Gravitation and Dark Matter

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March 20, , Dubna, 2026

Motivation for Our Study

The idea of constructing a theory of gravitation on the basis of field concept was put forward by Einstein and Grossmann in 1913 and the tensor gravitational field was introduced. Subsequent research by Einstein has been actually aimed at finding a field that could be carrier of gravitational charge in some way analogous to

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the electric charge. The search for a purely field theory of gravity continued until 1955 and was not completed. The idea of gauge or internal symmetry, first proposed by Weyl in 1918 on the basis of the general covariant concepts of Einstein's theory, gained fundamental importance in the Standard Model, but did not find heuristic application in general relativity itself. The internal symmetry transformations are inseparable from field representations, since they concern field functions and do not affect coordinates. Thus, internal symmetry is related with phenomena that cannot be described based on the classical concepts, for example, the concept of trajectory and, hence, point particle.
The guiding idea of purely field theory of gravity and electricity.
Basic concepts and equations of field-theoretical theory

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of electricity were formulated in the works:IP, TMP (1978); IP, Horizons of World Physics, Vol. 302, Chapter 4 (2020).

Fundamental Fields and Symmetries

There are three selection rules of unified physics: general covariance, reparametrization symmetry (R-symmetry) and concept of natural derivative. General covariance means that to formulate laws of Nature it is possible to use only fields with known laws of transformation under the transition from initial cartesian system of coordinates to any thinkable or general one. Example: GEF/
Main fundamental field of unified physics consists from

$$g_{ij}(x^1, x^2, x^3, x^4) = g_{ij}(x)$$

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consists from ten functions of four independent variables x^1, x^2, x^3, x^4. Space of these variables x^1, x^2, x^3, x^4, is denoted R^4 and is called space of references and variables x^1, x^2, x^3, x^4, are called cartesian coordinates,

$$-\infty < x^i < \infty.$$

A point of R^4 is set of four numbers

$$x = (x^1, x^2, x^3, x^4).$$

Potential of the gravitational electromagnetic field (GEF) and tensor of this field

$$P^i_{jk}, \quad H_{ijl}{}^k = \partial_i P^k_{jl} - \partial_j P^k_{il} + P^k_{in} P^n_{jl} - P^k_{jn} P^n_{il}.$$

Gravion field

$$V^j(x), \quad U_i(x).$$

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is field that carries gravitational charge and can be considered as fundamental representation of group of gauge symmetry of the gravitational physics.

$$\overline{V}^i(x) = S^i_j(x) V^j(x), \quad \overline{U}_i(x) = S^{-1}{}^j_i(x) U_j(x),$$

A product of two elements of this group is defined as follows

$$P^i_j(x) = S^i_k(x) T^k_j(x)$$

and

$$S^{-1}{}^j_i(x)$$

is inverse with respect to $S^j_i(x)$.
Law of transformation GEF

$$\overline{P}^i_{jk} = S^i_m P^m_{jn} S^{-1}{}^n_k + S^i_m \partial_j S^{-1}{}^m_k.$$

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Gauge covariant derivative of V^j and U_i

$$\mathrm{D}_j V^i = \partial_j V^i + P^i_{jk} V^k, \quad \mathrm{D}_j U_i = \partial_j U_i - P^k_{ji} U_k.$$

transforms like this fields

$$\overline{\mathrm{D}}_j \overline{V}^i = S^i_k \mathrm{D}_j V^k, \quad \overline{\mathrm{D}}_j \overline{U}_i = S^{-1}{}^k_i \mathrm{D}_j U_k.$$

Basic Equations

We introduce two fundamental gauge invariant covector fields and one scalar field putting

$$\xi_i = U_l \mathrm{D}_i V^l, \quad \eta_i = V^l \mathrm{D}_i U_l, \quad \varphi = U_l V^l.$$

We have $\xi_i + \eta_i = \partial_i \varphi$. Now we establish general covariant and gauge invariant Lagrangian for our general covariant gravion field (U_i, V^i) and derive equations for

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this field. Equations for the general covariant gauge field P^i_{jk} will be established in the next section. We put

$$\mathcal{L}_C = (\frac{1}{2}\xi_i\xi^i + \frac{1}{2}\eta_i\eta^i - \frac{1}{2}\mu^2\varphi^2),$$

where $\xi^i = g^{ij}\xi_j$, and μ are constants of the dimension of cm^{-1}, , (U_i, V^i) have dimension $cm^{-\frac{1}{2}}$ and action are dimensionless. Varying with respect to U_i and V^j we derive the following system of equations

$$\zeta^i \mathrm{D}_i U_l + (\nabla_i \xi^i + \mu^2\varphi)U_l = 0, \quad \zeta^i = \xi^i - \eta^i \tag{1}$$

$$\zeta^i \mathrm{D}_i V^j - (\nabla_i \eta^i + \mu^2\varphi)V^j = 0, \tag{2}$$

where ∇_i is covariant derivative with respect to connection belonging g_{ij}. From this system of equations

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it is not difficult to derive law of gravitational charge conservation

$$\nabla_i(\zeta^i\varphi) = 0, \tag{3}$$

$$\mathrm{D}_i(\sqrt{g}\zeta^i U_l V^j) = 0, \quad \sqrt{g} = -\mathrm{Det}(g_{ij}). \tag{4}$$

The antisymmetric tensor $F_{ij} = H_{ijk}{}^k$ (trace of the curvature tensor)

$$F_{ij} = \partial_i P^k_{jk} - \partial_j P^k_{ik}$$

is nontrivial here and should be considered separately from the irreducible (traceless) strength tensor

$$I_{ijl}{}^k = H_{ijl}{}^k - \frac{1}{4} H_{ijn}{}^n \delta^k_l, \quad I_{ijl}{}^l = 0.$$

It is evident that some indices have a double purpose. In view of this, in what follows we will use the natural

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matrix notation

$$\mathbf{S} = (S^k_l), \quad \mathbf{S}^{-1} = (S^{-1}{}^k_l), \quad \mathbf{P}_i = (P^k_{il}), \quad \mathbf{E} = (\delta^k_l),$$

$$\mathbf{H}_{ij} = (H_{ijl}{}^k), \mathrm{Tr}\,\mathbf{H}_{ij} = H_{ijk}{}^k, \mathbf{H}_{ij} = \partial_i \mathbf{P}_j - \partial_j \mathbf{P}_i + [\mathbf{P}_i, \mathbf{P}_j],$$

which allocates these indices.We also consider covector field U_i as matrix-row, vector field V^i as matrix-column, there product $U_l V^j$ as matrix and put

$$\mathbf{U} = (U_i), \quad \mathbf{V} = (V^i), \quad \mathbf{W} = (U_l V^j).$$

The transformations of the general covariant gauge field and the other fields take the simple form

$$\overline{\mathbf{P}}_i = \mathbf{S}\mathbf{P}_i\mathbf{S}^{-1} + \mathbf{S}\partial_i\mathbf{S}^{-1} = \mathbf{P}_i + \mathbf{S}\mathrm{D}_i\mathbf{S}^{-1}, \quad \overline{\mathbf{H}}_{ij} = \mathbf{S}\mathbf{H}_{ij}\mathbf{S}^{-1},$$

$$\overline{\mathrm{D}}_i\overline{\mathbf{H}}_{jk} = \mathbf{S}\mathrm{D}_i\mathbf{H}_{jk}\mathbf{S}^{-1}, \overline{\mathbf{U}} = \mathbf{U}\mathbf{S}^{-1}, \overline{\mathbf{V}} = \mathbf{V}\mathbf{S},$$

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where D_i is the gauge covariant derivative defined above and associated with general covariance and general covariant gauge symmetry, $\mathrm{D}_i\mathbf{S} = \partial_i\mathbf{S} + \mathbf{P}_i\mathbf{S} - \mathbf{S}\mathbf{P}_i = \partial_i\mathbf{S} + [\mathbf{P}_i, \mathbf{S}]$ is the tensor field, $\mathrm{D}_i\mathbf{H}_{jk} = \partial_i\mathbf{H}_{jk} + [\mathbf{P}_i, \mathbf{H}_{jk}]$ is not the tensor field but $\mathrm{D}_i\mathbf{H}_{jk} + \mathrm{D}_j\mathbf{H}_{ki} + \mathrm{D}_k\mathbf{H}_{ij}$ is the tensor field and the identity $\mathrm{D}_i\mathbf{H}_{jk} + \mathrm{D}_j\mathbf{H}_{ki} + \mathrm{D}_k\mathbf{H}_{ij} = 0$ is general covariant. To allocate the physical degrees of freedom of the GEF and disclose a mechanism of emergence of mass of this field, we introduce the important notion of the ground state. The ground state of the general covariant gauge field is defined as a solution of the equation $\mathbf{H}_{ij} = 0$. Let four linear independent vector fields E^i_μ be given. With this, one can construct purely algebraical components of the four covector fields E^μ_i, so that $E^i_\mu E^\mu_j = \delta^i_j$ holds valid. Setting $P^k_{il} = L^k_{il}$, where $L^k_{il} = E^k_\mu \partial_i E^\mu_l$ is a linear

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connection of the ground state, we get a general solution of the equation $\mathbf{H}_{ij} = 0$.
Thus, we can define the ground state as any quadruplet of linear independent vector fields E^i_μ associated with the connection $L^k_{il} = E^k_\mu \partial_i E^\mu_l$. The ground state is invariant under the general covariant gauge transformations. Indeed, if the quadruplet of vector fields E^i_μ represents the ground state, then $\overline{E}^i_\mu = S^i_j E^j_\mu$ is the ground state as well since $\overline{\mathbf{L}}_i = \mathbf{S}\mathbf{L}_i\mathbf{S}^{-1} + \mathbf{S}\partial_i\mathbf{S}^{-1}$.
A transition from the ground state to the excited one is characterized by the tensor of transition

$$T^i_{jk} = P^i_{jk} - L^i_{jk},$$

with a simple (homogeneous) law of transformation

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$\overline{\mathbf{T}}_i = \mathbf{S}\mathbf{T}_i\mathbf{S}^{-1}$ and the irreducible tensor

$$\mathbf{Q}_j = \mathbf{T}_j - \frac{1}{4}(\mathrm{Tr}\,\mathbf{T}_j)\,\mathbf{E}$$

with the trivial trace $\mathrm{Tr}\,\mathbf{Q}_j = 0$.
To derive equations of GEF we establish the Lagrangian putting

$$\mathcal{L}_P = -\frac{1}{4}\mathrm{Tr}(\mathbf{I}_{ij}\mathbf{I}^{ij}) + \frac{\mu^2}{2}\mathrm{Tr}\,(\mathbf{Q}_i\mathbf{Q}^i), \quad \mathcal{L}_{em} = -\frac{1}{4}F_{ij}F^{ij},$$

where $\mathbf{I}^{ij} = g^{ik}g^{jl}\mathbf{I}_{kl}$, $\mathbf{Q}^i = g^{ik}\mathbf{Q}_k$, μ is a constant of the dimension of cm^{-1}. Dimension of P^i_{jk} is equal to cm^{-1} and the action is dimensionless. The general covariant and gauge invariant Lagrangian $\mathcal{L}$ of all fields in question takes the form $\mathcal{L} = \mathcal{L}_P + \mathcal{L}_{em} + \mathcal{L}_s + \mathcal{L}_g$, where $\mathcal{L}_g = \frac{{l_g}^2}{2}R$ is the Lagrangian of the gravitational

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field and l_g is a constant of the dimension of cm^{-1}. By varying the Lagrangian $\mathcal{L}$ with respect to $\mathbf{P}_i$, the following equation holds

$$\frac{1}{\sqrt{g}}\mathrm{D}_i(\sqrt{g}\mathbf{I}^{ij})+\mu^2\mathbf{Q}^j+\frac{1}{\sqrt{g}}\partial_i(\sqrt{g}F^{ij})\,\mathbf{E}+\zeta^j\,\mathbf{W}=0. \quad (5)$$

Taking trace from equation (5) , we find that

$$\frac{1}{\sqrt{g}}\partial_i(\sqrt{g}F^{ij})+\frac{1}{4}\varphi\zeta^j=0,$$

since $\mathrm{Tr}\,\mathbf{I}^{\mathrm{ij}}=\mathrm{Tr}\,\mathbf{Q}^{\mathrm{j}}=0$. Hence,

$$\frac{1}{\sqrt{g}}\mathrm{D}_i(\sqrt{g}\mathbf{I}^{ij})+\mu^2\mathbf{Q}^j+\zeta^j(\mathbf{W}-\frac{1}{4}(\mathrm{Tr}\mathbf{W})\,\mathbf{E})=0. \quad (6)$$

From it follows that $\mathbf{Q}^i$ has to satisfy the equation $\mathrm{D}_i(\sqrt{g}\mathbf{Q}^i)=0$, because $\mathrm{D}_i\mathrm{D}_j(\sqrt{g}\mathbf{I}^{ij})=0$, $\mathrm{D}_i(\sqrt{g}\zeta^i\mathbf{W})=0$ and $\partial_i(\sqrt{g}\zeta^i\varphi)=0$.

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To make a clear and apparent conclusion from these results, we first of all mention that the strength tensor $\mathbf{I}_{ij}$ can be written in terms of the irreducible tensor $\mathbf{Q}_i$ only, since

$$\mathbf{I}_{ij} = \overset{o}{\mathrm{D}}_i \mathbf{Q}_j - \overset{o}{\mathrm{D}}_j \mathbf{Q}_i + \mathbf{Q}_i\mathbf{Q}_j - \mathbf{Q}_j\mathbf{Q}_i.$$

Here $\overset{o}{\mathrm{D}}_i$ denotes the gauge covariant derivative with respect to the connection $\mathbf{L}_i$ of the ground state and, hence, $[\overset{o}{\mathrm{D}}_i, \overset{o}{\mathrm{D}}_j] = 0$. For the antisymmetric tensor F_{ij} we obtain

$$F_{ij} = \partial_i P^k_{jk} - \partial_j P^k_{ik} = \partial_i(L^k_{jk} + T^k_{jk}) - \partial_j(L^k_{ik} + T^k_{ik}) = \partial_i T^k_{jk} - \partial$$

since $\mathrm{Tr}\mathbf{L}_i = \partial_i \ln|p|, \quad p = \mathrm{Det}(E^\mu_i)$. Thus, we can consider the tensor field Q^k_{il} with the constraints $Q^k_{ik} = 0$ and covariant vector field $A_i = T^k_{ik}$ as

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independent quantities, which obey the equations

$$\frac{1}{\sqrt{g}}\partial_i(\sqrt{g}F^{ij}) + \frac{1}{4}\varphi\zeta^j = 0, \quad F_{ij} = \partial_i A_j - \partial_j A_i,$$

$$\frac{1}{\sqrt{g}}\overset{o}{\mathrm{D}}_i(\sqrt{g}\mathbf{I}^{ij}) + [\mathbf{Q}_i, \mathbf{I}^{ij}] + \mu^2\mathbf{Q}^j + \zeta^j(\mathbf{W} - \frac{1}{4}(\mathrm{Tr}\mathbf{W})\,\mathbf{E}) = 0.$$

We conclude that Einstein equation of takes the form

$$l_g{}^2 G_{ij} + T_{ij} = 0,$$

where $T_{ij} = P_{ij} + E_{ij} + S_{ij}$ and

$$P_{ij} = -\mathrm{Tr}\,(\mathbf{I}_{ik}\mathbf{I}_j{}^k) - g_{ij}\mathcal{L}_P + \mu^2\mathrm{Tr}\,(\mathbf{Q}_i\mathbf{Q}_j)$$

is the energy-momentum tensor of the K-field, $\mathbf{I}_j{}^k = \mathbf{I}_{jl}g^{kl}$,

$$E_{ij} = -F_{il}F_j{}^l - g_{ij}\mathcal{L}_{em}, \quad S_{ij} = l_s^2(\xi_i\xi_j + \eta_i\eta_j) - g_{ij}\mathcal{L}_s$$

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are correspondingly the energy-momentum tensor of the electromagnetic field and the energy-momentum tensor of the gauge covariant bispinor field (U_i, V^j). It is evident that the tensors P_{ij}, S_{ij}, E_{ij} are invariant under the transformations of the general covariant gauge group. The mass term $\mu^2 \mathrm{Tr}\,(\mathbf{Q}_i \mathbf{Q}^i)$ is obtained by means which do not violate the general covariant gauge symmetry, and this is an important point for the renormalizability of the theory.

From our consideration it follows that the ground state E^i_μ represents nonphysical degrees of freedom and equation $S^i_j E^j_\mu = \delta^i_\mu$ is evident and natural gauge fixing. Thus, three fields in question form one whole with the gravitational field and represent the very fabric of the gravitational physics.

Now our goal is to establish the physical meaning of the

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constants l_g, μ. We can not to attribute $\mathbf{P}_i$ dimension different from acceptant, since in this case the tensor of this field

$$\mathbf{H}_{ij} = \partial_i \mathbf{P}_j - \partial_j \mathbf{P}_i + [\mathbf{P}_i, \mathbf{P}_j]$$

have not any sense. This observation we consider as sufficient founding to introduce gravitational Planck constant $\hbar_g$ and define parameter μ as number inverse to the Compton length

$$\mu = \frac{m_g c}{h_g},$$

where parameter m_g is mass of gravion and graviphoton. Let us introduce gravitational charge and coupling constant of the gravitational physics α_g simply putting

$$e_g = m_g \sqrt{G}, \quad \alpha_g = \frac{{e_g}^2}{\hbar_g c}.$$

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Hence,we can put

$$l_g{}^2 = \frac{c^4}{e_g{}^2 G} = \frac{\mu^2}{\alpha_g{}^2}.$$

To estimate numerical significance of m_g let us consider ratio $\frac{e^2}{e_g^2}$. Since

$$\frac{e^2}{e_g^2} = \frac{\alpha}{\alpha_g}\frac{\hbar}{\hbar_g},$$

then we can put

$$\frac{\hbar}{\hbar_g} = \alpha_g$$

and, hence,

$$\frac{e^2}{e_g^2} = \alpha.$$

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From this it follows that m_g is equal Planck mass

$$m_g = \sqrt{\frac{\hbar c}{G}} = 10^{-5}$$

gram.
Let us estimate now ration

$$\frac{\hbar}{\hbar_g} = \alpha_g.$$

The size of atomic nuclei is order 10^{-13} cm. The ratio of the size of nuclei and Planck length is equal 10^{20}. The size of the galactic nuclei is of the order 10^{15} cm. Let as accept that ratio of this size to the gravitational Planck length is also equal 10^{20}. Then gravitational Planck length will have order 10^{-5} cm and

$$\frac{\hbar}{\hbar_g} = 10^{-56}.$$

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From the formula

$$\hbar_g \mathcal{L}_g + \hbar \mathcal{L}_G$$

it follows that the matter with spin does not actually produce the the gravity.

Conclusion

Whilst the need for invisible matter was established almost a century ago, only its gravitational interaction has been confirmed so far. This is a very important observation, since it supports our evident statement that the the theory of this matter is theory of gravity.

THANK YOU FOR ATTENTION