SPIN EFFECTS RELATED TO ACCELERATION

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Outline

Rindler space is constructed to accomodate acceleration and seems to be a natural framework

1. Rindler space
2. Rindler space with conical singularity
3. Global structure of Rindler space
4. Miscellaneous

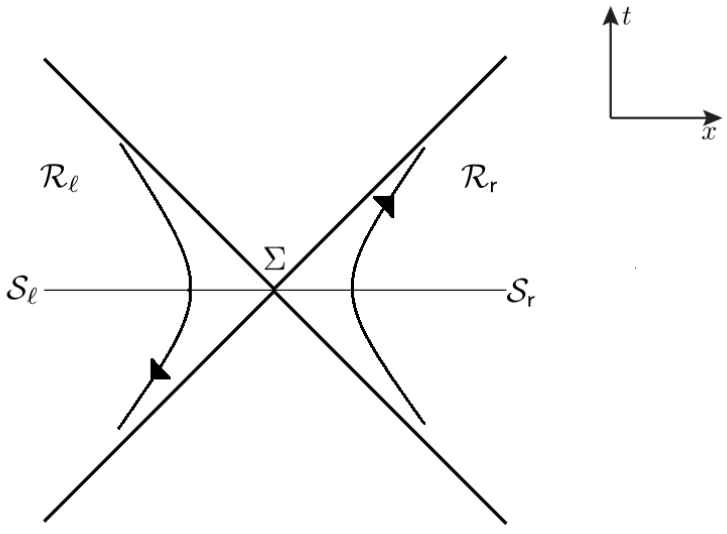
Mostly massless particles considered and " effects" are not of immediate interest for experiment

Practically no reference given

From trivial geometry to understanding physics

One starts with trajectories of particles moving with permanent acceleration (Landau & Lifshitz vol 2)
and ends up soon with basing features of Rindler space

Minkowski and Rindler coordinates

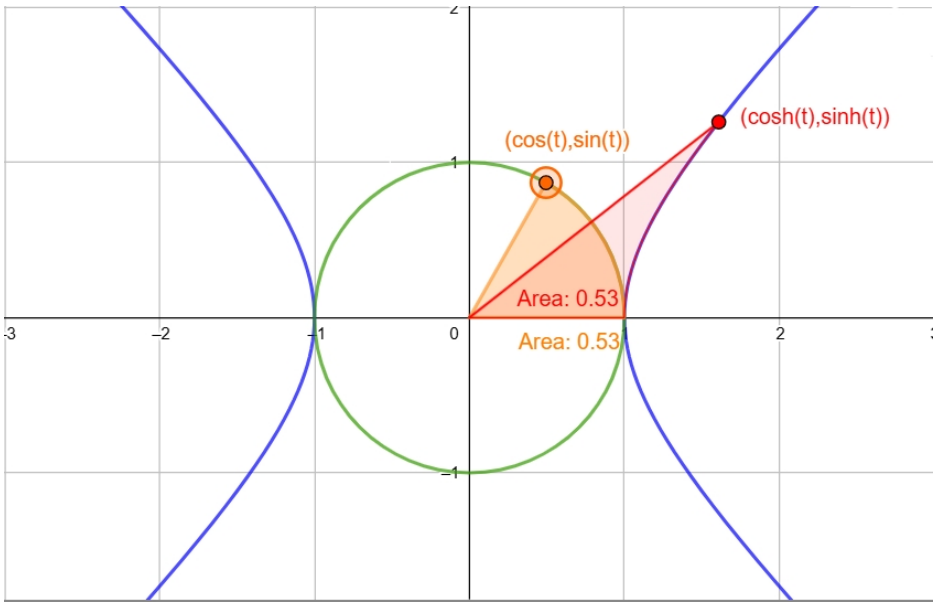


For the right, left Rindler wedge
(a is acceleration τ is proper time): $at\ t = 0, \quad x = \pm\frac{1}{a}$

$$t = \pm\rho\sinh(\eta), \qquad x = \pm\rho\cosh(\eta)$$
$$ds^2 = -\rho^2 d\eta^2 + d\rho^2, \quad \rho = 1/a, \quad \eta = a\tau$$

Wick rotation, towards Unruh temperature

$$\eta \to i\eta_E, \quad \eta_E = \tau_E/\rho : \quad t = \pm\rho \sin\frac{\tau_E}{\rho}, \quad x = \pm\rho\cos\frac{\tau_E}{\rho}$$



$$ds_E^2 = \rho^2 d\phi^2 + d\rho^2, \quad \tau_E \simeq \tau_E + \frac{2\pi}{a}, \quad (\phi = 2\pi T_U \cdot \tau_E)$$

Unruh effect

The Unruh effect is the statement that an accelerated observer will perceive Minkowski vacuum as a thermal bath of particles with temperature T_U

$$T_U = \frac{a\hbar}{2\pi c k_B}$$

where T_U is the Unruh temperature, a is the proper acceleration of the observer, $\hbar$ is the reduced Planck constant, c is the speed of light, and k_B is the Boltzmann constant. Assuming a proper choice of units, $T_U = a/(2\pi)$

Horizon

Turning back to Minkowski, and adding two dimensions:

$$ds^2 = -\rho^2 d\eta^2 + d\rho^2 + dy^2 + dz^2$$

We just have changed coordinates and did not change geometry of Minkowski space so far
However, at $\rho = 0$ we have $g_{00} = 0$, that is horizon which is 2d surface in 4d space
Making cut introduces “entanglement entropy"but does not heat up the space
If we normalize the vacuum energy, due to to quantum particles living on the manifold, to zero it should remain zero after making a cut

Relation between Unruh and Casimir effects

To bring Minkowskian and Euclidean views in agrement we need cancellation between energies associated with Unruh and Casimir effects:

$$\epsilon_{unruh} + \epsilon_{casimir} = 0$$

where $\epsilon_{casimir}$ is change of vacuum energy due existence of boudary, and

$$\epsilon_{unruh} \sim T_U^4$$

.

Apparently known from calculations by Candelas and Deutsch since long but not much emphasized

Conclusion on "Rindler space"

Vacuum energies ϵ_{unruh}, and $\epsilon_{casimir}$ do depend on spin of the quantum particles, as we discuss later.

However, cancellation of energies seems the only dynamical prediction. Otherwise, the Rindler space is about quantum mechanical entanglement.
It is clearly stated, e.g., by Unruh and Weiss.

Euclidean Rindler space with conical singularity

In search for more dynamics, one introduces temperature and acceleration as independ quantities

$$ds^2 = \frac{\rho^2}{\nu^2} d\eta^2 + d\rho^2 + dx^2 + dy^2 \quad . \tag{1}$$

The corresponding manifold contains a 2D cone with angular deficit $(\delta\phi) = 2\pi(1 - \nu^{-1})$. Both temperature and acceleration are geometric, $a = \rho^{-1}, T = (2\pi\nu)/\rho \quad (\nu > 1)$.

Cnt'd

This is a somewhat awkward construction. The singularity associated with the vertex is linked to an additional action and, in a sense, corresponds to extra burners.
For this reason other attempts were made to connect Rindler space with heated Minkowski space. Extension of Rindler space is not unique

Space could be cooled down below the Unruh temperature by introducing an external gravitational field.
Essentially, continuing to inside region of black hole.

Spin dependence of vacuum energy

For conformal scalar

$$< T_0^0 >_{R,S=0} = \frac{\pi^2 T^4}{30} - \frac{a^4}{480\pi^2}$$

For massless spin-1/2:

$$< T_0^0 >_{R,S=1} = \frac{7\pi^2 T^4}{60} + \frac{T^2 a^2}{24} - \frac{17 a^4}{960\pi^2}$$

For photon:

$$< T_0^0 >_{R,S=1} = \frac{\pi^2 T^4}{15} + \frac{T^2 a^2}{24} - \frac{11 a^4}{240\pi^2}$$

Remarks on $< T_0^0 >$

–The T^4 terms are fixed by Stefan-Boltzmann law

–The a^4 terms are fixed by condition that $< T_0^0 >$ vanishes at $T = T_U$ since this temperature corresponds to the Minkowski space

– for higher spins theory is not conformal

–Thus T^2 terms, or spin-dependent terms are crucial

Simple calculations

We are looking for the spectrum of free particles with account of perturbation:

$$\delta\hat{H} = \vec{a}\hat{\vec{K}}$$

where $\vec{a}$ is acceleration and $\hat{\vec{K}}$ is generator of boosts (consisting of spin and orbital parts)
and a similar problem, much better studied, in case of rotation:

$$\delta\hat{H} = \vec{\Omega}\hat{\vec{J}},$$

where $\vec{\Omega}$ is angular velotty and $\hat{\vec{J}}$ is generator of rotations (both spin and orbital parts)

Results for the spectrum

First, the orbital parts do not contribute because of the symmetry considerations (checked carefully in case of rotations)
As a result, the spectrum is very simple:

$$E = p \pm |\vec{\Omega}| S^3, \quad \textit{and} \quad E = p \pm |\vec{a}| S^3$$

where S^3 is spin projection, $S^3 = 1/2, 1$

Results of full-scale calculations in Euclidean space are perfectly reproduced.

Part 3. Miscellaneus

It is convenient to introduce "field of acceleration", like, say, "Electric field'' We considered Rindler space. Another well known example is provided by gravity.

Recently, in e-Print: 2604.23849 [hep-th]

"Weyl anomaly induced transport in hydrodynamics"
Shi-Zheng Yang, , Jian-Hua Gao , Zuo-Tang Liang , ,
Georgy Yu. Prokhorov , Shi Pu, O.V. Teryaev,
V.I. Zakharov
we introduced another universal field of acceleration, associated with Casimir boundary effect

Cnt'd

We rewrote a well-known expression for boundary current in terms of acceleration:

$$j^{\mu} = -4CF^{\mu\nu}a_{\nu}$$

and argued that there shoud be, as an echo of the equivalence principle, another novel current in hydrodynamic transport. And verified this prediction.

Moreover acceleration is leading source of force in any boundary effect

N. Yamamoto et al. 2604

$$J_i^{hHall} = \pm \frac{\zeta(3)}{4\pi^2}) T^3 (\vec{\nabla}\phi x \vec{u})_i$$