Centrality determination methods in the BM@N experiment

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The 16th Collaboration Meething of the BM@N Experiment at NICA

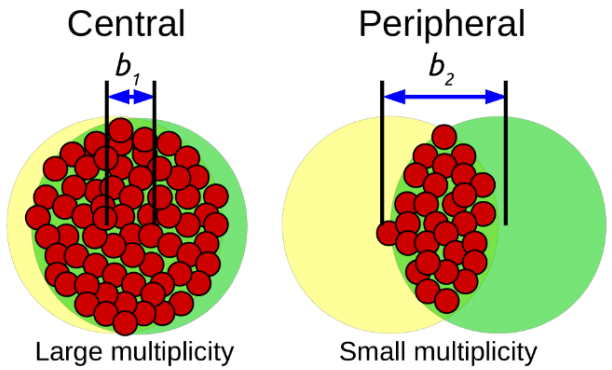
May 12 to 15, 2025 in Dubna, Russia

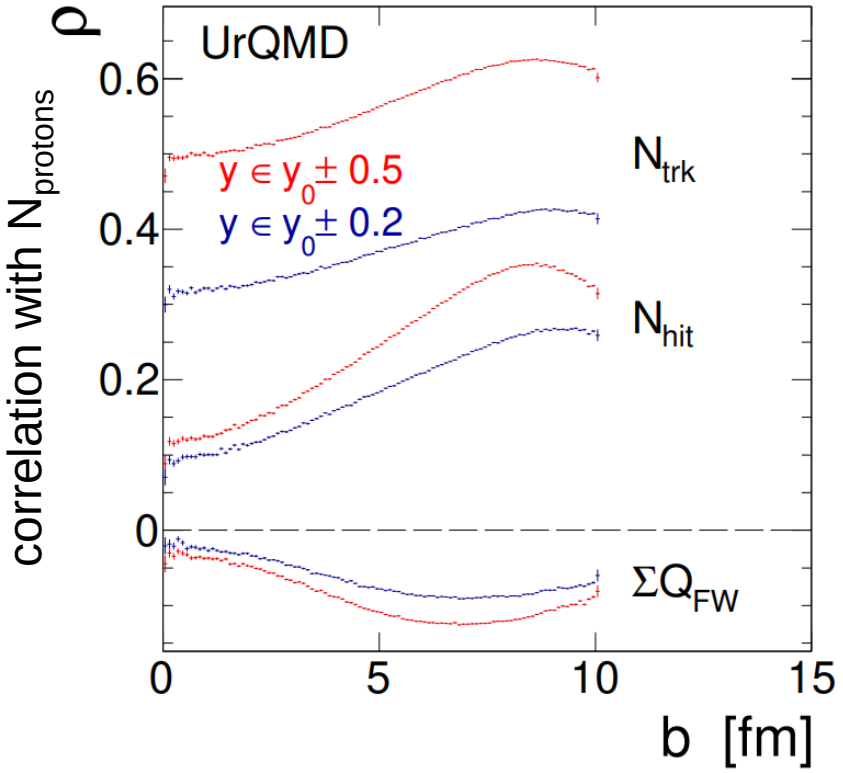


Centrality

- Evolution of matter produced in heavy-ion collisions depend on its initial geometry
- Centrality procedure maps initial geometry parameters with measurable quantities (multiplicity or energy of the spectators)
- **This allows comparison of the future BMAN results with the data from other experiments (STAR BES, NA49/NA61 scans) and theoretical models**

$$c(b) = \frac{\int_0^b \frac{d\sigma}{db'} db'}{\int_0^\infty \frac{d\sigma}{db'} db'} = \frac{1}{\sigma_{A-A}} \int_0^b \frac{d\sigma}{db'} db'$$



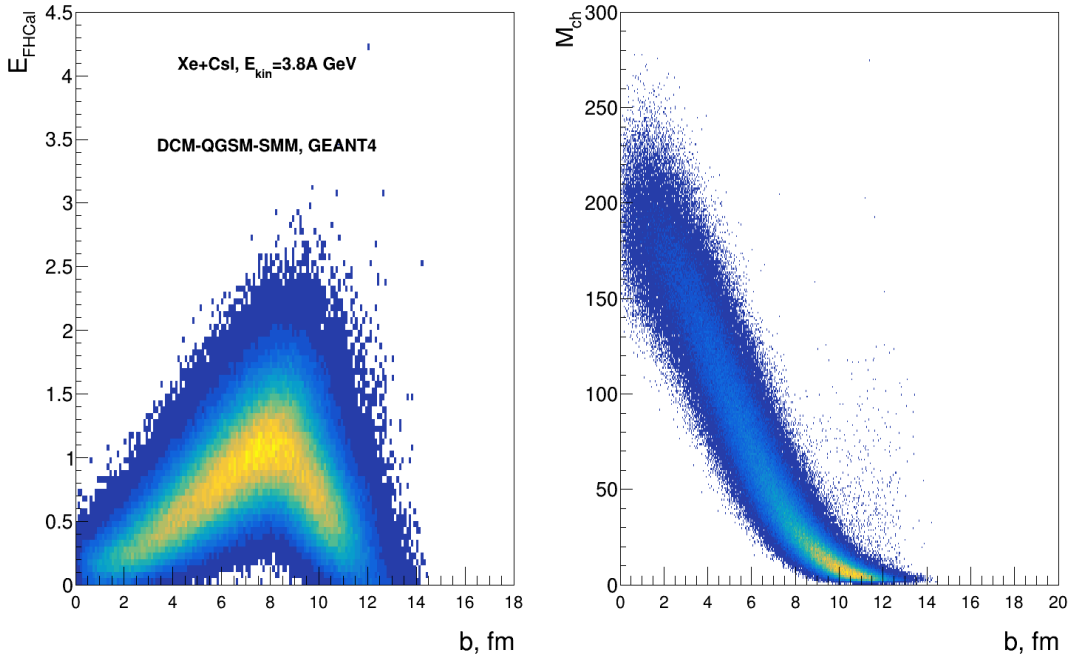


HADES; Phys.Rev.C 102 (2020) 2, 024914

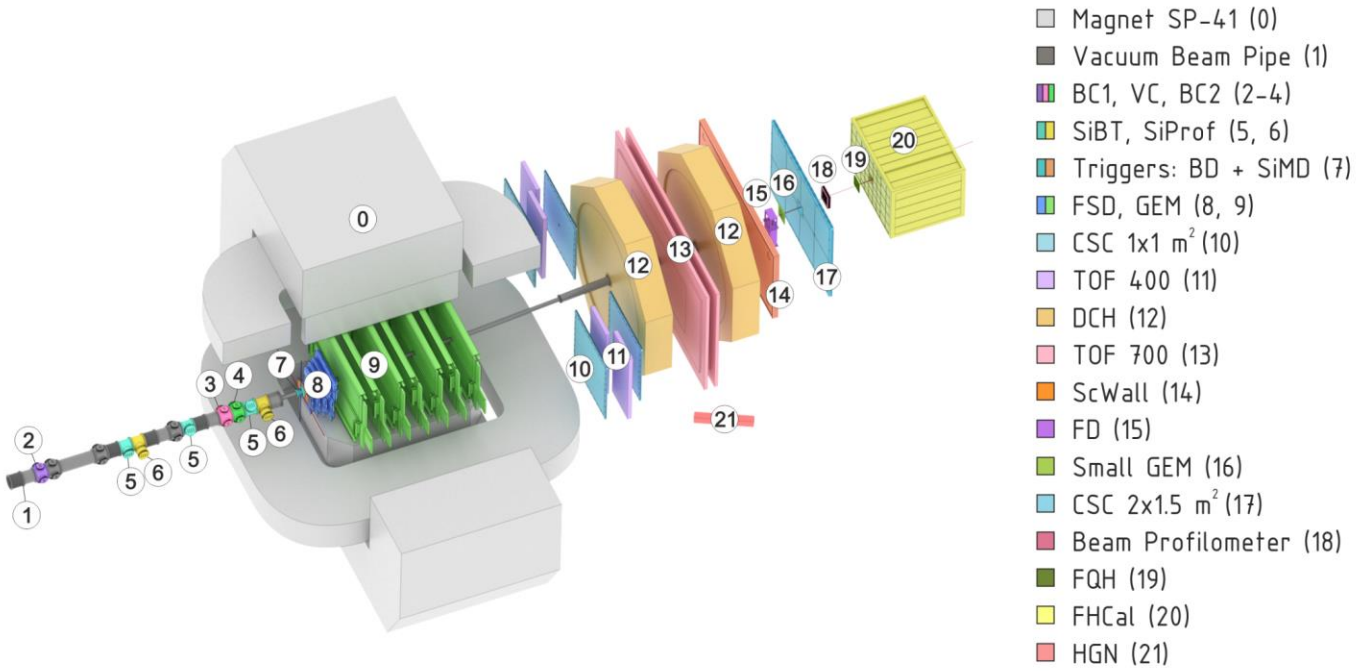
- A number of produced protons is stronger correlated with the number of produced particles (track & RPC+TOF hits) than with the total charge of spectator fragments (FW)
- to suppress self-correlation biases, it is necessary to use spectators fragments for centrality estimation

Centrality determination in BM@N

DCM-QGSM-SMM → GEANT4 → Reconstruction → Centrality determination



Dependence of energy in FHCal and track multiplicity on the impact parameter



BM@N setup overview

The Bayesian inversion method (Γ-fit): DCM-QSM-SMM based

- The fluctuation kernel for multiplicity at fixed impact parameter is Gamma distr.:

$$P(M \mid c_b) = \frac{1}{\Gamma(k(c_b))\theta^2} M^{k(c_b)-1} e^{-M/\theta}$$

$$c_b = \int_0^b P(b')db' \quad \text{– centrality based on impact parameter}$$

$$\theta = \frac{D(M)}{\langle M \rangle}, \quad k = \frac{\langle M \rangle}{\theta}$$

$\langle M \rangle, D(M)$ – average and variance of Multiplicty

$$P(M) = \int_0^1 P(M \mid c_b)dc_b$$

$$\langle M \rangle = m_1 \cdot \langle M' \rangle$$

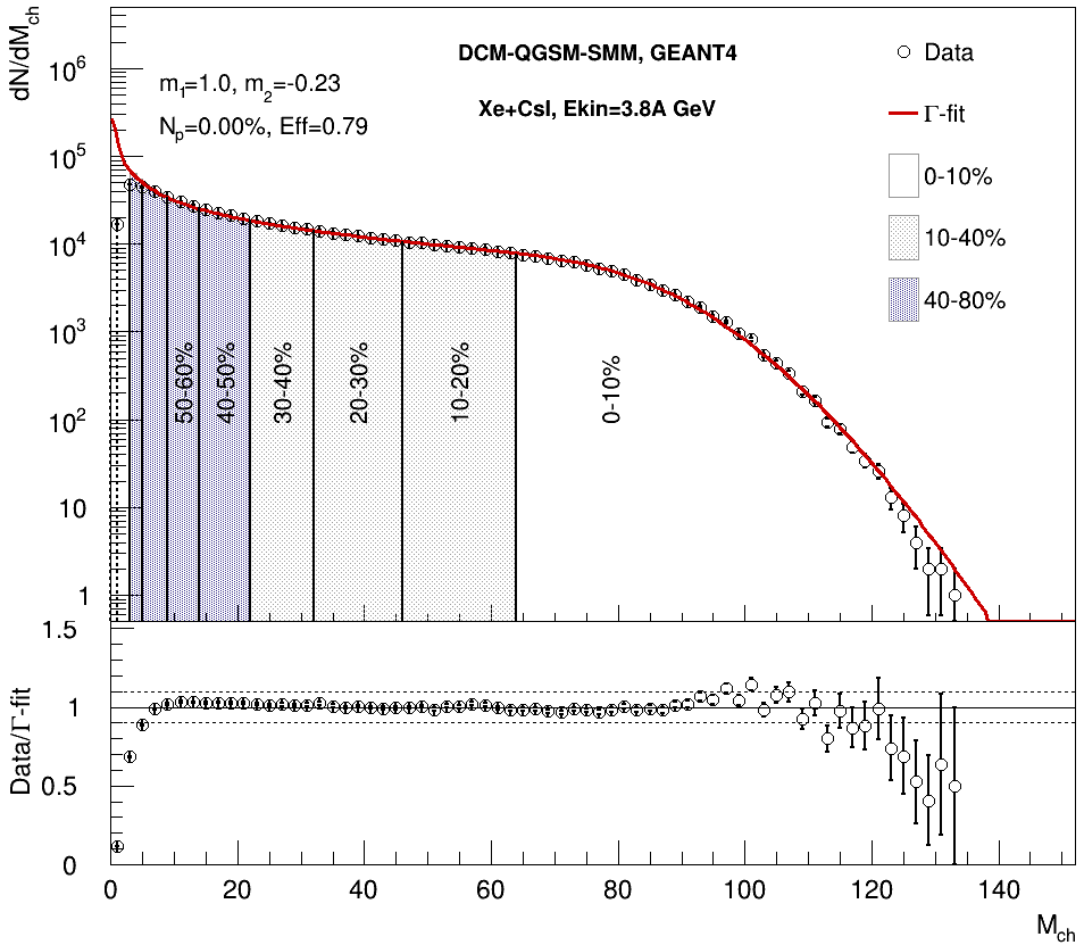
$$D(M) = m_1^2 \cdot D(M') + m_1 \cdot m_2 \langle M' \rangle$$

$\langle M'(c_b) \rangle$ – average value and var. of energy/mult.

$D(M'(c_b))$ from the rec. model data

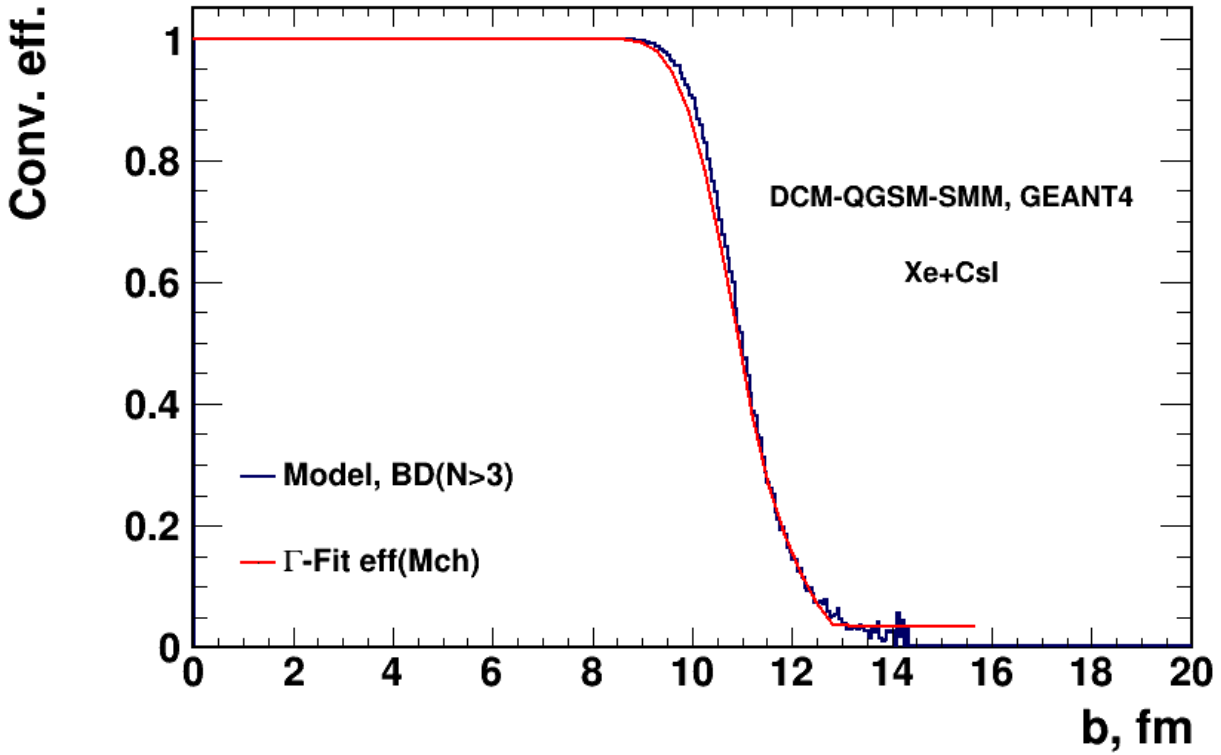
- can be approximated by polynomials or exponential polynomial

Fit results: DCM-QGSM-SMM, N(BD)≥3



Good agreement with fit

$$P_{eff}(b) = \int_0^{M_{max}} P_{eff}(M) P(M \mid b) dM$$



Convoluted trigger efficiency can be calculated using Bayes' theorem

The Bayesian inversion method (Γ-fit): 2D fit

- The fluctuation kernel for energy and multiplicity at fixed impact parameter can be describe by 2D Gamma distr.:

$$P(E,M \mid c_b) = G_{2D}(E,M,\langle E\rangle,\langle M\rangle,D(E),D(M),R)$$

$\langle E\rangle, D(E)$ – average value and variance of energy

$R(E,M)$ – Pirson correlation coefficient

$$P(E,M) = \int_0^1 P(E,M \mid c_b)\,dc_b$$

$$R(E,M) = \frac{\varepsilon_1^2 m_1^2}{\varepsilon_2 m_2} R(E',M') \qquad \varepsilon_1, \varepsilon_2, m_1, m_2 \text{ - fit parameters}$$

$\langle E'(c_b)\rangle$ – average value and var. of energy/mult.

$D(E'(c_b))$ from the rec. model data

$$\langle E\rangle = \varepsilon_1 \langle E'(c_b)\rangle, \quad D(E) = \varepsilon_2 D(E'(c_b))$$
$$\langle M\rangle = m_1 \langle M'\rangle, \quad D(M) = m_1^2 D(M') + m_2 \langle M'\rangle$$

$\langle E'(c_b)\rangle, \quad D(E'(c_b))$ - can be approximated by polynomials

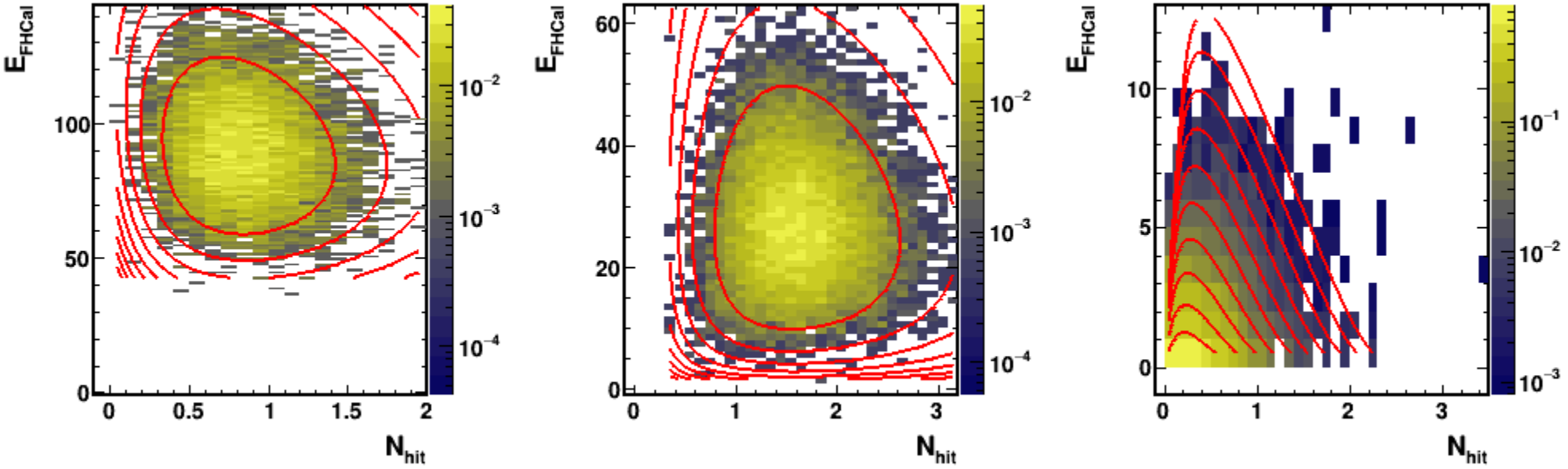
The fluctuation of energy and multiplicity at fixed impact parameter

It is possible to find such a rotation angle of the system that $\mathrm{cov}(x, y) = 0$

$$x = \cos(\alpha)E + \sin(\alpha)M,$$
$$y = -\sin(\alpha)E + \cos(\alpha)M$$

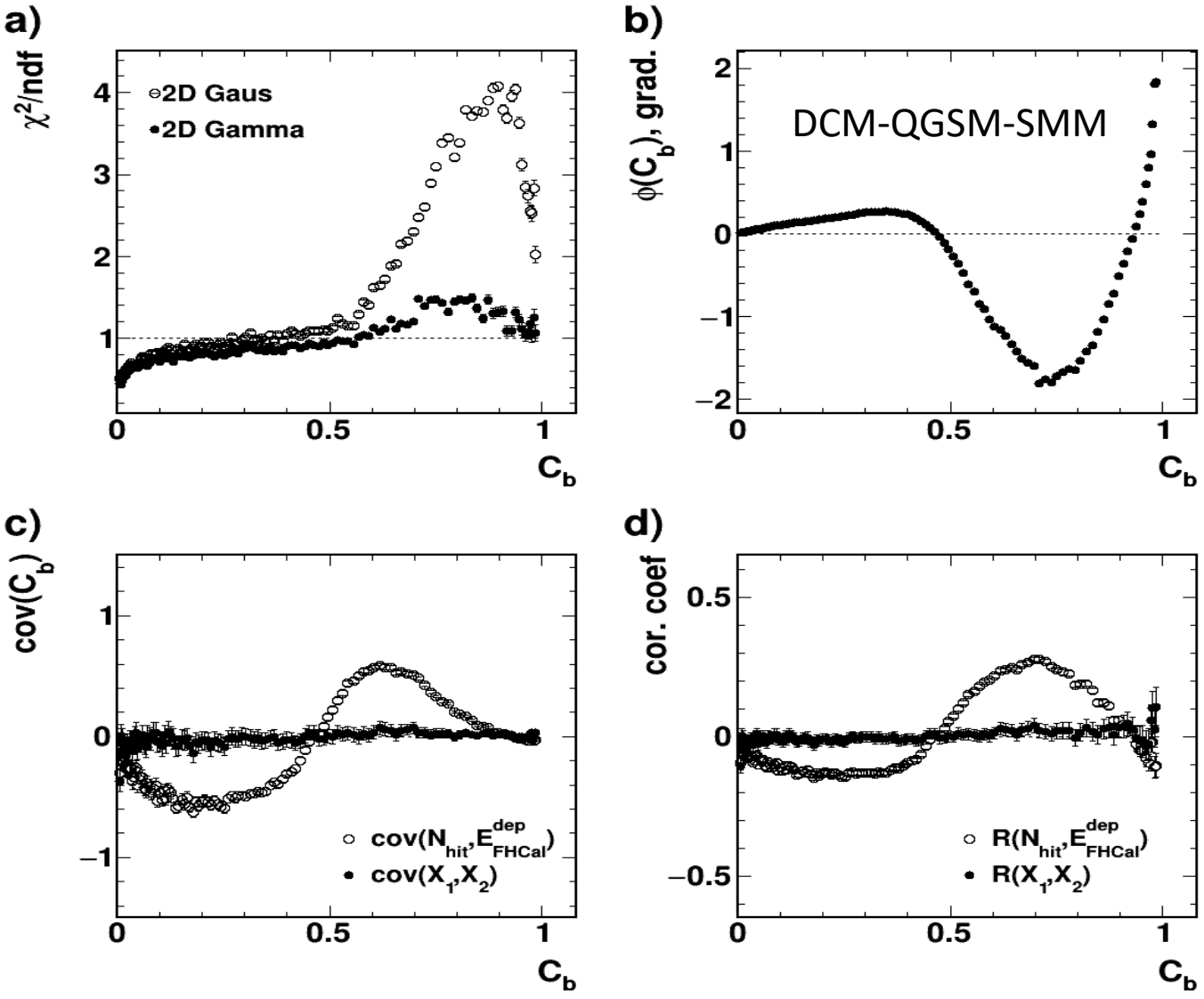
$$\alpha = \arctan\left(\frac{2\sqrt{D(E)D(M)}R(E,M)}{D(E) - D(M)}\right)$$

$$G_{2D}(E_{FH}, M_{ch}, \langle E \rangle, \langle M \rangle, D(E), D(M), R) = G(x, \theta_x, k_x) \cdot G(y, \theta_y, k_y)$$



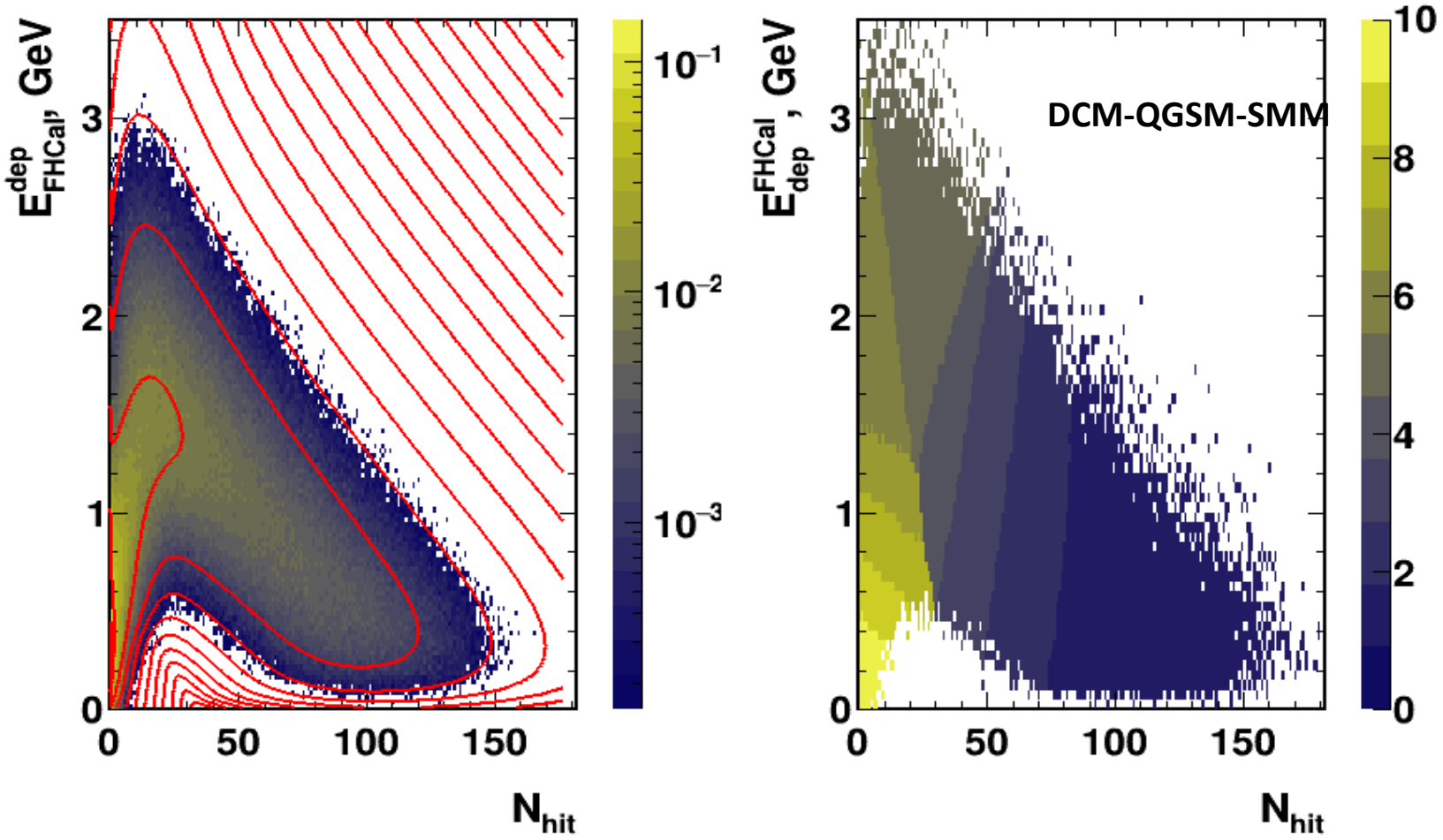
The distribution of energy and multiplicity at a fixed impact parameter is well described by the gamma distribution

Linear correlation whitening



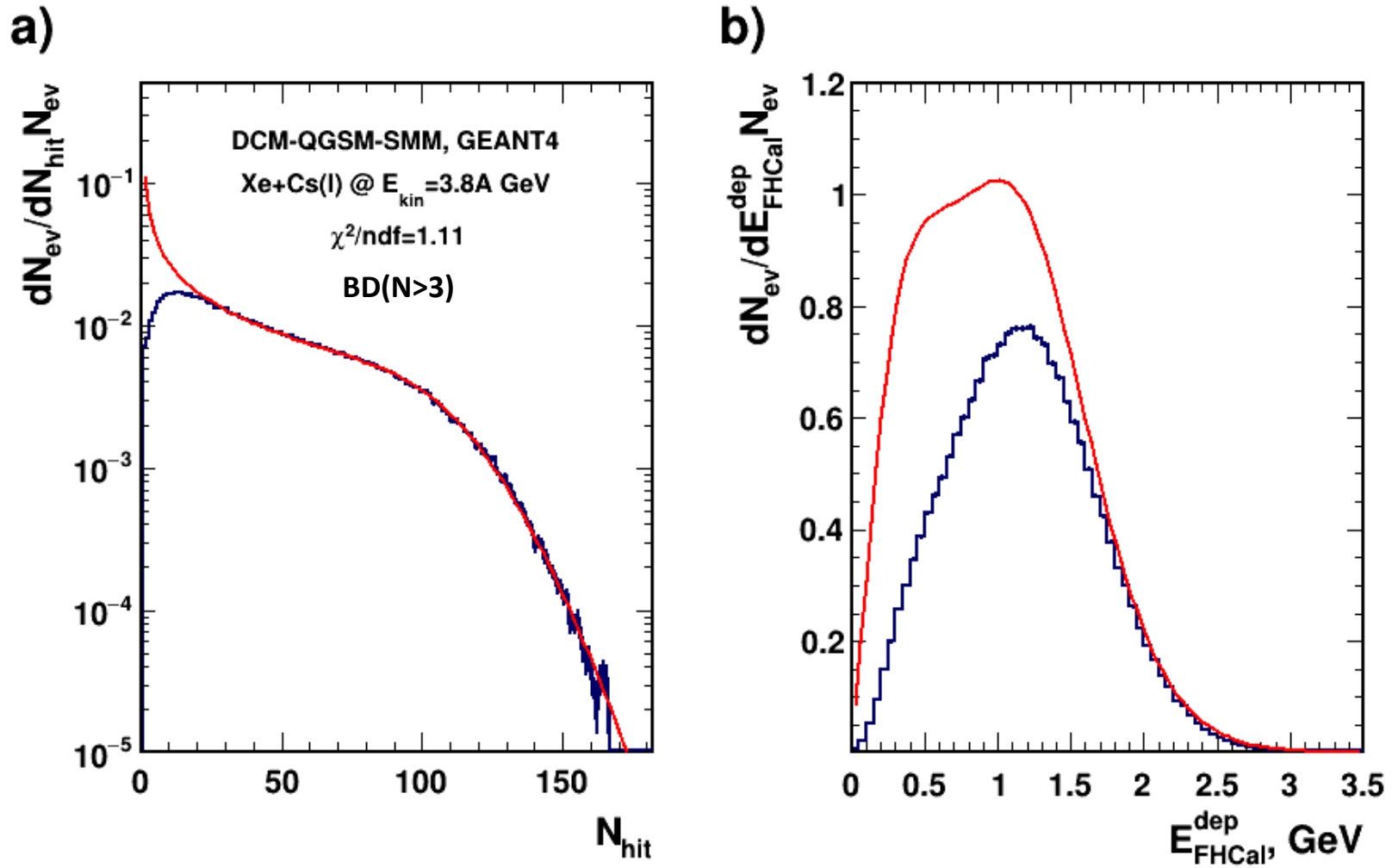
- The proposed approximation using a bivariate Gamma distribution fits the data better than the normal distribution.
- Rotation of the axes suppresses the linear component of correlation in the observations. This approach is used in statistics in the PCA method and when generating random variables with a given correlation coefficient.
- The rotation angle depends on the impact parameter and centrality.

2D fit and centrality classes



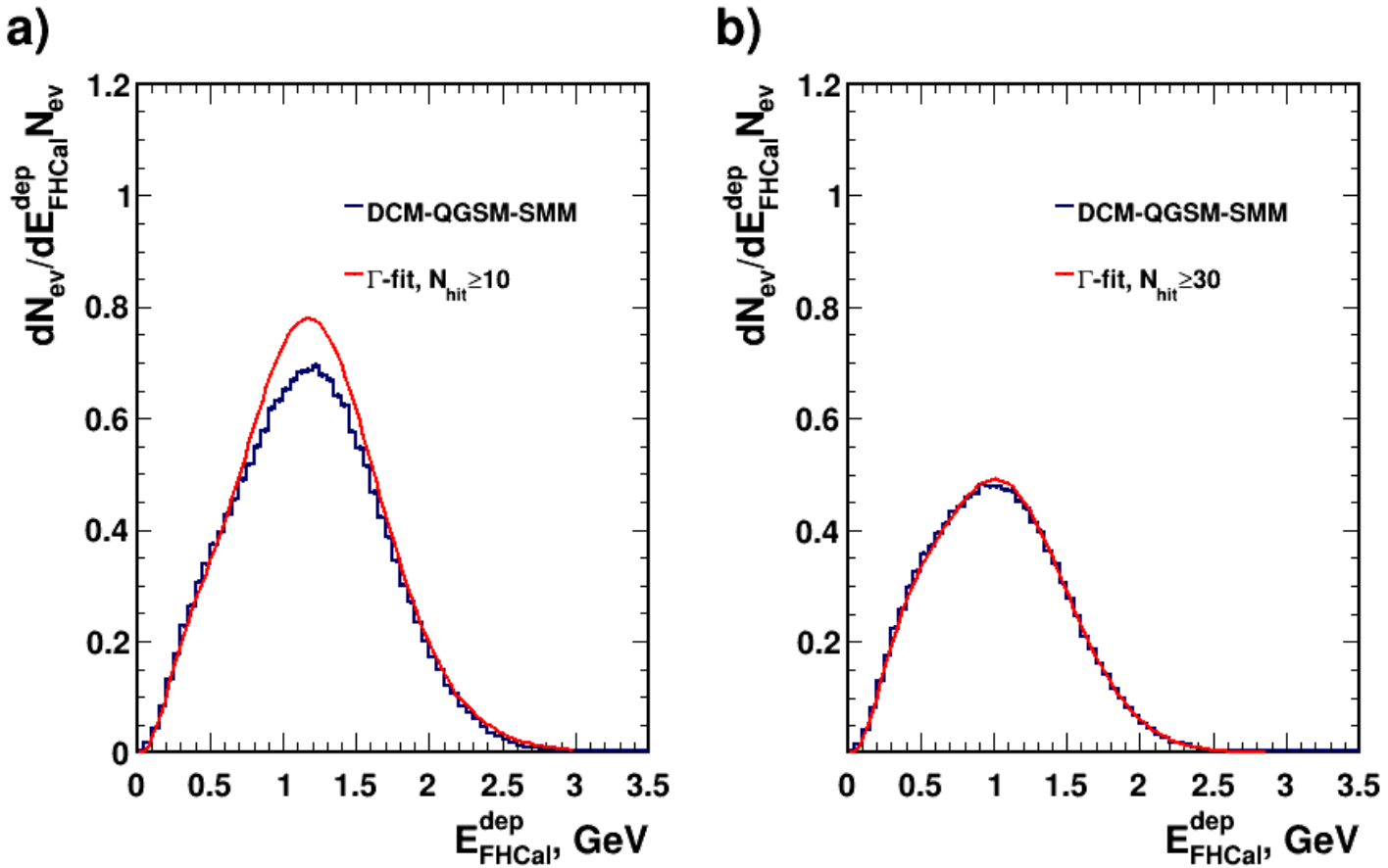
The fit function reproduces the two-dimensional distribution well.
Centrality classes are calculated using the k-means constrained method.

2D fit results



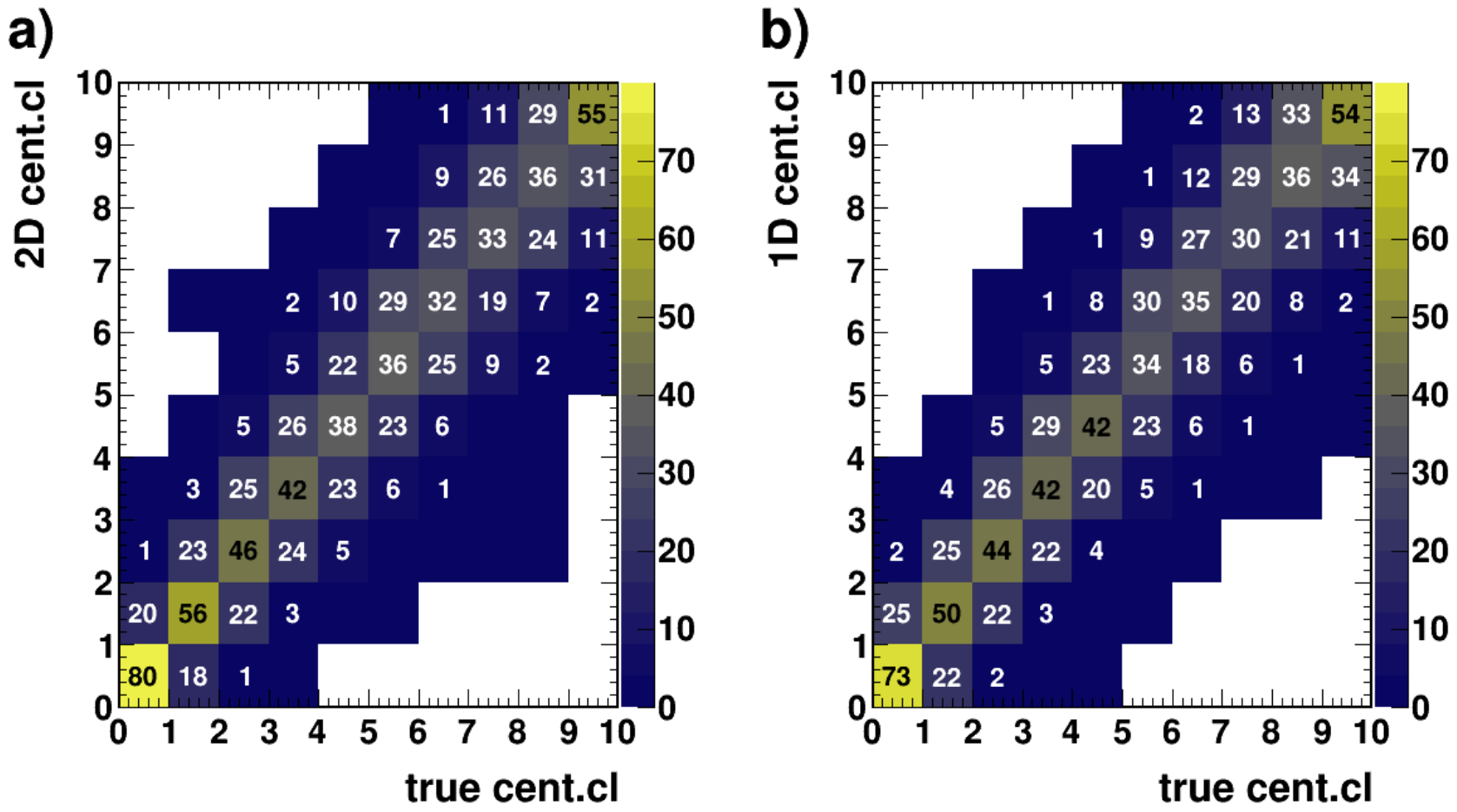
The two-dimensional fit function reproduces the distribution of the observables.

2D fit results



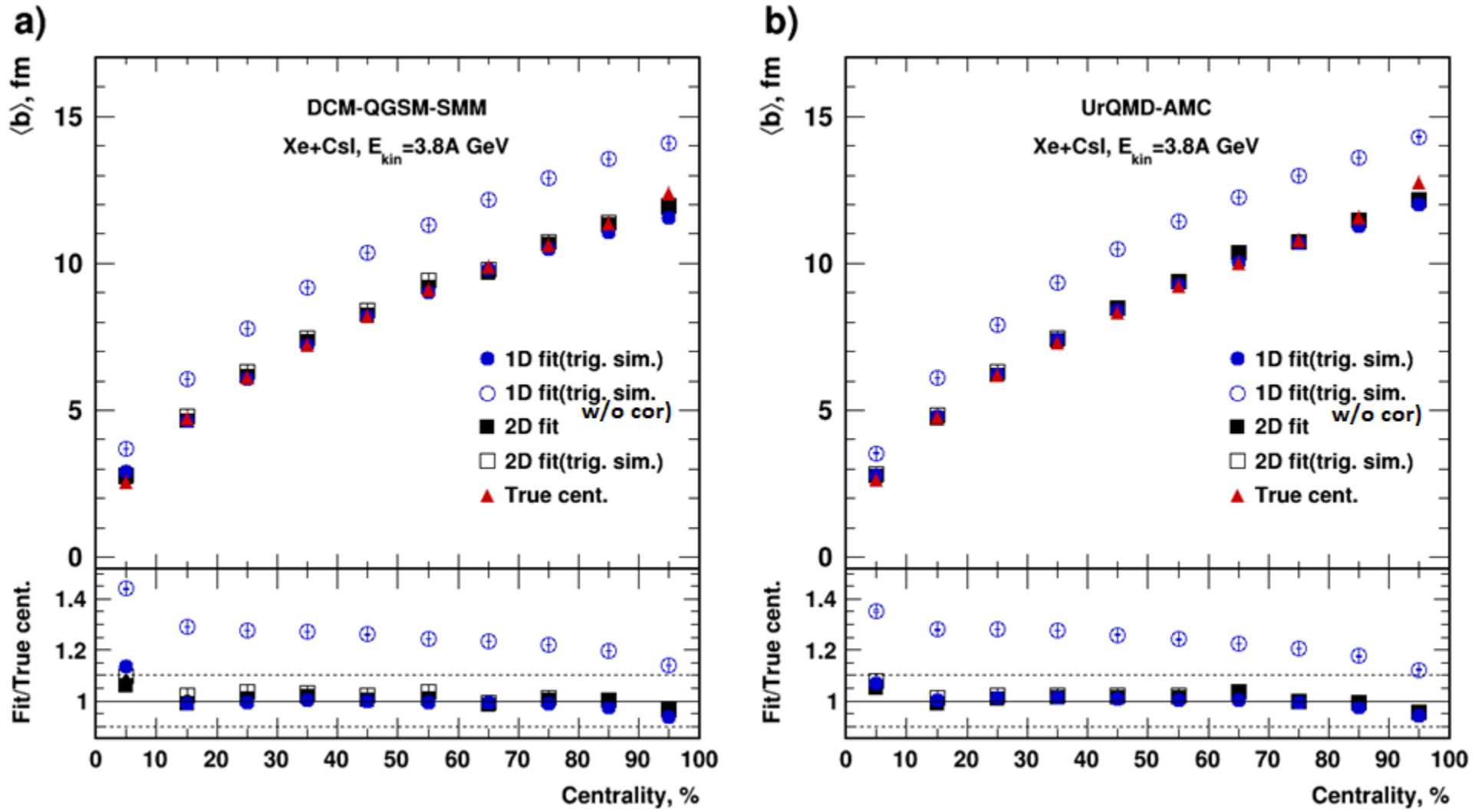
The difference in the energy distribution is explained by the non-monotonic dependence of energy on centrality. The energy distributions in the region below the anchor point agree well.

2D fit results

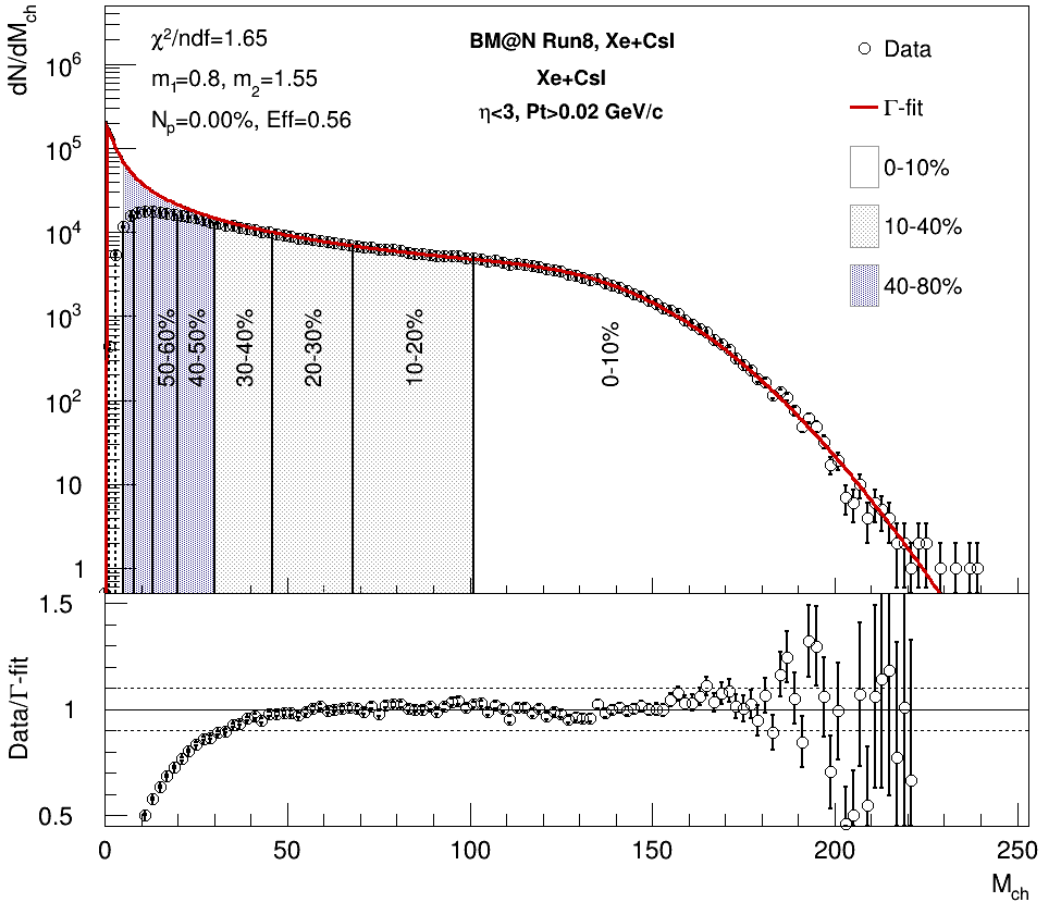


The two-dimensional approach determines the most central classes better.

Dependence of the mean value of b on centrality

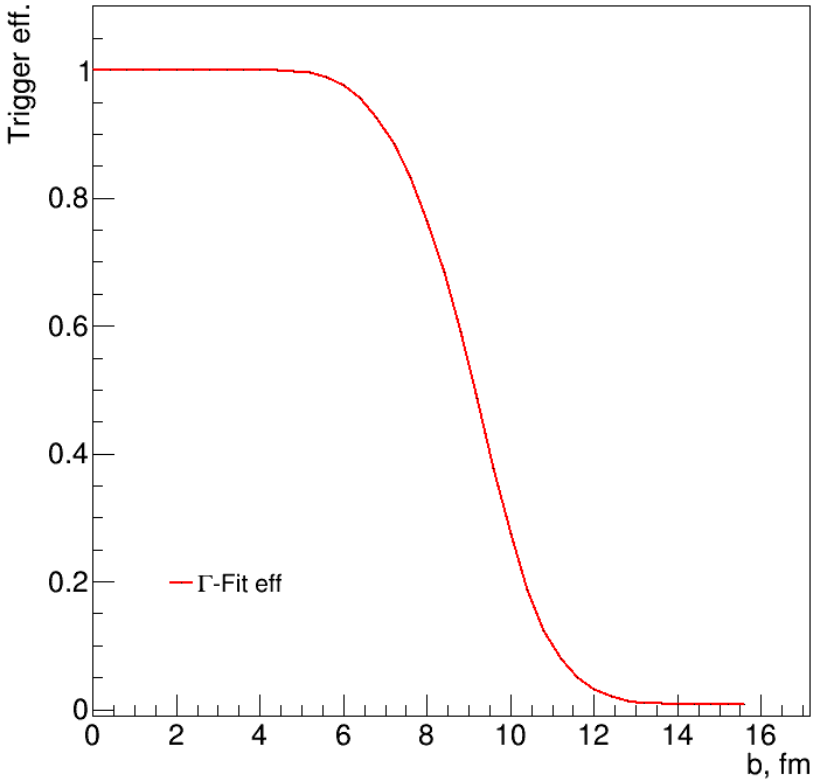


Fit results: experimental data



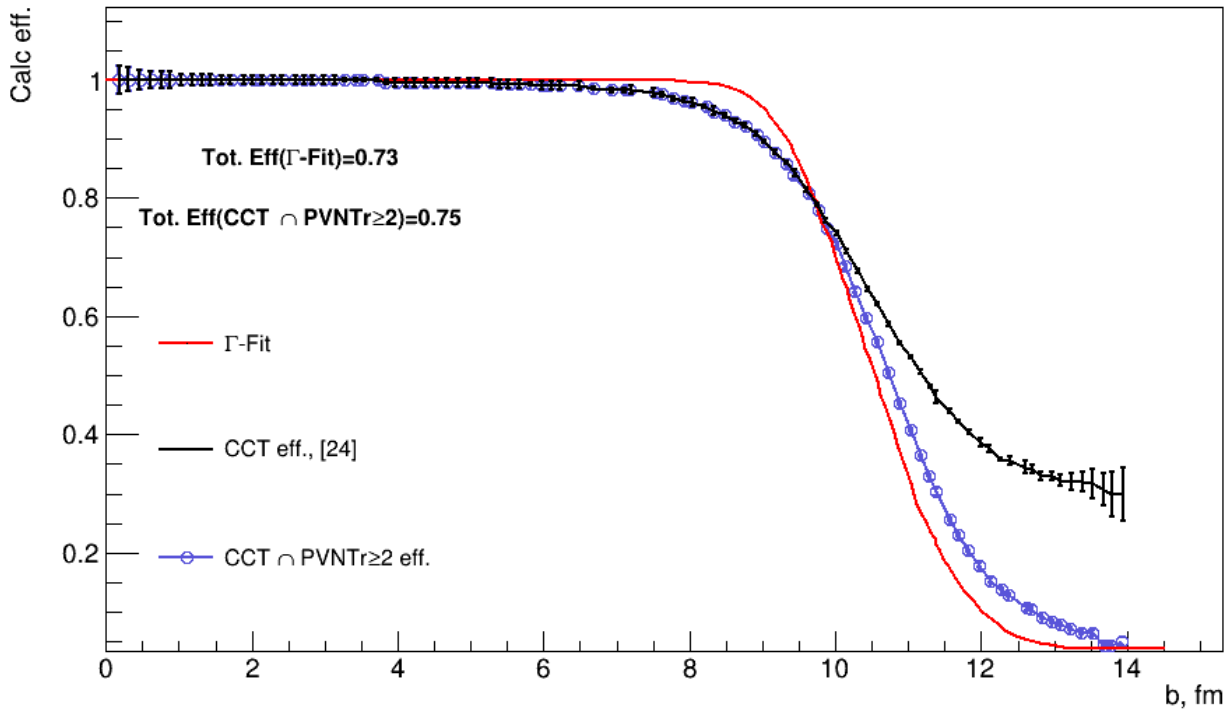
Good agreement with fit

$$P_{eff}(b) = \int_0^{M_{max}} P_{eff}(M) P(M \mid b) dM$$



Convoluted trigger efficiency can be calculated using Bayes' theorem

Comparison trigger efficiency



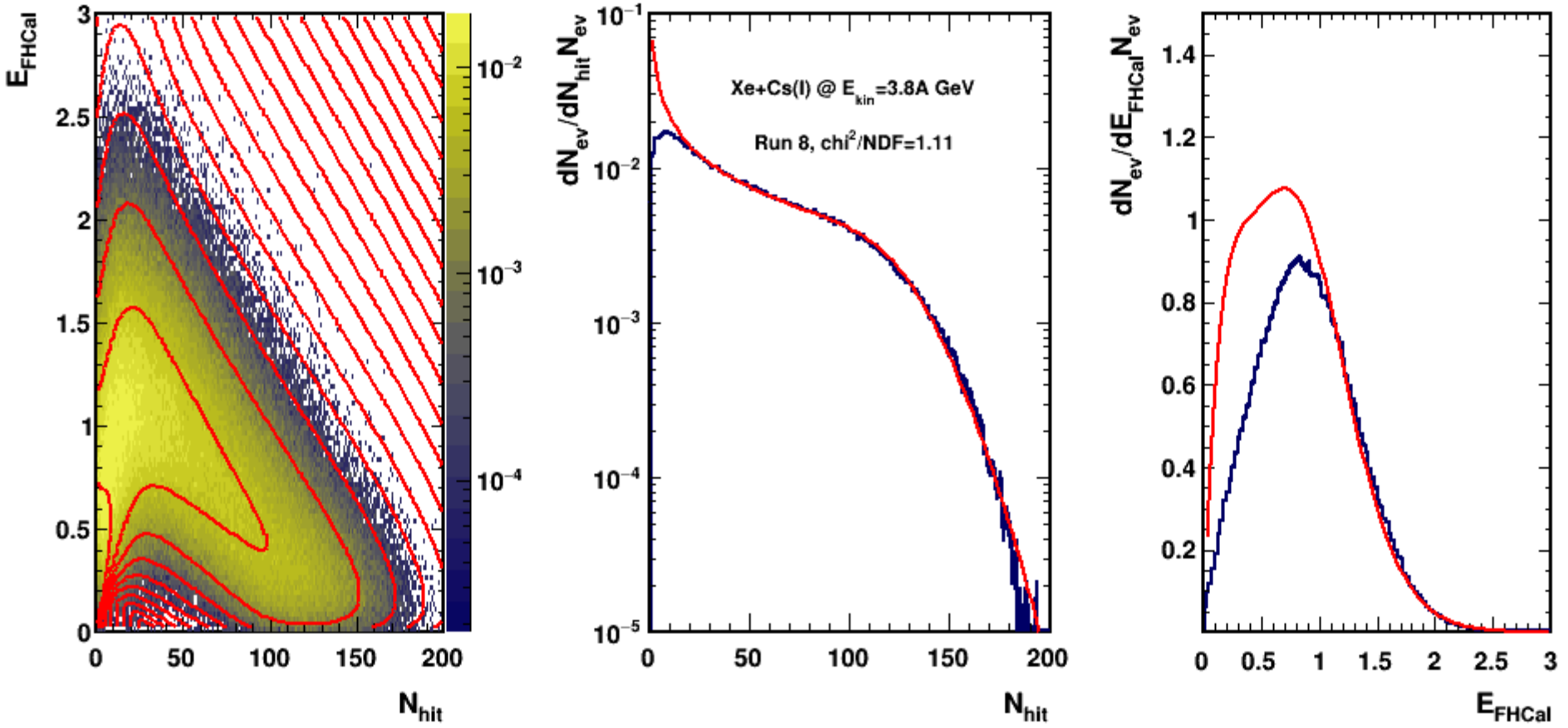
$$\varepsilon_{Plot}(CCT) \simeq \frac{N(CCT \wedge Ntr \geq 2)}{N(Ntr \geq 2)}$$

$$\varepsilon(CCT \wedge Ntr \geq 2) = \frac{N(CCT \wedge Ntr \geq 2)}{N(Ntr \geq 2)} \cdot \frac{N(Ntr \geq 2)}{N_{inel}}$$

$$\varepsilon_{\Gamma-\text{fit}}(CCT \wedge Ntr \geq 2) = \frac{N(CCT \wedge Ntr \geq 2)}{N_{inel}}$$

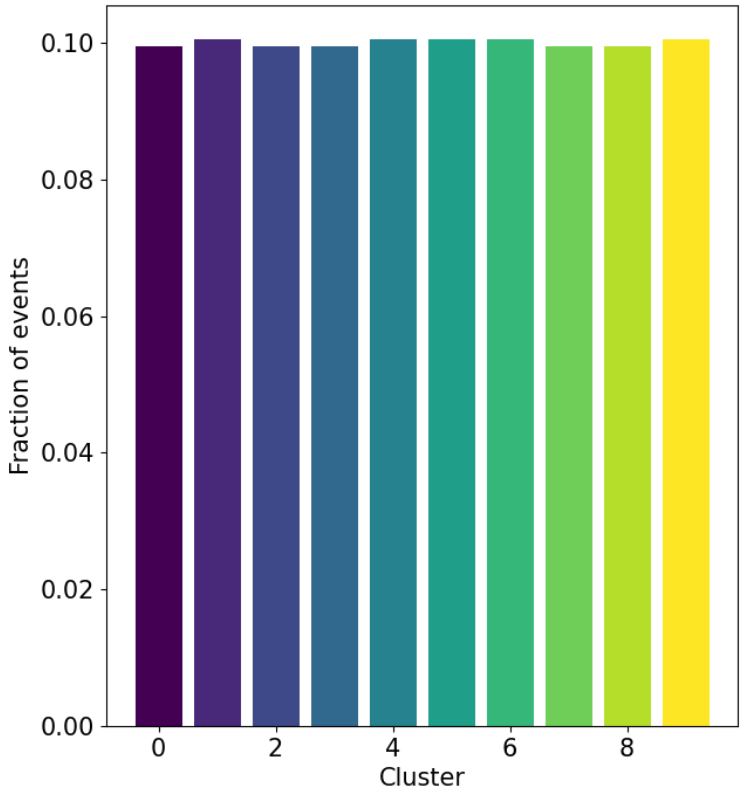
The efficiency of the CCT2 trigger in the experiment is calculated relative to events with PVntr ≥ 2.
For a correct comparison, it is necessary to take into account the registration efficiency of events with PVntr ≥ 2.

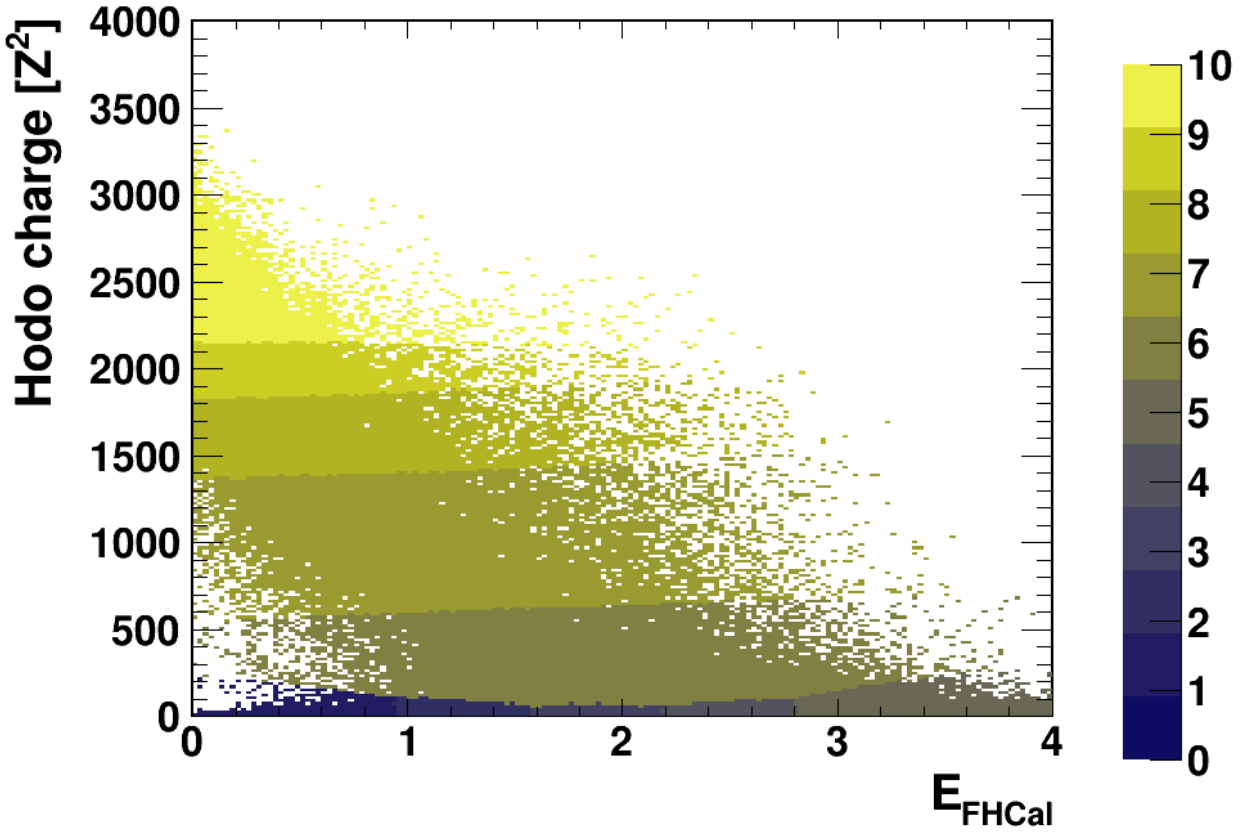
2D fit results



The fit function qualitatively reproduces the multiplicity-energy correlation from FHCal

Centrality determination using an forward calorimeter and hodoscope





The K-means method allows to divide a two-dimensional distribution into centrality classes.
In order to correctly apply the class boundaries, it is necessary to match the simulation results with the experiment

How to use 2D centrality with FHCal and hodoscope

```
    double hodo_sum = 0, En_sum = 0;
    for (int strip_id = 1; strip_id < 17; ++strip_id)
    {
        double val = HodoEv->GetStrip(strip_id)->GetSignal();
        hodo_sum += val;
    }
    for (int i = 1; i < 55; i++)
    {
        En = FHCalEv->GetModule(i)->GetEnergy() * 0.005;

        if (En < 0.03)
            continue;
        if (i==9 || i == 12 || i == 13 || i == 22 || i == 21 || i == 17)
            continue;
        else if (i == 49 || i == 50 || i == 51 || i == 52)
            continue;
        En_sum += En;
    }
    int binFHc = hClasses->GetXaxis()->FindBin(En_sum);
    int binHODO = hClasses->GetYaxis()->FindBin(hodo_sum);
    int CentClass = hClasses->GetBinContent(binFHc, binHODO) - 1; // 0-(0%-10%);1-(10%-20%);
//2-(20%-30%)..(9-(90%-100%))
    if (CentClass < 0)
        continue;
```

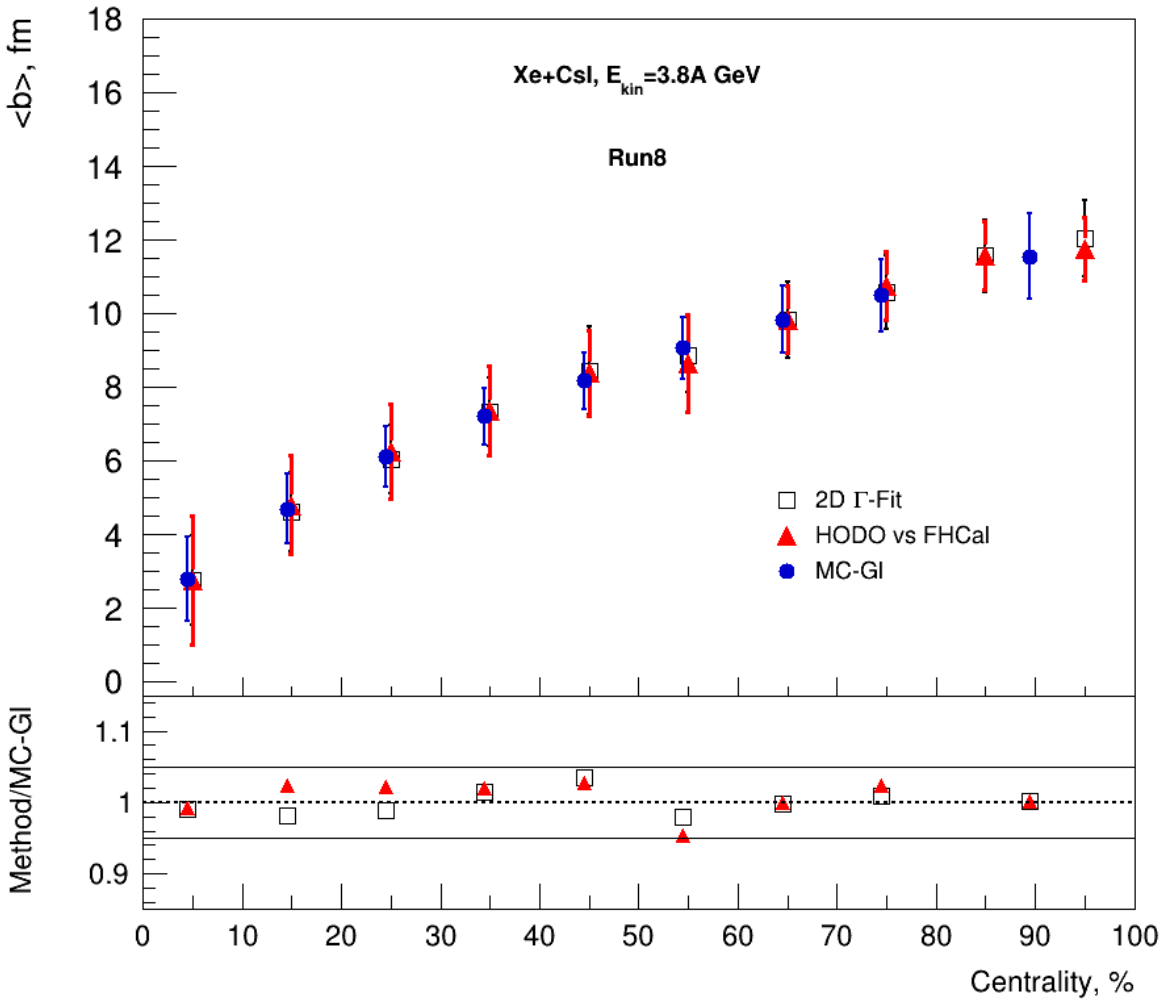
https://gitlab.com/idrisovdim13/2dcenthodoscpvsfhcal.git

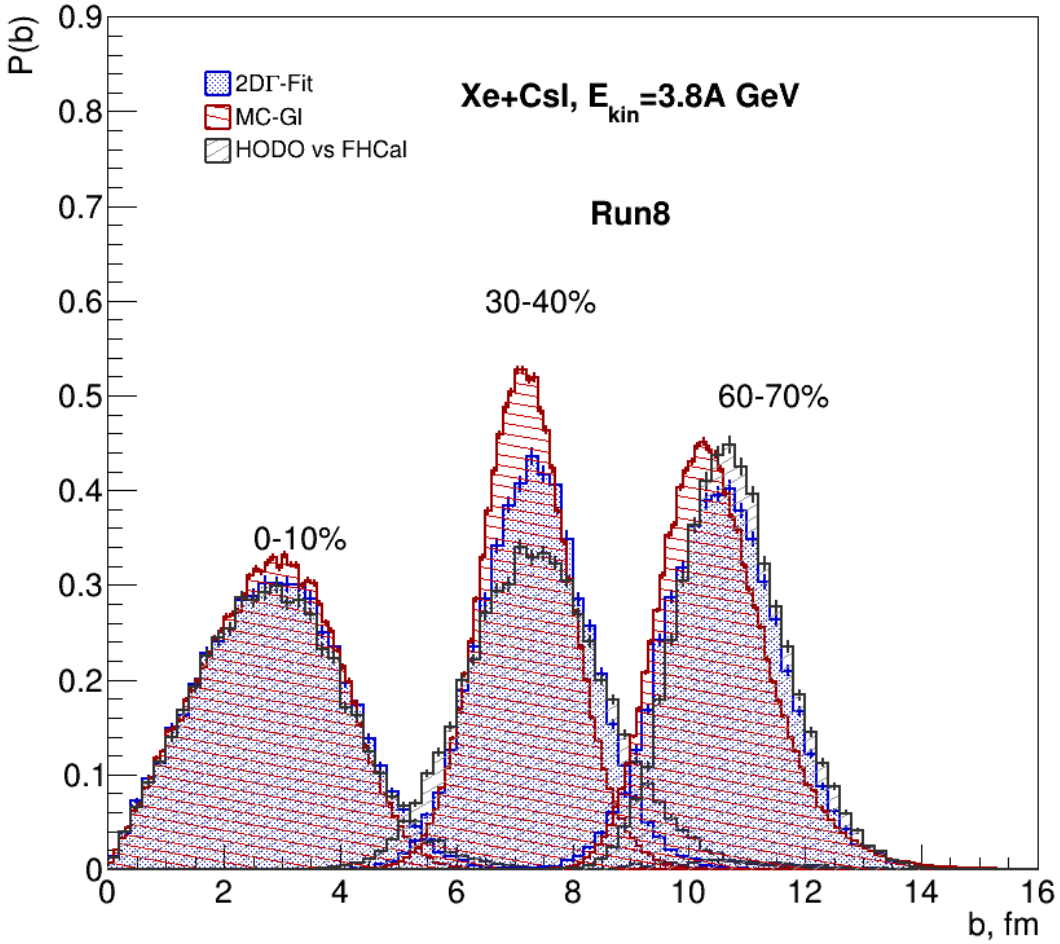
Summary and Outlook

- The procedure for estimating the calculated efficiency was validated on the DCM-QGSM-SMM model. The approach was applied to experimental data, and the obtained values are consistent with independent estimates.
- The Bayesian inversion method was applied to the BM@N data:
  - Multiplicity-based and 2D approaches using Q^2_{Hodo} and E_{FHCal} describe experimental data reasonably well.
- The procedure for determining centrality classes using forward detectors for all runs is developed.
- In the future, it is planned to study systematics uncertainties using different models (DCM, UrQMD, etc.) and observables (GEM hit multiplicity, etc.)

Thank you for your attention!

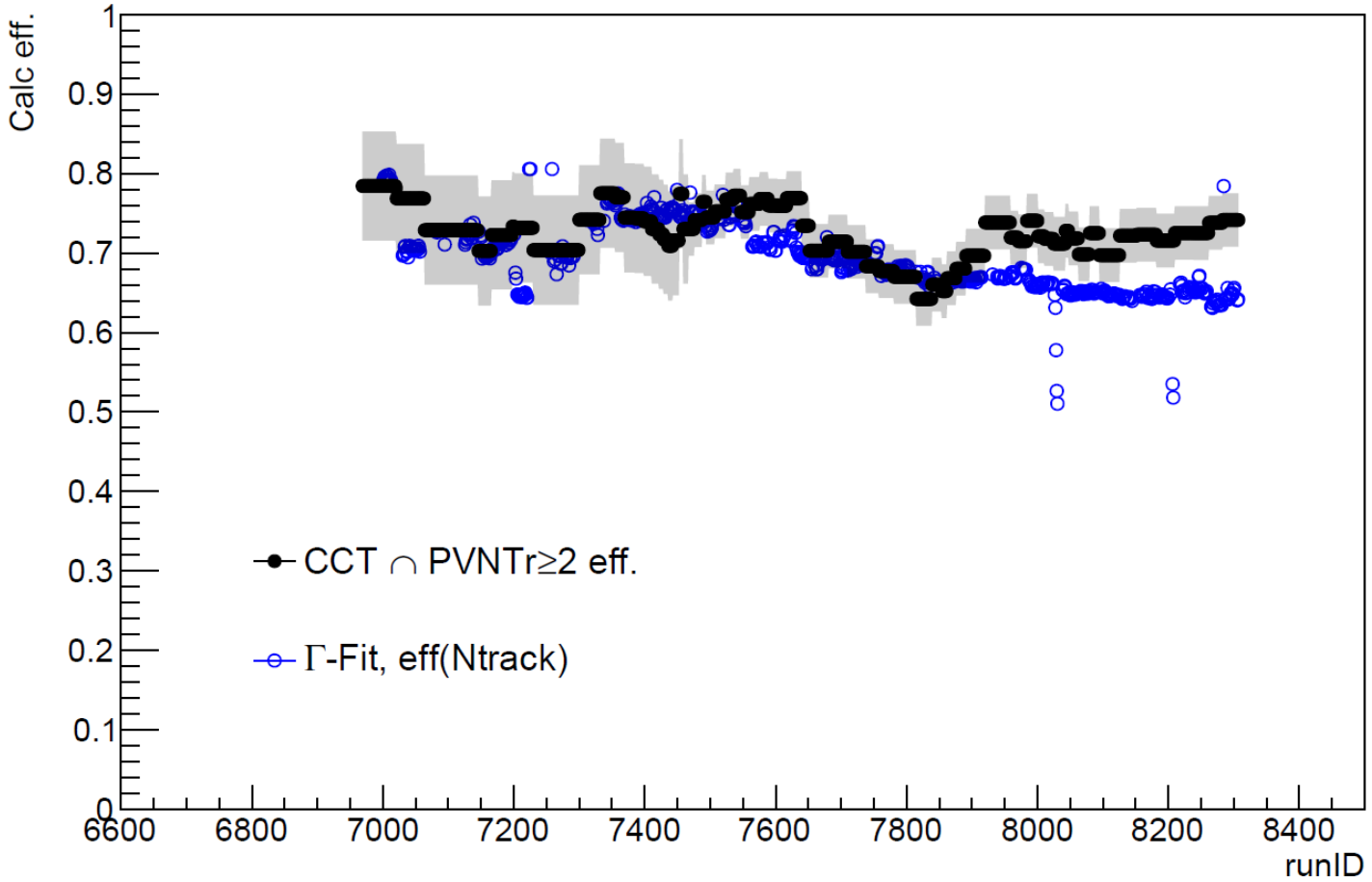
Comparison with Glauber MC fit





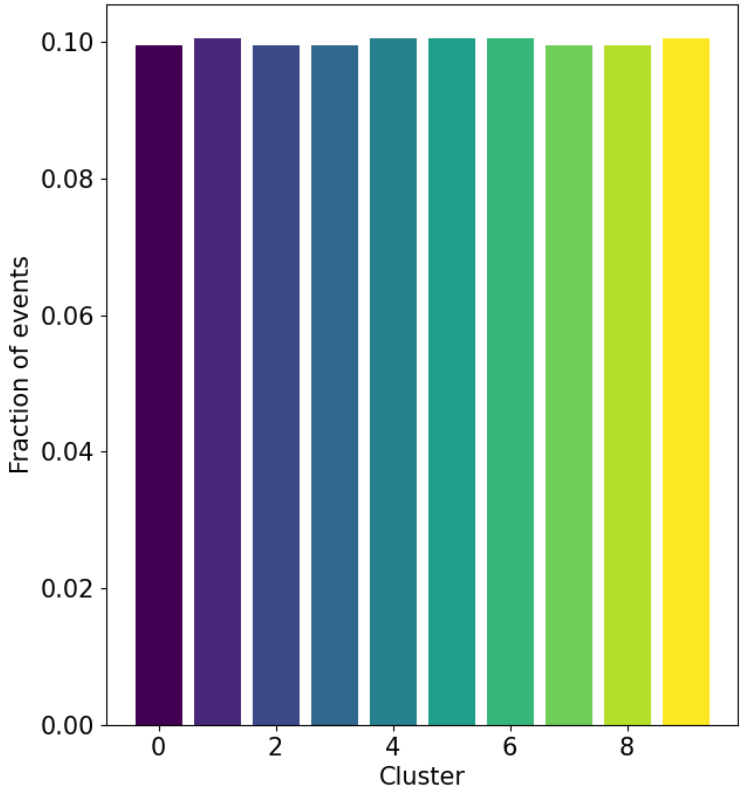
There is agreement within 5%.

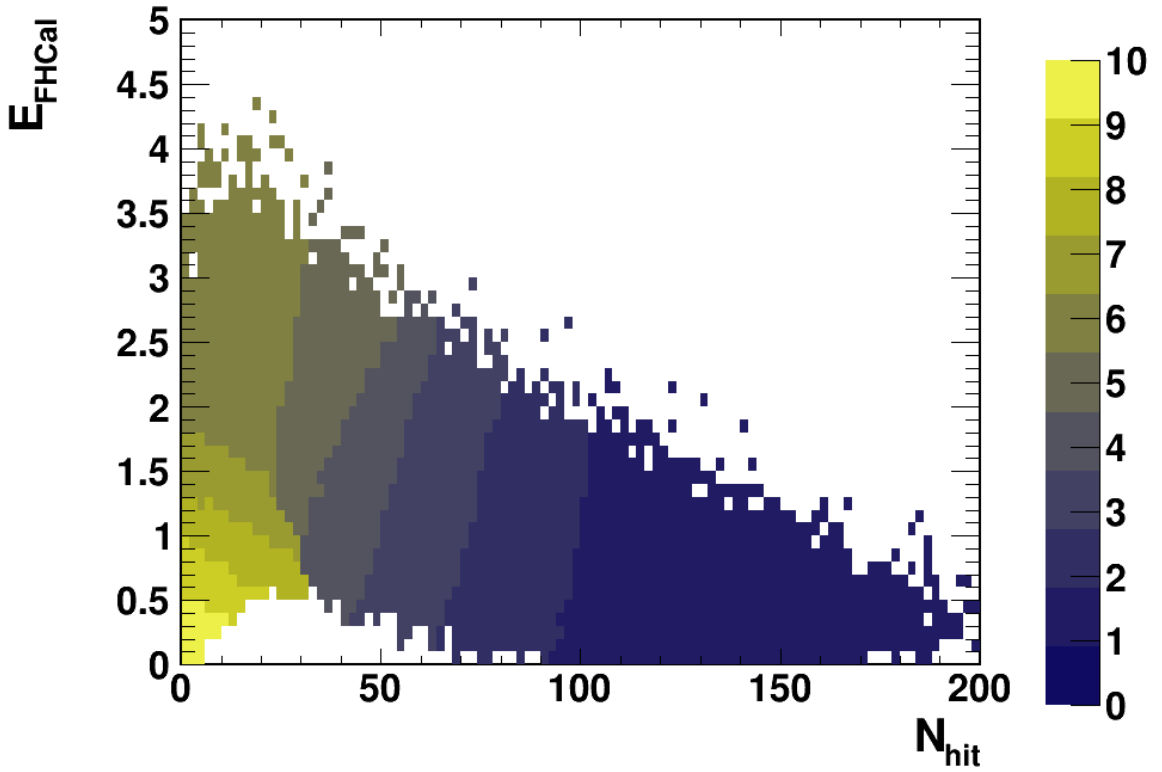
Comparison trigger efficiency: 2



The efficiency of the CCT2 trigger in the experiment is calculated relative to events with PVntr ≥ 2.
For a correct comparison, it is necessary to take into account the registration efficiency of events with PVntr ≥ 2.

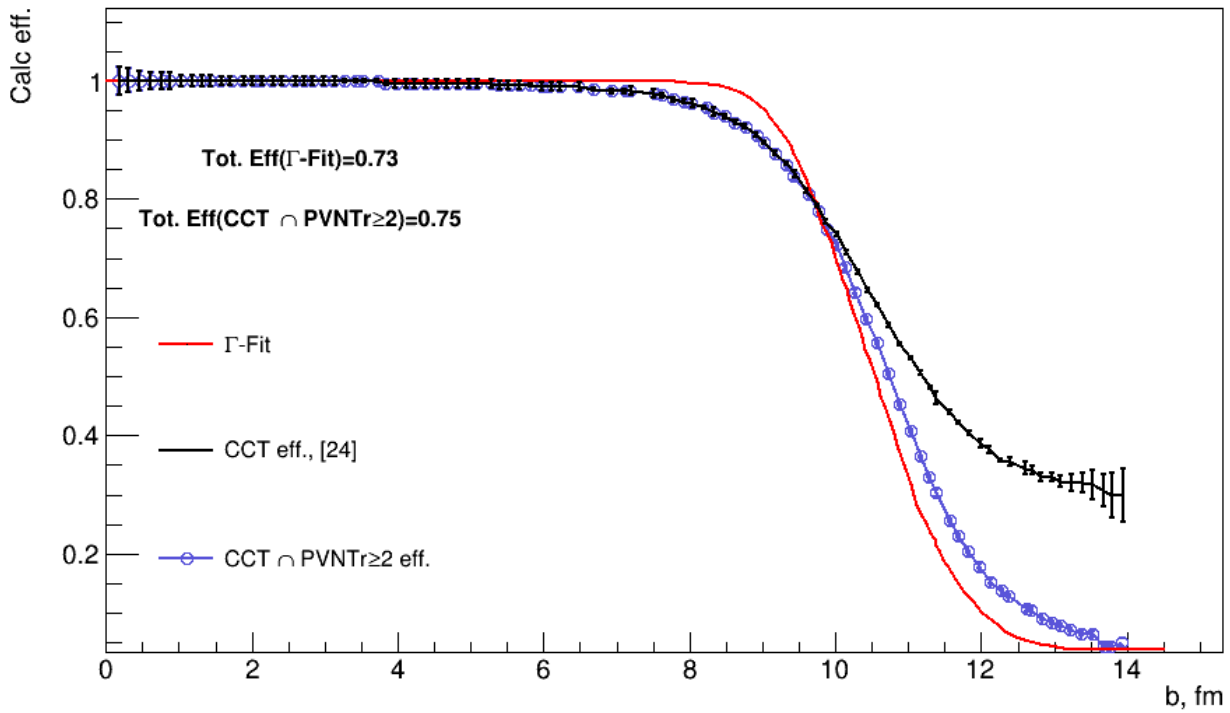
Centrality determination using forward calorimeter andNhit





The K-means method allows to divide a two-dimensional distribution into centrality classes.
In order to correctly apply the class boundaries, it is necessary to match the simulation results with the experiment

Comparison trigger efficiency



$$\varepsilon(CCT \wedge Ntr \geq 2) = \frac{N(CCT \wedge Ntr \geq 2)}{N_{inel}} =$$

$$= \frac{N(Ntr \geq 2)}{N_{inel}} \cdot \frac{N(CCT \wedge Ntr \geq 2)}{N(Ntr \geq 2)}$$

$$\varepsilon(CCT) = \frac{N(BD \geq 4 \wedge MBT \wedge Ntr \geq 2)}{N(MBT \wedge Ntr \geq 2)} \cdot \frac{N(FD \wedge CCT1 \wedge Ntr \geq 2)}{N(CCT1 \wedge Ntr \geq 2)}$$

$$\varepsilon(CCT) = \frac{N(CCT \wedge Ntr \geq 2)}{N(Ntr \geq 2)},$$

$$N(MBT) \simeq N_{inel}, N(CCT1) \simeq N(BD \geq 4 \wedge MBT)$$

The efficiency of the CCT2 trigger in the experiment is calculated relative to events with PVntr ≥ 2.
For a correct comparison, it is necessary to take into account the registration efficiency of events with PVntr ≥ 2.