

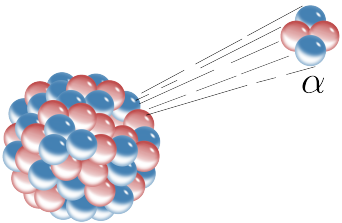
Joint Institute for Nuclear Research
Bogoliubov Laboratory of Theoretical Physics
Nuclear Dynamics: from Stars to Colliders

Cluster approach to spontaneous fission

I. S. Rogov

Dubna, 2026

Introduction



V. Rutherford, 1899; G. A. Gamov, 1930

Spontaneous Fission of Uranium

With 15 plates ionization chambers adjusted for detection of uranium fission products we observed 6 pulses per hour which we ascribe to spontaneous fission of uranium. A series of control experiments seem to exclude other possible explanations. Energy of pulses and absorption properties coincide with fission products of uranium bombarded by neutrons. No pulses were found with UX and Th. Mean lifetime of uranium follows ten to sixteen or seventeen years.

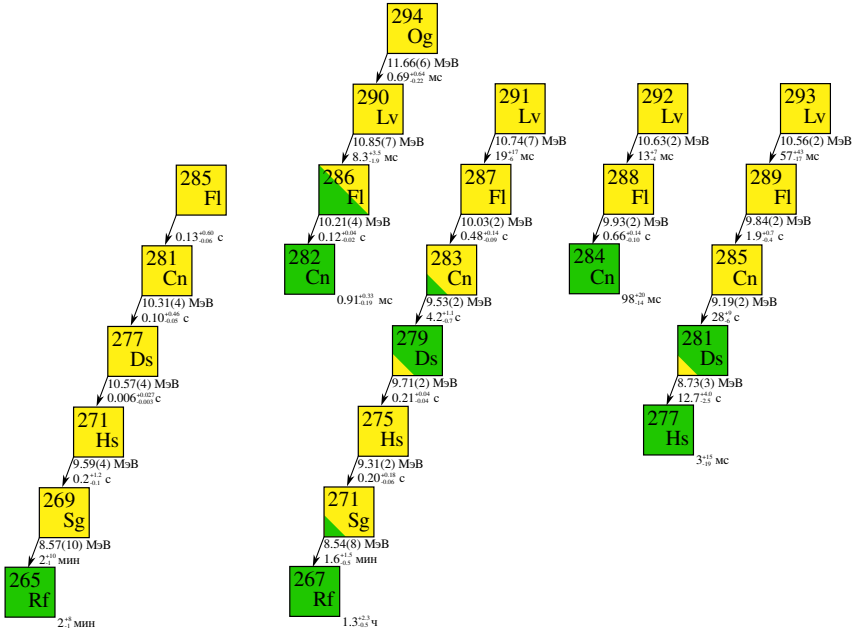
FLEROV
PETRJAK

Physico Technical Institute (F),
Radium Institute (P),
Leningrad, U. S. S. R.,
June 14, 1940 (by cable).

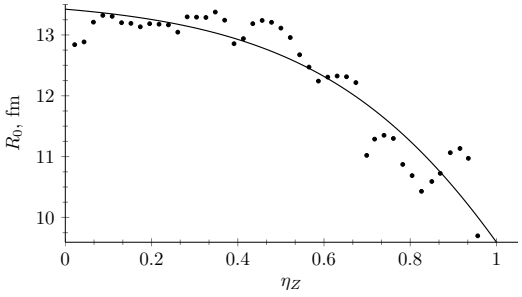
G. N. Flerov and K. A. Petrzak, 1940

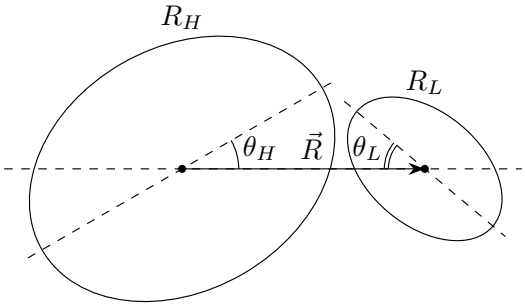
N. Bohr and J. A. Wheeler, 1939; G. T. Seaborg, 1952.
W. J. Swiatecki, 1955; ^{242f}Am, 1962; V. M. Strutinsky, 1967

$^{223}\mathrm{Ra} \rightarrow {}^{14}\mathrm{C} + {}^{209}\mathrm{Pb}$ Rose and Johnson, University of Oxford
Group of the Kurchatov Institute of Atomic Energy, 1984


294 Og
11.66(6) МэВ
0.69 +0.64 −0.22 мс
290 Lv
10.85(7) МэВ
8.3 +3.5 −1.9 мс
291 Lv
10.74(7) МэВ
19 +17 −6 мс
292 Lv
10.63(2) МэВ
13 +7 −4 мс
293 Lv
10.56(2) МэВ
57 +43 −17 мс
285 Fl
0.13 +0.60 −0.06 с
286 Fl
10.21(4) МэВ
0.12 +0.04 −0.02 с
287 Fl
10.03(2) МэВ
0.48 +0.14 −0.09 с
288 Fl
9.93(2) МэВ
0.66 +0.14 −0.10 с
289 Fl
9.84(2) МэВ
1.9 +0.7 −0.4 с
281 Cn
10.31(4) МэВ
0.10 +0.06 −0.03 с
282 Cn
0.91 +0.33 −0.19 мс
283 Cn
9.53(2) МэВ
4.2 +1.1 −0.7 с
284 Cn
98 +20 −14 мс
285 Cn
9.19(2) МэВ
28 +9 −6 с
277 Ds
10.57(4) МэВ
0.006 +0.027 −0.003 с
279 Ds
9.71(2) МэВ
0.21 +0.04 −0.04 с
281 Ds
8.73(3) МэВ
12.7 +4.0 −2.5 с
271 Hs
9.59(4) МэВ
0.2 +1.2 −0.1 с
275 Hs
9.31(2) МэВ
0.20 +0.18 −0.06 с
277 Hs
3 +15 −10 мс
269 Sg
8.57(10) МэВ
2 +10 −1 мин
271 Sg
8.54(8) МэВ
1.6 +1.5 −0.5 мин
265 Rf
2 +8 −1 мин
267 Rf
1.3 +2.3 −0.5 ч

Dinuclear system (DNS)

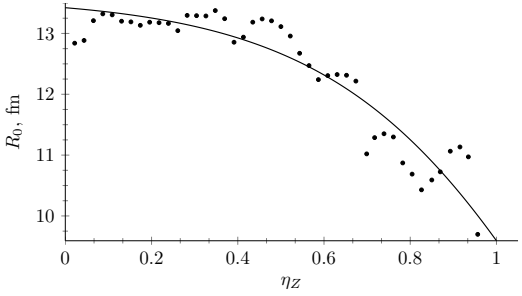




$$\eta_Z = \frac{Z_H - Z_L}{Z_H + Z_L}, \qquad \eta = \frac{A_H - A_L}{A_H + A_L}$$

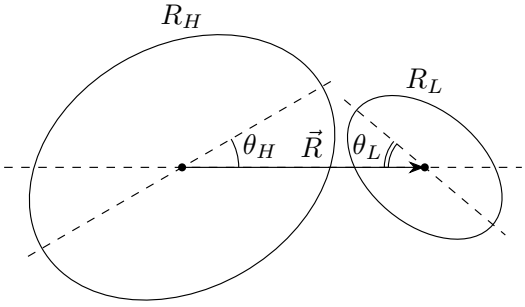
Dinuclear system (DNS)

DNS preformation (η_Z)



Spectroscopic factor S_L

DNS decay (R)



Penetration probability P_L

$$\Gamma_L \sim P_L S_L$$

DNS state

The system is described by stationary wave function $\Psi(\eta_Z)$:

$$\hat{H}\Psi_n(\eta_Z) = E_n\Psi_n(\eta_Z),$$

where

$$\hat{H} = \hat{T}_{\eta_Z} + U(\eta_Z)$$

Kinetic energy

$$\hat{T}_{\eta_Z} = \frac{\hbar^2}{2}\frac{\partial}{\partial\eta_Z}B_{\eta_Z}^{-1}\frac{\partial}{\partial\eta_Z}$$

Potential energy

[G. Adamian *et al.* Int. J. Mod. Phys. A, 1996]

$$U(R,\eta_Z) = V(R,\eta_Z) - (Q_M - Q_L - Q_H)$$

$$V(R,\eta_Z) = V_{\rm C}(R,\eta_Z) + V_N(R,\eta_Z) + V_r(R,\eta_Z)$$

Driving potential

Coulomb potential

[C.Y. Wong, Phys. Rev. Lett., 1973]

$$V_C = \frac{e^2 Z_L Z_H}{R}\left(1 + \frac{3}{5R^2}\sum_{i=L,H} R_i^2 \beta_{2i} Y_{20}(\theta_i) + \frac{12}{35R^2}\sum_{i=L,H}\left(R_i \beta_{2i} Y_{20}(\theta_i)\right)^2\right)$$

Nucleus shape

$$R_i(\theta) = r_{0i} A_i^{1/3}\left(1 + \beta_{2i} Y_{20}(\theta)\right)$$

Centrifugal potential

[Dudek J. 1992, T. M. Shneidman 2000]

$$V_r = \hbar^2 \Omega(\Omega+1)/(2\Im),\ \Im = 0.85(j_L + j_H + \mu R^2)$$

Driving potential

Nuclear potential

$$V_N = \int \rho_H(\boldsymbol{r}_H)\rho_L(\boldsymbol{R}-\boldsymbol{r}_L)F(\boldsymbol{r}_H-\boldsymbol{r}_L)d\boldsymbol{r}_L d\boldsymbol{r}_H$$

$$F(\boldsymbol{r}_H-\boldsymbol{r}_L) = C_0\left[F_{in}\frac{\rho(\boldsymbol{r}_H)}{\rho_0} + F_{ex}\left(1-\frac{\rho(\boldsymbol{r}_H)}{\rho_0}\right)\right]\delta(\boldsymbol{r}_H-\boldsymbol{r}_L)$$

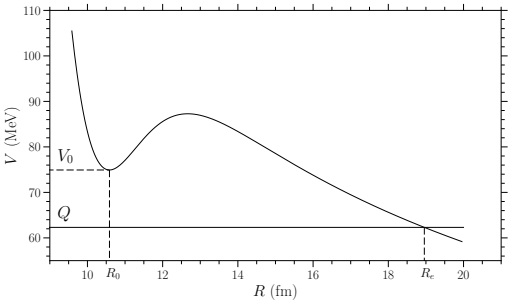
$$F_{\rm in,ex} = \xi_{\rm in,ex} + \xi'_{\rm in,ex}\frac{A_L-2Z_L}{A_L}\frac{A_H-2Z_H}{A_H}$$

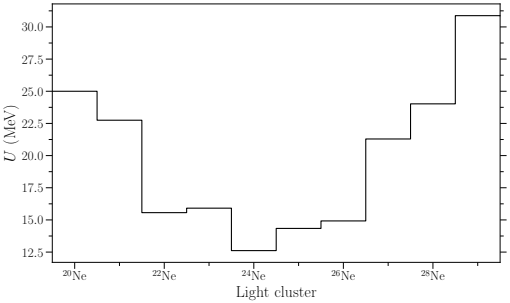
$$\rho(\mathbf{r}_H) = \rho_H(\mathbf{r}_H) + \rho_L(\mathbf{R}-\mathbf{r}_L)$$

The constants $\xi_{\rm in} = 0.09$, $\xi_{\rm ex} = -2.59$, $\xi'_{\rm in} = 0.42$, $\xi'_{\rm ex} = 0.54$, $C_0 = 300$ MeV·fm^3

$$\rho_i(\boldsymbol{r}) = \frac{\rho_0}{1+\exp(|\boldsymbol{r}-\boldsymbol{R_i}|/a_{0i})}.$$

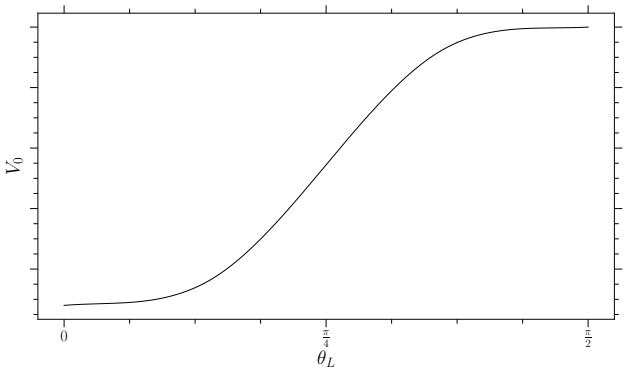
Driving potential

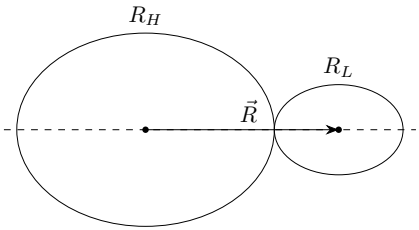




$$a_0 = 0.47 - 0.56 \text{ fm}; \quad r_0 = 1.00 - 1.16 \text{ fm}$$

Driving potential





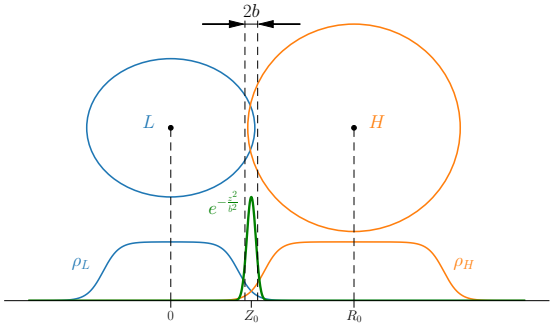
Inertia parameter

Inertia parameter:

[G. Adamian *et al.* Nucl. Phys. A. 1995]

$$B_{\eta_z}^{-1} = \frac{1}{2m_0} \frac{A_{neck}}{2\sqrt{2\pi} b^2 A^2}$$

where b – the DNS neck size



A_{neck} – is number of nucleons in the neck:

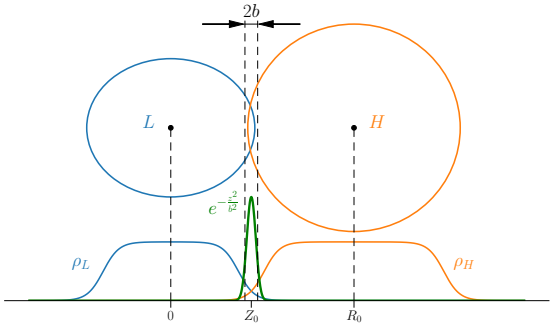
$$A_{neck} = \int \left[\rho_L(\boldsymbol{r}) + \rho_H(\boldsymbol{R} - \boldsymbol{r})\right] \exp\left(-\frac{((z - Z_0)^2}{b^2}\right) d\boldsymbol{r}$$

A_{neck} calculation

$$\rho_L(z) = \rho_H(R_0 - z)$$

$$Z_0 = \frac{R_0 a_H - R_L(0) a_H + R_H(0) a_L}{a_L + a_H}$$

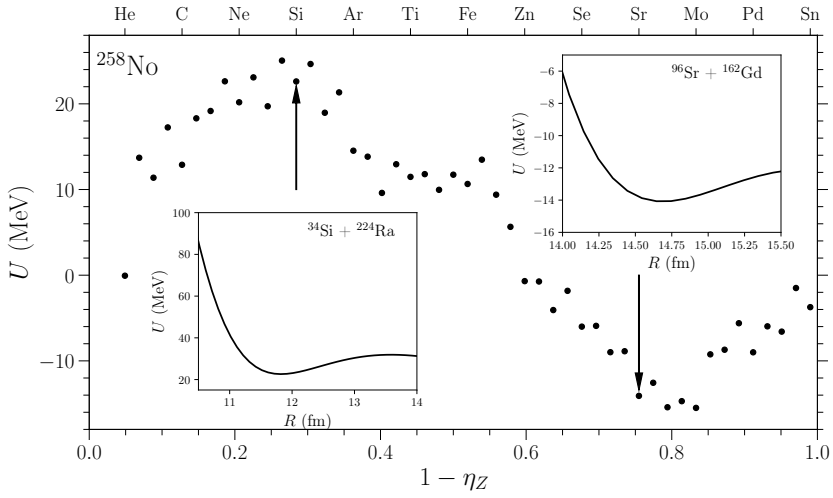
$$A_{neck} = A^L_{neck} + A^H_{neck}$$



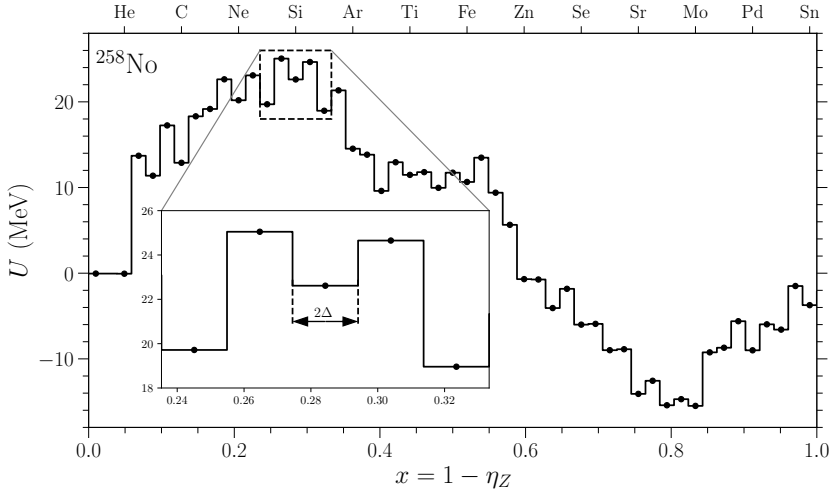
$$A^{L,H}_{neck} = 2\pi \int\int \rho^{cyl}_{L,H}(z,r) \exp\left(-\frac{(z - Z_{L,H})^2}{b^2}\right) r dr dz$$

$$Z_L = Z_0,\ Z_H = R_0 - Z_0$$

Formation of the driving potential

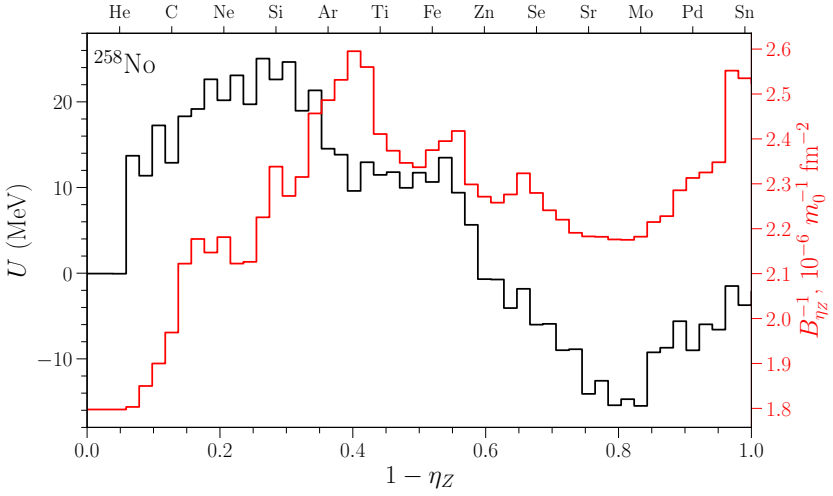


Formation of the driving potential



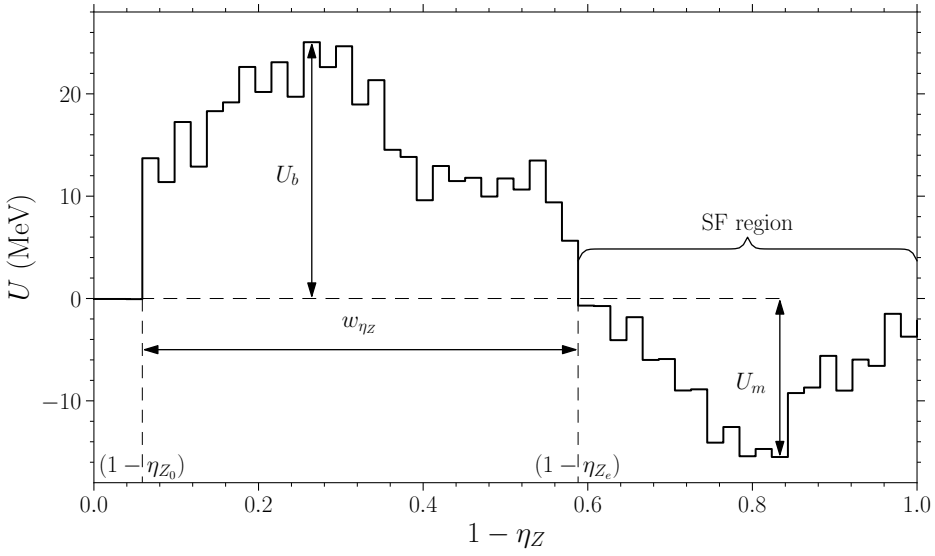
$$\eta_Z \to x = 1 - \eta_Z; \quad \Delta = 1/Z;$$

Formation of the driving potential



$$\eta_Z \to x = 1-\eta_Z; \quad \Delta = 1/Z; \quad \Psi_n = \sum_i \psi_{ni}$$

Driving potential characteristics



Schrödinger equation solution

$$\hat{H} \sum_i \psi_{ni}(x) = E_n \sum_i \psi_{ni}(x)$$

$$-\frac{\hbar^2}{2} B^{-1}_{\eta z_j} \frac{\partial^2}{\partial x^2} \psi_j(x) + U_j \psi_j(x) = E_n \psi_j(x)$$

Schrödinger equation solution

$$\hat{H} \sum_i \psi_{ni}(x) = E_n \sum_i \psi_{ni}(x)$$

$$-\frac{\hbar^2}{2} B^{-1}_{\eta z_j} \frac{\partial^2}{\partial x^2} \psi_j(x) + U_j \psi_j(x) = E_n \psi_j(x)$$

$$\psi_j(x) = a_j e^{ik_j x} + b_j e^{-ik_j x};\ k_j = \sqrt{\frac{2}{\hbar^2 B^{-1}_{\eta z_j}} (E_n - U_j)}$$

Schrödinger equation solution

$$\hat{H} \sum_i \psi_{ni}(x) = E_n \sum_i \psi_{ni}(x)$$

$$-\frac{\hbar^2}{2} B^{-1}_{\eta z_j} \frac{\partial^2}{\partial x^2} \psi_j(x) + U_j \psi_j(x) = E_n \psi_j(x)$$

$$\psi_j(x) = a_j e^{ik_j x} + b_j e^{-ik_j x};\ k_j = \sqrt{\frac{2}{\hbar^2 B^{-1}_{\eta z_j}} (E_n - U_j)}$$

Joining conditions:

$$\left\{ \begin{array}{c} \psi_j(x_j) = \psi_{j+1}(x_j) \\ \frac{\partial \psi_j(x_j)}{\partial x} = \frac{\partial \psi_{j+1}(x_j)}{\partial x} \end{array} \right\}_{j=0\ldots(N-1)}$$

Boundary conditions:

$$\frac{\partial \psi_0(0)}{\partial x} = 0,\ \frac{\partial \psi_N(1)}{\partial x} = 0$$

Schrödinger equation solution

$$
\begin{gathered}
a_0 k_0 - b_0 k_0 = 0 \\
\left\{ \begin{array}{c}
a_j e^{ik_j x_j} + b_j e^{-ik_j x_j} - a_{j+1} e^{ik_{j+1} x_j} - b_{j+1} e^{-ik_{j+1} x_j} = 0 \\
a_j k_j e^{ik_j x_j} - b_j k_j e^{-ik_j x_j} - a_{j+1} k_{j+1} e^{ik_{j+1} x_j} + b_{j+1} k_{j+1} e^{-ik_{j+1} x_j} = 0
\end{array} \right\}_{j=0..(N-1)} \\
a_N k_N e^{ik_N} - b_N k_N e^{-ik_N} = 0
\end{gathered}
$$

Solution 1

$$D_e = \begin{bmatrix} k_0 & -k_0 & 0 & ... & 0 \\ e^{ik_0 4\Delta} & e^{-ik_0 4\Delta} & -e^{ik_1 4\Delta} & ... & 0 \\ k_0 e^{ik_0 4\Delta} & -k_0 e^{-ik_0 4\Delta} & -k_1 e^{ik_1 4\Delta} & ... & 0 \\ 0 & 0 & e^{ik_1 5\Delta} & ... & 0 \\ 0 & 0 & k_1 e^{ik_1 5\Delta} & ... & 0 \\ ... & ... & ... & ... & ... \\ 0 & 0 & 0 & ... & -k_N b_N e^{-ik_N} \end{bmatrix}$$

$$C = [a_0 \ b_0 \ a_1 \ b_1 \ . \ . \ a_N \ b_N]^T \ , \ \Delta = 1/Z$$

$$D_e \cdot C = 0$$

Solution 1

$$k_0 = \sqrt{\frac{2}{\hbar^2 B_{\eta z 0}^{-1}} \left(E_n - U_0\right)}$$

$$\det\left[D_e(U_0, E_0 = 0)\right] = 0 \Rightarrow U_0$$

$$D_e(U_0) \cdot C = 0 \Rightarrow \{a_j\}, \{b_j\}$$

Normalization:

$$\int_0^1 |\Psi(\eta_z)|^2 d\eta_z = 1$$

Solution 2

$$a_j e^{ik_j x_j} + b_j e^{-ik_j x_j} = a_{j+1} e^{ik_{j+1} x_j} + b_{j+1} e^{-ik_{j+1} x_j}$$
$$a_j k_j e^{ik_j x_j} - b_j k_j e^{-ik_j x_j} = a_{j+1} k_{j+1} e^{ik_{j+1} x_j} - b_{j+1} k_{j+1} e^{-ik_{j+1} x_j}$$

Solution 2

$$a_j e^{ik_j x_j} + b_j e^{-ik_j x_j} = a_{j+1} e^{ik_{j+1} x_j} + b_{j+1} e^{-ik_{j+1} x_j}$$
$$a_j k_j e^{ik_j x_j} - b_j k_j e^{-ik_j x_j} = a_{j+1} k_{j+1} e^{ik_{j+1} x_j} - b_{j+1} k_{j+1} e^{-ik_{j+1} x_j}$$

$$\underset{\uparrow\, L_j}{\begin{pmatrix} e^{ik_j x_j} & e^{-ik_j x_j} \\ k_j e^{ik_j x_j} & -k_j e^{-ik_j x_j} \end{pmatrix}} \quad \underset{\uparrow\, C_j}{\begin{pmatrix} a_j \\ b_j \end{pmatrix}} \quad = \quad \underset{\uparrow\, L_{j+1}}{\begin{pmatrix} e^{ik_{j+1} x_j} & e^{-ik_{j+1} x_j} \\ k_{j+1} e^{ik_{j+1} x_j} & -k_{j+1} e^{-ik_{j+1} x_j} \end{pmatrix}} \quad \underset{\uparrow\, C_{j+1}}{\begin{pmatrix} a_{j+1} \\ b_{j+1} \end{pmatrix}}$$

Solution 2

$$a_j e^{ik_j x_j} + b_j e^{-ik_j x_j} = a_{j+1} e^{ik_{j+1} x_j} + b_{j+1} e^{-ik_{j+1} x_j}$$
$$a_j k_j e^{ik_j x_j} - b_j k_j e^{-ik_j x_j} = a_{j+1} k_{j+1} e^{ik_{j+1} x_j} - b_{j+1} k_{j+1} e^{-ik_{j+1} x_j}$$

$$\underset{\uparrow\\ L_j}{\begin{pmatrix} e^{ik_j x_j} & e^{-ik_j x_j} \\ k_j e^{ik_j x_j} & -k_j e^{-ik_j x_j} \end{pmatrix}} \quad \underset{\uparrow\\ C_j}{\begin{pmatrix} a_j \\ b_j \end{pmatrix}} \quad = \quad \underset{\uparrow\\ L_{j+1}}{\begin{pmatrix} e^{ik_{j+1} x_j} & e^{-ik_{j+1} x_j} \\ k_{j+1} e^{ik_{j+1} x_j} & -k_{j+1} e^{-ik_{j+1} x_j} \end{pmatrix}} \quad \underset{\uparrow\\ C_{j+1}}{\begin{pmatrix} a_{j+1} \\ b_{j+1} \end{pmatrix}}$$

$$C_j = L_j^{-1} L_{j+1} C_{j+1}$$

$$T_j^{j+1} = L_j^{-1} L_{j+1} \Rightarrow C_j = T_j^{j+1} C_{j+1}$$

Solution 2

$$\left.\frac{\partial \Psi_n}{\partial x}\right|_{x=1} = 0 \Rightarrow a_n k_n e^{ik_n} - b_n k_n e^{-ik_n} = 0$$

$$b_n = a_n e^{2ik_n}$$

$$k_n\,(E=0) = i\sqrt{\frac{2}{\hbar^2 B_n^{-1}} U_n}$$

$$b_n = \exp\left[-\sqrt{\frac{8U_n}{\hbar^2 B_n^{-1}}}\right] a_n \Rightarrow C_n = \gamma \left(\exp\left[-\sqrt{\frac{8U_n}{\hbar^2 B_n^{-1}}}\right]^{1}\right)$$

$$C_{n-1} = T_{n-1}^n C_n,$$

$$\ldots$$

$$C_1 = T_1^2 C_2$$

Solution 2

$$C_0 = T_0^1\left(U_0\right) C_1$$

$$\left.\begin{array}{l} C_0 = T_0^1\left(U_0, E=0\right) C_1 \\ \frac{\partial \Psi_0}{\partial x}\big|_{x=0} = 0 \Rightarrow a_0 = b_0 \end{array}\right\} \Rightarrow \begin{pmatrix} a_0 \\ a_0 \end{pmatrix} = T_0^1(U_0, E=0) C_1$$

$$\begin{pmatrix} a_0 \\ a_0 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} a_1(-k_0-k_1)e^{-ik_0x_0}e^{ik_1x_1} + b_1(k_1-k_0)e^{-ik_0x_0}e^{-ik_1x_1} \\ a_1(k_1-k_0)e^{ik_0x_0}e^{ik_1x_1} + b_1(-k_0-k_1)e^{ik_0x_0}e^{-ik_1x_1} \end{pmatrix}$$

Solution 2

Solving equation

$$\frac{(k_0+k_1)e^{-2ik_0x_0}-k_0+k_1}{k_0+k_1-(k_0-k_1)e^{-2ik_0x_0}}=\frac{b_1}{a_1}e^{-2ik_1x_1}$$

with respect to k_0 we obtain mononucleus energy U_0

$$U_0=-\frac{\hbar^2\left(B_0^{-1}\right)_{\eta_z}}{2}k_0^2.$$

Solution 2

Normalization of a_j, b_j coefficients:

$$\int_0^1 |\Psi(\eta_Z)|^2 d\eta_Z = 1 \to \gamma$$

Spectroscopic factor:

$$S_L = \int_{\eta_Z(Z_L)-\Delta}^{\eta_Z(Z_L)+\Delta} |\Psi(\eta_Z)|^2 d\eta_Z$$

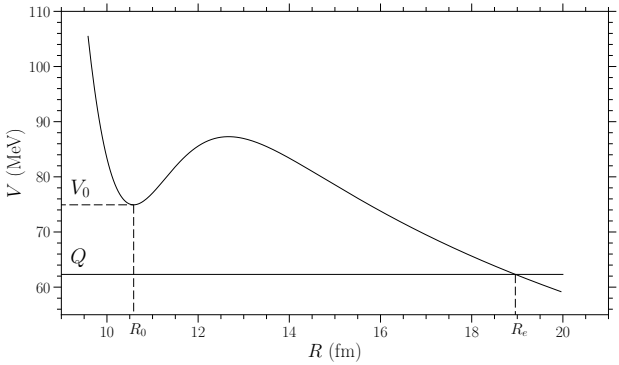
$$\Delta = 1/Z$$

Half-lives

$$S_L = \int\limits_{\eta_Z(Z_L)-\Delta}^{\eta_Z(Z_L)+\Delta} |\Psi(\eta_Z)|^2 d\eta_Z$$

$$\Gamma_L = \frac{\hbar\omega_0}{\pi} S_L P_L,\ T_{1/2} = \frac{\hbar \ln 2}{\Gamma_L}$$

$$T_{1/2} = \frac{\pi \ln 2}{\omega_0 S_L P_L}$$



$$P_L = \left(1 + \exp\left[\frac{2}{\hbar}\int\limits_{R_0}^{R_e} \sqrt{2\mu(V(R,\eta_Z)-Q)}dR\right]\right)^{-1}$$

Spectroscopic factor for SF

The SF decay width is

$$\Gamma_{SF} = \sum_L \Gamma_L = \frac{\hbar\omega_0}{\pi} \sum_L S_L P_L$$

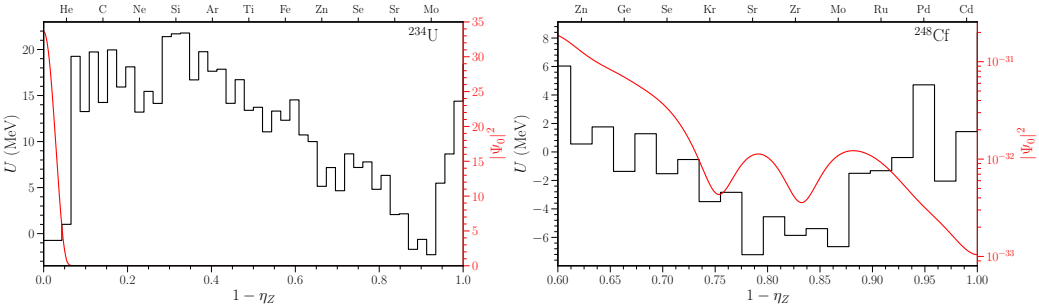
For SF region $P_L = 1$, so:

$$\Gamma_{SF} = \frac{\hbar\omega_0}{\pi} \sum_L S_L = \frac{\hbar\omega_0}{\pi} S_{SF},$$

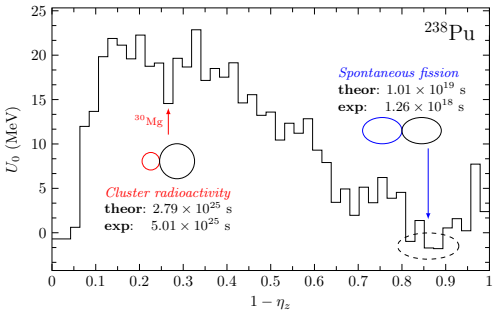
and the spectroscopic factor S_{SF} for SF:

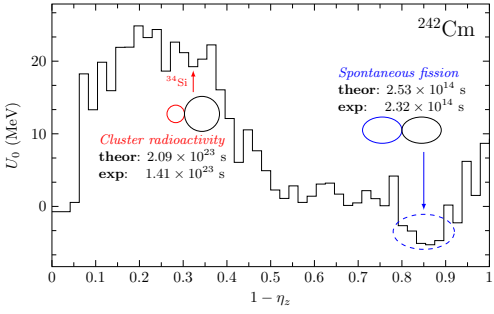
$$S_{SF} = \sum_L S_L,$$

Wave function

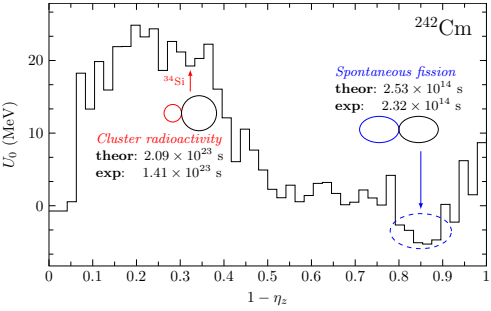


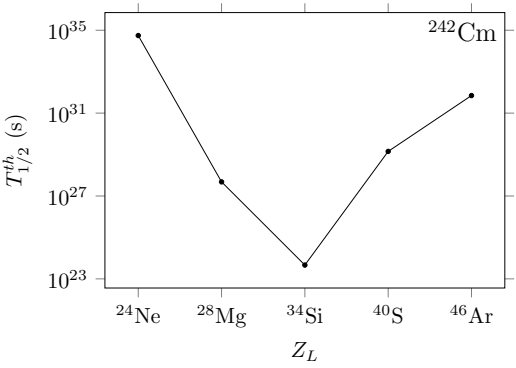
Decay zones


^{238}Pu
^{30}Mg
Cluster radioactivity
theor: 2.79×10^{25} s
exp: 5.01×10^{25} s
Spontaneous fission
theor: 1.01×10^{19} s
exp: 1.26×10^{18} s
U_0 (MeV)
$1 - \eta_z$

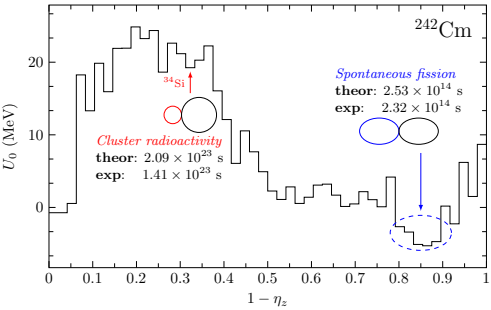

^{242}Cm
^{34}Si
Cluster radioactivity
theor: 2.09×10^{23} s
exp: 1.41×10^{23} s
Spontaneous fission
theor: 2.53×10^{14} s
exp: 2.32×10^{14} s
U_0 (MeV)
$1 - \eta_z$

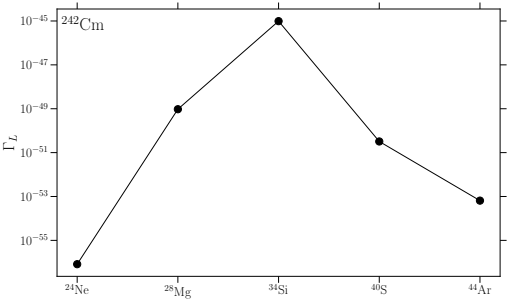
Decay zones


^{242}Cm
^{34}Si
Cluster radioactivity
theor: 2.09×10^{23} s
exp: 1.41×10^{23} s
Spontaneous fission
theor: 2.53×10^{14} s
exp: 2.32×10^{14} s
U_0 (MeV)
$1 - \eta_z$

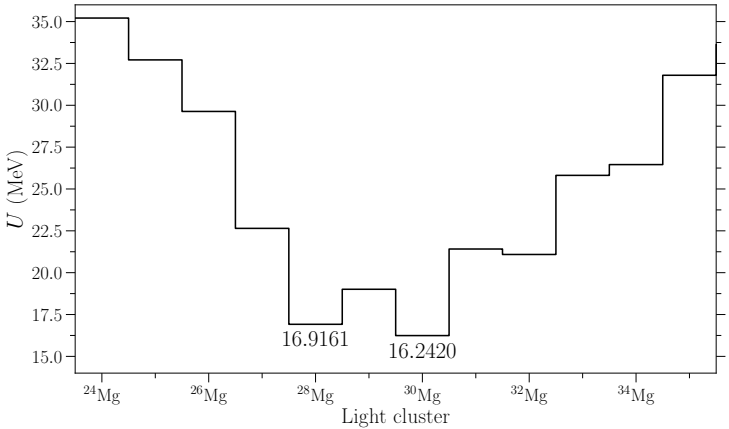

^{242}Cm
$T^{th}_{1/2}$ (s)
^{24}Ne
^{28}Mg
^{34}Si
^{40}S
^{46}Ar
Z_L

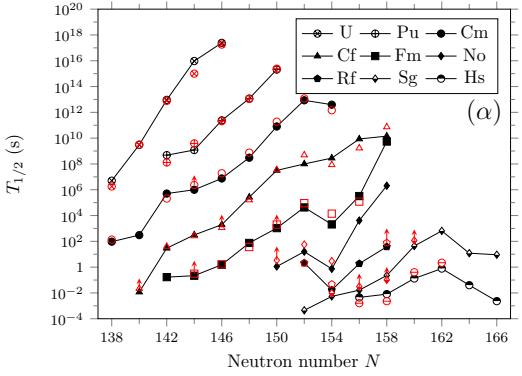
Decay zones

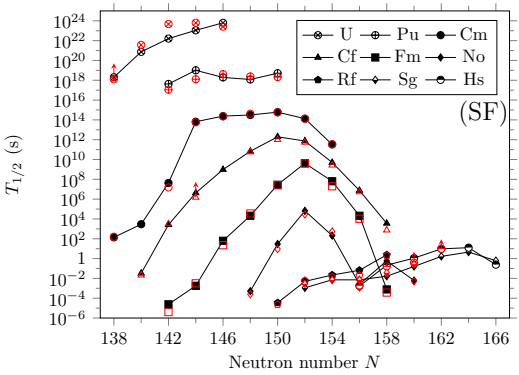

^{242}Cm
^{34}Si
Spontaneous fission
theor: 2.53×10^{14} s
exp: 2.32×10^{14} s
Cluster radioactivity
theor: 2.09×10^{23} s
exp: 1.41×10^{23} s
U_0 (MeV)
$1 - \eta_z$


^{242}Cm
Γ_L
^{24}Ne
^{28}Mg
^{34}Si
^{40}S
^{44}Ar

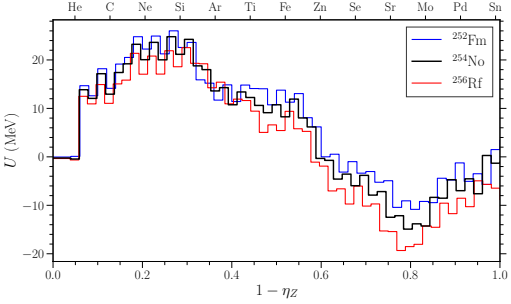
Multiple clusters (^{236}U)

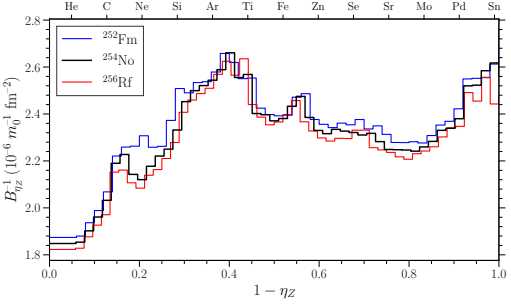


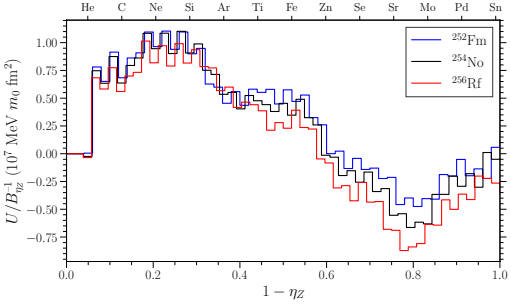


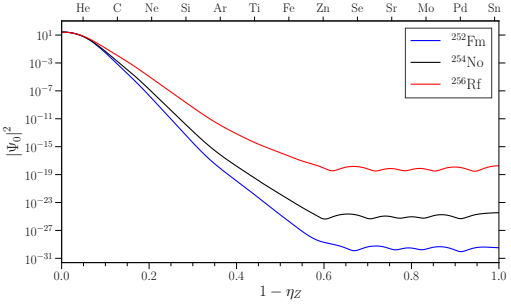


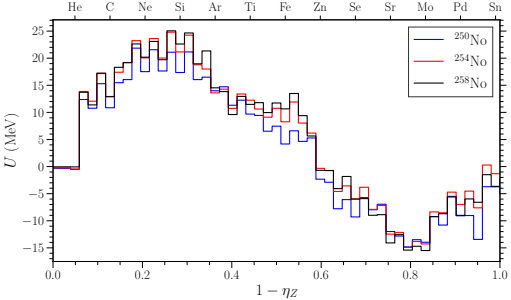
	^{232}U $\rightarrow$ ^{24}Ne	^{234}U $\rightarrow$ ^{26}Ne	^{234}U $\rightarrow$ ^{28}Mg	^{236}U $\rightarrow$ ^{30}Mg
$T_{1/2}^{theor}$ (s)	4.07×10^{21}	1.29×10^{25}	4.33×10^{25}	1.85×10^{26}
$T_{1/2}^{exp}$ (s)	1.89×10^{21}	1.20×10^{25}	3.47×10^{25}	1.89×10^{26}
	^{236}Pu $\rightarrow$ ^{28}Mg	^{238}Pu $\rightarrow$ ^{30}Mg	^{242}Cm $\rightarrow$ ^{34}Si	^{248}Cf $\rightarrow$ ^{40}S
$T_{1/2}^{theor}$ (s)	2.06×10^{21}	2.79×10^{25}	4.64×10^{23}	9.86×10^{25}
$T_{1/2}^{exp}$ (s)	4.67×10^{21}	5.01×10^{25}	1.41×10^{23}	

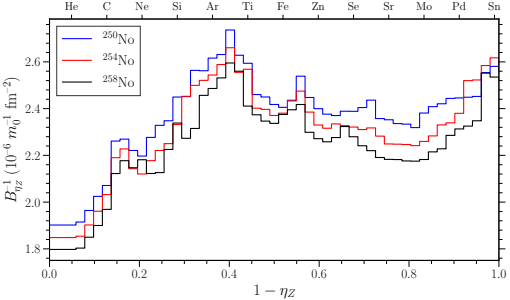

He C Ne Si Ar Ti Fe Zn Se Sr Mo Pd Sn
^{252}Fm
^{254}No
^{256}Rf
U (MeV)
$1-\eta_Z$

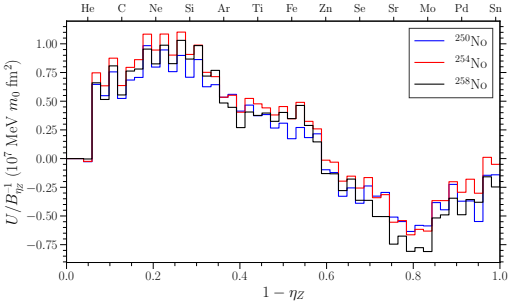

He C Ne Si Ar Ti Fe Zn Se Sr Mo Pd Sn
^{252}Fm
^{254}No
^{256}Rf
$B_{\eta Z}^{-1}$ (10^{-6} m_0^{-1} fm^{-2})
$1-\eta_Z$

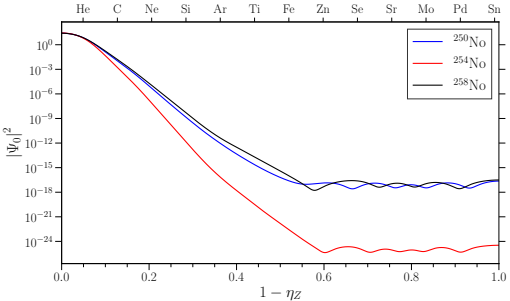

He C Ne Si Ar Ti Fe Zn Se Sr Mo Pd Sn
^{252}Fm
^{254}No
^{256}Rf
$U/B_{\eta z}^{-1}$ (10^7 MeV m_0 fm^2)
$1-\eta_Z$

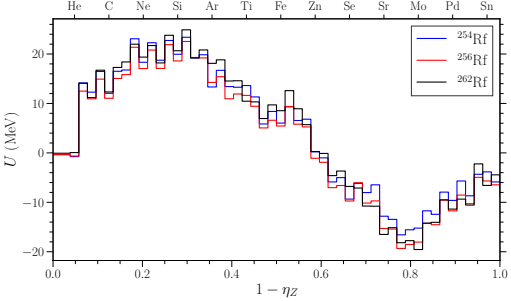

He C Ne Si Ar Ti Fe Zn Se Sr Mo Pd Sn
^{252}Fm
^{254}No
^{256}Rf
$|\Psi_0|^2$
$1-\eta_Z$

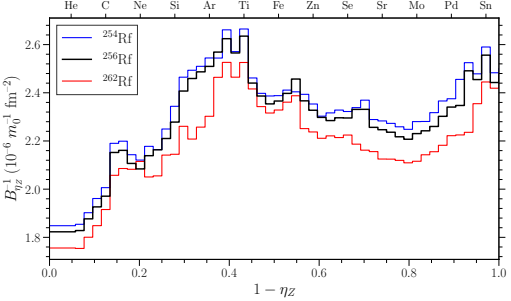

He C Ne Si Ar Ti Fe Zn Se Sr Mo Pd Sn
^{250}No
^{254}No
^{258}No
U (MeV)
$1-\eta_Z$

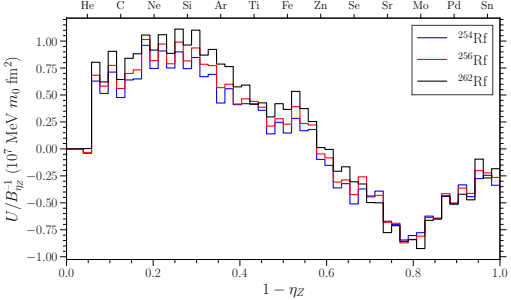

He C Ne Si Ar Ti Fe Zn Se Sr Mo Pd Sn
^{250}No
^{254}No
^{258}No
$B_{\eta_Z}^{-1}$ (10^{-6} m_0^{-1} fm^{-2})
$1-\eta_Z$

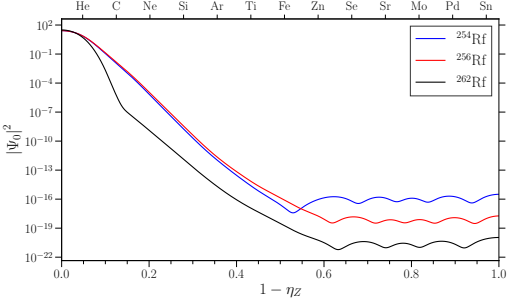

He C Ne Si Ar Ti Fe Zn Se Sr Mo Pd Sn
^{250}No
^{254}No
^{258}No
$U/B_{\eta_Z}^{-1}$ (10^7 MeV m_0 fm^2)
$1-\eta_Z$


He C Ne Si Ar Ti Fe Zn Se Sr Mo Pd Sn
^{250}No
^{254}No
^{258}No
$|\Psi_0|^2$
$1-\eta_Z$

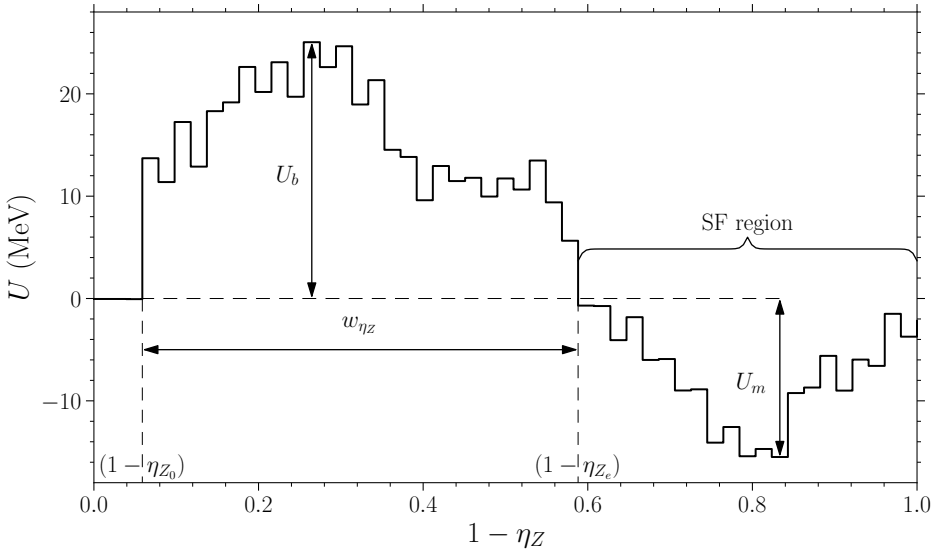

He C Ne Si Ar Ti Fe Zn Se Sr Mo Pd Sn
^{254}Rf
^{256}Rf
^{262}Rf
U (MeV)
$1-\eta_Z$

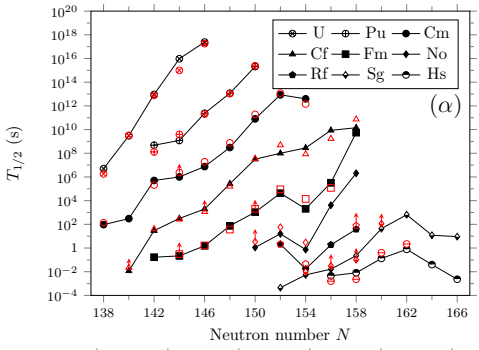

He C Ne Si Ar Ti Fe Zn Se Sr Mo Pd Sn
^{254}Rf
^{256}Rf
^{262}Rf
$B_{\eta_Z}^{-1}$ (10^{-6} m_0^{-1} fm^{-2})
$1-\eta_Z$

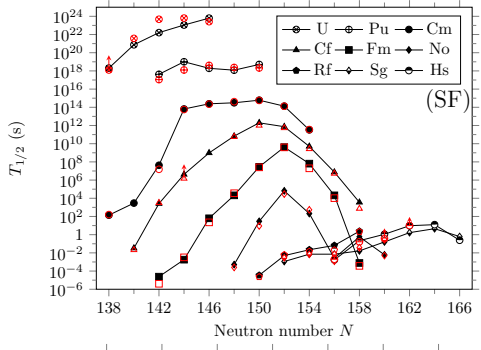

He C Ne Si Ar Ti Fe Zn Se Sr Mo Pd Sn
^{254}Rf
^{256}Rf
^{262}Rf
$U/B_{\eta_Z}^{-1}$ (10^7 MeV m_0 fm^2)
$1-\eta_Z$

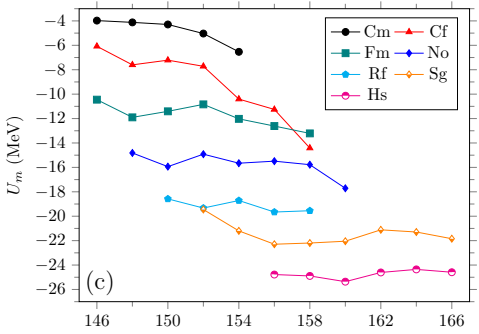

He C Ne Si Ar Ti Fe Zn Se Sr Mo Pd Sn
^{254}Rf
^{256}Rf
^{262}Rf
$|\Psi_0|^2$
$1-\eta_Z$

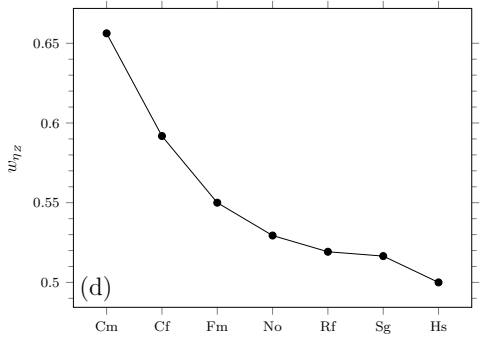
Driving potential characteristics

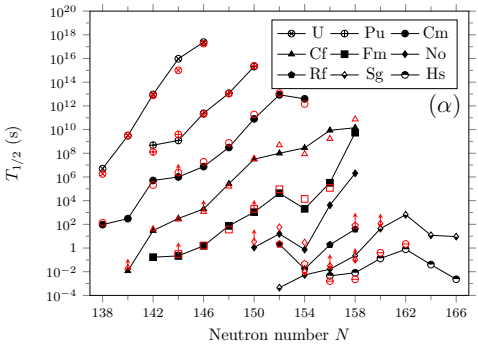


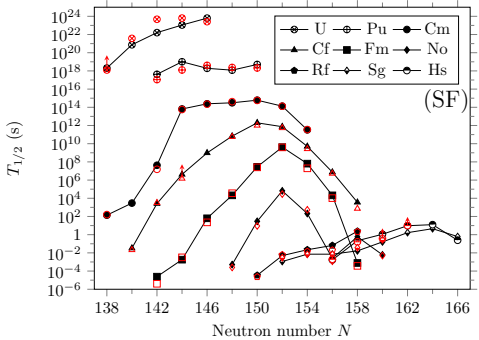

U
Pu
Cm
Cf
Fm
No
Rf
Sg
Hs
(α)
$T_{1/2}$ (s)
Neutron number N

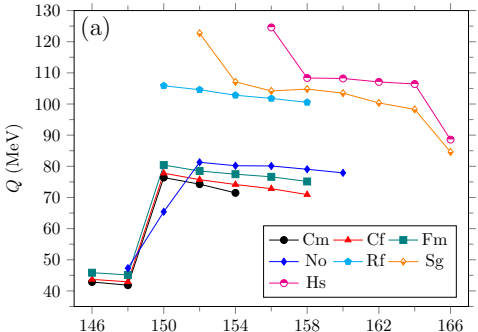

U
Pu
Cm
Cf
Fm
No
Rf
Sg
Hs
(SF)
$T_{1/2}$ (s)
Neutron number N

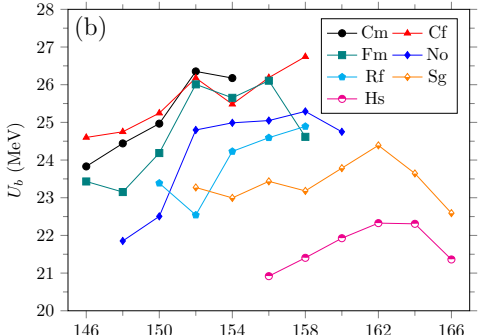

Cm
Cf
Fm
No
Rf
Sg
Hs
(c)
U_m (MeV)


(d)
$w_{\eta z}$
Cm
Cf
Fm
No
Rf
Sg
Hs

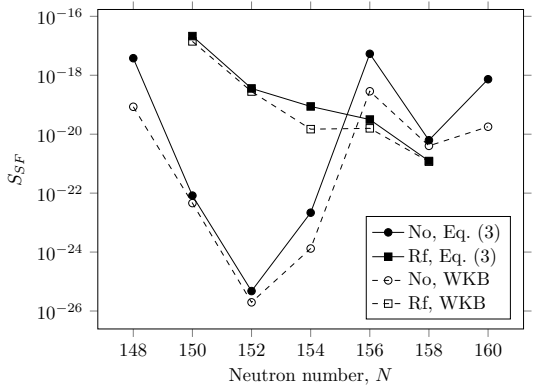

U
Pu
Cm
Cf
Fm
No
Rf
Sg
Hs
(α)
$T_{1/2}$ (s)
Neutron number N


U
Pu
Cm
Cf
Fm
No
Rf
Sg
Hs
(SF)
$T_{1/2}$ (s)
Neutron number N


(a)
Cm
Cf
Fm
No
Rf
Sg
Hs
Q (MeV)

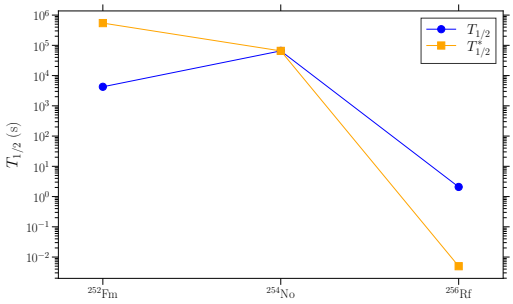

(b)
Cm
Cf
Fm
No
Rf
Sg
Hs
U_b (MeV)

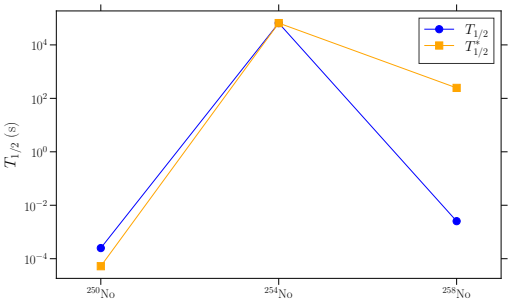
WKB-approximation

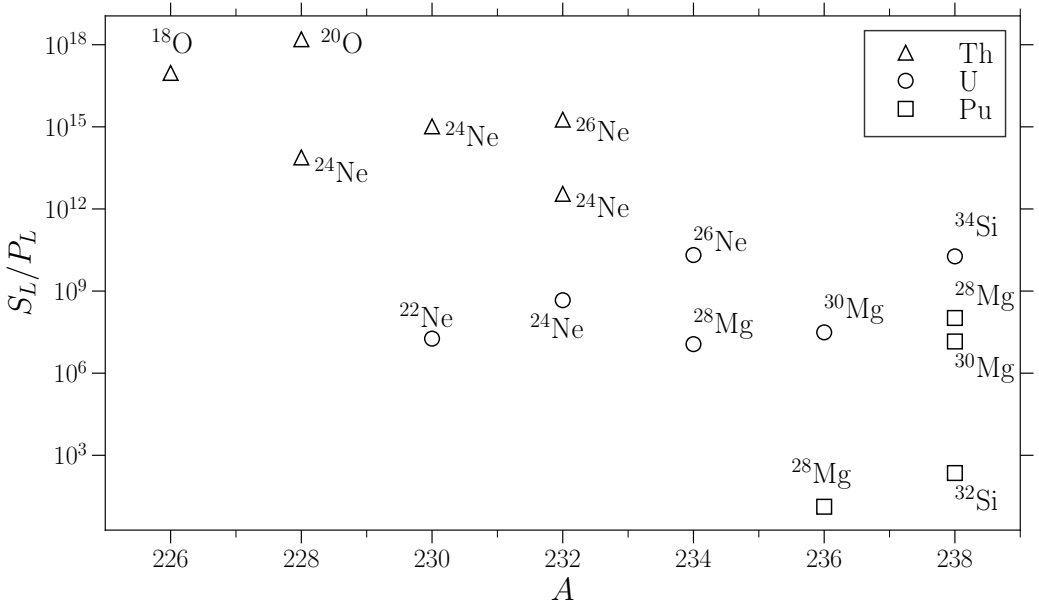


$$S_{SF} = \left(1 + \exp\left[\frac{2}{\hbar}\int_{\eta_{Z0}}^{\eta_Z} \sqrt{2U(R,\eta_Z,\Omega)/\left(B^{-1}\right)_{\eta_Z}} d\eta_Z\right]\right)^{-1}$$

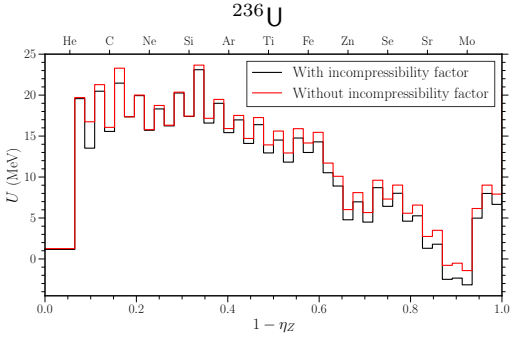
Inertia parameter influence

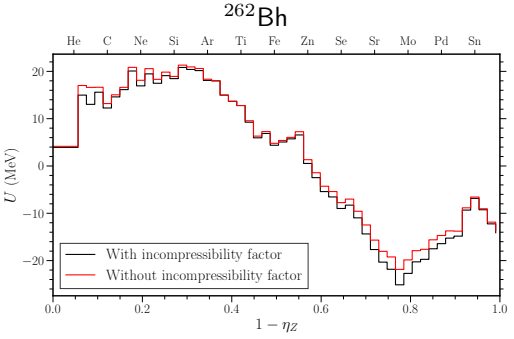





Th
U
Pu
^{18}O
^{20}O
^{24}Ne
^{24}Ne
^{26}Ne
^{24}Ne
^{26}Ne
^{34}Si
^{22}Ne
^{24}Ne
^{28}Mg
^{30}Mg
^{28}Mg
^{30}Mg
^{28}Mg
^{32}Si
10^{18}
10^{15}
10^{12}
10^{9}
10^{6}
10^{3}
S_L/P_L
226
228
230
232
234
236
238
A

Incompressibility effect

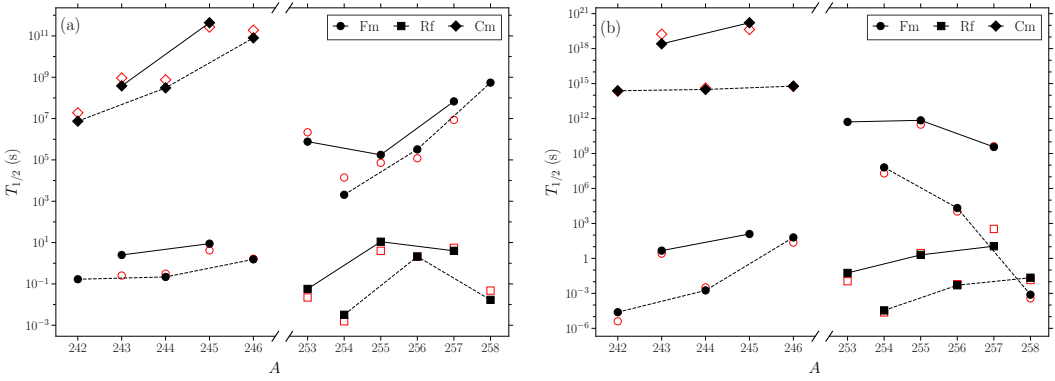




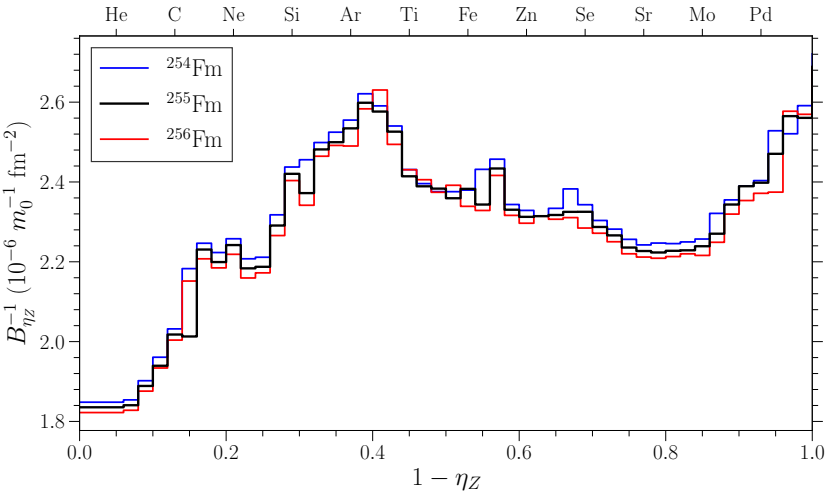
Even-odd nuclei half-lives

	Ω	T_α (s)	$T_\alpha^{\rm exp}$ (s)	$T_{\rm SF}$ (s)	$T_{\rm SF}^{\rm exp}$ (s)
^{235}U	$\frac{7}{2}$	6.49×10^{16}	2.23×10^{16}	1.65×10^{26}	3.19×10^{26}
^{239}Pu	$\frac{1}{2}$	2.73×10^{11}	7.61×10^{11}	5.39×10^{23}	2.54×10^{23}
^{241}Pu	$\frac{5}{2}$	2.60×10^{13}	4.52×10^{13}	3.04×10^{24}	1.89×10^{24}
^{265}Rf	$\frac{1}{2}$	8.91×10^{4}	–	96.8	60
^{267}Rf	$\frac{3}{2}$	2.79×10^{5}	–	7.12×10^{2}	2.9×10^{3}
^{271}Sg	$\frac{1}{2}$	1.4×10^{2}	2.88×10^{2}	4.24×10^{2}	2.88×10^{2}
^{275}Hs	$\frac{1}{2}$	1.35×10^{-1}	1.5×10^{-1}	1.22	> 4
^{277}Hs	$\frac{1}{2}$	98.7	–	1.57×10^{-3}	3×10^{-3}
^{279}Ds	$\frac{1}{2}$	2.96	1.8	1.1×10^{-1}	2×10^{-1}
^{281}Ds	$\frac{5}{2}$	3.97×10^{2}	73.8	5.62	9.6
^{283}Cn	$\frac{5}{2}$	9.07	4.44	19	40

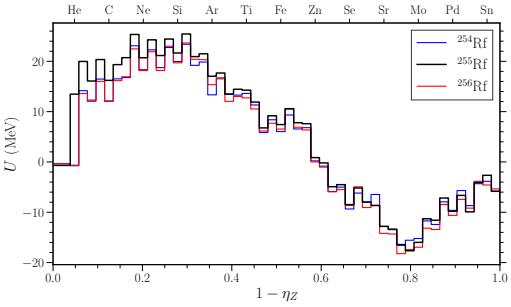
Hindrance factor

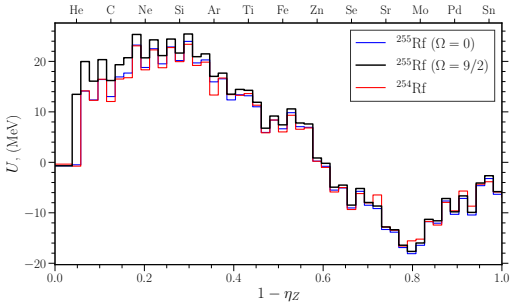


Inertia parameter influence



Driving potential influence





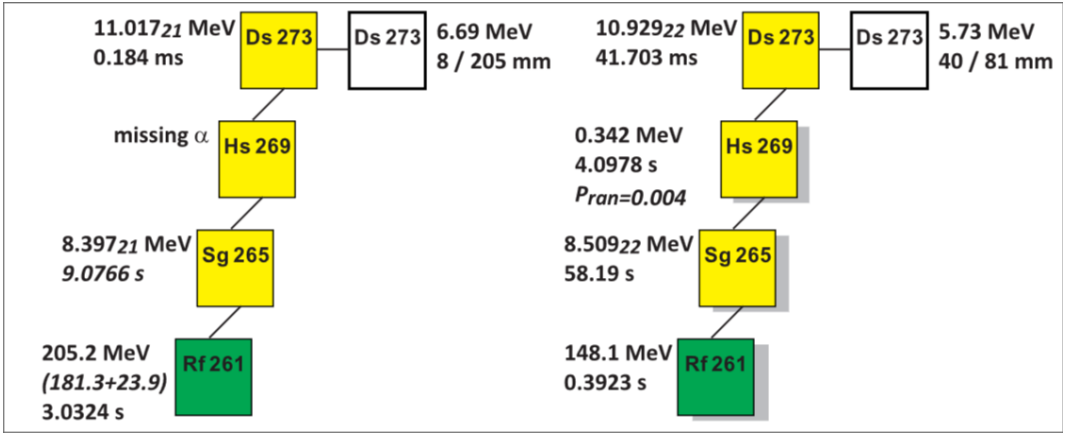
	Ω	S_α	S^*_α	$T_{1/2}$, s	$T^*_{1/2}$, s	$T^{\text{est}}_{1/2}$, s
^{243}Cm	5/2	0.0526	0.0707	2.57×10^{18}	1.02×10^{14}	2.75×10^{14}
^{245}Cm	7/2	0.0428	0.0947	1.65×10^{20}	7.34×10^{14}	4.35×10^{14}
^{243}Fm	7/2	0.0712	0.0904	3.51	3.14×10^{-4}	2.08×10^{-4}
^{255}Fm	7/2	0.0527	0.0816	3.01×10^{11}	1.62×10^{6}	1.16×10^{6}
^{257}Fm	9/2	0.0481	0.0888	4.13×10^{9}	1.19	4.02
^{255}Rf	9/2	0.0691	0.0930	2.00	2.95×10^{-3}	4.14×10^{-4}
^{257}Rf	1/2	0.0893	0.0918	1.11×10^{1}	3.99×10^{-2}	1.05×10^{-2}

Approximate half-lives estimation

$$T_{1/2} = F \cdot T_{1/2}(\Omega = 0),\ F = \exp\left[\frac{c_0 \cdot \Omega(\Omega+1)}{\sqrt{B_\eta^{-1} Z_\alpha}}\right],\ c = 0.086\ \text{MeV}^{-1/2}\ \text{s}^{-1}$$

Ядро	Ω	$T_{1/2}(\Omega = 0)$ (s)	F	$T_{1/2}^{\text{fit}}$ (s)	$T_{1/2}^{exp}$ (s)
^{243}Cm	5/2	1.02×10^{14}	3.47×10^{3}	3.54×10^{17}	2.57×10^{18}
^{243}Fm	7/2	3.14×10^{-4}	1.20×10^{4}	3.77	4.64
^{245}Cm	7/2	7.34×10^{14}	2.63×10^{6}	1.93×10^{21}	1.65×10^{20}
^{253}Rf	1/2	2.21×10^{-5}	1.54	3.40×10^{-5}	5.64×10^{-2}
^{255}Fm	7/2	1.62×10^{6}	7.72×10^{5}	1.25×10^{12}	7.04×10^{11}
^{255}Rf	9/2	2.95×10^{-3}	3.17×10^{4}	9.38×10^{1}	2.00
^{257}Fm	9/2	1.19	3.33×10^{8}	3.96×10^{8}	3.64×10^{9}
^{257}Rf	1/2	2.02×10^{-2}	1.59	3.21×10^{-2}	1.15×10^{1}

K-isomeric states



[Phys. Rev. C, 109, 054307 (2024)]

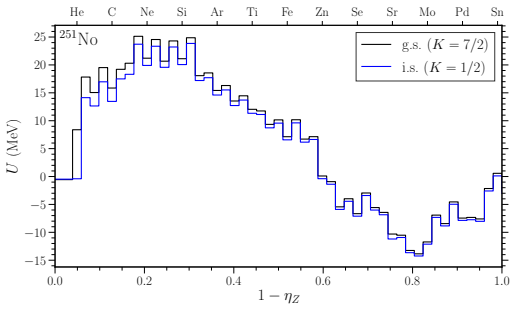
K-isomeric states

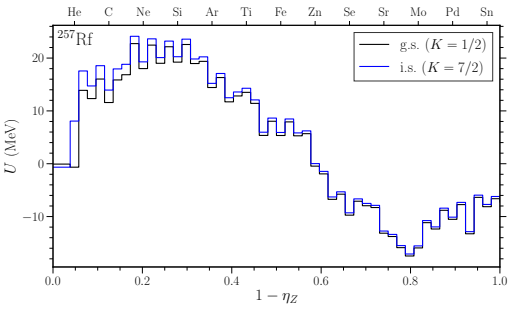
Nucleus	K^π	E_{is} (MeV)	S_α	$T^\alpha_{1/2}$(th.) (s)	$T^\alpha_{1/2}$(exp.) (s)	$T^{\rm sf}_{1/2}$(th.) (s)	$T^{\rm sf}_{1/2}$(exp.) (s)
^{243}Cm	$\frac{5}{2}^+$	0	5.26×10^{-2}	3.81×10^{8}	9.21×10^{8}	2.57×10^{18}	1.73×10^{19}
^{243}Cm	$\frac{1}{2}^+$	0.087	7.42×10^{-2}	1.30×10^{6}		2.27×10^{12}	
^{249}No	$\left(\frac{7}{2}^+\right)$	0	8.50×10^{-2}	1.94×10^{-3}		5.42×10^{2}	> 19
^{249}No	$\left(\frac{1}{2}^-\right)$	0.100	8.60×10^{-2}	9.84×10^{-4}		45.1	
^{251}No	$\left(\frac{7}{2}^+\right)$	0	7.14×10^{-2}	7.59		3.46×10^{2}	
^{251}No	$\left(\frac{1}{2}^+\right)$	0.106	8.81×10^{-2}	1.31×10^{-1}		6.60	1.7
^{253}Rf	$\left(\frac{7}{2}^+\right)$	0	7.77×10^{-2}	5.70×10^{-2}	2.20×10^{-2}	5.64×10^{-2}	1.46×10^{-2}
^{253}Rf	$\left(\frac{1}{2}^+\right)$	0.200	4.31×10^{-2}	4.34×10^{-3}	6.00×10^{-3}	8.32×10^{-5}	4.4×10^{-5}
^{255}Rf	$\left(\frac{7}{2}^+\right)$	0	6.91×10^{-2}	11	4	2	2.9
^{255}Rf	$\left(\frac{1}{2}^+\right)$	0.100	4.07×10^{-2}	2.94×10^{-2}		2.44×10^{-2}	$> 3 \times 10^{-5}$

Nucleus	K^π	E_{is} (MeV)	S_α	$T^{\alpha}_{1/2}$(th.) (s)	$T^{\alpha}_{1/2}$(exp.) (s)	$T^{\rm sf}_{1/2}$(th.) (s)	$T^{\rm sf}_{1/2}$(exp.) (s)
^{257}Rf	$\left(\frac{1}{2}^+\right)$	0	8.93×10^{-2}	4.00	5.55	11.1	338
^{257}Rf	$\left(\frac{7}{2}^+\right)$	0.073	1.59×10^{-2}	21.90		1.07×10^{3}	> 490
^{261}Rf	$\left(\frac{3}{2}^+\right)$	0	9.17×10^{-2}	9.84×10^{-2}		3.22×10^{-2}	
^{261}Rf	$\left(\frac{11}{2}^+\right)$	0.070	8.91×10^{-2}	3.44		6.10	3.17
^{259}Sg	$\left(\frac{1}{2}^+\right)$	0	9.26×10^{-2}	2.25×10^{-4}		8.45×10^{-3}	$> 1.4 \times 10^{-3}$
^{259}Sg	$\left(\frac{9}{2}^+\right)$	0.100	6.83×10^{-2}	2.28×10^{-1}		9.72	8
^{265}Sg	$\left(\frac{9}{2}^+\right)$	0	8.71×10^{-2}	1.39		7.72×10^{2}	≥ 17
^{265}Sg	$\left(\frac{3}{2}^+\right)$	0.070	8.33×10^{-2}	4.85		42	≥ 28

$$K \to V_r,\ E_{is} \to Q^{is}_M = Q_M + E_{is}$$

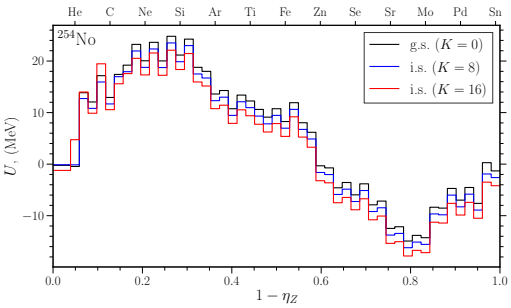
Driving potentials

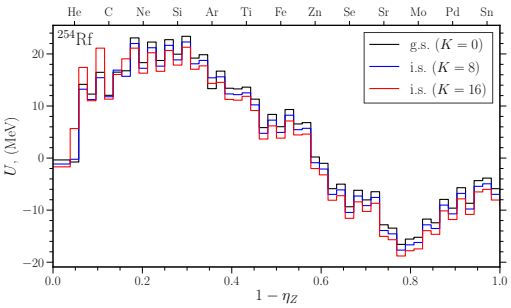




Nucleus	K^π	E_{is} (MeV)	S_α	$T^\alpha_{1/2}$(th.) (s)	$T^\alpha_{1/2}$(exp.) (s)	$T^{\rm sf}_{1/2}$(th.) (s)	$T^{\rm sf}_{1/2}$(exp.) (s)
^{250}Fm	0^+	0	5.99×10^{-2}	1.06×10^{3}	2.00×10^{3}	2.86×10^{7}	2.52×10^{7}
^{250}Fm	(8^-)	1.199	3.89×10^{-2}	3.39×10^{5}		2.63×10^{4}	
^{256}Fm	0^+	0	6.56×10^{-2}	3.23×10^{5}	1.20×10^{5}	2.10×10^{4}	1.04×10^{4}
^{256}Fm	7^-	1.425	3.95×10^{-3}	6.23×10^{6}		3.6×10^{-1}	
^{250}No	0^+	0	7.60×10^{-2}	1.85×10^{-3}	$> 2.1 \times 10^{-4}$	7.02×10^{-5}	4.7×10^{-6}
^{250}No	(6^+)	1.050	6.14×10^{-3}	1.20×10^{1}		2.20×10^{-5}	$4.50^{+2.2}_{-1.5} \times 10^{-5}$
^{252}No	0^+	0	7.07×10^{-2}	15.8	56.7	29.4	9
^{252}No	(8^-)	1.255	5.30×10^{-2}	2.18×10^{3}		2.05×10^{-1}	
^{254}No	0^+	0	6.50×10^{-2}	7.39×10^{-1}	2.93	6.55×10^{4}	2.88×10^{4}
^{254}No	(8^-)	1.297	3.10×10^{-3}	7.84×10^{3}	2.80×10^{3}	1.41×10^{3}	1.40×10^{3}
^{254}No	(16^+)	2.917	1.99×10^{-2}	1.47×10^{6}		8.36×10^{3}	≥ 1.65
^{254}Rf	0^+	0	8.07×10^{-2}	3.22×10^{-2}	$> 1.55 \times 10^{-3}$	3.45×10^{-5}	2.30×10^{-5}
^{254}Rf	(8^-)	1.100	5.77×10^{-2}	2.19×10^{2}		1.14×10^{-4}	$> 4.70 \times 10^{-5}$
^{254}Rf	(16^+)	2.250	2.63×10^{-2}	5.38×10^{4}		1.36×10^{-1}	$> 6.02 \times 10^{-4}$
^{256}Rf	0^+	0	7.76×10^{-2}	2.11	2.08	4.98×10^{-3}	6.20×10^{-3}
^{256}Rf	(8^-)	1.400	6.01×10^{-2}	1.27×10^{2}		5.37×10^{-4}	$1.4^{+0.6}_{-0.4} \times 10^{-5}$

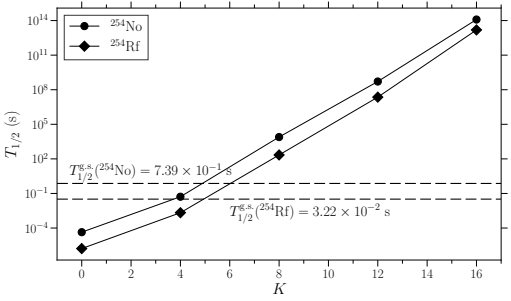
Driving potentials



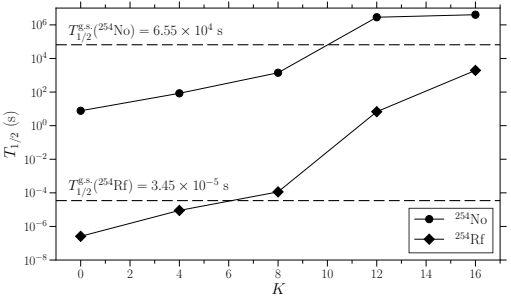


K effect

$^{254}\mathrm{No} \rightarrow E_{is} = 1.297$ MeV

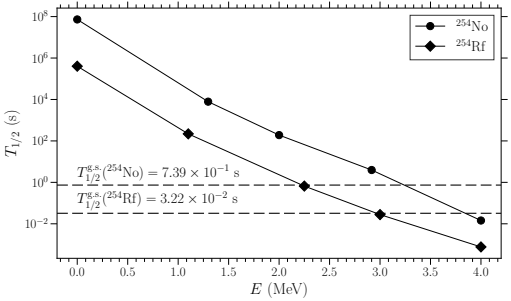


$^{254}\mathrm{Rf} \rightarrow E_{is} = 1.1$ MeV

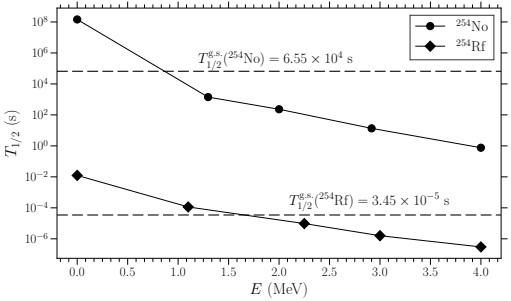


E_{is} effect

^{254}No $\rightarrow K = 8$

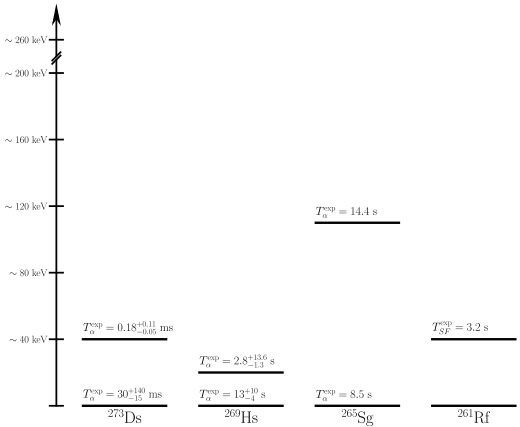


^{254}Rf $\rightarrow K = 8$



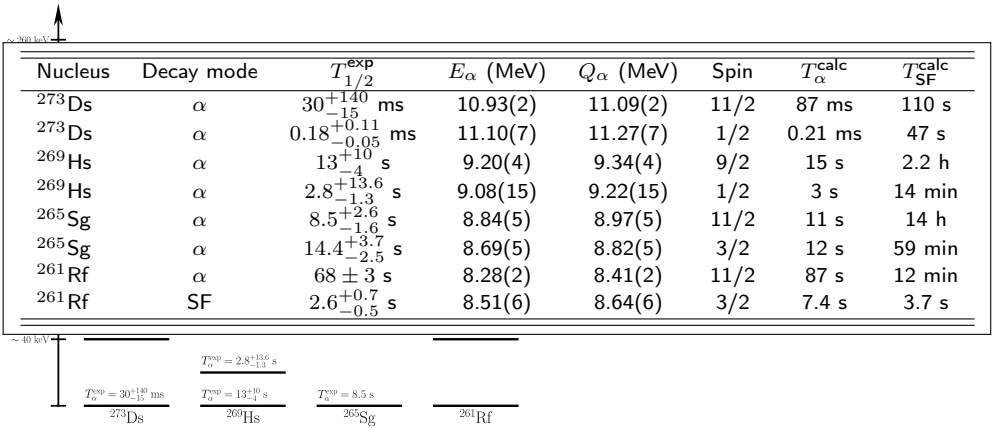
Decay chains ^{273,275}Ds

[Yu. Ts. Oganessian, V. K. Utyonkov *et al.* Phys. Rev C 109, 054307 (2024)]



Decay chains ^{273,275}Ds

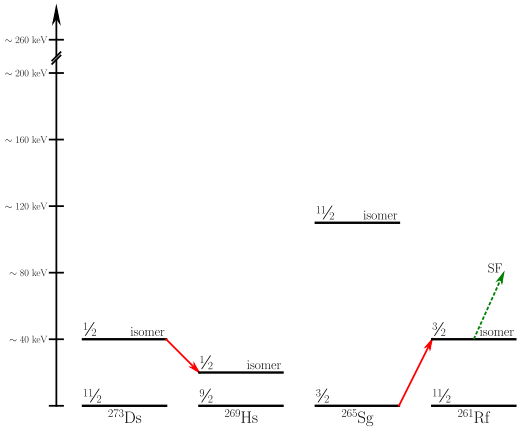
[Yu. Ts. Oganessian, V. K. Utyonkov *et al.* Phys. Rev C 109, 054307 (2024)]



Nucleus	Decay mode	$T_{1/2}^{\rm exp}$	E_α (MeV)	Q_α (MeV)	Spin	$T_\alpha^{\rm calc}$	$T_{\rm SF}^{\rm calc}$
^{273}Ds	α	30^{+140}_{-15} ms	10.93(2)	11.09(2)	11/2	87 ms	110 s
^{273}Ds	α	$0.18^{+0.11}_{-0.05}$ ms	11.10(7)	11.27(7)	1/2	0.21 ms	47 s
^{269}Hs	α	13^{+10}_{-4} s	9.20(4)	9.34(4)	9/2	15 s	2.2 h
^{269}Hs	α	$2.8^{+13.6}_{-1.3}$ s	9.08(15)	9.22(15)	1/2	3 s	14 min
^{265}Sg	α	$8.5^{+2.6}_{-1.6}$ s	8.84(5)	8.97(5)	11/2	11 s	14 h
^{265}Sg	α	$14.4^{+3.7}_{-2.5}$ s	8.69(5)	8.82(5)	3/2	12 s	59 min
^{261}Rf	α	68 ± 3 s	8.28(2)	8.41(2)	11/2	87 s	12 min
^{261}Rf	SF	$2.6^{+0.7}_{-0.5}$ s	8.51(6)	8.64(6)	3/2	7.4 s	3.7 s

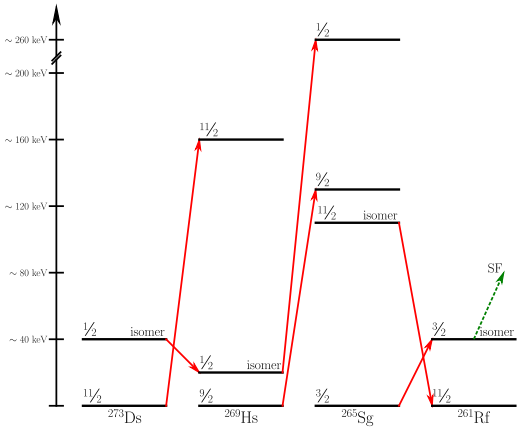
Decay chains ^{273,275}Ds

[Yu. Ts. Oganessian, V. K. Utyonkov *et al.* Phys. Rev C 109, 054307 (2024)]



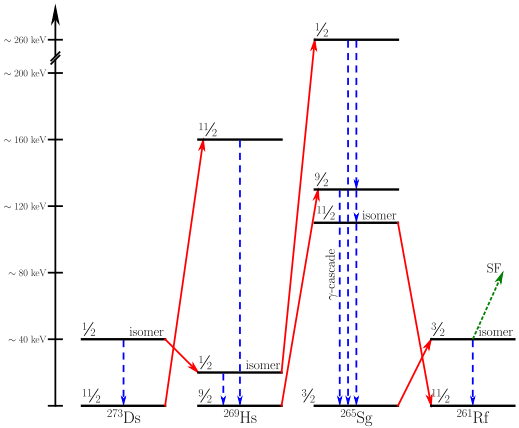
Decay chains ^{273,275}Ds

[Yu. Ts. Oganessian, V. K. Utyonkov *et al.* Phys. Rev C 109, 054307 (2024)]



Decay chains ^{273,275}Ds

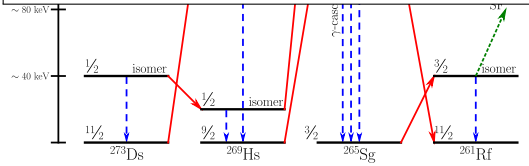
[Yu. Ts. Oganessian, V. K. Utyonkov *et al.* Phys. Rev C 109, 054307 (2024)]



Decay chains ^{273,275}Ds

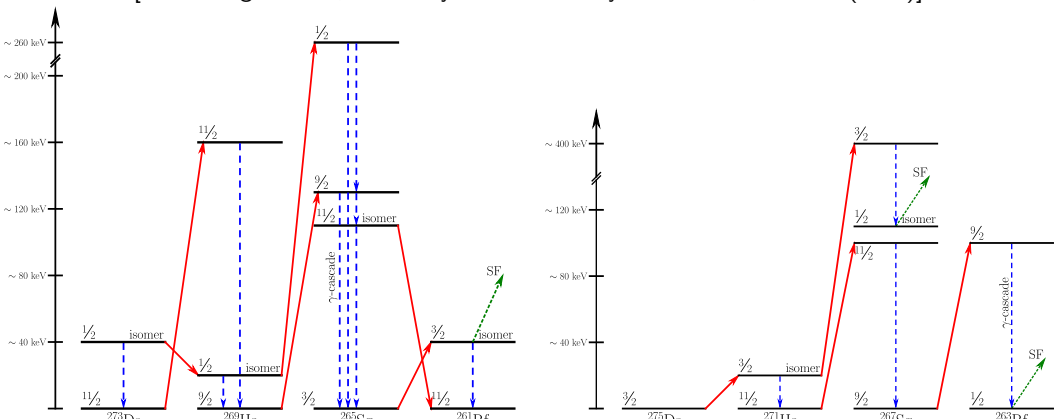
[Yu. Ts. Oganessian, V. K. Utyonkov *et al.* Phys. Rev C 109, 054307 (2024)]

Nucleus	Decay mode	$T_{1/2}^{\text{exp}}$	E_α (MeV)	Q_α (MeV)	Spin	T_α^{calc}	$T_{\text{SF}}^{\text{calc}}$
^{275}Ds	α	$0.43^{+0.29}_{-0.12}$ ms	11.20(2)	11.37(2)	3/2	0.22 ms	2.0 s
^{271}Hs	α	$7.1^{+8.4}_{-2.5}$ s	9.05(2)	9.18(2)	3/2	5.1 s	6.0 min
^{271}Hs	α	46^{+56}_{-16} s	9.34(2)	9.48(2)	11/2	63 s	21 h
^{267}Sg	SF	100^{+92}_{-39} s			1/2	16 h	140 s
^{267}Sg	α	$9.8^{+11.3}_{-4.5}$ min	8.27(2)	8.40(2)	9/2	6 min	2.9 h
^{263}Rf	SF	$5.1^{+4.6}_{-1.7}$ s			1/2	0.5 h	6.4 s

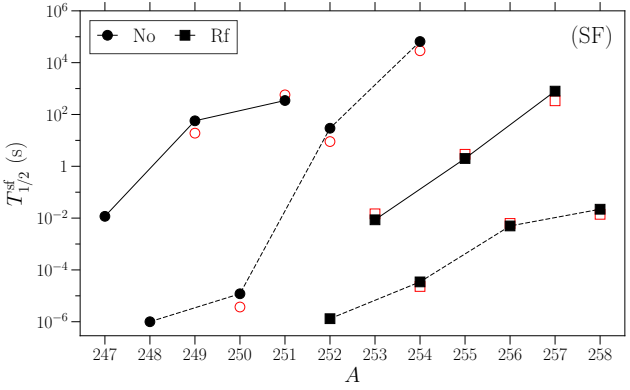


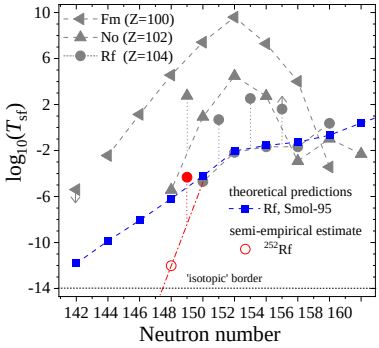
Decay chains ^{273,275}Ds

[Yu. Ts. Oganessian, V. K. Utyonkov *et al.* Phys. Rev C 109, 054307 (2024)]



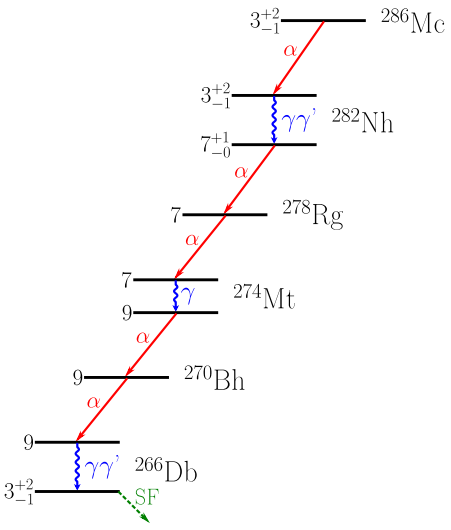
Proton-rich nuclei



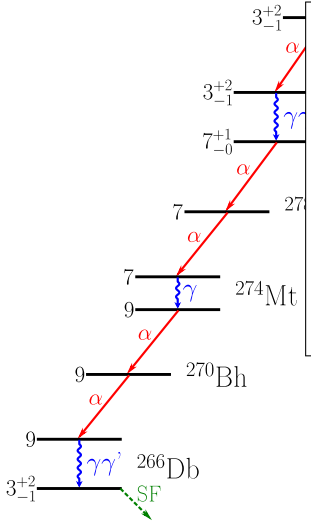


[Khuyagbaatar et al., Phys. Rev. C 104, L031303 (2021)]

^{286}Mc decay chain

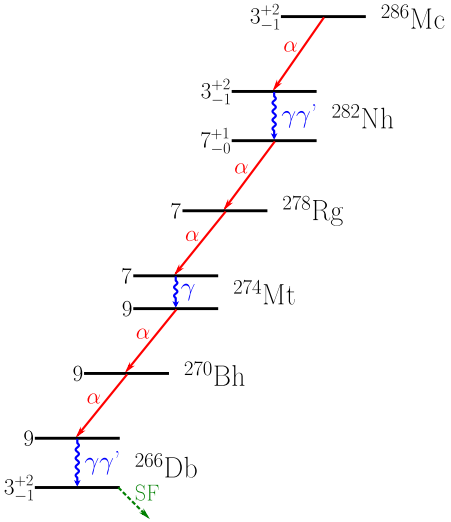


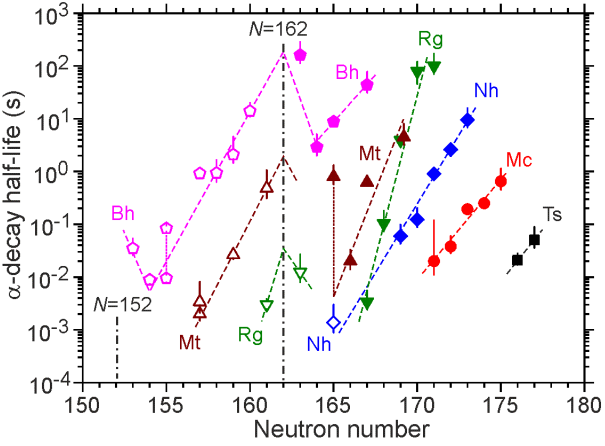
^{286}Mc decay chain



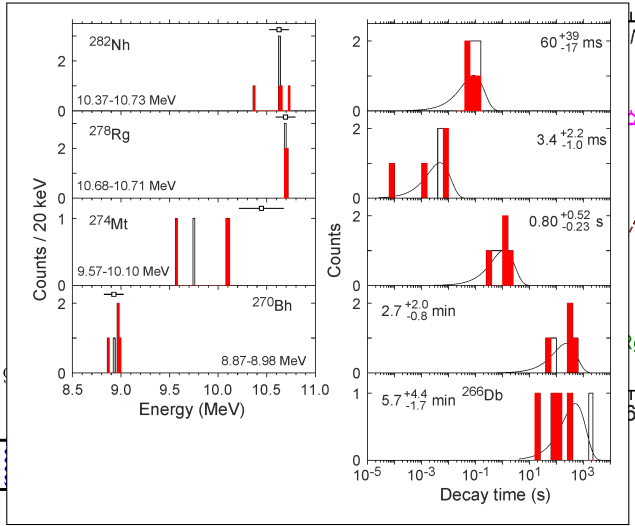
Nucleus	Q_M (MeV)	$T_\alpha^{\rm exp}$ (s)	K	$T_\alpha^{\rm th}$ (s)	$T_{\rm SF}^{\rm th}$ (s)
^{286}Mc	174.59	$2^{+9.8}_{-0.9} \times 10^{-2}$	5	1.12×10^{-1}	2.90×10^{5}
			3	2.46×10^{-2}	1.98×10^{5}
			2	1.04×10^{-2}	1.60×10^{5}
^{282}Nh	161.31	$6^{+3.9}_{-1.7} \times 10^{-2}$	8	1.81×10^{-1}	5.28×10^{1}
			7	5.30×10^{-2}	2.02×10^{1}
^{278}Rg	148.13	$3.4^{+2.2}_{-1} \times 10^{-3}$	7	3.67×10^{-3}	4.63×10^{2}
^{274}Mt	134.83	$0.8^{+0.52}_{-0.23}$	9	0.896	6.78×10^{5}
^{270}Bh	122.35	$2.62^{+1.2}_{-0.48} \times 10^{2}$	9	2.41×10^{2}	2.22×10^{6}
^{266}Db			5	2.99×10^{5}	5.72×10^{2}
			3	4.47×10^{4}	1.04×10^{2}

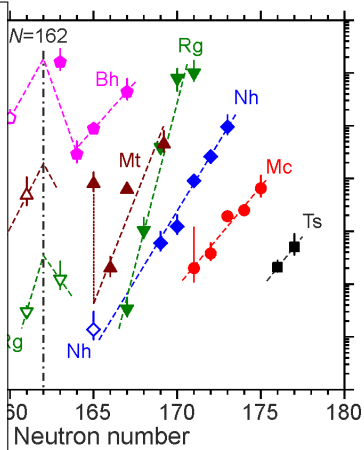
^{286}Mc decay chain



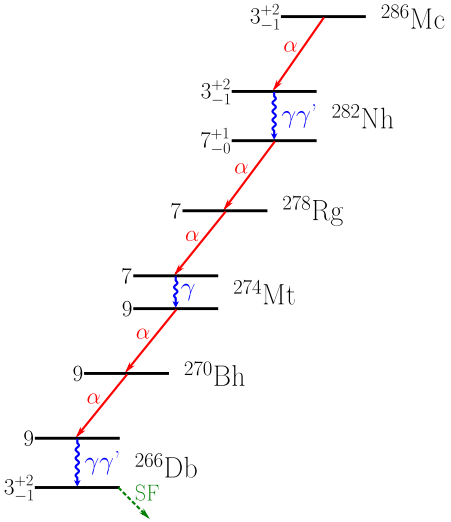


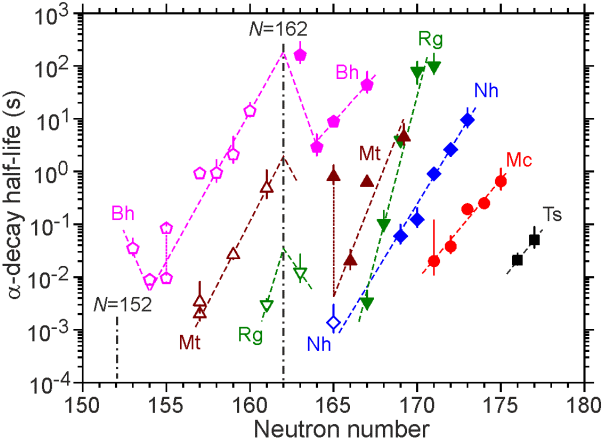
^{286}Mc decay chain



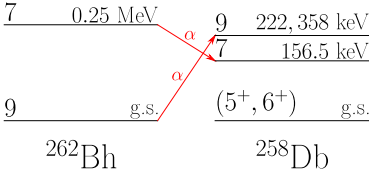


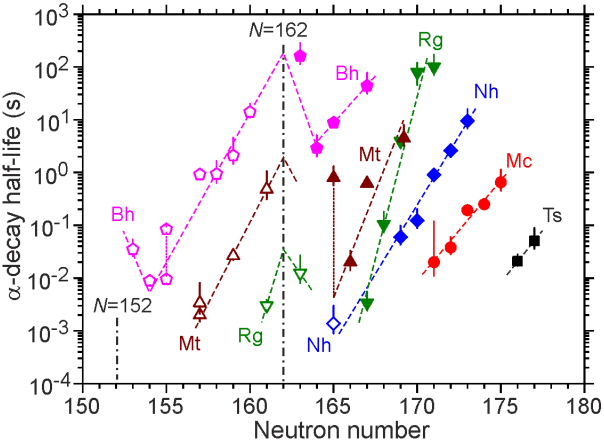
^{286}Mc decay chain



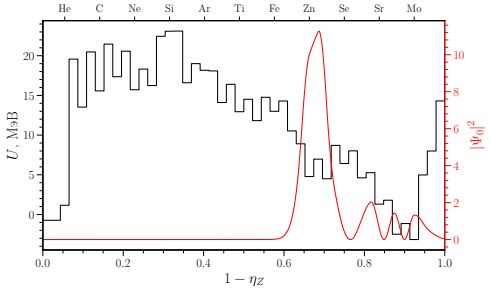


^{286}Mc decay chain

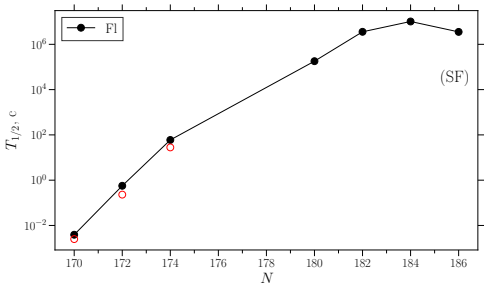




Perspectives



shape isomers, $E_n \to B_M$



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Thank you for your attention