Enhancement of order in the FitzHugh–Nagumo neurons ring by Lévy noise

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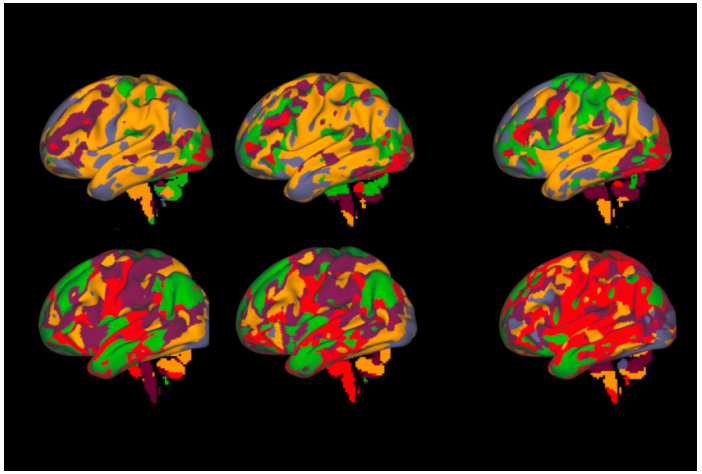
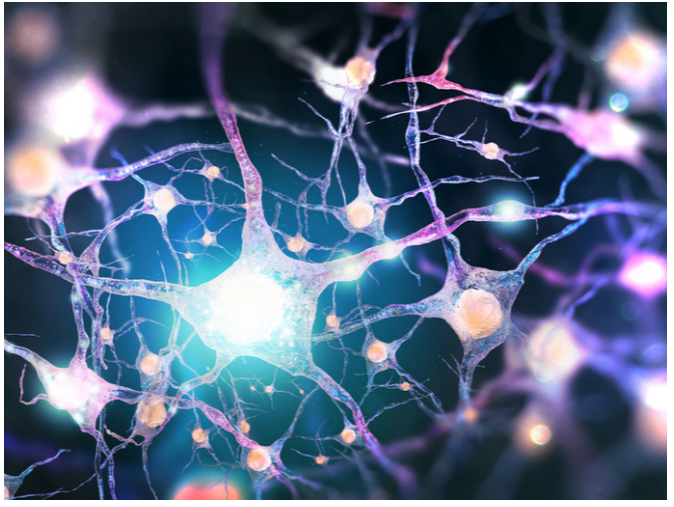
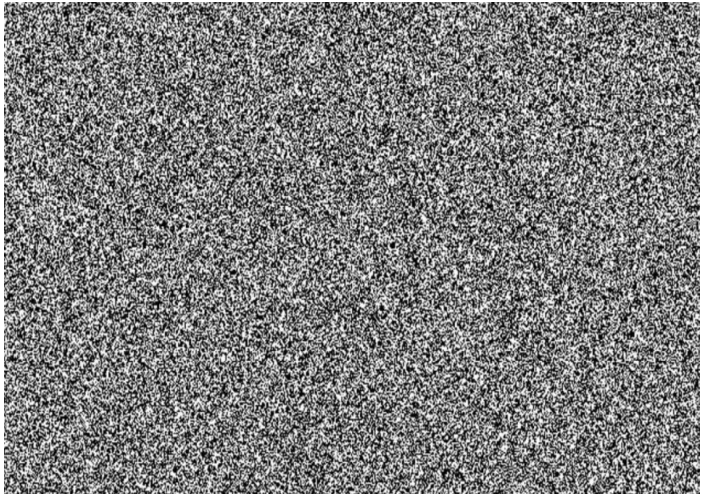


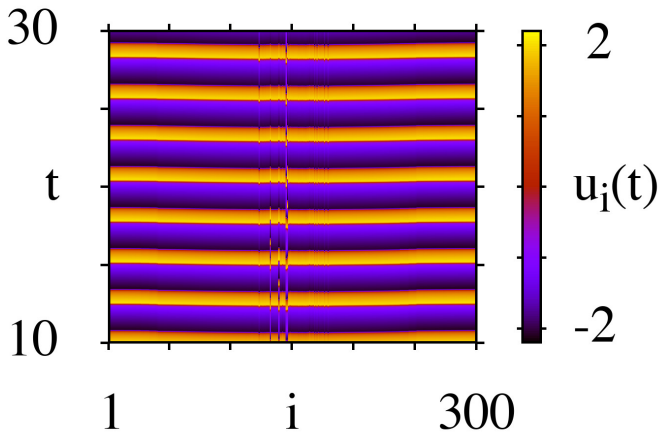


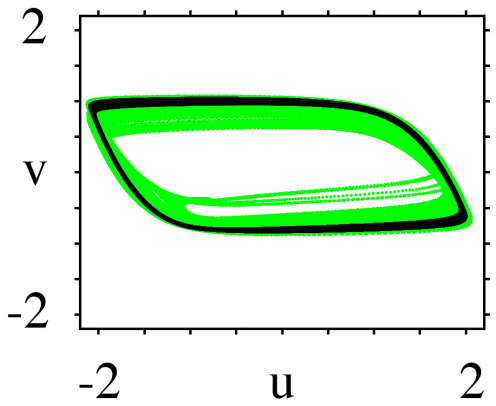


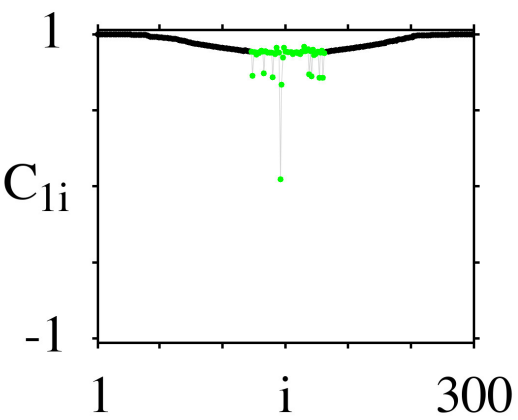


Chimera state

- Chimera states represent a regime in which clusters with coherent and incoherent dynamics coexist in an ensemble. This regime corresponds to normal brain function. But also chimera state correspond to pathological activities too.

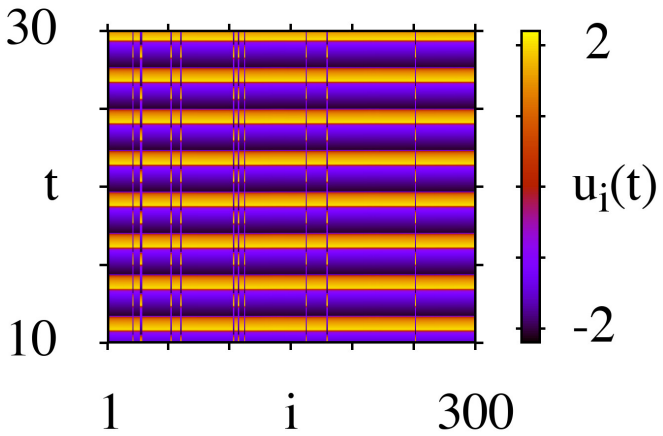


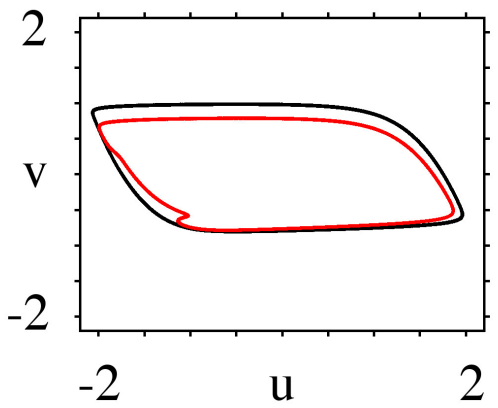


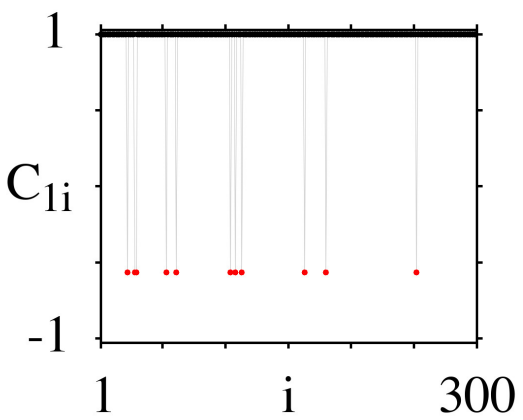


Solitary state

- Solitary states represent a cluster synchronization regime in which individual nodes whose dynamics differ from the dynamics of the rest of the system can be distinguished.





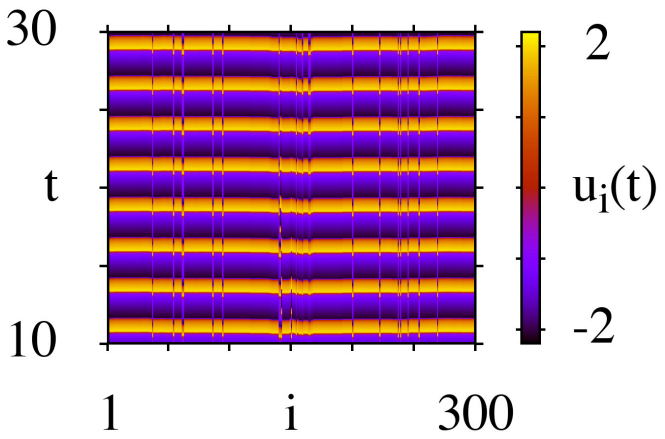


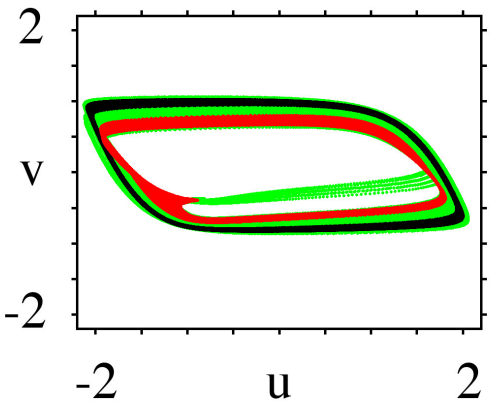
• Chimera states:
- power-grids networks [Menck, 2014; Motter, 2013; Wang, 2016]
- social systems [Hong, 2010; González-Avella, 2014]
- Parkinson's disease [Levy, 2000]
- unihemispheric slow-wave sleep [Rattenborg, 2000]
- eye movement [Funahashi, 1991; Swindale, 1980]
- epileptic seizures [Andrzejak, 2016]

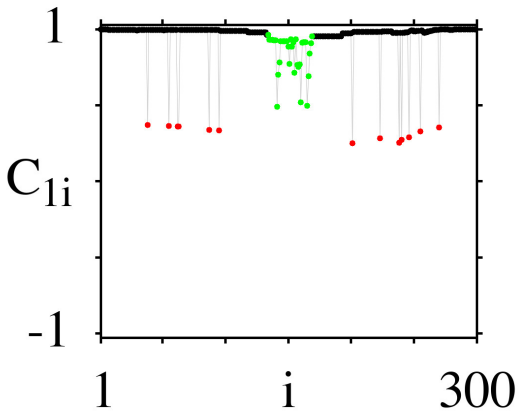
• Solitary states:
- response of individual neurons to specific stimuli [Fried, 1997; Kreiman, 2000]
- grandmother cells (neurons) [Rose, 1996; Quiroga, 2005]
- categorization tasks [Xin, 2019]

Combined state

The combined structure represents the coexistence of chimera and solitary states, wherein coherence and incoherence clusters along with solitary nodes can be clearly distinguished.

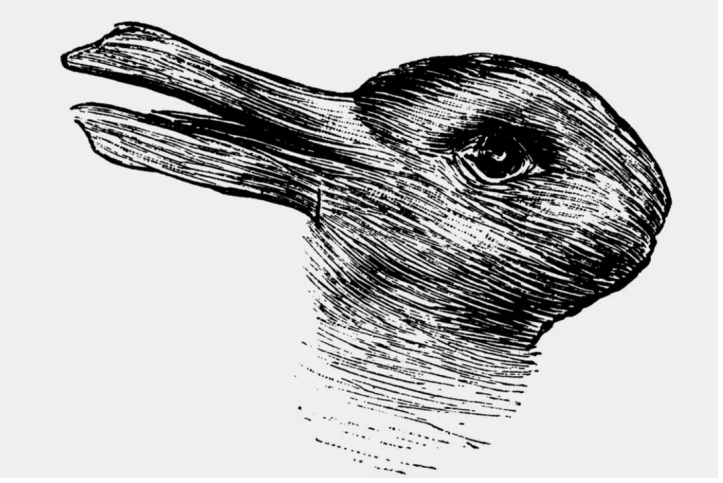






Multi-stability

Depending on the initial conditions, the system can exhibit chimera states, solitary states, or combined states. The entropy is used to determine whether a system is monostable (entropy is zero) or multistable (entropy is non-zero) — multiple regimes can arise in the system depending on the initial conditions.



Lévy noise

Janicki A., Weron A. Simulation and chaotic behavior of α-Stable stochastic processes. *New York: Marcel Dekker,* 1994. 376 p.

When $\alpha \neq 1$,

$$\eta_i = D_{\alpha,\beta,\sigma} \frac{\sin\left(\alpha\left(V + C_{\alpha,\beta}\right)\right)}{\left(\cos\left(V\right)\right)^{1/\alpha}} \times \left[\frac{\cos\left(V - \alpha\left(V + C_{\alpha,\beta}\right)\right)}{W}\right]^{\frac{1-\alpha}{\alpha}} + \mu$$

where,

$$C_{\alpha,\beta} = \frac{\arctan\left(\beta \tan\left(\frac{\pi\alpha}{2}\right)\right)}{\alpha}$$

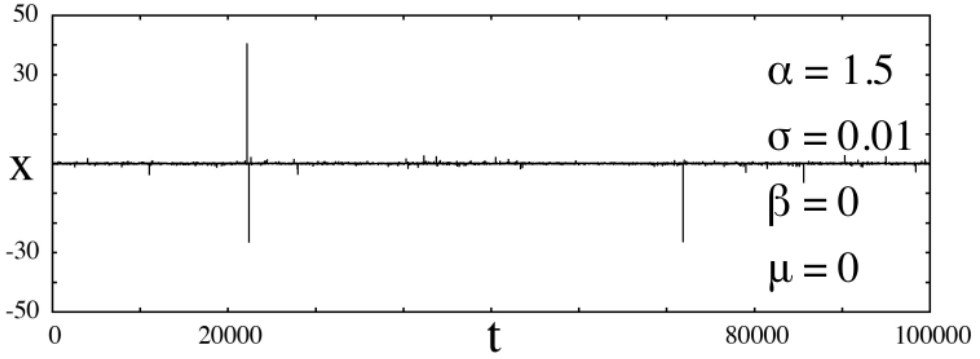
$$D_{\alpha,\beta,\sigma} = \sigma\left[\cos\left(\arctan\left(\beta \tan\left(\frac{\pi\alpha}{2}\right)\right)\right)\right]^{-1/\alpha}$$

When $\alpha = 1$,

$$\eta_i = \frac{2\sigma}{\pi}\left[\left(\frac{\pi}{2} + \beta V\right)\tan(V) - \beta \ln \frac{\frac{\pi}{2} W \cos(V)}{\frac{\pi}{2} + \beta V}\right] + \mu$$

V is a random variable following the uniform distribution on $(-\pi/2, \pi/2)$.

W represents a standard exponential random variable.



α ($0 < \alpha \leq 2$) is called the stability index, representing flatness of the tail of the distribution curve (a α=2, the distribution is equal to the Gaussian distribution).

β ($\beta \in [-1, 1]$) stands for the skewness parameter characterizing the skewness of the distribution.

σ ($\sigma > 0$) determines the width of the distribution.

Location parameters μ ($\mu \in R$) indicates the center of distribution.

$Q = \sigma^{\alpha}$ is the noise intensity.

Goals

- investigate the spatiotemporal dynamics of a ring of non-locally coupled FitzHugh–Nagumo neurons under the influence of Lévy noise;

- demonstrate how non-Gaussian noise parameters can be controlled to manipulate system order, suppress solitary states, and induce chimera states in neural networks.

Model under study

$$\varepsilon \frac{du_i}{dt} = u_i - \frac{u_i^3}{3} - v_i + \frac{c}{2R} \sum_{j=i-R}^{i+R} [b_{uu}(u_j - u_i) + b_{uv}(v_j - v_i)],$$

$$\frac{dv_i}{dt} = u_i + a + \frac{c}{2R} \sum_{j=i-R}^{i+R} [b_{vu}(u_j - u_i) + b_{vv}(v_j - v_i)] + \xi_i(t).$$

u_i and v_i — dynamic variables of element;

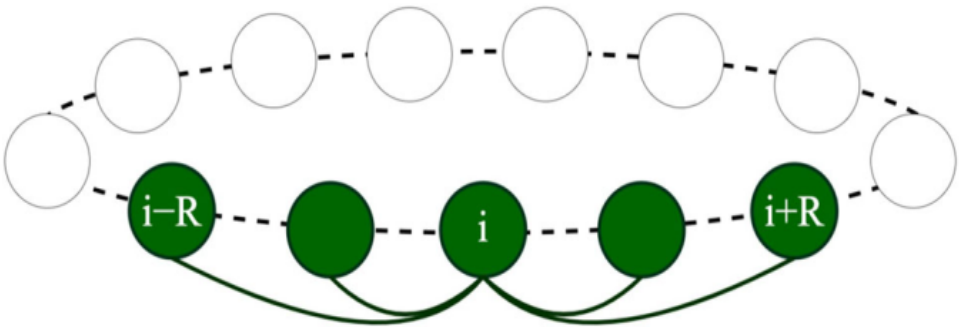
i = 1, 2, ..., N = 300 — the element number;

ε = 0.05 — scale parameter;

a = 0.5 — the excitability threshold value.

c — coupling strength;

R = 105 — coupling range.



Order parameters

The space-averaged cross-correlation coefficient

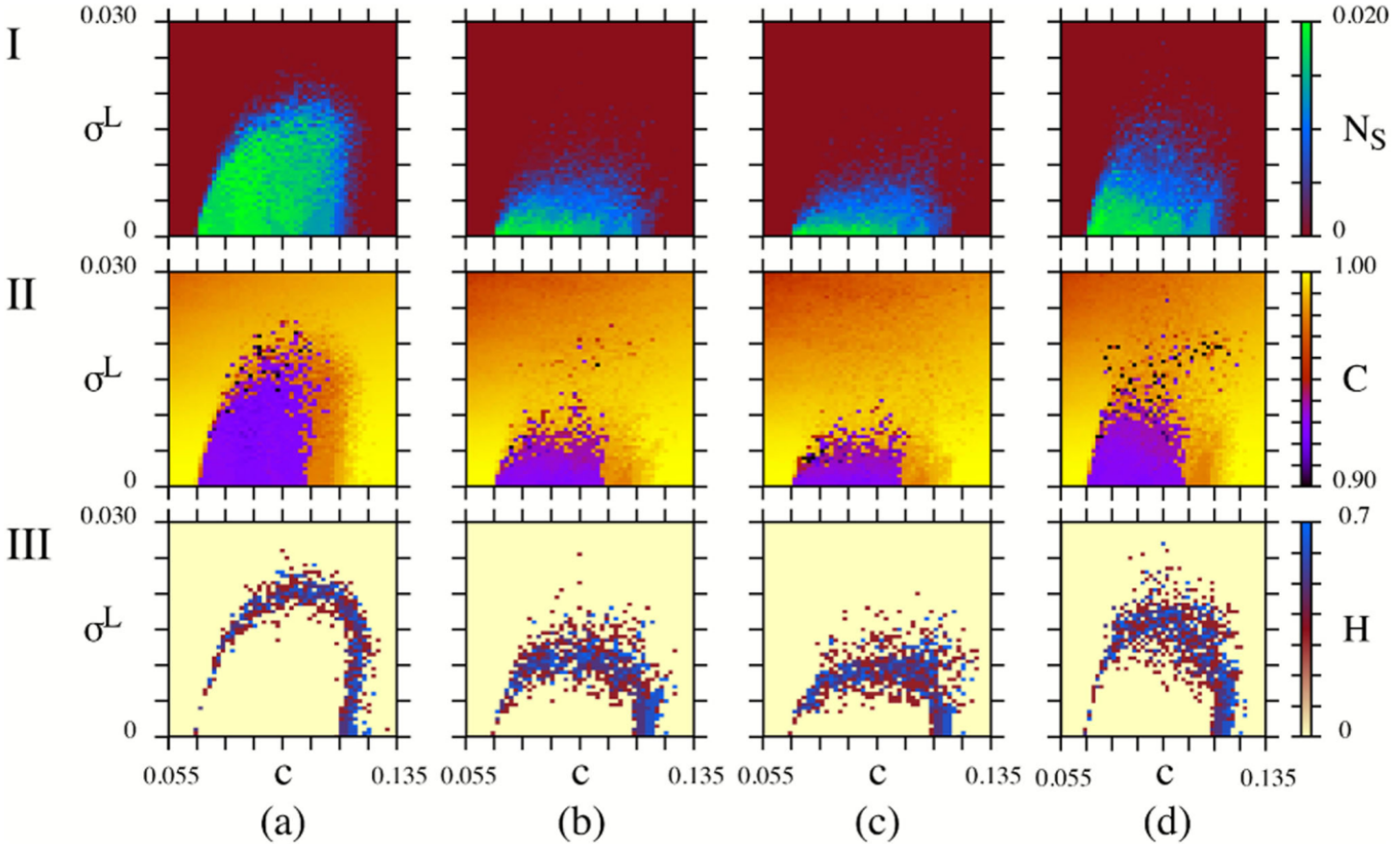
$$C = \frac{1}{N} \sum_{i=1}^{N} C_{1i},$$

The Shannon entropy

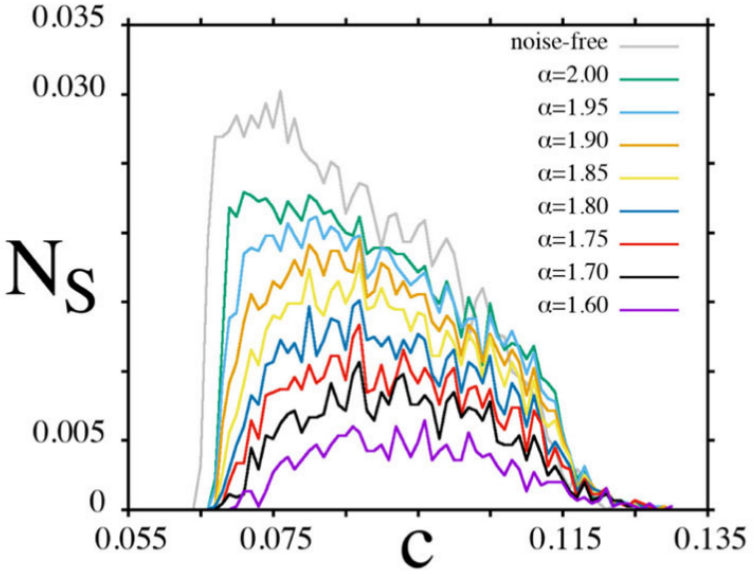
$$H = -\sum_{l} p_l \log p_l,$$

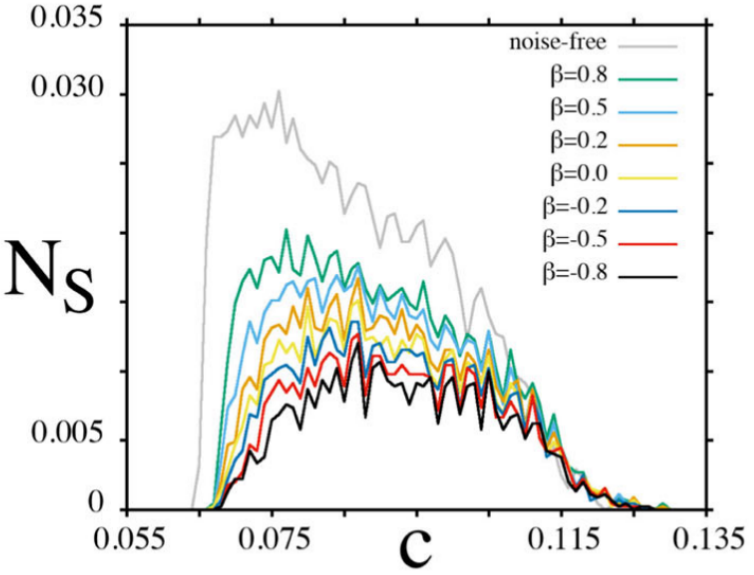
The normalized number of solitary nodes

$$N_S = \frac{1}{M} \sum_{m} S_m / N$$

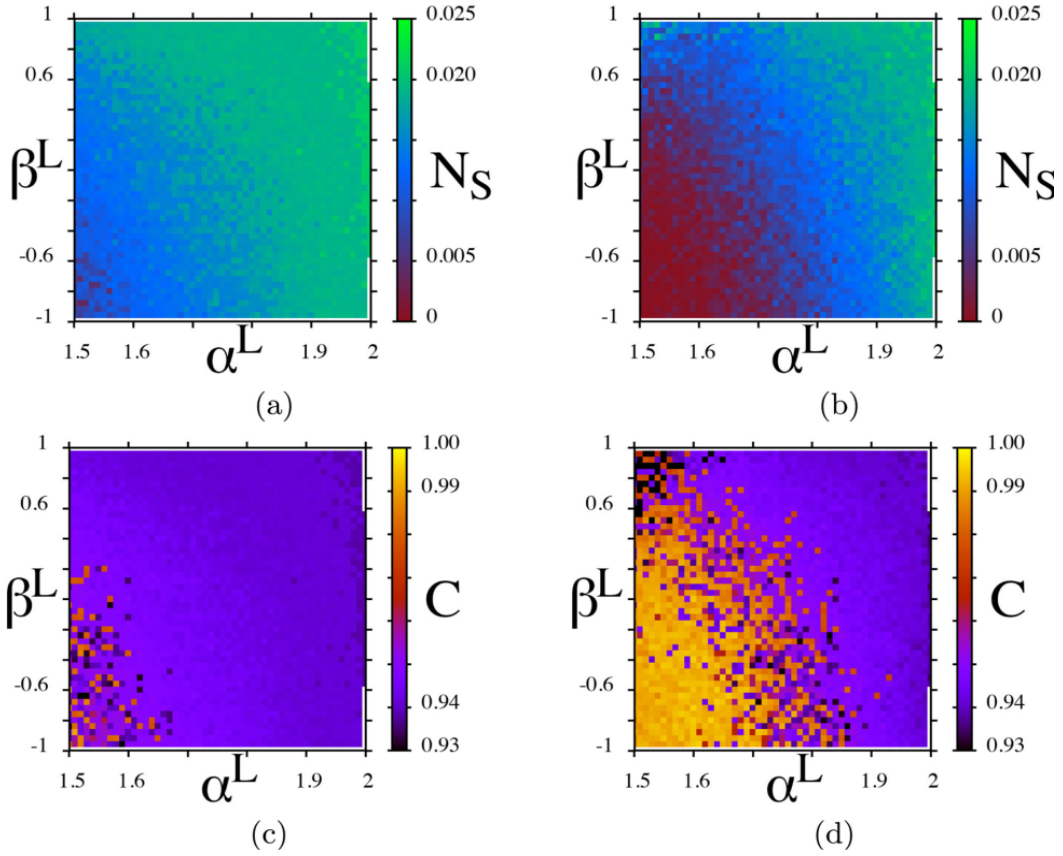


Distributions: (I) normalized number of solitary nodes, (II) space-averaged cross-correlation coefficient, and (III) entropy in the "coupling strength c – noise intensity σ^L" parameter plane for different values of the noise source parameters: a) α^L = 2.0, b) α^L = 1.8, β^L = 0.0, c) α^L = 1.8, β^L = −0.8, d) α^L = 1.8, β^L = 0.8.

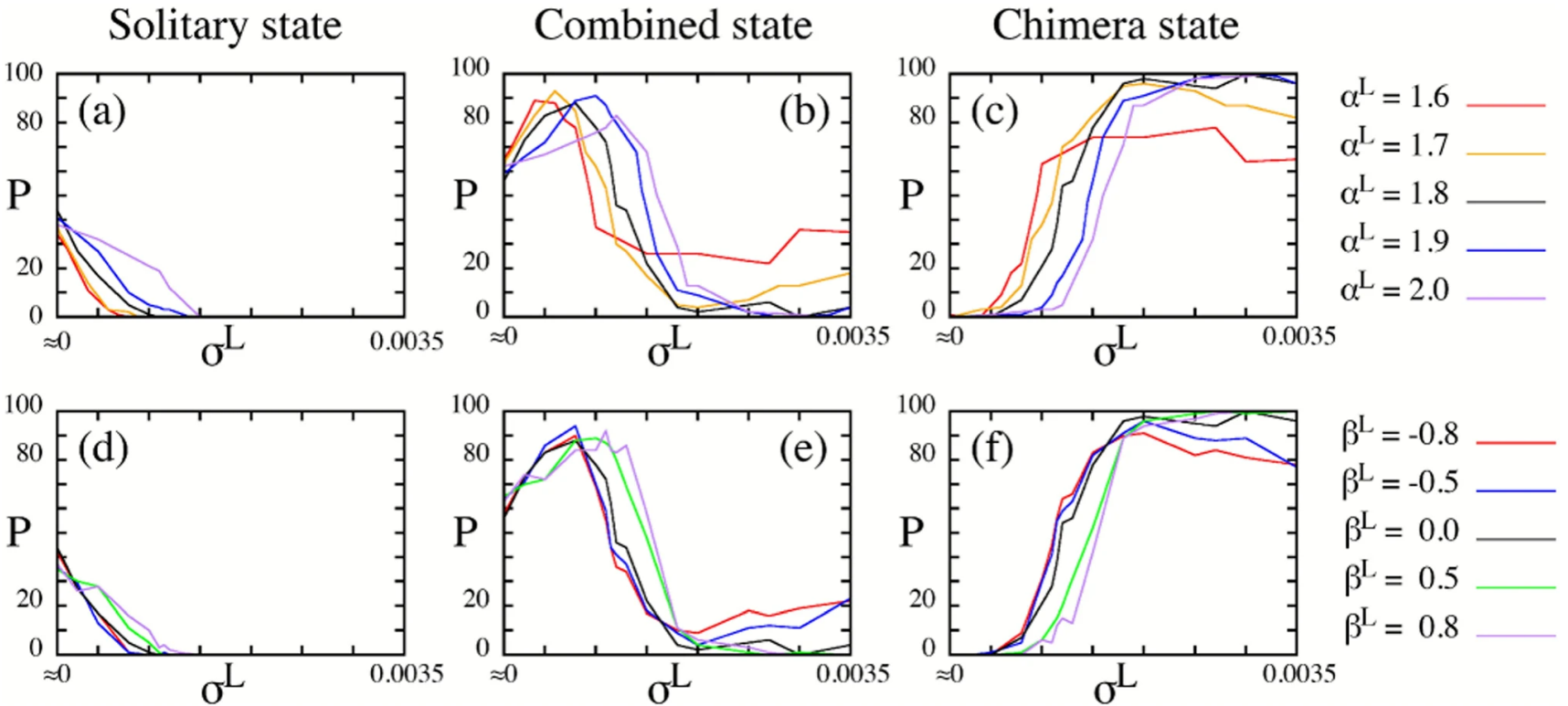




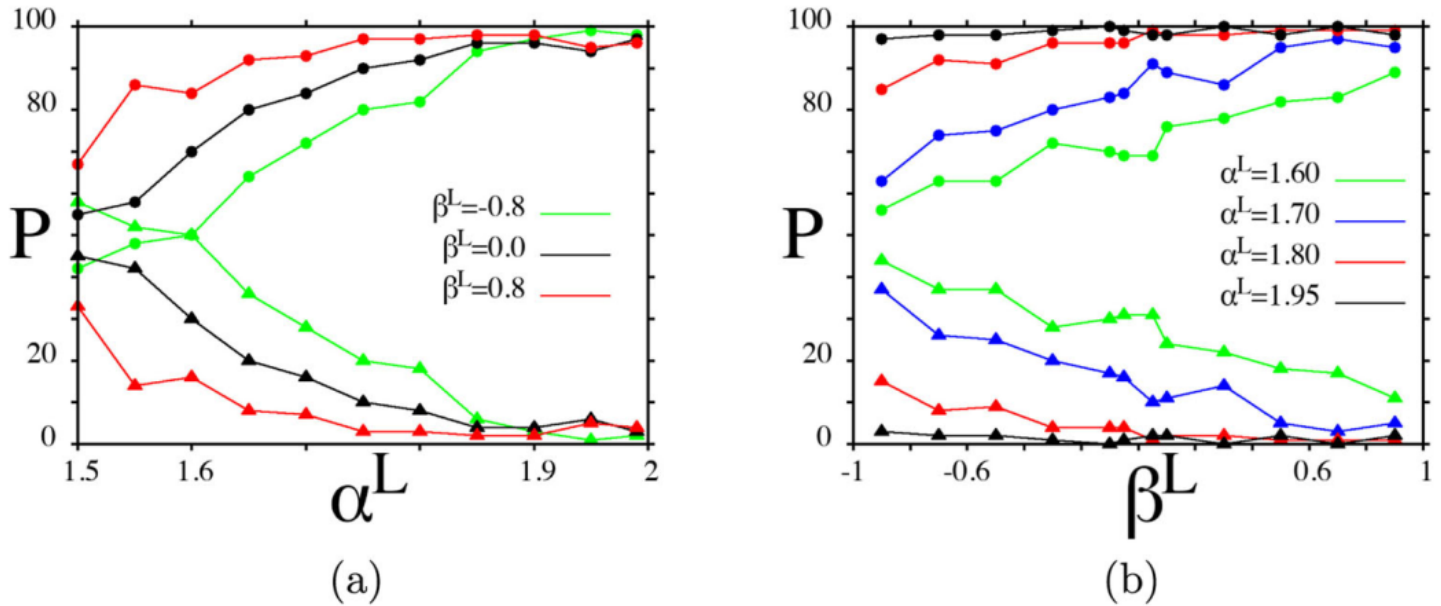
Dependencies of the normalized number of solitary nodes N_S on the coupling strength c: a) for different values of the parameter α^L at $\beta^L = 0$ and b) for different values of the parameter β^L at $\alpha^L = 1.8$



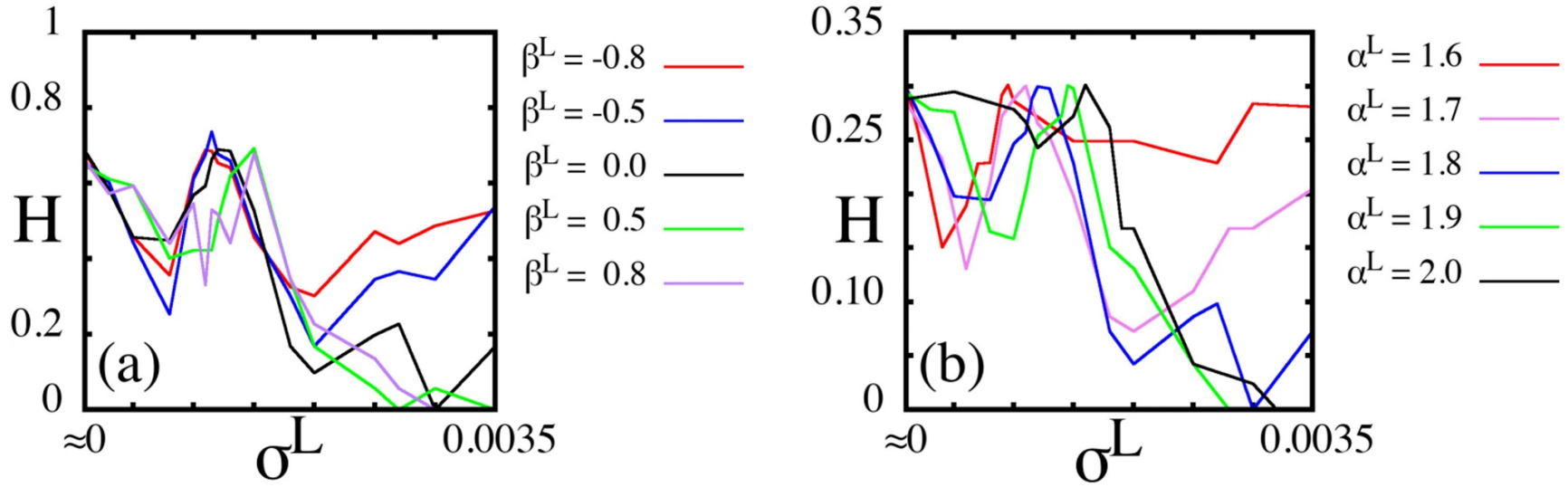
Distribution of a), b) normalized number of solitary nodes and c),d) space-averaged cross-correlation coefficient in the "stability index α^L – skewness of the distribution β^L " parameter plane at a),c) σ^L = 0.002; b),d) σ^L = 0.005.



Dependencies of the probability of structure appearance on the noise intensity for different values of the stability index α^L at $\beta^L = 0$ (a)–(c) and of the skewness of the distribution β^L at $\alpha^L = 1.8$ (d)–(f):(a, d) probability for solitary states, (b, e) probability for combined states, and (c, f) probability for chimera states



Dependencies of the probability for chimera states (lines with circles) and combined states (lines with triangles) versus (a) the stability index α^L for different values of the skewness of the distribution β^L and (b) the distribution β^L for different values of α^L



Entropy versus the noise intensity for different values of the skewness of the distribution β^L for $\alpha^L = 1.8$ (a) and the stability index α^L for $\beta^L = 0$ (b)

Conclusion

- The formation of solitary nodes N_s is observed within a strictly defined range of coupling strength. There is a critical noise intensity threshold above which solitary structures cannot exist.
- A decrease in the stability index α^L and in the skewness β^L exert an identical destructive effect on solitary nodes, but an increase in the stability index α^L and skewness β^L raises the probability of chimera states, suppressing combined states.
- Lévy noise can suppress all dynamic regimes except for chimera states, thereby contributing to a significant reduction in the system’s entropy.

Thank you!