex configurations is also important.

Fragmentation (dissipation) Landau

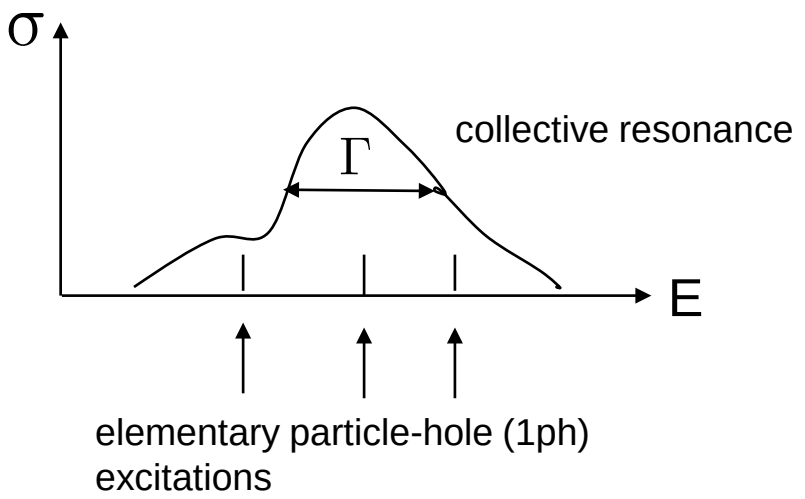
$V << \Delta E$

Resonance does not interact with 1ph excitations and so does not decay.
Resonance width Γ is small.



$V > \Delta E$

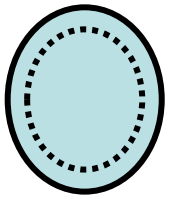
Resonance interacts with c 1ph states and rapidly decays.
Resonance width Γ is large.



Landau damping means:
decay of the collective state into nearby elementary states.

$$\tau \propto \frac{1}{\Gamma}$$

Giant monopole resonance (K_∞) (GMR) [breathing mode]



E0
(T=0,1)

GMR is the main source of information on nuclear incompressibility

of nuclear matter K_∞

and finite atomic nuclei K_A

Five problems with GMR

Giant monopole resonance E0(T=0) - GMR

GMR is the main source of information on nuclear incompressibility K_∞ and m_0^*

GMR energy of atomic nuclei

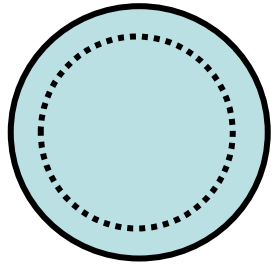
$$E_M = \sqrt{\frac{\hbar^2 K_A}{m_0^* \langle r^2 \rangle_0}}$$

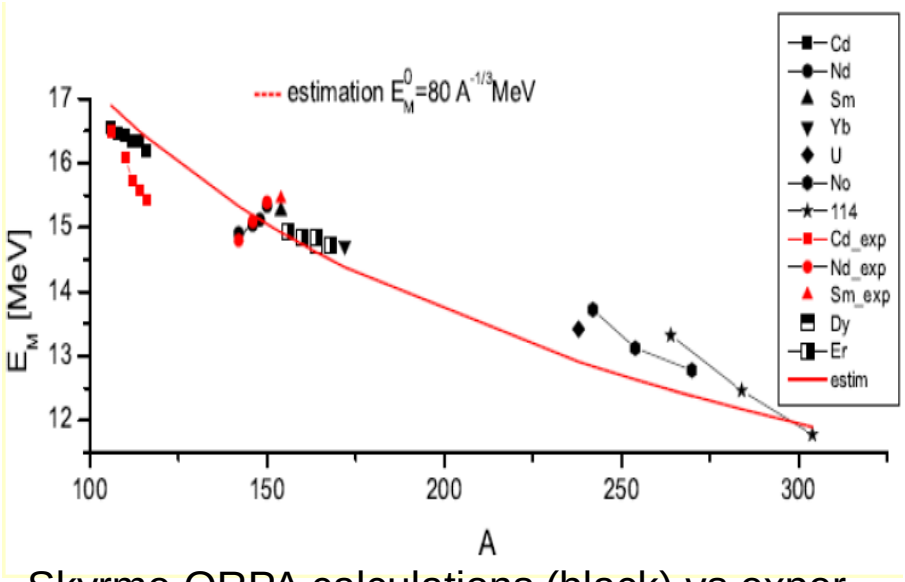
incompressibility of nuclear matter

$$K_\infty = 9\rho^2 \frac{\partial^2}{\partial \rho^2}\left[\frac{E(\rho)}{A}\right]_{\rho=\rho_0}$$

Each Skyrme force have fixed K_∞ and m_0^*

GMR





Skyrme QRPA calculations (black) vs exper. data (red) and empirical estimation

$$E_{GMR} = 80\,A^{-1/3}\ \text{MeV}$$

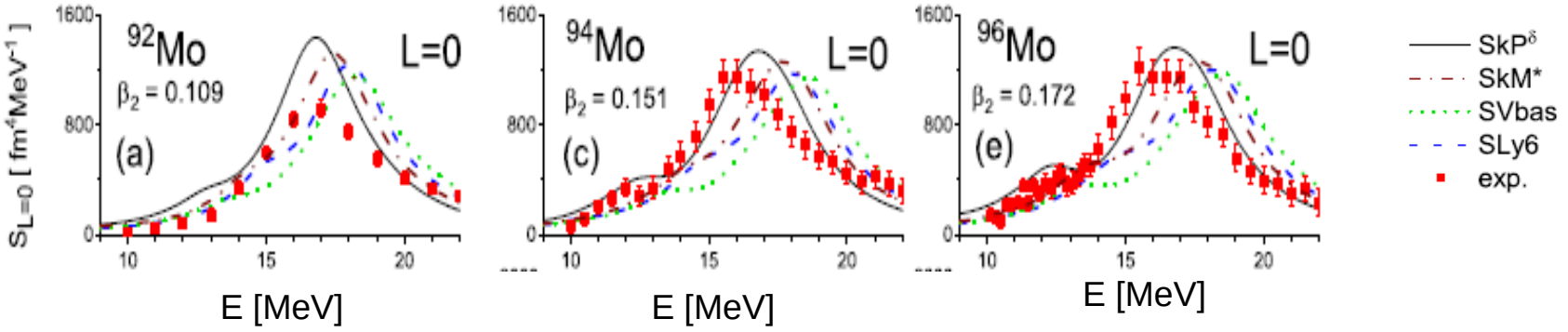
$K_\infty = 200 - 240$ MeV ???
Skyrme RMF

J. Kvasil , V. O. Nesterenko., A. Repko, W. Kleinig, and P.-G. Reinhard, PRC,94, 064302 (2016)

GMR problems:

G. Colo, D. Gambacurta, W. Kleinig, J. Kvasil, V.O. Nesterenko, and A. Pastore, Phys. Lett. B 811, 135940 (2020).

1) Skyrme DFT calculations often give **low** incompressibility $K_\infty = 202$ MeV



while relativistic DFT gives much larger values $K_\infty \sim 240$ MeV

The reason: Skyrme and relativistic DFT functionals have essentially density-dependent terms!

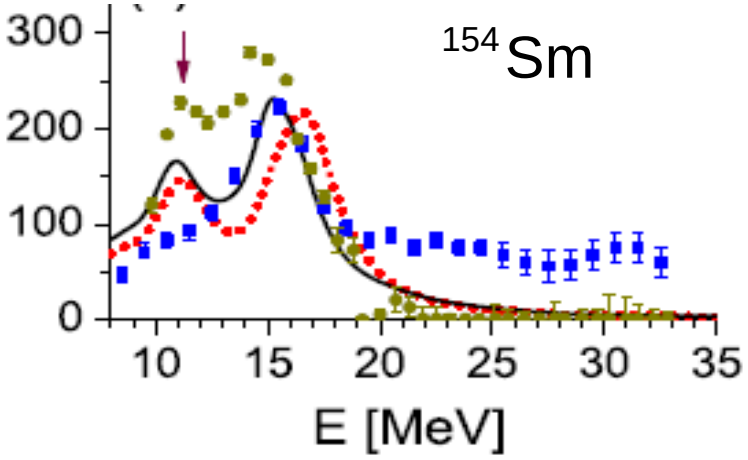
$$K_\infty = 9\rho^2 \frac{\partial^2}{\partial \rho^2}\left[\frac{E(\rho)}{A}\right]_{\rho=\rho_0}$$

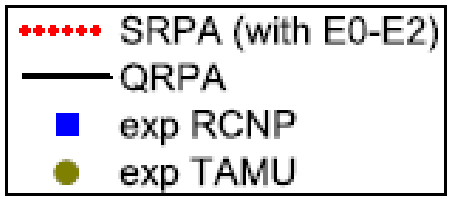
2) The main experimental facilities

- RCNP: Research Center for Nuclear Physics (Osaka, Japan)
- TAMU: Texas A&M University (Texas, USA)

give essentially different results

despite they use the same reaction (α,α')





J. Kvasil , V. O. Nesterenko., A. Repko, W. Kleinig, and P.-G. Reinhard, PRC,94, 064302 (2016)

The reason of the discrepancy: different prescriptions for extraction of the background

3) Coupling with complex configurations (CCC)

There was long-standing problem of simultaneous description of GMR in some spherical nuclei: Sn isotopes and ^{208}Pb.

The problem was recently solved by taking into account CCC.
So, in some nuclei CCC can be important for GMR description.

Z.Z. Li, N.F. Niu, G. Colo, PRL, 181, 082501 (2023)

4) The incompressibility K_∞ and IS effective mass m_0^* are coupled $E_M = \sqrt{\frac{\hbar^2 K_A}{m_0^* \langle r^2 \rangle_0}}$

nuclear matter: $K_\infty = B + C\sigma + D(1 - \frac{3}{2}\sigma)\Theta, \quad \frac{m^*}{m} = [1 + \frac{m\rho_0}{8\hbar^2}\Theta]^{-1}$

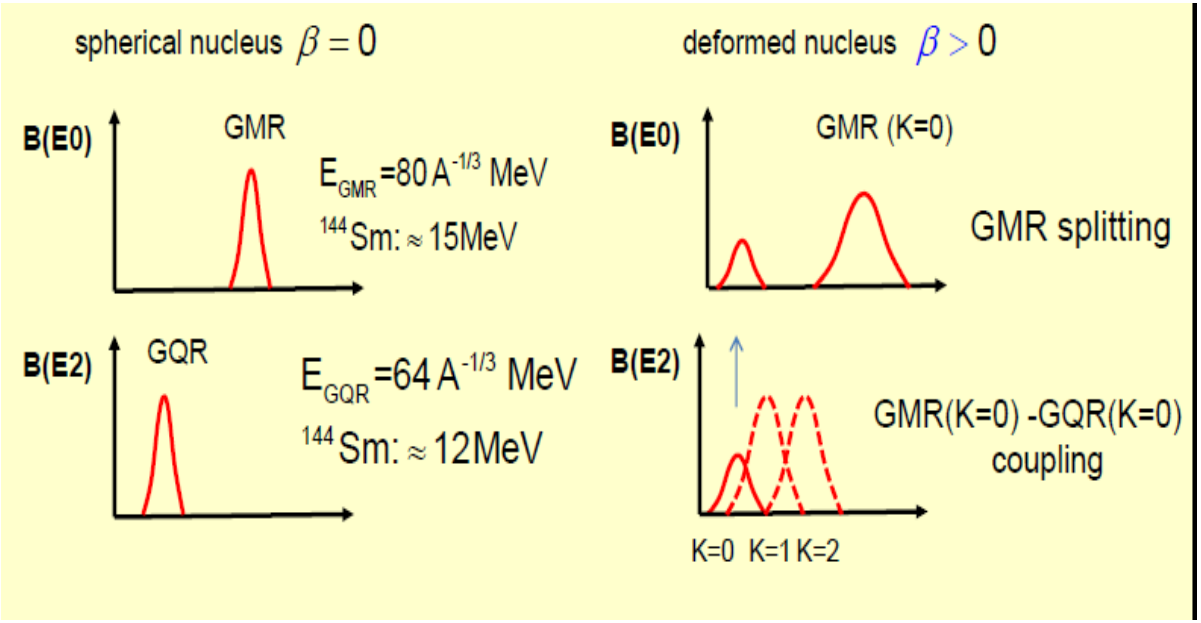
So both **GMR** and **GQR** (sensitive to) resonances should be described simultaneously.

Moreover, this coupling is crucial in deformed nuclei.

5) Deformation effect in GMR

J. Kvasil , V. O. N., A. Repko, W. Kleinig, and P.-G. Reinhard, PRC,94, 064302 (2016)

Till now mainly GMR in spherical nuclei was used to get K_∞
But most of nuclei are deformed! Is it possible to use deformed nuclei?



In deformed nuclei, the states with the same K^π are coupled and mixed; LK=00, LK=20

GMR and QMR are independent in spherical nuclei but coupled in axial deformed nuclei. This leads to double-peak structure of GMR. This is maybe the strongest deformation-induced coupling of different multipole modes!

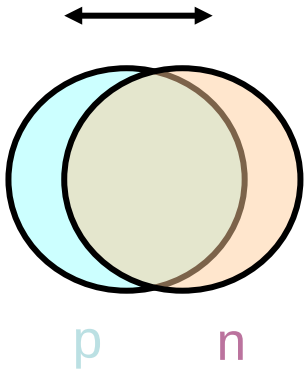
- GMR strength in deformed nuclei has two humps!

- Both GMR and GQR should be described simultaneously!

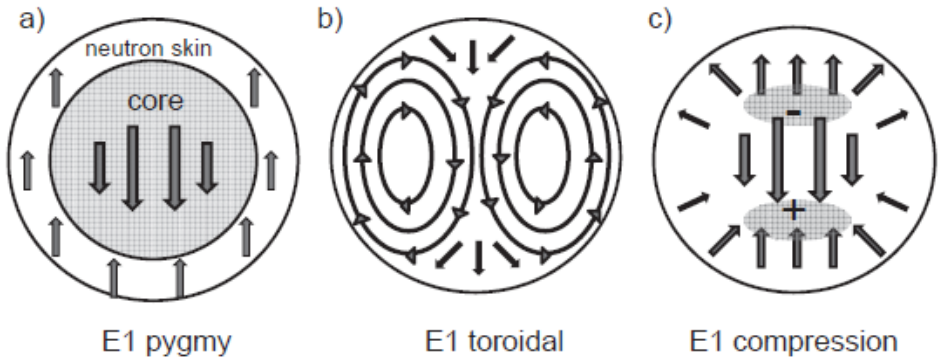
Exotic E1 dipole resonances:

- pygmy
- toroidal
- compression





+



E1 GDR (IV)

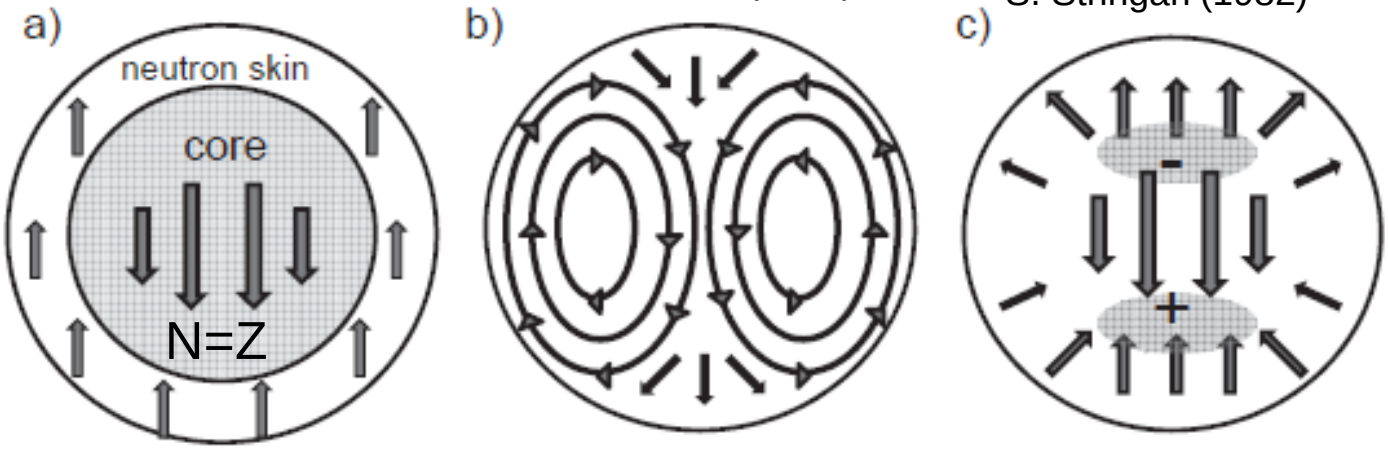
- center of mass is fixed
- both IS and IV version

In addition to GDR, there are exotic dipole resonances

R. Mohan et al (1971),

V.M. Dubovik (1975)
S.F. Semenko (1981)

M.N. Harakeh (1977)
S. Stringari (1982)



Alternative source of information on nuclear incompressibility

E1 pygmy

E1 toroidal

E1 compression

irrotational,

$\vec{v} \sim \vec{\nabla}\phi$

vortical

$\vec{v}(\vec{r}) = \vec{\nabla} \times \vec{\nabla} \times [\vec{r}\,\chi(\vec{r})]$

irrotational

$\vec{v} \sim \vec{\nabla}\phi$

$E = 50 \div 60\, A^{-1/3} MeV$

$E = 50 \div 70\, A^{-1/3} MeV$

$E = 132\, A^{-1/3} MeV$

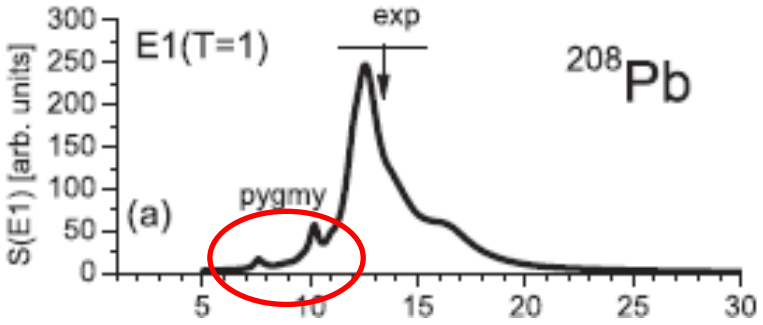
Dominate in E1(T=0) channel if the dominant E1(T=1) motion is suppressed

Reviews:
N. Paar et al, Rep. Prog. Phys. 70 691 (2007);
D. Savran et al, Prog. Part. Nucl. Phys. 70, 210 (2013)
V.O. Nesterenko, Phys. Atom. Nucl. 79, 842 (2016).

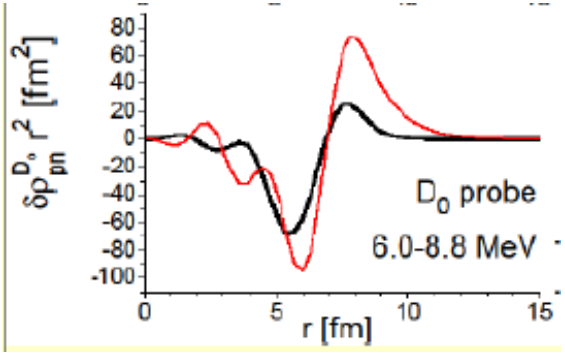
Pygmy E1 resonance

A. Repko, P.G. Reinhard, VON, J. Kvasil,
PRC, 87, 024305 (2013)

GDR

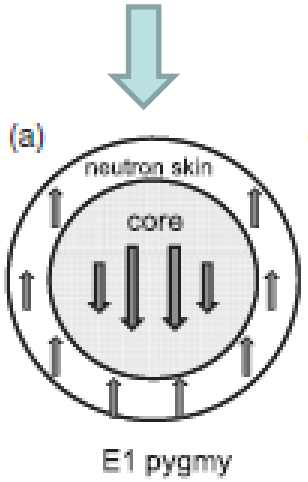


Typical PDR transition density:



PDR:

- is not formally GR since it carries a small E1 strength
- however, is extremely important for astrophysical applications since it lies near particle thresholds and so its E1 strength affects astrophysics-relevant nuclear reactions.

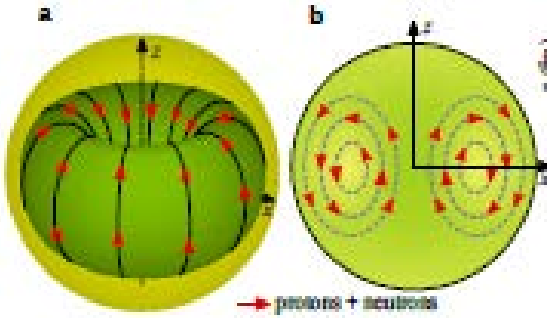


$^{208}_{82}$Pb

Abundance (%)	Separation Energies (MeV)							
	γ,n	γ,p	γ,t	γ,^{3}He	γ,α	γ,2n	γ,np	γ,2p
52.40	7.4	8.0	12.9	14.4	-0.5	14.1	14.9	15.4

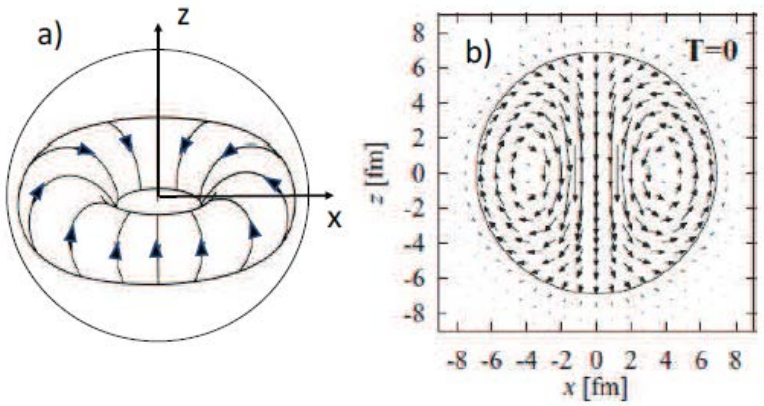
See for recent review:
A. Bracco, E.G. Lanza, A. Tamii,
Prog. Part. Nucl. Phys. 106, 360 (2019)

Toroidal E1 resonance as a Hill's vortex ring



Hill's vortex ring:

- was suggested by Hill in 1894 as one of the exact solutions of Euler equations
- is one of the most simple cases of a vortical flow caused by a turbulence
- this is a vortex ring
- very stable, may not consume the external energy
- is very common in liquids and gases but rarely noticed unless the motion of the fluid is revealed by suspended particles (smoke rings)





colored by tobacco particles and so are visible

Each listener of my talk permanently produces (invisible) Hill's vortex rings in the surrounding air when breathing out!

Hill's vortex is very general phenomenon. It must exist in nuclei as TDR but still was not safely observed. TDR is a general phenomenon in atomic nuclei, independently on nuclear size, Z/A ratio and nuclear shape.

Unlike a conventional hydrodynamics, nuclear TDR is caused not by turbulence but by mean-field vortical flow of nucleons.

Being mainly vortical, TDR does not almost contribute to the continuity equation (CE)

$$\dot{\rho} + \vec{\nabla} \cdot \vec{j}_{nuc} = 0$$

So TDR is a remarkable example of nuclear dynamics beyond CE.

This is still a challenge to observe electric vortical excitations in general and TDR in particular in nuclei So the experimental search of TDR is a useful first step in solution of this general experimental problem.

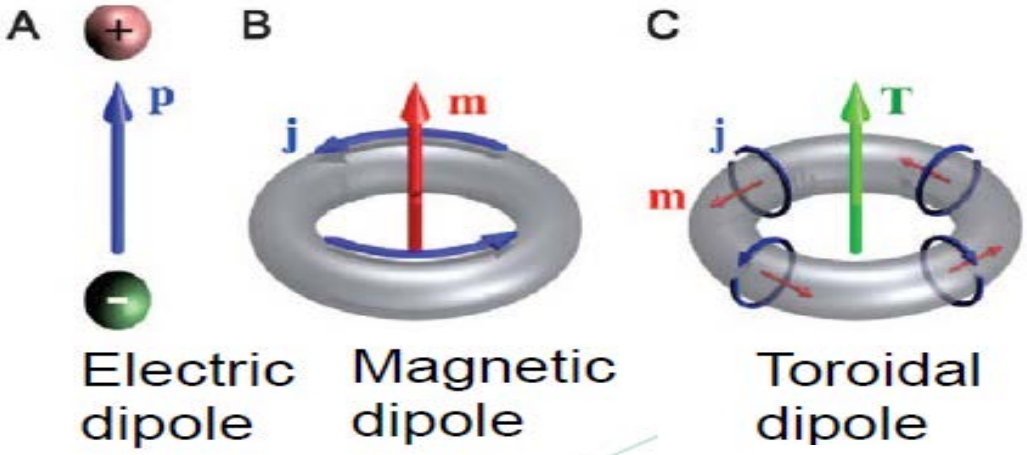
TDR should exist in PDR energy region near nuclear thresholds. It can affect E1 strength in this energy range. So it can be important for nucleosynthesis and astrophysical problems.

Interesting crossover with other areas of physics (heavy-ion collisions, metamaterials, condensed matter physics…).

V. M. Dubovik and V. M. Tugushev,
Toroid moments in electrodynamics and solid-state physics,
Phys. Rep. 187, 145 (1990).

T. Kaelberer, V.A. Fedotov, Papasimakis, D.P. Tsai, N.I. Zheludev, Science 330, 1510 (2010).

Static dipole moments



$$T_i = \frac{1}{10c}\int [r_i(\mathbf{r}\cdot\mathbf{J}) - 2r^2 J_i]\mathrm{d}^3x.$$

spatial inversion

$(\mathrm{P}: \mathbf{r} \mapsto -\mathbf{r})$

time reversal

$(\mathrm{T}: t \mapsto -t)$

Symmetry properties

	P	T
electric dipole	-1	+1
magnetic dipole	+1	-1
toroidal dipole	-1	-1

← PT-invariance

Toroidal vortical mode appears in:

V.M. Dubovik and A.A. Cheshkov,
Sov. J. Part. Nucl. v.5, 318 (1975).

- nuclear current density

Following theorems of Helmholtz and Chandrasekhar/Moffat, the current distribution can be decomposed as

$$\vec{j}(\vec{r}) = \vec{\nabla}\phi(\vec{r}) + \vec{\nabla}\times[\vec{r}\,\psi(\vec{r})] + \vec{\nabla}\times\vec{\nabla}\times[\vec{r}\,\chi(\vec{r})]$$

electric moments | magnetic moments | electric **toroidal** moments

E1 GDR, compression — transversal

- Multipole electric operator (probe external field) :

$$\hat{M}(Ek\lambda\mu) = \frac{(2\lambda+1)!!}{ck^{\lambda+1}}\sqrt{\frac{\lambda}{\lambda+1}}\int d\vec{r}\;\;\hat{\vec{j}}_{nuc}(\vec{r})\cdot[\vec{\nabla}\times(\vec{r}\times\vec{\nabla})\,(j_\lambda(kr)Y_{\lambda\mu})]$$

$$j_\lambda(kr) = \frac{(kr)^\lambda}{(2\lambda+1)!!}\left[1-\frac{(kr)^2}{2(2\lambda+3)}+\ldots\right]$$

So, the toroidal operator is the second order term in long-wave expansion of the electric operator

$$\hat{M}(Ek\lambda\mu) = \hat{M}(E\lambda\mu) + k\hat{M}_{tor}(E\lambda\mu)$$

$$\hat{M}(E\lambda\mu) = \int d\vec{r}\,\rho(\vec{r})r^\lambda Y_{\lambda\mu}$$ ← standard electric operator In long wave approximation

Toroidal vortical mode appears in:

V.M. Dubovik and A.A. Cheshkov,
Sov. J. Part. Nucl. v.5, 318 (1975).

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- Multipole electric operator (probe external field) :

$$\hat{M}(Ek\lambda\mu) = \frac{(2\lambda+1)!!}{ck^{\lambda+1}}\sqrt{\frac{\lambda}{\lambda+1}}\int d\vec{r}\;\;\hat{\vec{j}}_{nuc}(\vec{r})\cdot[\vec{\nabla}\times(\vec{r}\times\vec{\nabla})\,(j_\lambda(kr)Y_{\lambda\mu})]$$

$$j_\lambda(kr) = \frac{(kr)^\lambda}{(2\lambda+1)!!}\left[1-\frac{(kr)^2}{2(2\lambda+3)}+\ldots\right]$$

So, the toroidal operator is the second order term in long-wave expansion of the electric operator

$$\hat{M}(Ek\lambda\mu) = \hat{M}(E\lambda\mu) + k\hat{M}_{tor}(E\lambda\mu)$$

$$\hat{M}(E\lambda\mu) = \int d\vec{r}\,\rho(\vec{r})r^\lambda Y_{\lambda\mu}$$ ← standard electric operator In long wave approximation

J. Kvasil, VON, W. Kleinig, P.-G. Reinhard, P. Vesely, PRC, 84, 034303 (2011)

Toroidal E1 operator:

$$\hat{M}_{tor}(E1\mu) = \frac{1}{10\sqrt{2}c}\int d\vec{r}\,[r^3 + \frac{5}{3}r < r^2 >_0]\,\vec{Y}_{11\mu}(\hat{\vec{r}})\cdot[\vec{\nabla}\times\hat{\vec{j}}_{nuc}(\vec{r})]$$

cmc

mainly vortical flow

Compression E1 operator:

$$\hat{M}_{com}(E1\mu) = -\frac{i}{10c}\int d\vec{r}\,[r^3 - \frac{5}{3}r < r^2 >_0]Y_{1\mu}\,[\vec{\nabla}\cdot\hat{\vec{j}}_{nuc}(\vec{r})]$$

$$\dot{\rho} + \vec{\nabla}\cdot\vec{j}_{nuc} = 0$$

irrotational flow

$$\hat{M}'_{com}(E1\mu) = \int d\vec{r}\,\hat{\rho}(\vec{r})\,[r^3 - \frac{5}{3}r < r^2 >_0]Y_{1\mu}$$

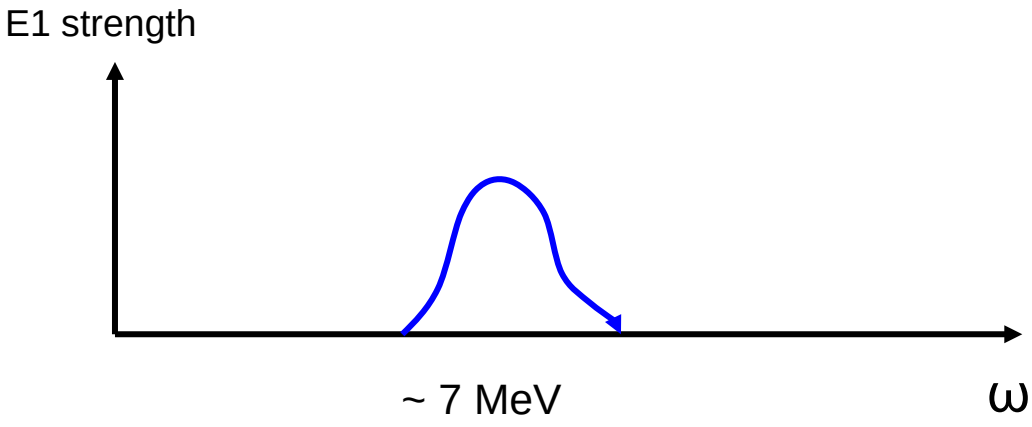
Toroidal and compression operators are coupled:

$$\hat{M}_{tor}(E1\mu) = -\frac{i}{2\sqrt{3}c}\int d\vec{r}\;\hat{\vec{j}}_{nuc}(\vec{r})\cdot\vec{\nabla}\times(\vec{r}\times\vec{\nabla})[r^3 - \frac{5}{3}r < r^2 >_0]\,Y_{1\mu}(\hat{\vec{r}})$$

Origin of E1 vortial toroidal strength

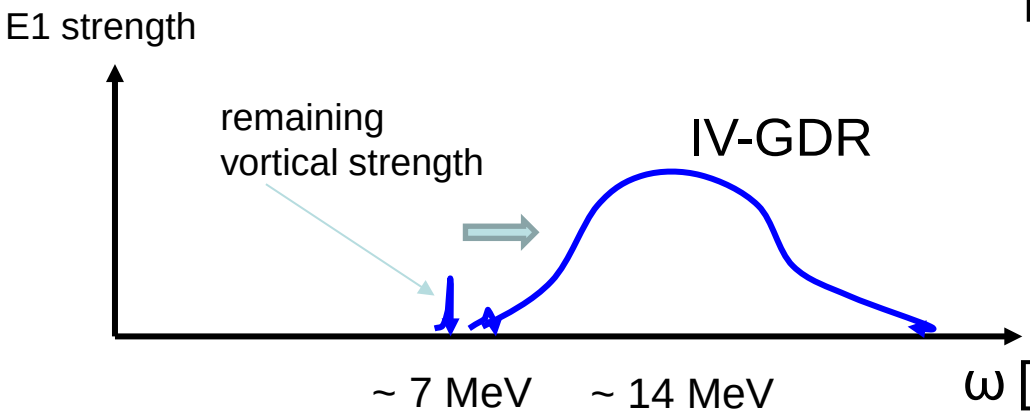
A. Repko, VON, J. Kvasil and P.-G. Reinhard, EPJA, 55, 242 (2019)

E1 toroidal strength must exist in all nuclei at the energy ~ E($\Delta N = 1$).



$H = H_{sp}$

1ph E1 strength from $\Delta N = 1$ dipole transitions.
Strength is mainly irrotational
Vortical fraction is small



$H = H_{sp} + H_{res}$

IV residual interaction upshifts the irrotational E1 strength to form GDR but leaves at 7 MeV the vortical (toroidal) strength.

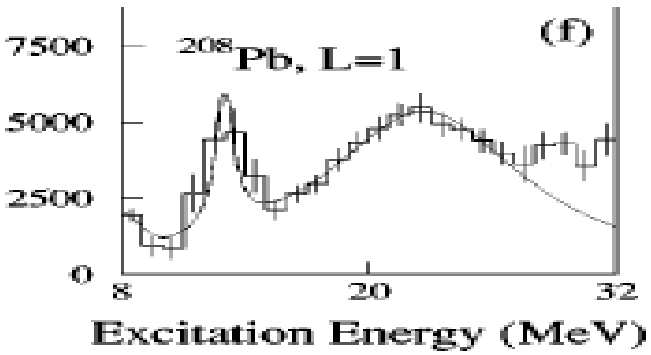
So TDR is indeed a general feature of atomic nuclei

TDR and CDR constitute low- and high-energy ISGDR branches (?)

Experiment: (α,α')

Familiar treatment → LE (toroidal) HE (compression)

^{208}Pb
D.Y. Youngblood et al, 1977
H.P. Morsch et al, 1980
G.S. Adams et al, 1986
B.A. Devis et al, 1997
H.L. Clark et al, 2001
D.Y. Youngblood et al, 2004
M.Uchida et al, PRC 69, 051301(R) (2004)



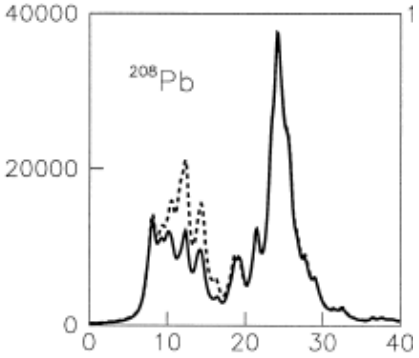
There are also exp ISGDR data in
^{56}Fe, ^{58,60}Ni, ^{90}Zr, ^{116}Sn, ^{144}Sm, ...

TDR
CDR

Theory:

G. Colo et al, PLB 485, 362 (2000)
D. Vretenar et al, PRC, 65, 021301(R) (2002)
N. Paar et al, Rep. Prog. Phys. 70 691 (2007);

A.Repko, P.-G. Reinhard, V.O.N. and J. Kvasil, PRC 87, 024305 (2013).



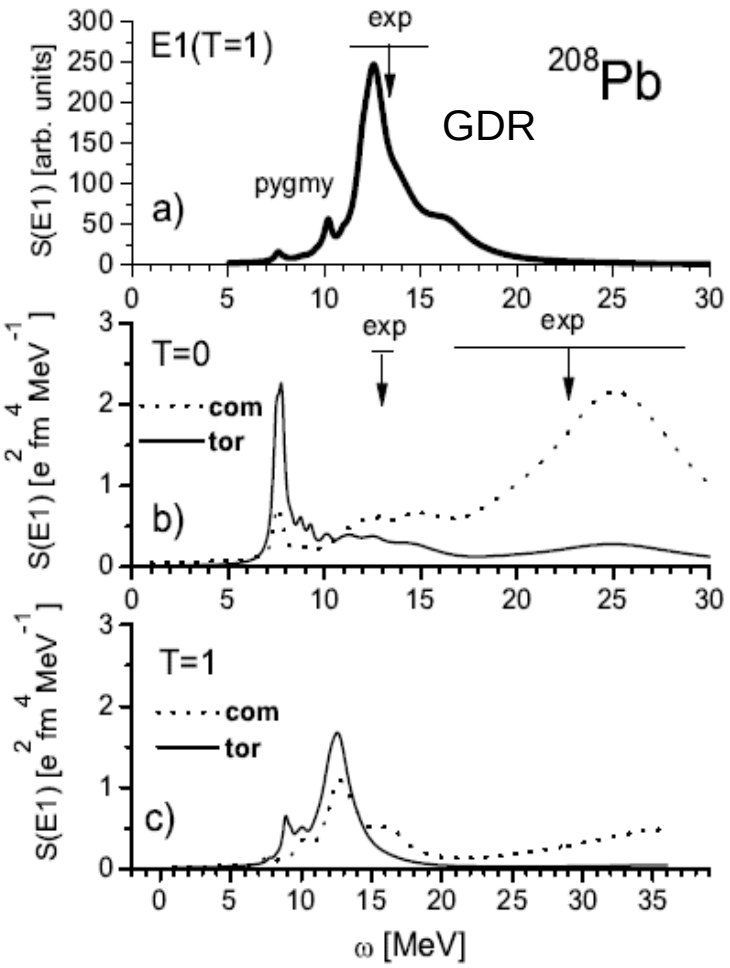
Perhaps Uchida observed at 10-17 MeV not TDR but CDR fraction coupled to TDR. Main TDR peak should lie lower at ~ 7-9 MeV. The direct observation of TDR in (α,α') can be disputed in general since (α,α') is mainly determined by transition density while toroid mainly depends on the vortical transition current.

NEED IN NEW EXPERIMENTS!

Skyrme QRPA calculations

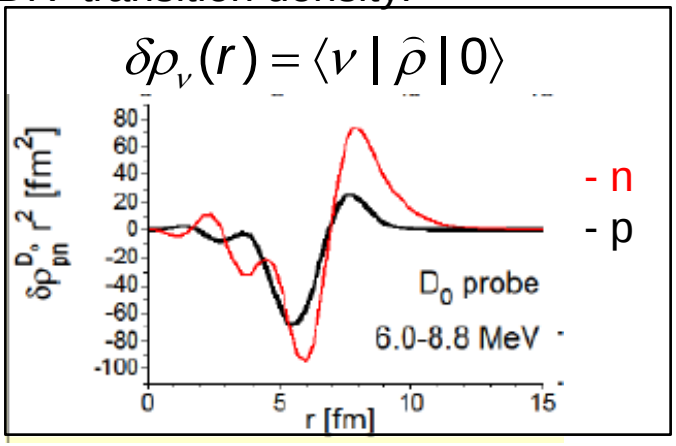
Strength functions SLy6

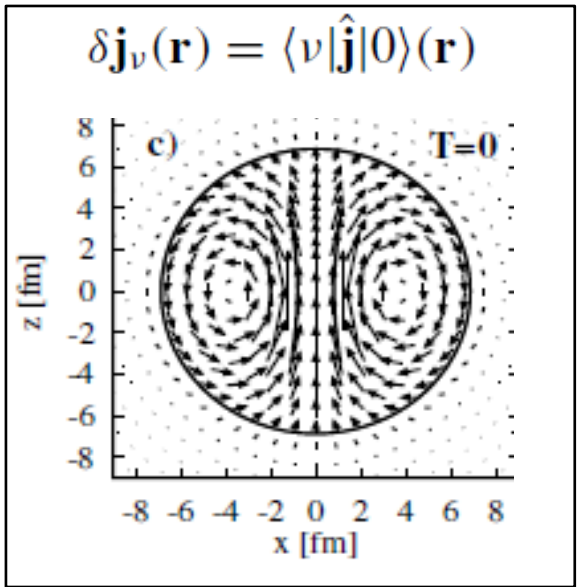
A. Repko, P.G. Reinhard, VON, J. Kvasil, PRC, 87, 024305 (2013)

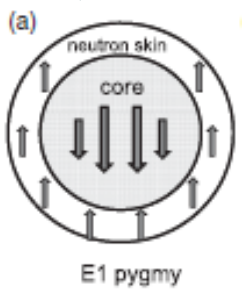


PDR region hosts TDR and CR!

Typical PDR transition density:



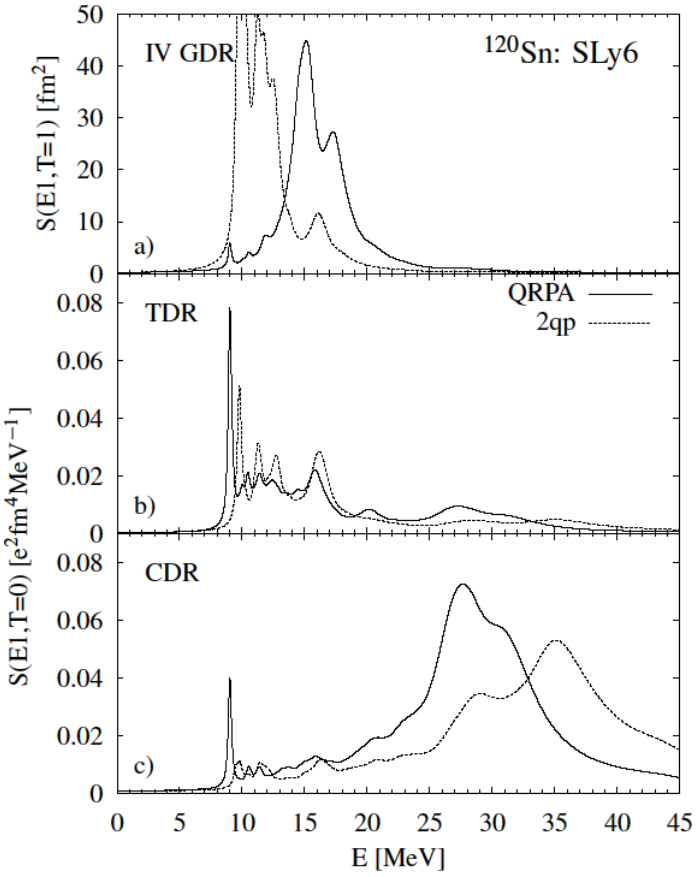


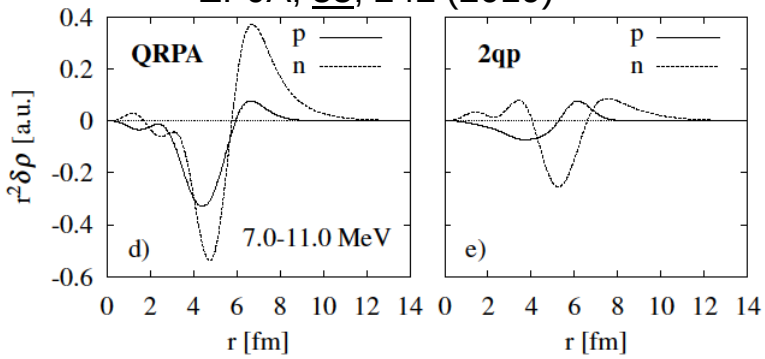


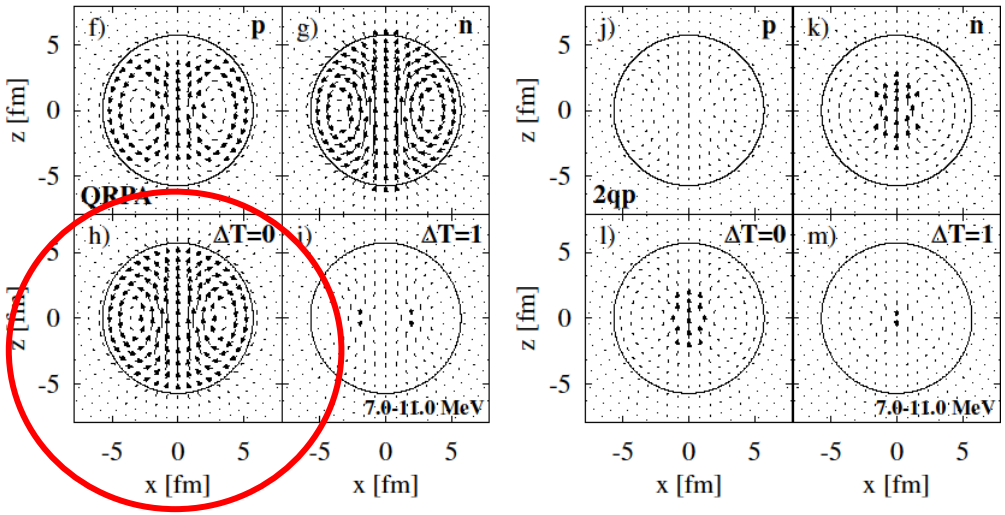
Nucleon current in the PDR region is mainly toroidal!

120Sn SLy6

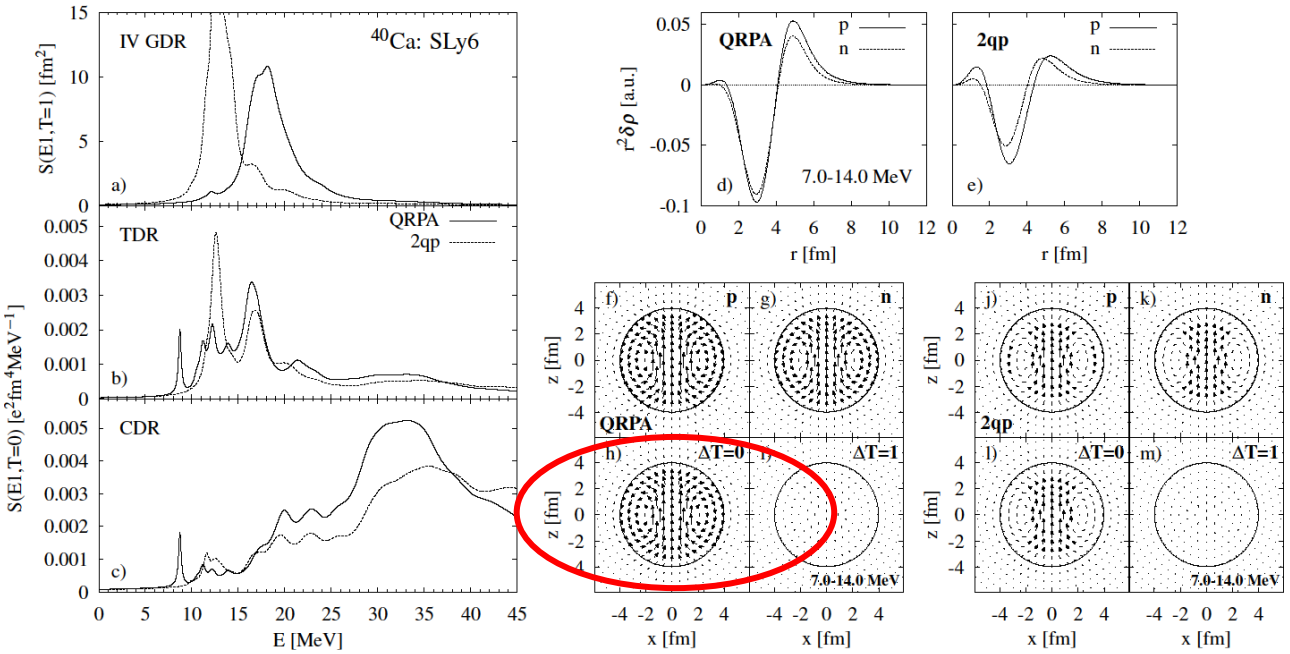
A. Repko, VON, J. Kvasil and P.-G. Reinhard, EPJA, 55, 242 (2019)





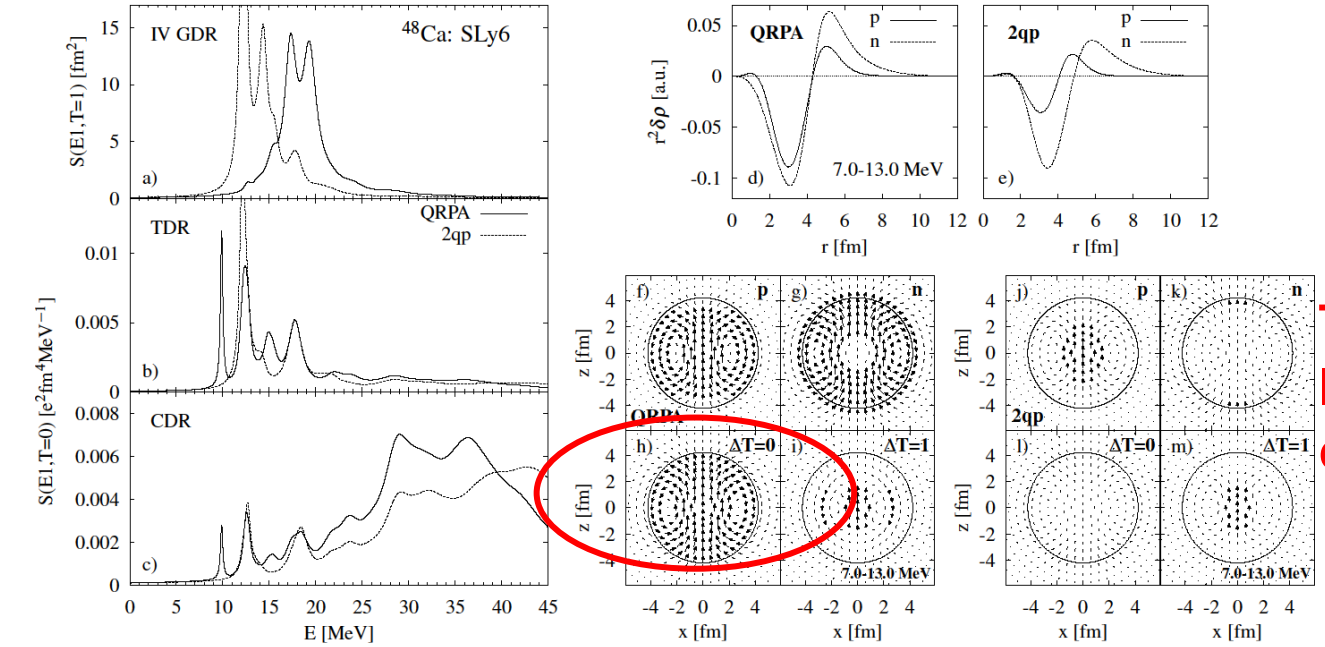


A. Repko, VON, J. Kvasil and P.-G. Reinhard, EPJA, 55, 242 (2019)



40Ca, SLy6

- no PDR but clear TDR
- residual interaction enforces toroidal flow
- protons and neutrons equally contribute
- squeezed toroidal flow

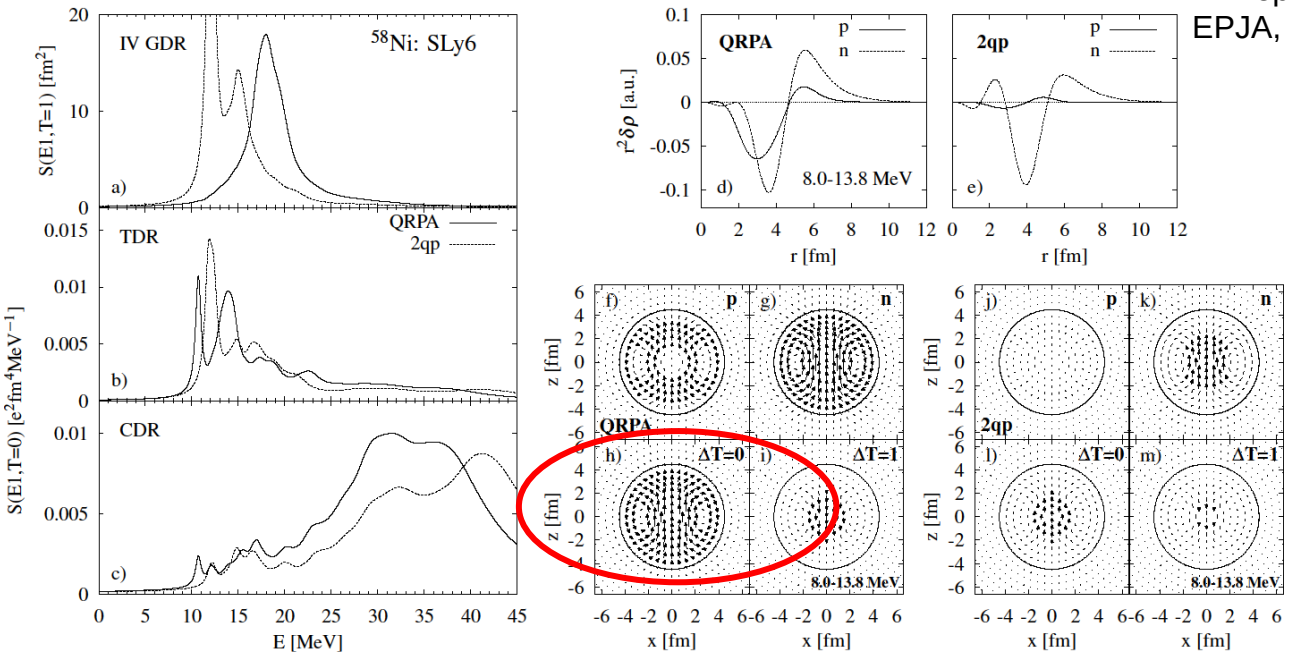


48Ca, SLy6

Basically the same results as for 120Sn, SLy6.

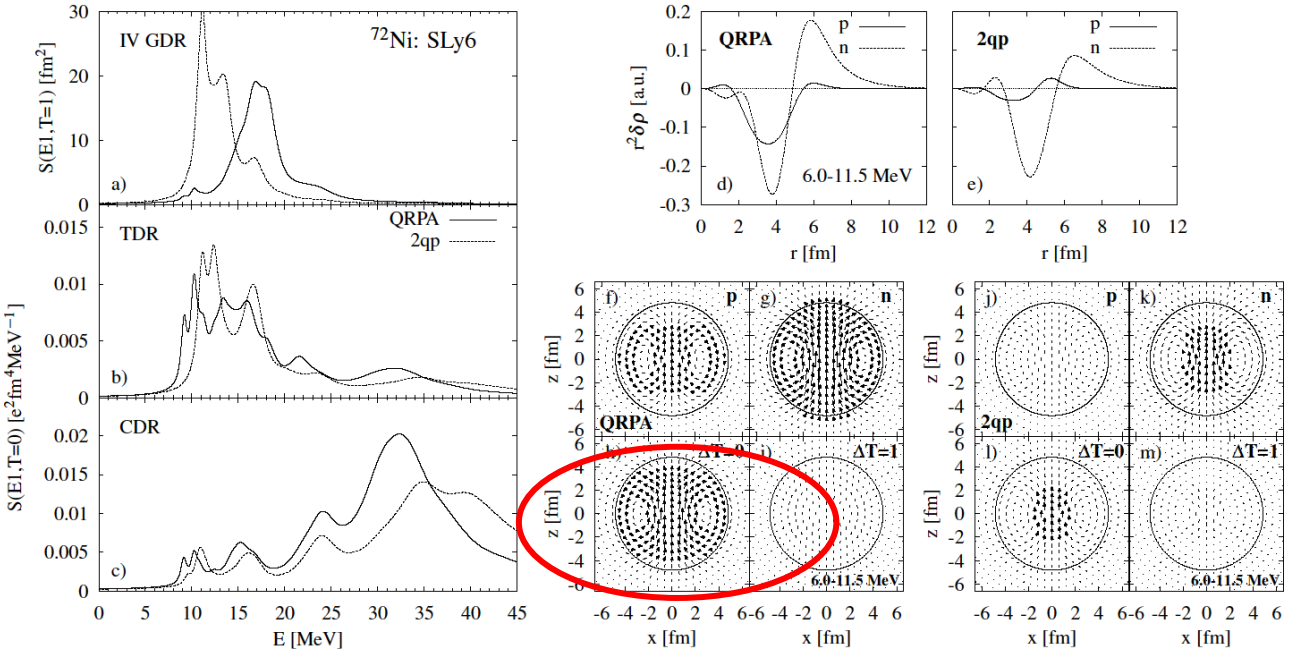
Toroidal mode persists In nuclei independently on the neutron excess

A. Repko, VON, J. Kvasil and P.-G. Reinhard, EPJA, 55, 242 (2019)



58Ni, SLy6

- small neutron excess, almost no PDR but strong TDR

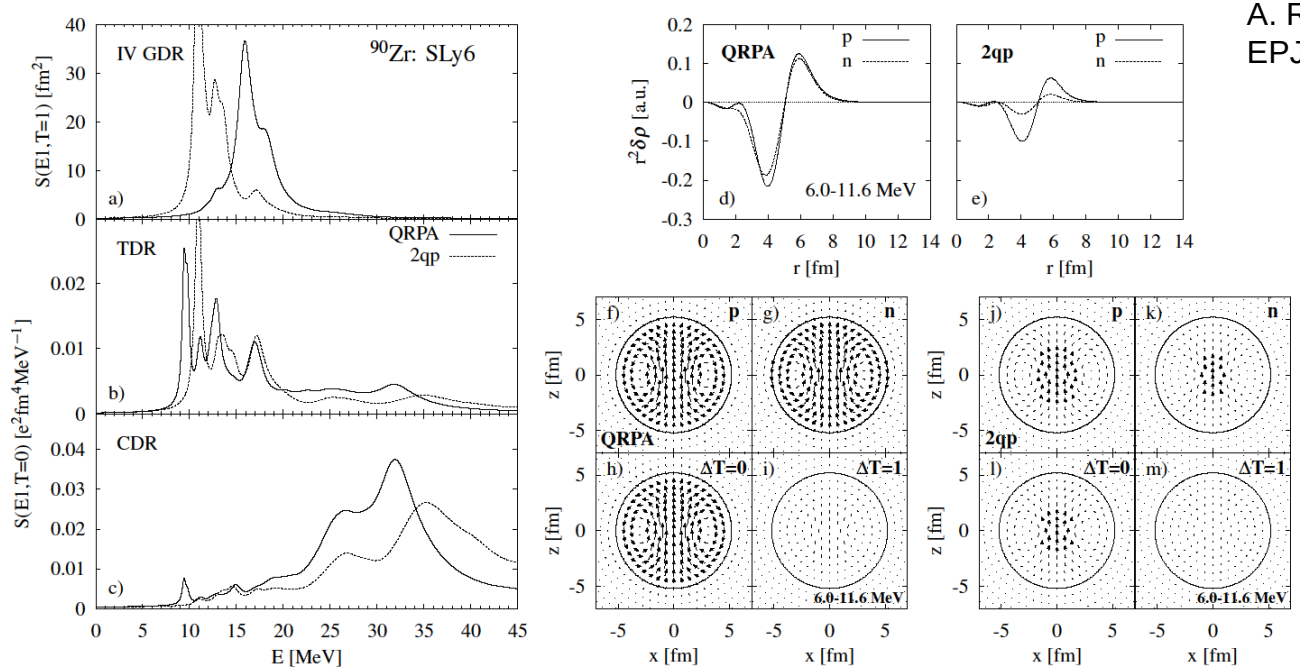


72Ni, SLy6

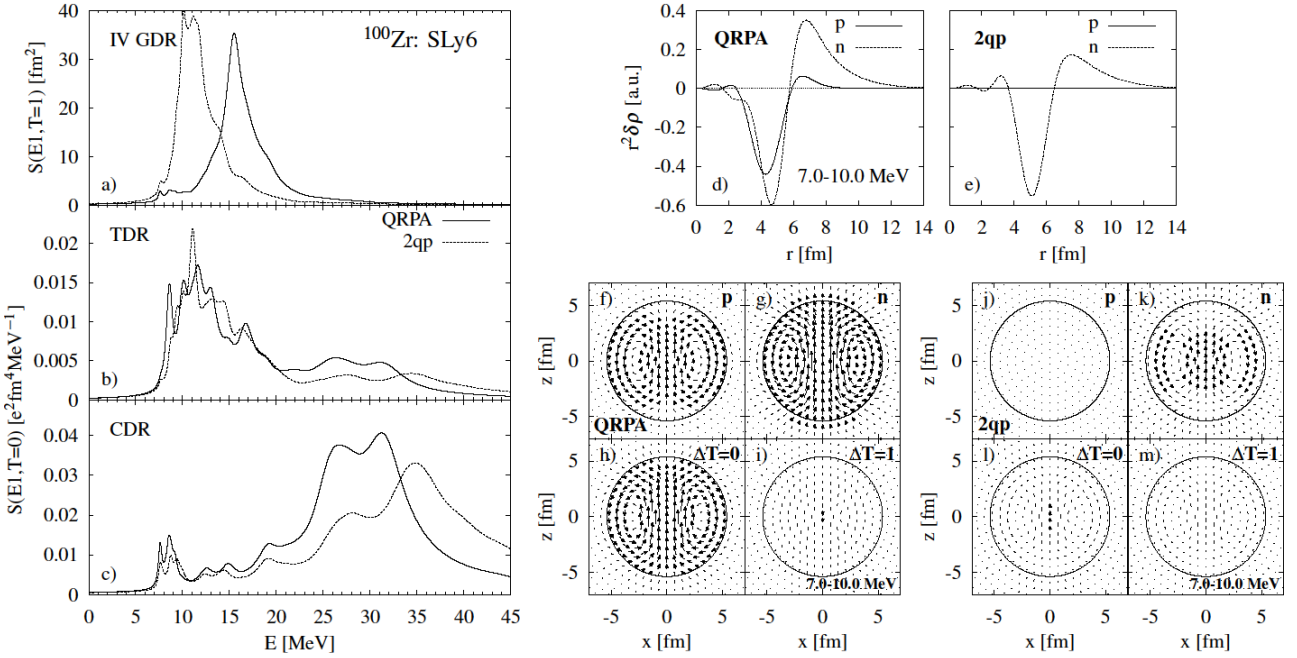
- large neutron excess
- dominant neutron, contribution to toroidal current,
- RPA enforces the flow
- basically the same results as for 120Sn, SLy6.

A. Repko, VON, J. Kvasil and P.-G. Reinhard, EPJA, 55, 242 (2019)

90Zr, SLy6



100Zr, SLy6

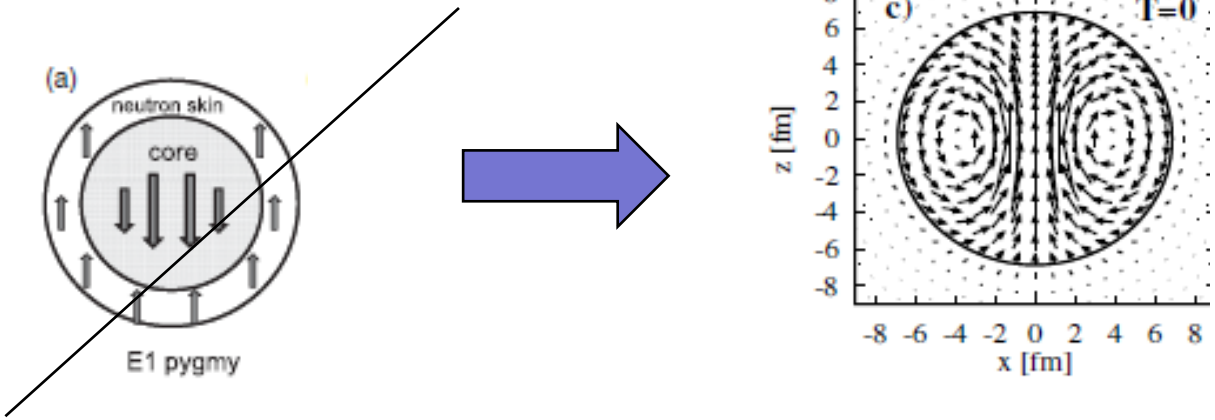


Basically the same results as for 120Sn, SLy6.

Altogether,

- TDR as a Hill's vortex is a general feature of atomic nuclei

- PDR scheme is a rough oversimplification of the actual toroidal nuclear flow.



Despite the toroidal contribution dominates in E1 states at PDR energy region, these states:

- have generally a complicated structure,
- include some irrotational fraction resulting in E1 strength.

There is some interplay between E1 vortical toroidal and E1 irrotational fractions. We should not skip the numerous previous results for E1 PDR strength. However, investigation of the dominant toroidal fraction will allow to understand better the origin and structure of low-energy E2 strength.

PHYSICAL REVIEW LETTERS **120,** 182501 (2018)

Individual Low-Energy Toroidal Dipole State in ^{24}Mg

V. O. Nesterenko,[1*] A. Repko,[2] J. Kvasil,[3] and P.-G. Reinhard[4]
[1]*Laboratory of Theoretical Physics, Joint Institute for Nuclear Research, Dubna, Moscow region 141980, Russia*
[2]*Department of Nuclear Physics, Institute of Physics SAS, 84511 Bratislava, Slovakia*
[3]*Institute of Particle and Nuclear Physics, Charles University, CZ-18000 Prague, Czech Republic*
[4]*Institut für Theoretische Physik II, Universität Erlangen, D-91058 Erlangen, Germany*

First observation of toroidal states in (e,e') reaction

PHYSICAL REVIEW LETTERS **133,** 232502 (2024)

Editors' Suggestion Featured in Physics

Candidate Toroidal Electric Dipole Mode in the Spherical Nucleus ^{58}Ni

P. von Neumann-Cosel,[1,*] V. O. Nesterenko,[2,3,†] I. Brandherm,[1] P. I. Vishnevskiy,[2,4] P.-G. Reinhard,[5] J. Kvasil,[6] H. Matsubara,[7,8] A. Repko,[9] A. Richter,[1] M. Scheck,[10,11] and A. Tamii[7]

The direct and unambiguous identification of vortical toroidal E1 states in nuclei is still an open experimental problem (e,e' γ) ???

Some interesting crossover points:

1) Toroid vs Zel'dovich anapole ?

Anapole is a system of currents that does not radiate into the far field.

P=-1, T=+1

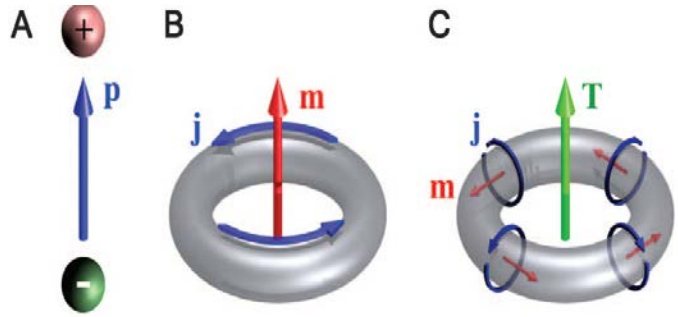
Introduced by Zel'dovich to describe parity non-conservation (PNC) caused by a weak interaction.

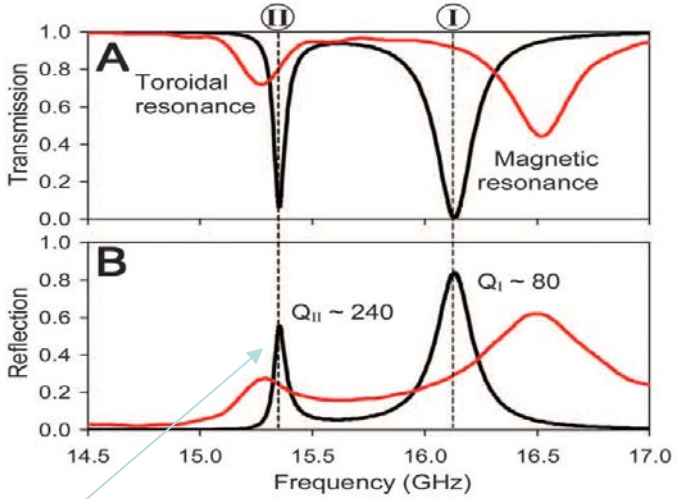
2) Interplay with toroid in metamaterials

Metamaterials are engineered materials whose properties arise from their deliberately designed internal structure. These properties are often rare or absent in naturally occurring materials.

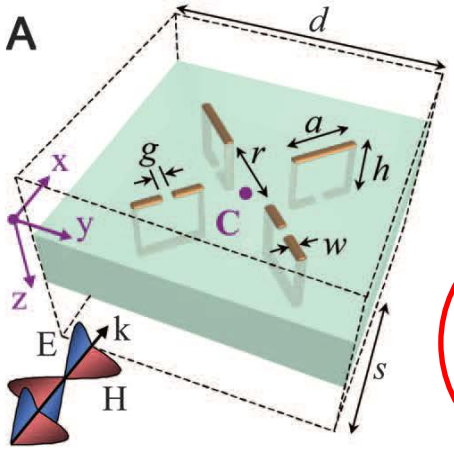
First experimental discovery of toroidal dipole response in metamaterial

T. Kaelberer, V.A. Fedotov, Papasimakis, D.P. Tsai, N.I. Zheludev, Science 330, 1510 (2010).

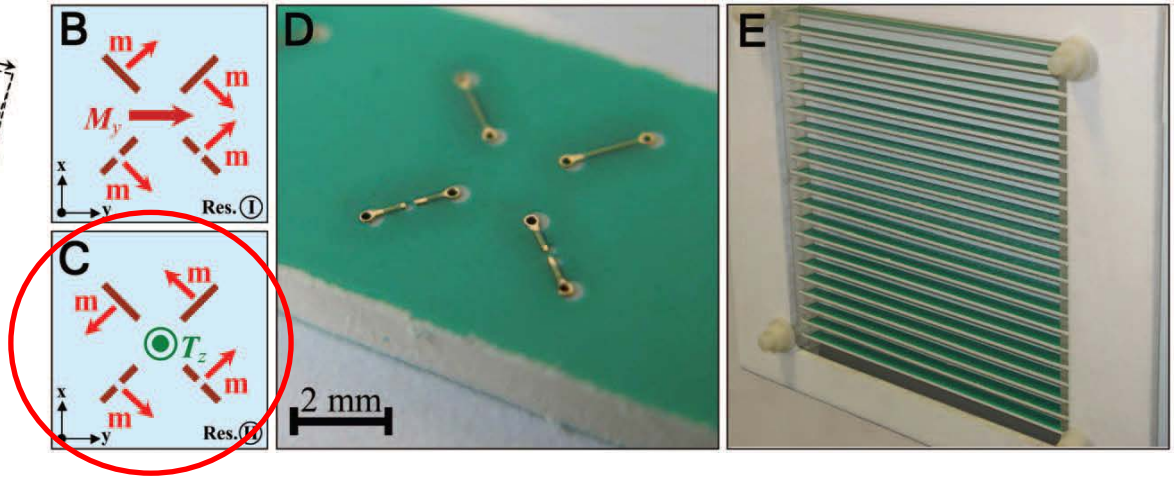




Toroidal dipole resonance at 15.4 GHz



metacell



Toroidal case

Metamaterial is irradiated by polarized microwave field at in x-direction

Metamaterials:

G.N. Afanasjev and Yu. P. Stepanovsky,
"The electromagnetic field of elementary time dependent toroidal sources",
J.Phys. A, 28, 4565 (1995).

Attempts to create artificial materials simulating the time-dependent analog of Aharonov-Bohm effect:

The **Aharonov–Bohm effect**:
electrically charged particle is affected by an electromagnetic vector-potential in the region where both the magnetic and electric fields are zero.

- This can be done by E1 toroidal-based sources using the currents with
- a particular time dependence.

Topologically non-trivial (unremovable by a **gauge** transformation) electromagnetic potentials can be created outside sources.

- Can be used for effective and save information transfer.

- *Skolkovo Institute of Science and Technology, Moscow*
- *Moscow Institute of Physics and Technology*
- *Moscow institute of Steel and Alloys*

Application to toroidal E1 excitations in nuclei?

M1 giant resonances



- M1 orbital scissors (exists only in deformed nuclei)

- M1 spin-flip (exists only in both spherical and deformed nuclei)

$$\hat{\Gamma}(M1, K) = \mu_N \sqrt{\frac{3}{4\pi}} \sum_{q \in p,n} \sum_{k \in q} [g_s^q \hat{s}_k(K) + g_l^q \hat{l}_k(K)].$$

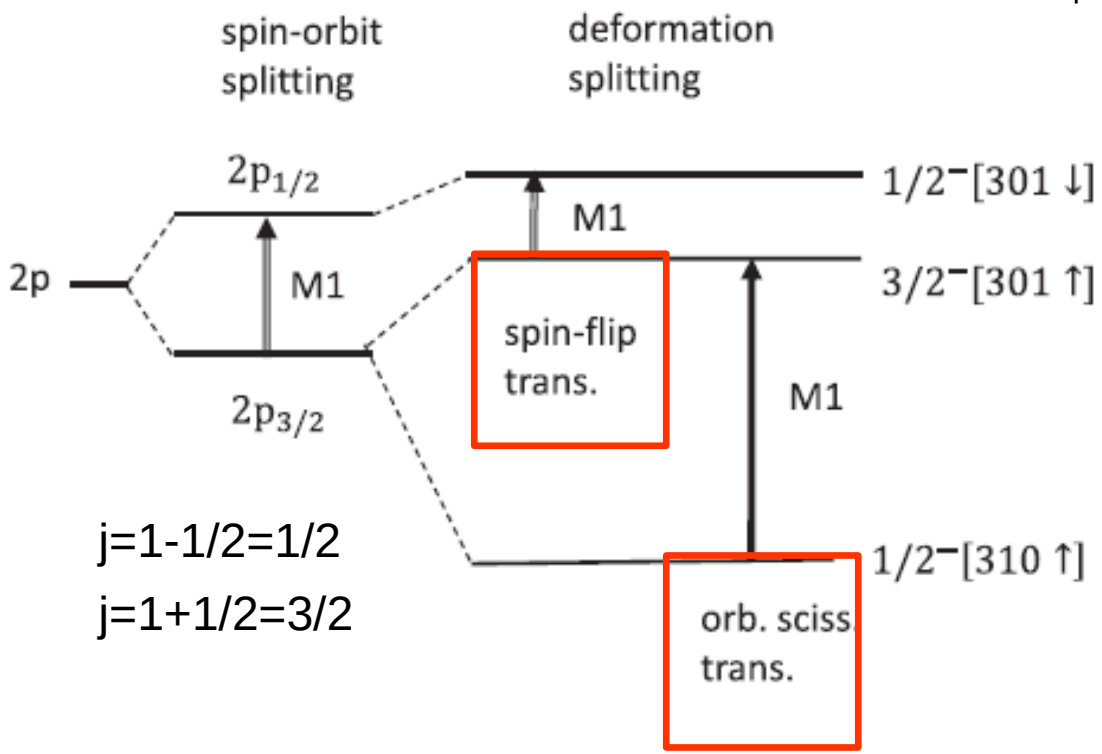
Operator of M1 transition has two terms:

- orbital with gyromagnetic factors $g_s^p = g_s^n = 0$
- spin with gyromagnetic factors $g_s^p = 5.56,\ g_s^n = -3.83,$

$g_l^p = g_l^n = 0$

Origin of orbital (scissors) and spin-flip excitations

VON, P.I. Vishnevskiy, P.G. Reinhard,
A. Repko, J. Kvasil,PRC, 103, 064313 (2021)



Both M1 modes are valuable tests for:

- orbital magnetism in nuclei,
- spin-dependent parameters in Skyrme functional
- impact of tensor (non-central) nuclear forces

Production of Spin-Flip Giant Resonance (SFGR)

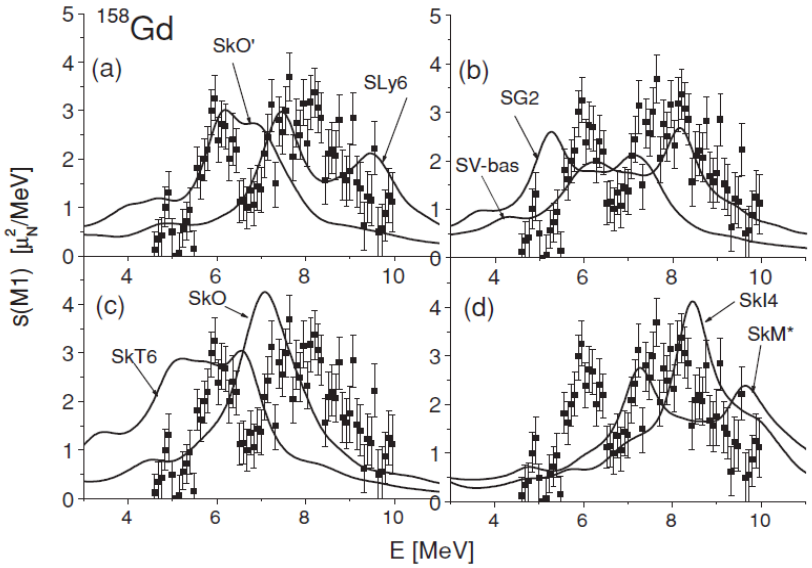
**In medium/heavy nuclei, N>Z and so we should have two well separated peaks (proton and neutron) arising from spin-orbit splitting

$$\Delta E_{so} \propto (l \cdot s)$$

**The proton peak should lie at a lower energy than neutron one

**However, sometimes the strong residual interaction $V > \Delta E_{so}$ can mix these two peak and thus provide one-hump SFGR structure.

**Different Skyrme parameterizations have different relations $V \leftrightarrow \Delta E_{so}$ and so provide a different gross-structure of SFGR



The final SFGR gross-structure is determined by competition between spin-orbit splitting and value of residual interaction

- So SFGR is the robust test for spin-isospin channel in DFT Skyrme functional
- Impact of tensor (non-crntral)forces!

P. Vesely et al, PRC, 80, 031302(R) (2009)

There are still absent universal Skyrme parametrizations suitable for description of all GR in all finite nuclei

IV-GDR prefers Skyrme forces (SLy4, …) with a low IS effective mass

$m^*/m=0.7\text{-}0.9$

IS-GQR prefers Skyrme forces (SkT6, …) with a high IS effective mass

$m^*/m=0.9\text{-}1.0$

Nuclear masses and low-energy states are well described with $m^*/m=1$

IS-GMR: Skyrme DFT and RMF give different values of nuclear incompressibility

Depending on the Skyrme force, we get one- or two-hump structure of spin-flip M1 GR.

So, for the quantitative description of the particular GR, we should choose the proper Skyrme parametrization.
Unfortunately, this is still a common practice.

Conclusions

Features of different GR have been studied for many years and now, as a rule, are basically known.

However, investigation of GR remains to be very important because:

- **GR are used for fitting modern self-consistent models to nuclear dynamics**
- **provide valuable information on nuclear matter characteristics**
- **have many open problems**

  - fine structure of GDR and GQR, wavelet analysis,
  - double hump structure of GMR in deformed nuclei, different K_∞
  - to search direct signatures of toroidal E1(T=0) in experiment
  - impact of tensor forces
  - …….

Still very hot topic!

Thank you for your attention!