Probing Gluon Sivers Function through photon + jet/hadron production in pp collision

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Outline

- Transverse Momentum Dependent (TMD) factorization
- Sivers function and Transverse Single Spin Asymmetries (TSSAs)
- $p^{\uparrow} p \to \gamma + X$
- $p^{\uparrow} p \to \gamma + \text{jet/hadron} + X$

Transverse Momentum Dependent (TMD) factorization

- Single-scale process ($Q^2 >> \Lambda_{\rm QCD}$)
  - Inclusive DIS:
    $$d\sigma^{lp \to l'X} \approx \sum_q \int dx\ f(x, Q^2) d\hat{\sigma}^{lq \to lq}$$
- Two-scale processes ($Q^2 >> q_T \sim \Lambda_{\rm QCD}$)
  - Semi-Inclusive DIS:
    $$d\sigma^{lp \to l'hX} \approx \sum_q \int dx\ dz\ d^2\mathbf{k}_\perp f(x, k_\perp) d\hat{\sigma}^{lq \to lq} D_q^h(z, p_\perp)$$
  - Drell-Yan:
    $$d\sigma^{pp \to l^+l^-X} \approx \sum_{ab} \int dx_a\ dx_b\ d^2\mathbf{k}_{\perp a}\mathbf{k}_{\perp b} f(x_a, k_{\perp a}) f(x_b, k_{\perp b}) d\hat{\sigma}^{ab \to l^+l^-}$$
- $f(x, k_\perp)$, $\quad D_q^h(z, p_\perp)$ are TMD PDFs and FFs
- TMDs encodes 3D structure of the proton

TMD correlator

- Correlator: $\Phi_{ij}(k;P,S) := \int \frac{d^4\xi}{(2\pi)^4}\, e^{ik\cdot\xi}\, \langle P,S|\bar{\psi}_j(0)\psi_i(\xi)|P,S\rangle$.
- Integrate over k^-: TMD correlator

$$\Phi(x,\mathbf{k}_\perp;P,S) = \int dk^-\, \Phi(k;P,S) = \int \frac{d\xi^- d^2\boldsymbol{\xi}_\perp}{(2\pi)^3} e^{ik\cdot\xi} \langle P,S|\bar{\psi}(0)\psi(\xi)|P,S\rangle\Big|_{\xi^+=0} .$$

- Further integrate over $\mathbf{k}_\perp$: Correlator for collinear PDFs:

$$\Phi(x;P,S) = \int dk^- d^2\mathbf{k}_\perp\, \Phi(k;P,S) = \int \frac{d\xi^-}{2\pi} e^{ik\cdot\xi} \langle P,S|\bar{\psi}(0)\psi(\xi)|P,S\rangle\Big|_{\xi^+=\boldsymbol{\xi}_\perp=0} .$$

- Wilson line $\mathcal{U}_{(\xi_1,\xi_2)} := \mathcal{P}\exp\left[-ig\int_{\xi_1}^{\xi_2} d\eta^\mu A_\mu(\eta)\right]$
  $\Longrightarrow$ TMDs are not universal (i.e. process-dependent)!

Leading-twist expansion of TMD correlator

- $n_+ = (1, 0, \mathbf{0}_\perp)$ $\quad, \epsilon_\perp^{12} = -\epsilon_\perp^{21} = 1\,, \quad \epsilon_\perp^{11} = \epsilon_\perp^{22} = 0$
- $\Phi(k; P, S) = \int \frac{d^4\xi}{(2\pi)^4}\, e^{ik\cdot\xi}\, \langle P, S|\bar{\psi}(0)\psi(\xi)|P, S\rangle$
- The correlator matrix satisfies the following relations arising from requirements of
  - $\Phi_{ij}^\dagger(p, P, S) = \gamma^0 \Phi_{ij}(p, P, S)\gamma^0$ (hermiticity),
  - $\Phi_{ij}(p, P, S) = \gamma^0 \Phi_{ij}(\tilde{p}, \tilde{P}, -\tilde{S})\gamma^0$ (parity invariance),
  - $\Phi_{ij}^\star(p, P, S) = \gamma^5 C \Phi_{ij}(\tilde{p}, \tilde{P}, \tilde{S}) C^\dagger \gamma^5$ (time-reversal invariance)

  where $C = i\gamma^2\gamma^0$ and $\tilde{p} = (p^0, -\mathbf{p})$
- Expanding at Leading-twist approximation,

$$
\begin{aligned}
\Phi \approx \frac{1}{2}\Bigg[& f_1^q \not{n}_+ - \frac{\epsilon_\perp^{ij} k_\perp^i S_\perp^j}{M} f_{1T}^{\perp q} \not{n}_+ \\
& + S_z g_{1L}^q \gamma^5 \not{n}_+ + \frac{\mathbf{k}_\perp \cdot \mathbf{S}_\perp}{M} g_{1T}^q \gamma^5 \not{n}_+ \\
& + \frac{[\not{S}_\perp, \not{n}_+]}{2} \gamma^5 h_{1T}^q + S_z \frac{[\not{k}_\perp, \not{n}_+]}{2M} \gamma^5 h_{1L}^{\perp q} + \frac{\mathbf{k}_\perp \cdot \mathbf{S}_\perp [\not{k}_\perp, \not{n}_+]}{2M^2} \gamma^5 h_{1T}^{\perp q} + \frac{i[\not{k}_\perp, \not{n}_+]}{2M} h_1^{\perp q} \Bigg]
\end{aligned}
$$

- Quark TMDs

Table: The eight independent leading twist quark TMDs. Columns correspond to U(unpolarized), L(longitudinally), T(transversely) polarized quarks and rows correspond to nucleons polarizations. TMDs showed in blue f_1, g_{1L} and h_1 survive upon integration over $k_\perp$ i.e they have a collinear PDF counterpart. TMDs shown in red are T-odd.

Nucl. \ Quark	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	h_1, $h_{1T}^\perp$

- Gluon TMDs

Table: The eight independent leading twist gluon TMDs. Columns correspond to U(unpolarized), Linearly, and transvers polarized gluons and rows correspond to U (unpolarized), L (longitudinally)and T (transversely) polarized nucleons respectiv TMDs shown in blue f_1^g and g_{1L}^g survive upon integration over $k_\perp$ i.e they have a collinear PDF counterpart. TMDs shown red are T-odd.

Nucl. \ Gluon	Unpolarized	Circular	Linear
U	f_1^g		$h_1^{\perp g}$
L		g_{1L}^g	$h_{1L}^{\perp g}$
T	$f_{1T}^{\perp g}$	g_{1T}^g	h_1^g, $h_{1T}^{\perp g}$

Sivers function and Transverse single spin asymmetries (TSSAs)

- $f_{q/p^\uparrow} = f_{q/p}(x, k_\perp) - \frac{\epsilon_T^{\alpha\beta} k_{\perp\alpha} S_{\perp\beta}}{M} f_{1T}^\perp(x, k_\perp)$
  $= f_{q/p}(x, k_\perp) - \frac{(\hat{\mathbf{P}} \times \mathbf{k}_\perp).\mathbf{S}}{M} f_{1T}^\perp(x, k_\perp)$
- $-2\frac{k_\perp}{M} f_{1T}^\perp(x, k_\perp) = f_{q/p^\uparrow} - f_{q/p^\downarrow}$
- Another notation $\Delta^N f(x, k_\perp) = -2\frac{p_T}{M} f_{1T}^\perp(x, k_\perp)$
- Transverse single spin asymmetry (TSSA) is defined as the ratio of the difference and the sum of the cross sections when the hadron's spin vector $S_\perp$ is flipped.

$$A_N = \frac{d\sigma^\uparrow - d\sigma^\downarrow}{d\sigma^\uparrow + d\sigma^\downarrow} = \frac{d\Delta\sigma}{2d\sigma}$$

D. W. Sivers, Phys. Rev. D41, 83 (1990); 261 (1991)

Status of QSF and GSF

- QSF - well-studied
  - SIDIS ($ep \to e'hX$) data available from COMPASS (CERN), HERMES (DESY-HERA), Hall A (JLab) collabs - Two-scale process - TMD factorization applicable
- GSF - not so much
  - $pp \to \pi^0 X$ - GSF contribution dominates at LO - Data from STAR, PHENIX and BRAHMS collabs (RHIC) - Single-scale process
  - However, data points are less. Also, TMD not applicable - Indirect fits in GPM by
    U. D'Alesio, F. Murgia, C. Pisano, JHEP, 09 (2015)
- Upcoming experiments
  - Spin Physics Detector (SPD) experiment at NICA-JINR, Dubna, Russia
  - Electron Ion Collider (EIC) at Brookhaven Lab

Parametrization of TMDs

For the unpolarized TMDs, the commonly used form of collinear PDF multiplied by a Gaussian transverse momentum dependence is adopted

$$f_{i/p}(x, k_\perp; Q) = f_{i/p}(x, Q) \frac{1}{\pi \langle k_\perp^2 \rangle} e^{-k_\perp^2/\langle k_\perp^2 \rangle}$$

with $\langle k_\perp^2 \rangle = 0.25$ GeV^2.
Sivers function parametrization:

$$\Delta^N f_{i/p^\uparrow}(x_a, \mathbf{k}_{\perp a}; Q) = 2\mathcal{N}_i(x) f_{i/p}(x, Q) \frac{\sqrt{2e}}{\pi} \sqrt{\frac{1-\rho}{\rho}} k_\perp \frac{e^{-k_\perp^2/\rho\langle k_\perp^2 \rangle}}{\langle k_\perp^2 \rangle^{3/2}}$$

D'Alesio , Murgia and Pisano, J. High Energ. Phys. (2015) 2015: 119.
Sivers function satisfies the positivity bound

$$\frac{|\Delta^N f_{i/p^\uparrow}(x, k_\perp)|}{2 f_{i/p}(x, k_\perp)} \leq 1 \, \forall \, x, k_\perp$$

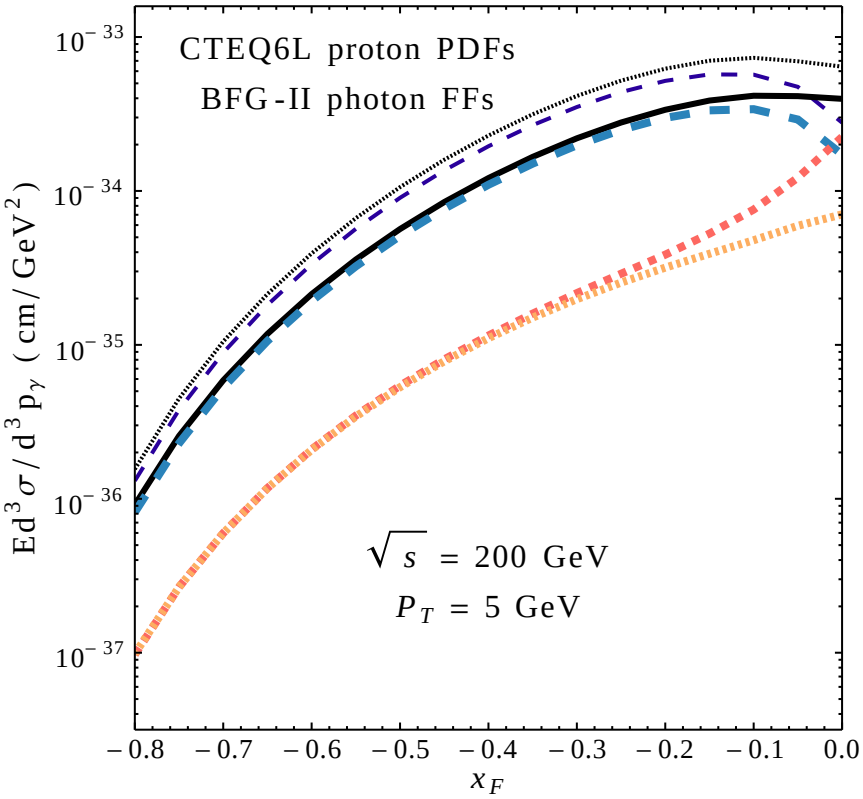
Quark and gluon Sivers functions considered with the positivity bound saturated, viz. $\mathcal{N}_i(x) = 1$.

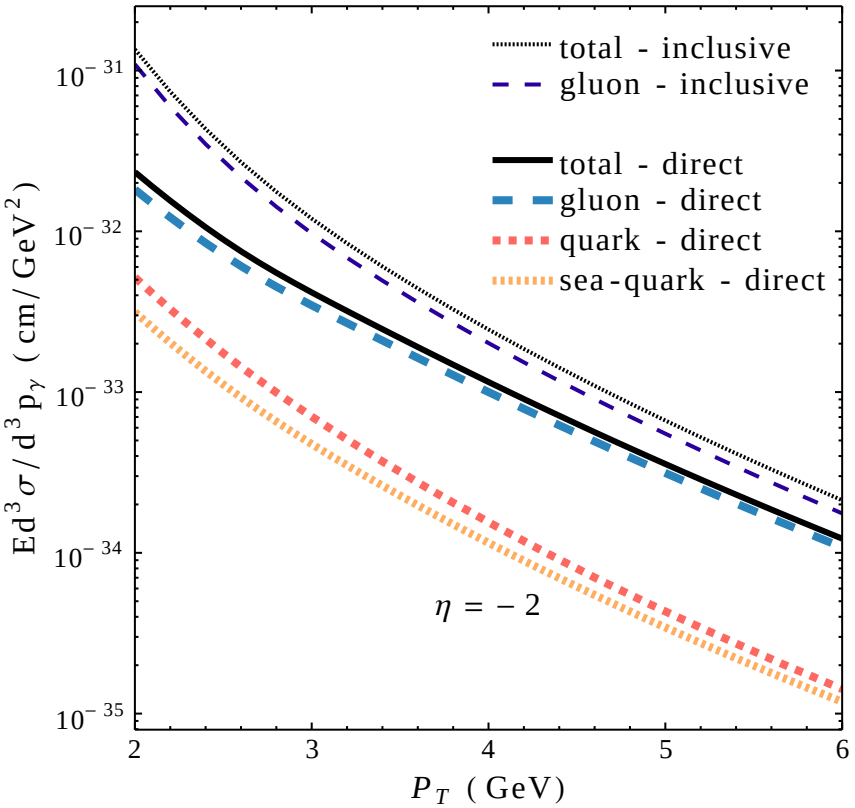
Using saturated Sivers functions for quarks and gluons facilitates the following studies –

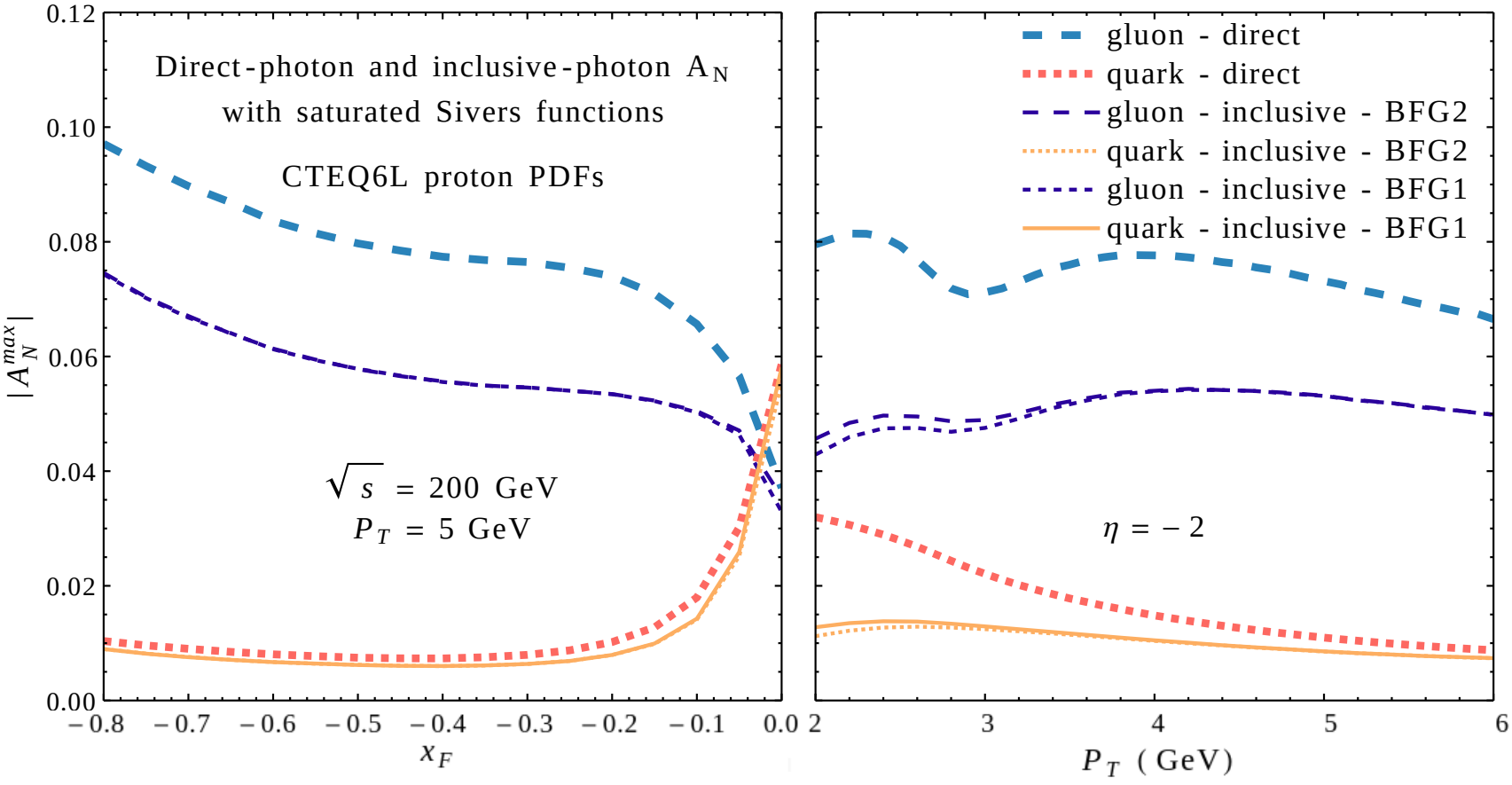
- The general *kinematic dependencies of the asymmetry* and the *relative importances of quark and gluon contributions* to the asymmetry.
- How *uncertainties in the knowledge of the collinear, unpolarized gluon and sea quark densities* might impact the probe.

Results for SSA in $p^{\uparrow}p \to \gamma + X$ in GPM

R. M. Godbole, A. Kaushik, A. Misra, and S. Padval, Phys. Rev. D 99 (2019) 014003







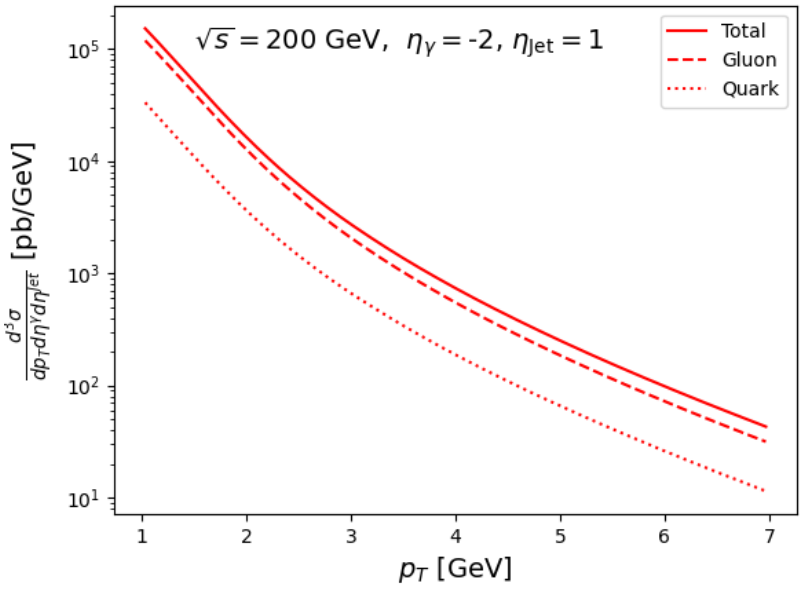
Similar work carried out for J/Ψ, D-meson production

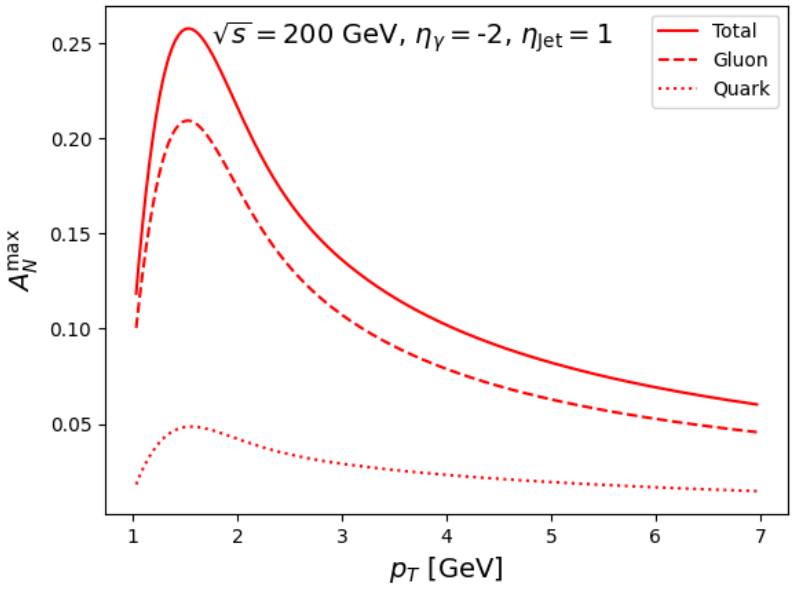
$p^{\uparrow} p \to \gamma + \text{jet} + X$

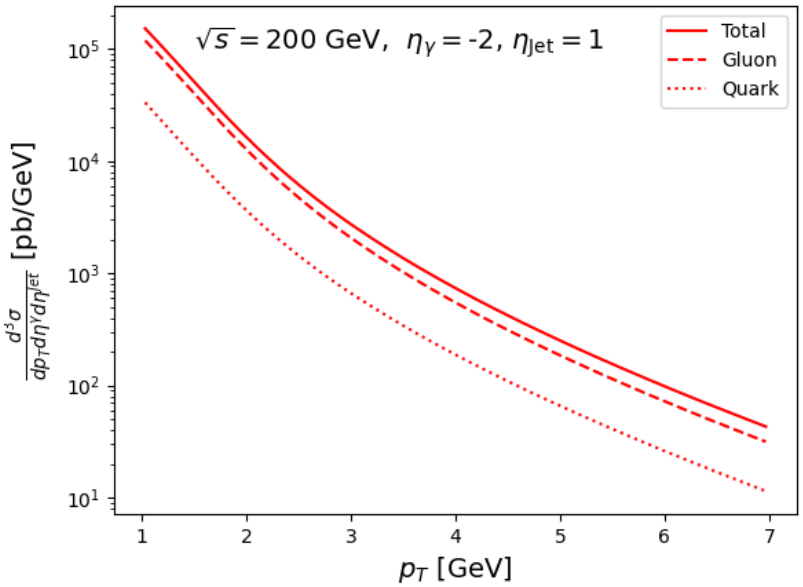
- $\vec{q}_{\perp} \equiv \vec{p}_{\gamma\perp} + \vec{p}_{J\perp}, \quad \vec{p}_{\perp} \equiv |\vec{p}_{\gamma\perp} - \vec{p}_{J\perp}|/2; \quad \vec{q}_{\perp} << \vec{p}_{\perp}$
- 2-scale process - TMD factorization applicable

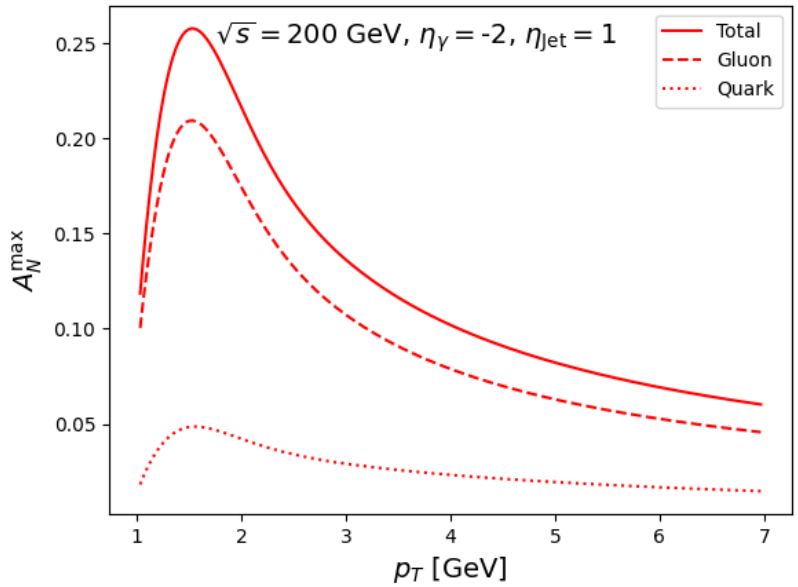
$$d\sigma^{p^{\uparrow} p \to \gamma + \text{Jet} + X} = \frac{1}{2s} \sum_{ab} \int dx_a d^2\mathbf{k}_{\perp a} dx_b d^2\mathbf{k}_{\perp b} \Delta^N f_{a/p^{\uparrow}}(x_a, \mathbf{k}_{\perp a}) \hat{f}_{b/p}(x_b, \mathbf{k}_{\perp b})$$
$$\times \sum \frac{|M(ab \to \gamma d)|^2}{x_a x_b} (2\pi)^4 \delta^4 (p_a + p_b - p_{\gamma} - p_d) \frac{d^3\mathbf{p}_{\gamma}}{(2\pi)^3 2E_{\gamma}} \frac{d^3\mathbf{p}_d}{(2\pi)^3 2E_d}$$

where $\Delta^N f_{a/p^{\uparrow}}(x_a, \mathbf{k}_{\perp a})$ is the Sivers function.


$\sqrt{s} = 200$ GeV, $\eta_\gamma = -2$, $\eta_{\rm Jet} = 1$
Total
Gluon
Quark
$\frac{d^3\sigma}{dp_T d\eta^\gamma d\eta^{\rm Jet}}$ [pb/GeV]
p_T [GeV]


$\sqrt{s} = 200$ GeV, $\eta_\gamma = -2$, $\eta_{\rm Jet} = 1$
Total
Gluon
Quark
$A_N^{\rm max}$
p_T [GeV]





- Next task: $p^{\uparrow} p \to \gamma + \pi(\text{hadron}) + X$ at SPD-NICA energies

(We thank Profs. Alexey Guskov and Amaresh Datta for suggesting this study.)

Thanks a lot for your attention!