Lattice study of rotating gluodynamics and QCD

V. Braguta[1]

in collaboration with

M. Chernodub, E. Eremeev, I. Kudrov, A. Roenko, D. Sychev

[1]Joint Institute for Nuclear Research
Bogoliubov Laboratory of Theoretical Physics

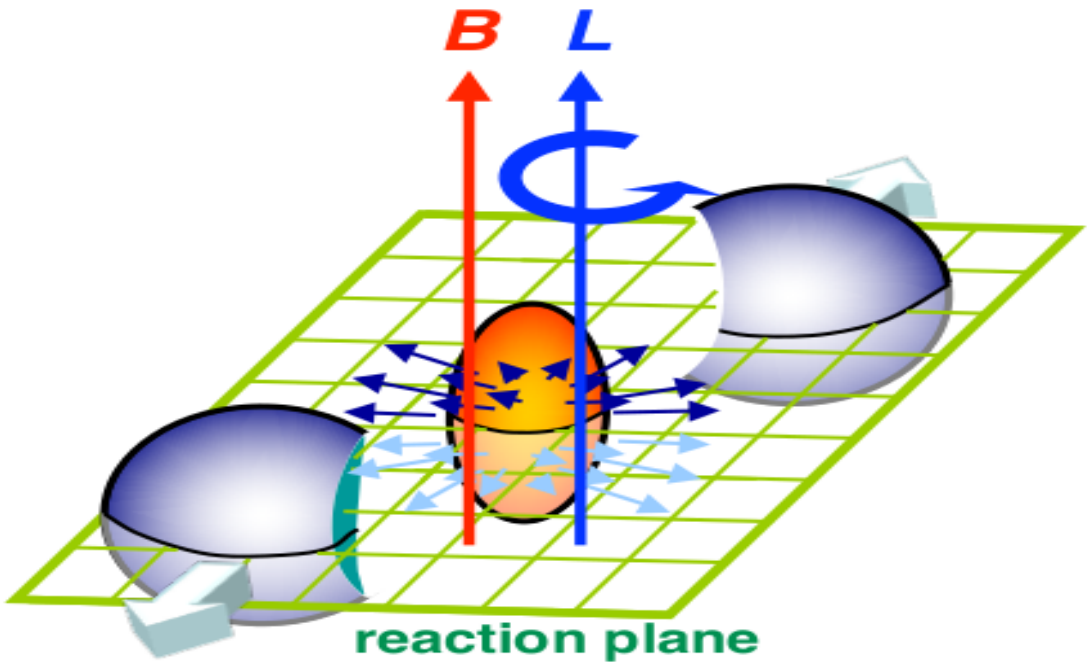
The XXth Workshop on High Energy Spin Physics

7-11 September 2026

Outline:

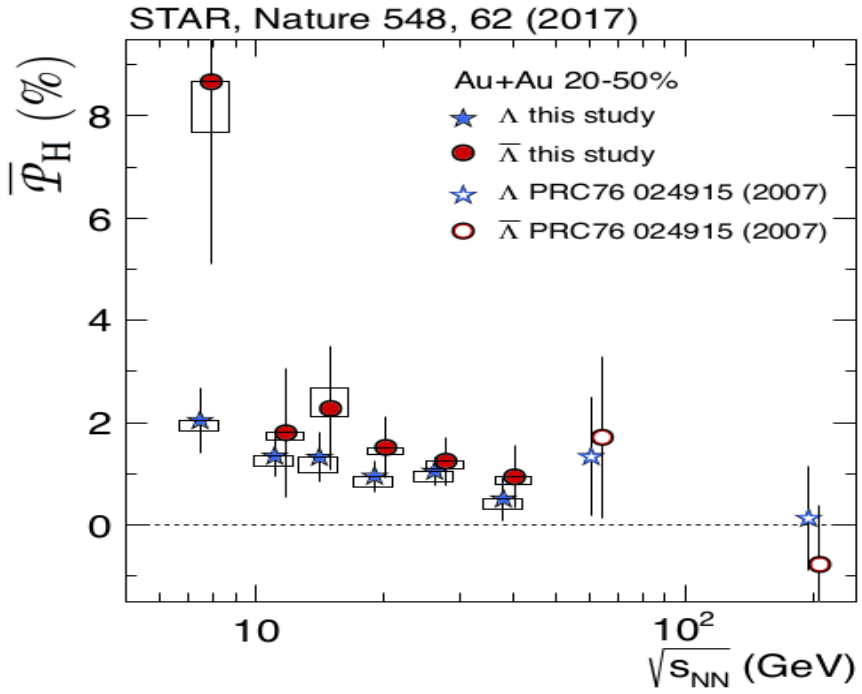
- Introduction
- Equation of state and the moment of inertia
- Spatial confinement/deconfinement transition
- Free energy of rotating static quark
- Spin potential
- Conclusion

QCD under extreme conditions



- High temperatures
- Large baryon density
- Intense magnetic fields
- Strong acceleration (up to $a \sim 0.1 - 1.0$ GeV)
- **Relativistic rotation**

Rotation of QGP in heavy ion collisions



Angular velocity from STAR (Nature 548, 62 (2017))

- $\Omega = (P_\Lambda + P_{\bar{\Lambda}})\frac{k_B T}{\hbar}$ (Phys. Rev. C 95, 054902 (2017))
- $\Omega \sim 10$ MeV ($v \sim c$ at distances 10-20 fm, $\sim 10^{22}\, s^{-1}$)
- Relativistic rotation of QGP

How relativistic rotation influences QCD?

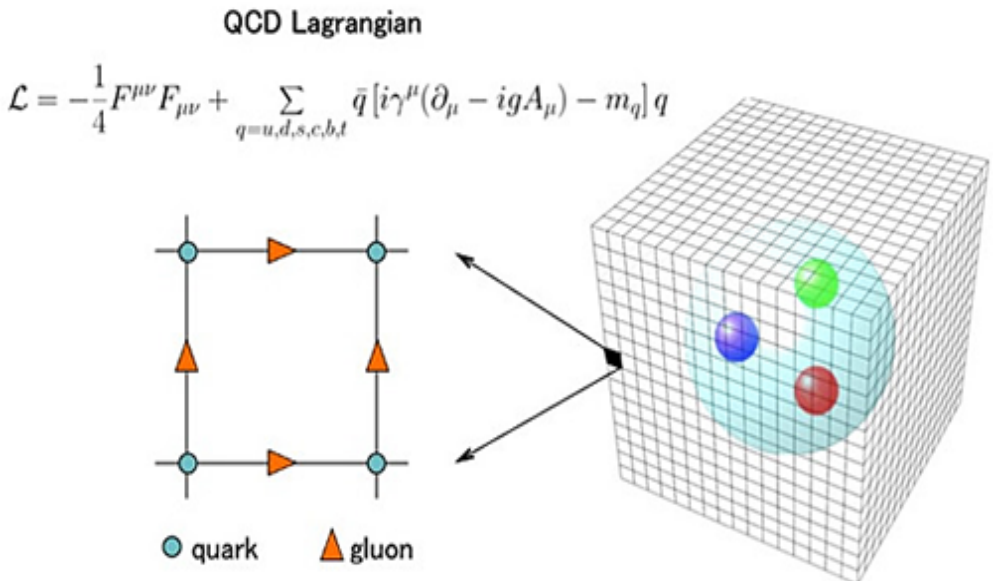
Quantum Chromodynamics(QCD)

- Degrees of freedom
  - Quarks q
  - Gluons A
- The QCD Lagrangian is well known

$$L = -\frac{1}{4}\sum_{a=1}^{8} F_a^{\mu\nu} F_{\mu\nu}^a + \sum_{f=u,d,s,\ldots} \bar{q}_f (i\gamma^\mu \partial_\mu - m) q_f + g \sum_{f=1}^{N_f} \bar{q}_f \gamma^\mu \hat{A}_\mu q_f$$

- Non-linear equations of motion with $g \sim 1$
- The main problem: calculation of observables based on the QCD Lagrangian
- Theoretical approaches contain assumptions with systematic errors which are difficult to estimate

Building lattice QCD



- Introduce regular cubic four dimensional lattice $N_s \times N_s \times N_s \times N_t = N_s^3 \times N_t$
- Lattice spacing–a
- Degrees of freedom
  - **Gluon fields:** 3x3 matrices $U \in SU(3)$, live on links
  - **Quarks fields:** column $q, \bar{q}$, live on sites

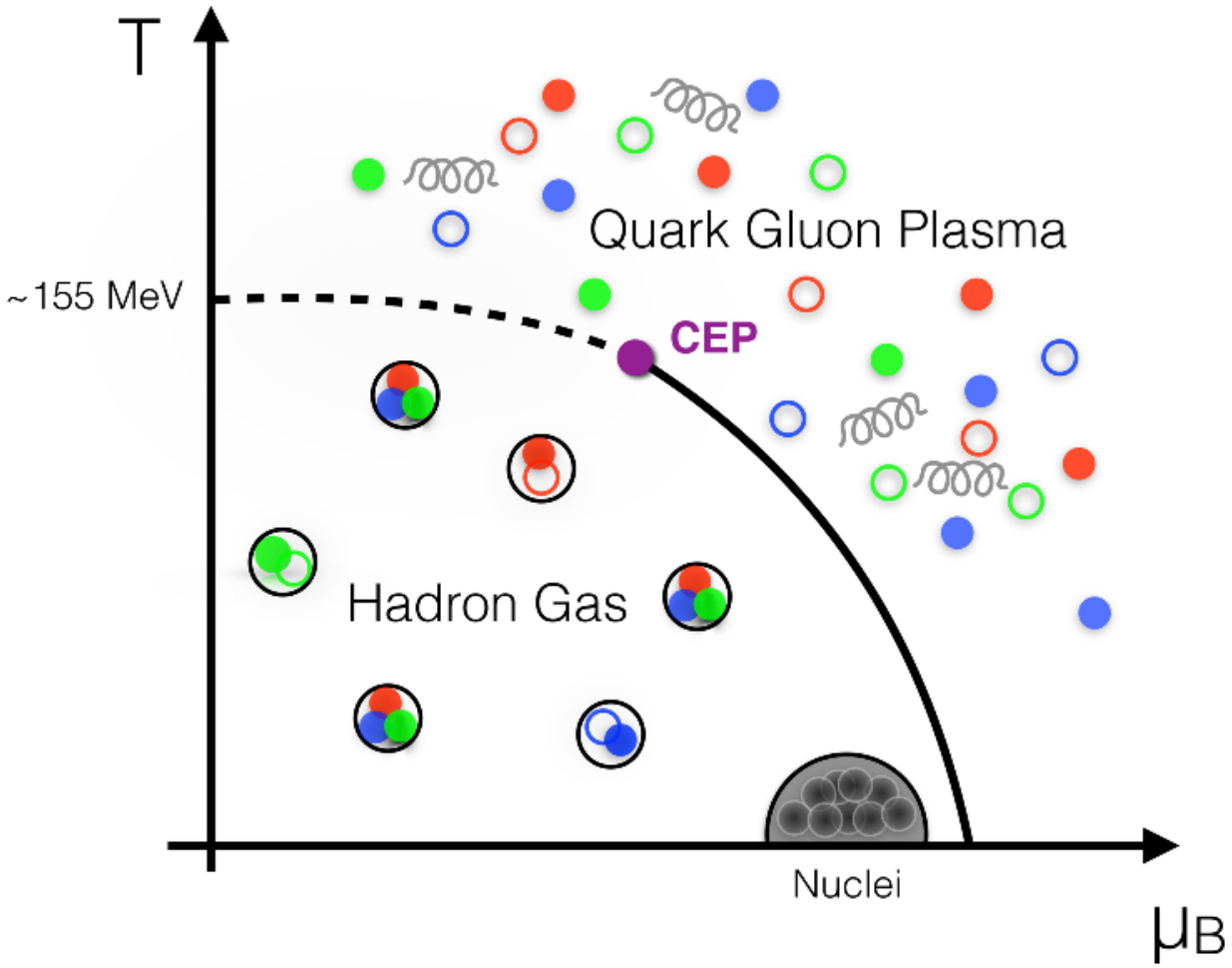
Lattice QCD

- QCD partition function (thermodynamic equilibrium!)

  $$Z = \int DU D\psi D\bar{\psi} \exp\left(-S_G - S_Q\right) = \int DU e^{-S_{eff}(U)}$$

- Hybrid Monte Carlo simulations
  (generation of gluon configurations with weight $\sim e^{-S_{eff}(U)}$)
- In continuum lattice partition function exactly reproduces QCD partition function
  - Gluon contribution: $S_G\big|_{a\to 0} = -\frac{1}{4}\sum_{a=1}^{8} F_a^{\mu\nu} F_{\mu\nu}^a$
  - Quark contribution: $S_Q\big|_{a\to 0} = \bar{q}(\gamma^\mu \partial_\mu + ig\gamma^\mu A_\mu + m)q$
- Carry out continuum extrapolation $a \to 0$
- Uncertainties (discretization and finite volume effects) can be systematically reduced
- The first principles based approach. No assumptions!
- Parameters: coupling constant $g(a)$ and masses of quarks $m_q(a)$

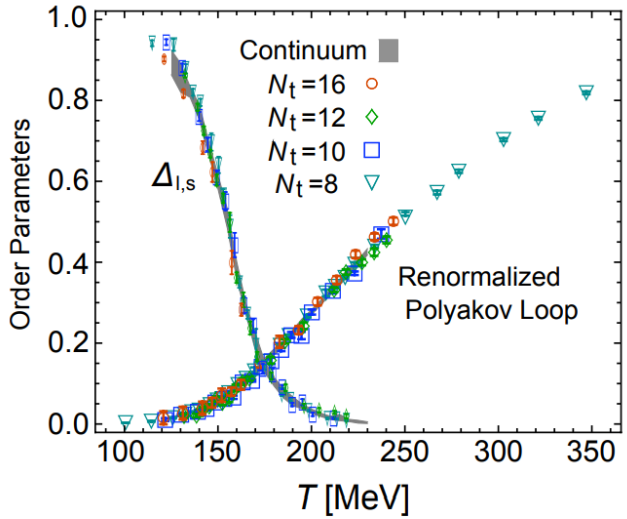
Phase transitions in QCD

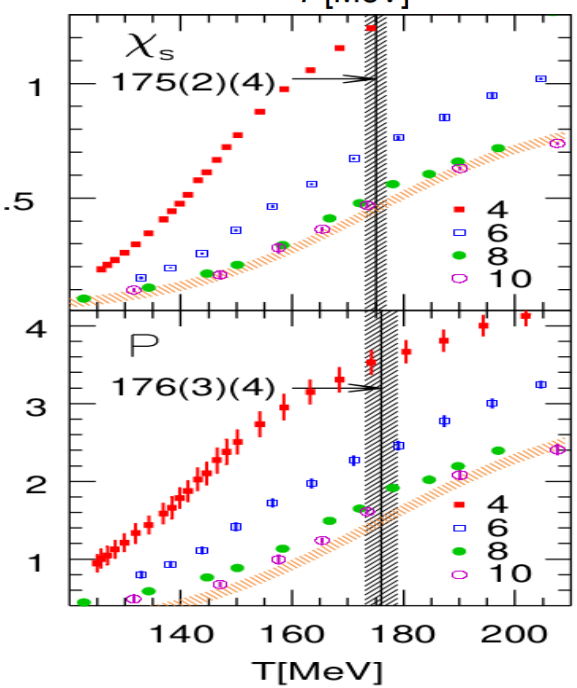


- Confinement/deconfinement transtion
- Chiral symmetry breaking/restoration transition

Confinement/deconfinement transition in QCD

- Order parameter Polyakov line:
  $P(\vec{x}) = \langle TrP \exp\left(ig \int_0^T dx^4 \hat{A}_4(\vec{x}, x^4)\right)\rangle$
  $P = e^{-F_Q/T}$
  - Confinement: $F_Q = \infty \Rightarrow P = 0$
  - Deconfinement: $F_Q < \infty \Rightarrow P \neq 0$
- Z_3 symmetry of gluodynamics
  but $P \to e^{2\pi k/3i} P$, $k = 0, 1, 2$
- P is similar to magnetization in ferromagnetic
- First order phase transition in SU(3) gluodynamics
- Quarks violate Z_3 symmetry
- Confinement/deconfinement is crossover
  $T_c = (176 \pm 5)$ MeV
  Z. Fodor, Phys.Lett.B 643 (2006)

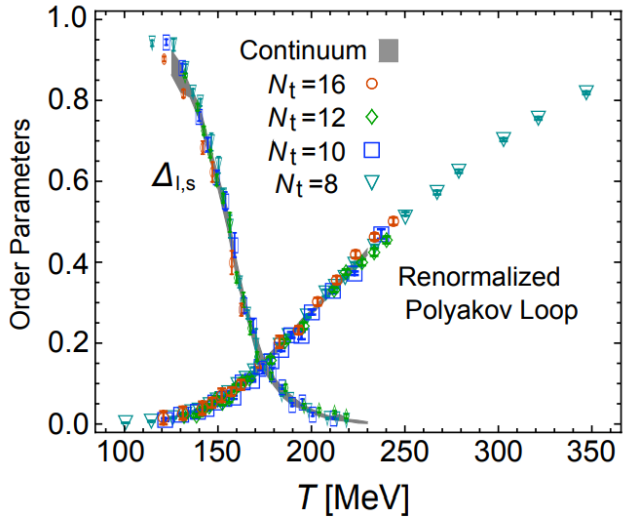


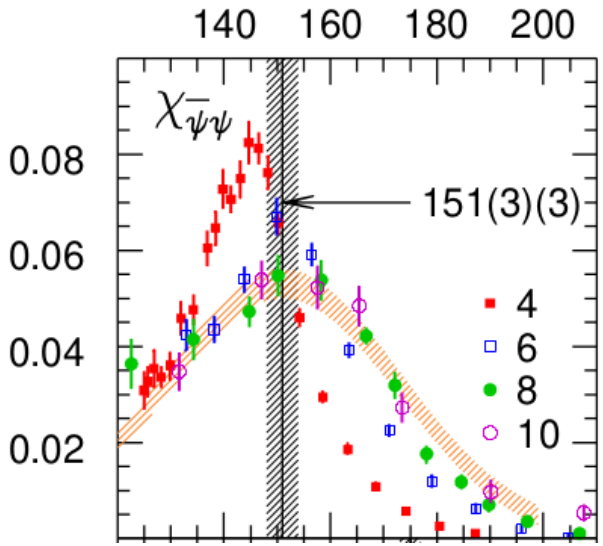


Chiral symmetry breaking in QCD

- Massless QCD Lagrangian
  $\mathcal{L} = \bar{\Psi} i\hat{D}\Psi = \bar{\Psi}_R i\hat{D}\Psi_R + \bar{\Psi}_L i\hat{D}\Psi_L$
- For N_f quarks chiral symmetry is
  $SU_L(N_f) \times SU_R(N_f) \times U_V(1) \times U_A(1)$
- Order parameter: chiral condensate
  $\langle\bar{\Psi}\Psi\rangle = \langle\bar{\Psi}_L\Psi_R\rangle + \langle\bar{\Psi}_R\Psi_L\rangle$
  - CS is broken: $\langle\bar{\Psi}\Psi\rangle \neq 0$, $M_q \neq 0$
  - CS is restored: $\langle\bar{\Psi}\Psi\rangle = 0$, $M_q = 0$
- Dynamical chiral symmetry breaking
  $SU_L(N_f) \times SU_R(N_f) \to SU_V(N_f)$
- In massless QCD the first or the second order phase transition
- But for $m_q \neq 0$ CS symmetry is explicitly broken. $M_q \sim 10 \to 300$ MeV
- CS breaking/restoration is crossover
  $T_c = (151 \pm 4)$ MeV

  Z. Fodor, Phys.Lett.B 643 (2006)





Study of rotating QGP

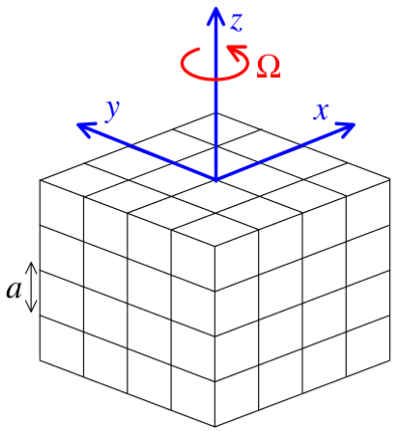
- Our aim: study rotating QCD within lattice simulations
- Rotating QCD at thermodynamic equilibrium
  - At the equilibrium the system rotates with some Ω
  - The study is conducted in **the reference frame which rotates with QCD matter**
  - QCD in external gravitational field
- **Boundary conditions are very important!**
  Causality condition: v<c

Details of the simulations

- Gluodynamics is studied at thermodynamic equilibrium in external gravitational field
- The metric tensor

$$g_{\mu\nu} = \begin{pmatrix} 1 - r^2\Omega^2 & \Omega y & -\Omega x & 0 \\ \Omega y & -1 & 0 & 0 \\ -\Omega x & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

- Geometry of the system: $N_t \times N_z \times N_x \times N_y = N_t \times N_z \times N_s^2$



Details of the simulations

- Partition function ($\hat{H}$ is conserved)

$$Z = \mathrm{Tr}\, \exp\left[-\beta \hat{H}\right] = \int DU D\psi D\bar{\psi}\ \exp\left[-S\right]$$

- Euclidean action(in the cylindrical coordinates)

$$\begin{aligned} S &= S_0 + S_1\Omega + S_2\Omega^2 \\ S_1 &= \int d^4x \left[r\left(\frac{i}{g^2}\left[F^a_{r\hat{\varphi}}F^a_{\tau r} - F^a_{\hat{\varphi}z}F^a_{\tau z}\right] + \bar{\psi}\gamma_4 D_{\hat{\varphi}}\psi \right) + \frac{i}{2}\bar{\psi}\gamma_4\sigma_{12}\psi \right] \\ S_2 &= -\frac{1}{2g^2}\int d^4x\ r^2\left[(F^a_{\hat{\varphi}z})^2 + (F^a_{r\hat{\varphi}})^2\right] \end{aligned}$$

spin potential

- S_1 total momentum, S_2 centrifugal force
- Competition of S_1 and S_2
- S_2 gives the dominant contribution
- Rotation generates effective spin potential $\mu_S \sim \Omega$

Details of the simulations

Boundary conditions

- **Periodic b.c.:**
  - $U_{x,\mu} = U_{x+N_i,\mu}$
  - Not appropriate for the field of velocities of rotating body
- **Dirichlet b.c.:**
  - $U_{x,\mu}\big|_{x\in\Gamma} = 1, \quad A_\mu\big|_{x\in\Gamma} = 0$
  - Violate Z_3 symmetry
- **Neumann b.c.:**
  - Outside the volume $U_P = 1, \quad F_{\mu\nu} = 0$
- The dependence on boundary conditions is the property of all approaches
- One can expect that boundary conditions influence our results considerably, but their influence is restricted due to the screening

Details of the simulations

Sign problem

$$
\begin{aligned}
S_G \;=\; & \frac{1}{2g^2}\int d^4x \Big[(F^a_{\tau r})^2 + (F^a_{\tau\hat{\varphi}})^2 + (F^a_{\tau z})^2 + \\
& + (F^a_{rz})^2 + \left(1-(\Omega r)^2\right)(F^a_{\hat{\varphi} z})^2 + \left(1-(\Omega r)^2\right)(F^a_{r\hat{\varphi}})^2 + \\
& + 2ir\Omega(F^a_{r\hat{\varphi}}F^a_{\tau r} - F^a_{\hat{\varphi} z}F^a_{\tau z})\Big]
\end{aligned}
$$

- The Euclidean action has imaginary part (**sign problem**)
- Simulations are carried out at imaginary angular velocities $\Omega \to i\Omega_I$
- The results are analytically continued to real angular velocities
- This approach works up to sufficiently large Ω
- Sometimes instead of Ω^2 we use $v^2 = (\Omega R)^2$ and $v_I^2 = (\Omega_I R)^2$

Ehrenfest–Tolman law

- In gravitational field the temperature is not constant in space at thermal equilibrium

$$T(r)\sqrt{g_{00}} = const = T_0$$

$$T(r) = \frac{T_0}{\sqrt{1-(\Omega r)^2}} = \frac{T_0}{\sqrt{1+(\Omega_I r)^2}}$$

- We use the designation $T = T(r=0) = T_0$
- Rotation effectively heats the system: $T(r) > T(r=0)$
- Inhomogeneous phase: confinement in the center and deconfinement in the periphery

  M. Chernodub, Phys.Rev.D 103 (2021) 5, 054027
- For imaginary rotation: deconfinement/confinement in the center/periphery
- We observe this phenomenon for accelerated observer

  talk of Jayanta Dey, Tue, 16.10

EoS of rotating plasma

- Free energy of rotating QGP

  $F(T,R,\Omega) = F_0(T,R) - \frac{F_2}{2}\Omega^2 - \frac{F_4}{4}\Omega^4 + ...$

- Total momentum

  $$J_z = -\left(\frac{\partial F}{\partial \Omega}\right)_T = (F_2 + F_4\Omega^2)\cdot\Omega$$

- Moment of inertia

  $$I = \frac{J_z}{\Omega} = (F_2 + F_4\Omega^2)$$

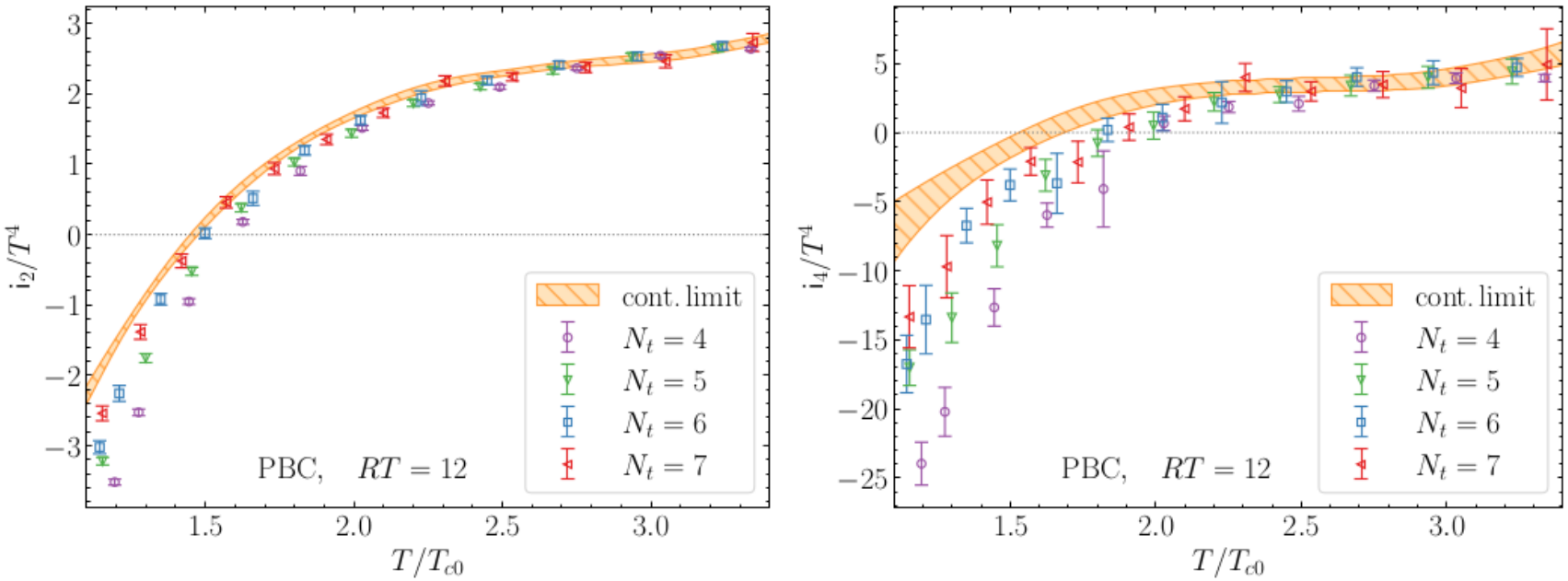
- F_2–moment of inertia at vanishing Ω
  F_4–deformation due to rotation, $F_4 > 0$

- We calculate: $i_2 = \frac{F_2}{VR^2}$, $\quad i_4 = \frac{F_4}{VR^4}$

  Talk of Artem Roenko, Tue, 15.45



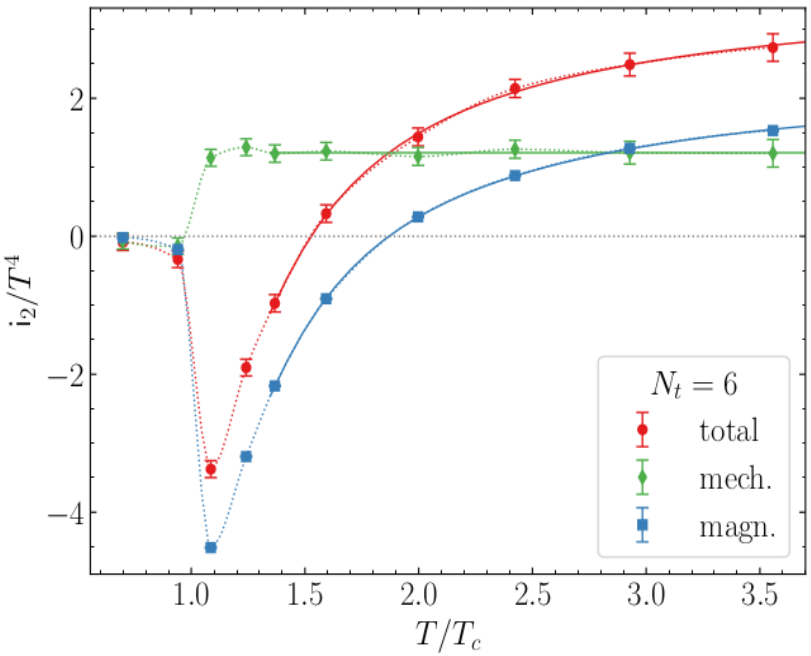
Moment of inertia of gluon plasma

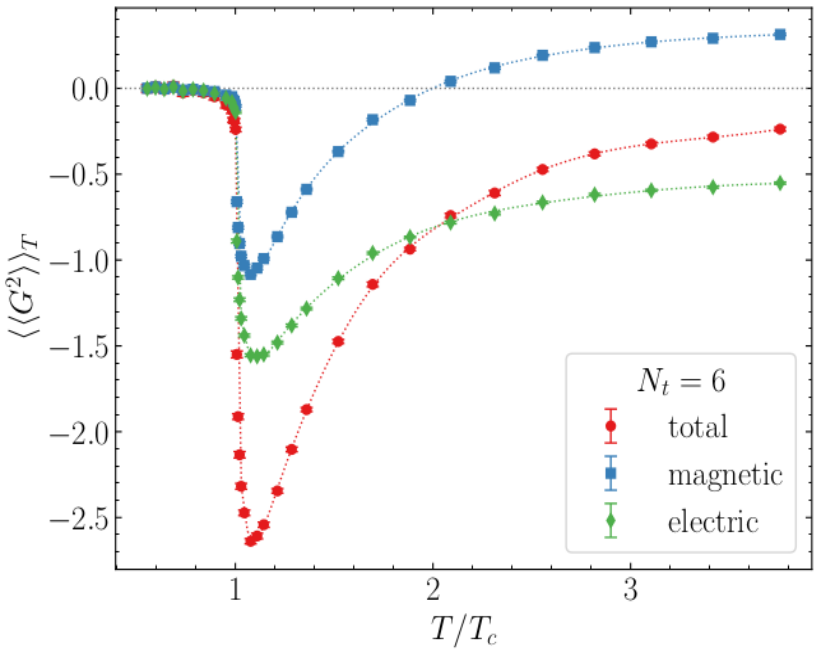


- EoS of gluon plasma

  V. Braguta et al., JETP Lett. 117 (2023) 9; 639-644, Phys.Lett.B 852 (2024) 138604; Phys.Part.Nucl. 57 (2026) 1146; to be published
- $i_2 < 0$ for $T < 1.5T_c$ and $i_2 > 0$ for $T > 1.5T_c$
- $i_2 < 0$ is related to magnetic condensate and the scale anomaly
- $i_4 < 0$ for $T < (1.6 - 1.8) \cdot T_c$ and $i_4 > 0$ for $T > (1.6 - 1.8) \cdot T_c$
- For $(\Omega R)^2 \sim 1/3, \quad \delta I/I_0 \sim 80\%$

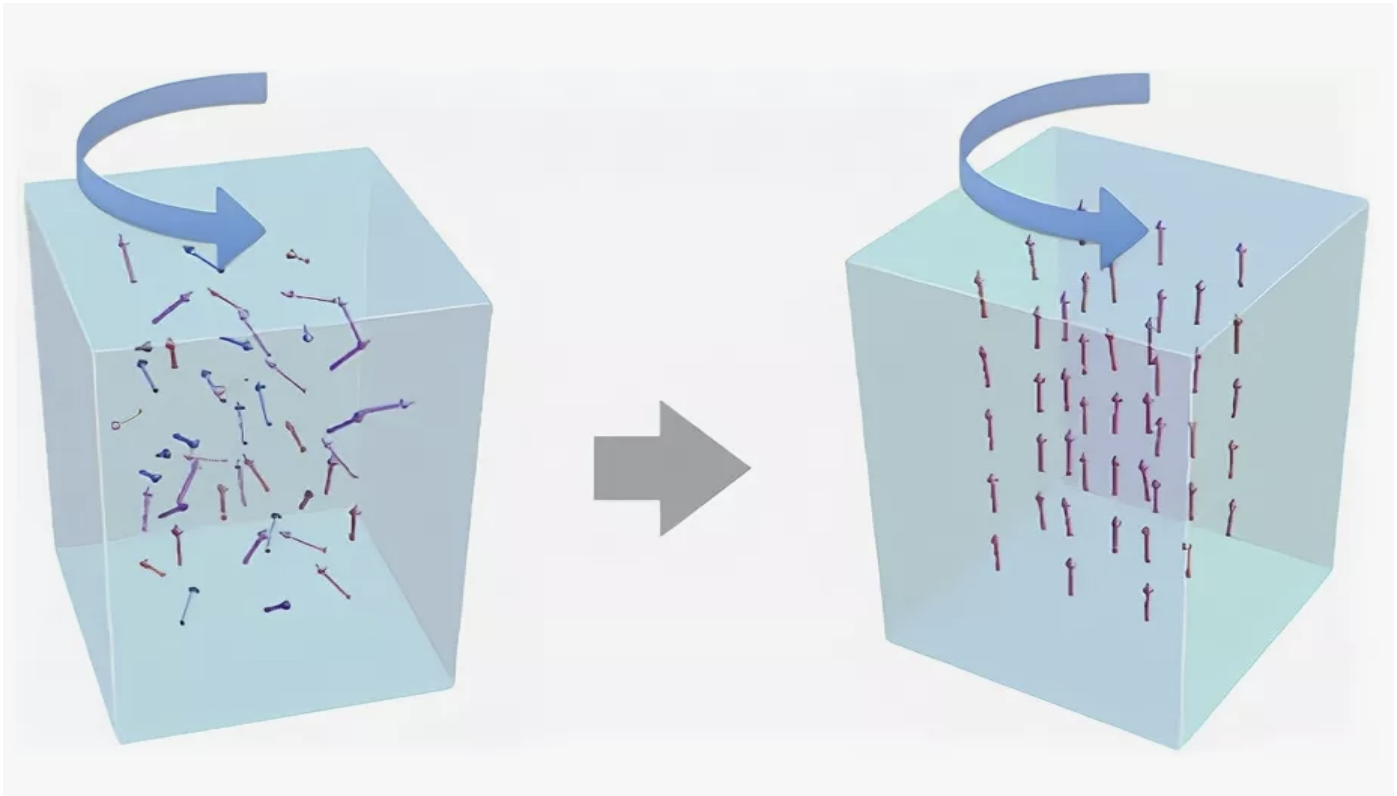
Moment of inertia of gluon plasma





- Decomposition of moment of inertia
  V. Braguta et al. Phys.Rev.D 110 (2024) 1, 014511
- $i_2 = \frac{I_0}{V R_\perp^2}, \quad I_0 = I_{mech} + I_{magn}$
  $I_{mech} = \langle J_z^2 \rangle - (\langle J_z \rangle)^2 \sim \langle {S_1}^2 \rangle$
  $I_{magn} = \frac{1}{3} \int d^3x r^2 \langle H^2 \rangle \sim \langle S_2 \rangle$
- Gluon condensate: $\langle G^2 \rangle = \langle E^2 \rangle + \langle H^2 \rangle$
- $\langle T^\mu_\mu \rangle = \epsilon - 3p = \frac{\beta_s(\alpha_s)}{4\alpha_s} \langle G^2 \rangle, \quad \beta_s(\alpha_s) = -(33 - 2n_f)\frac{\alpha_s^2}{6\pi}$

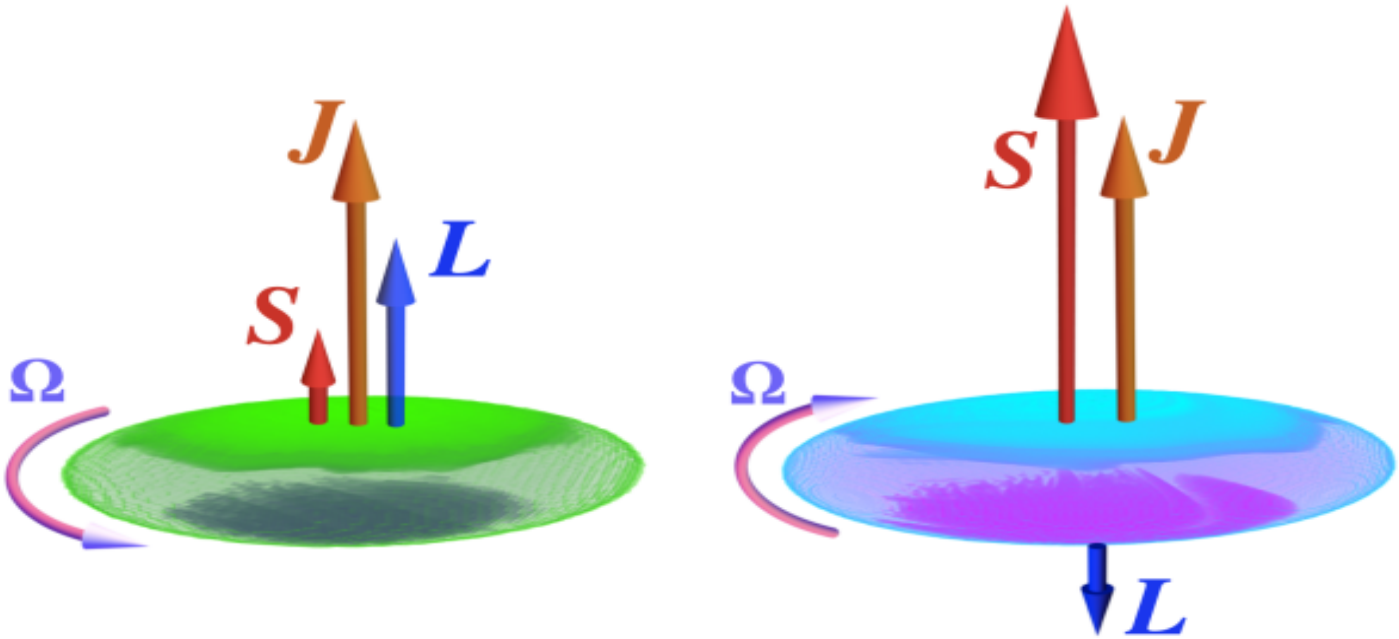
Barnett effect



- Rotation polarizes spins of ferromagnetic

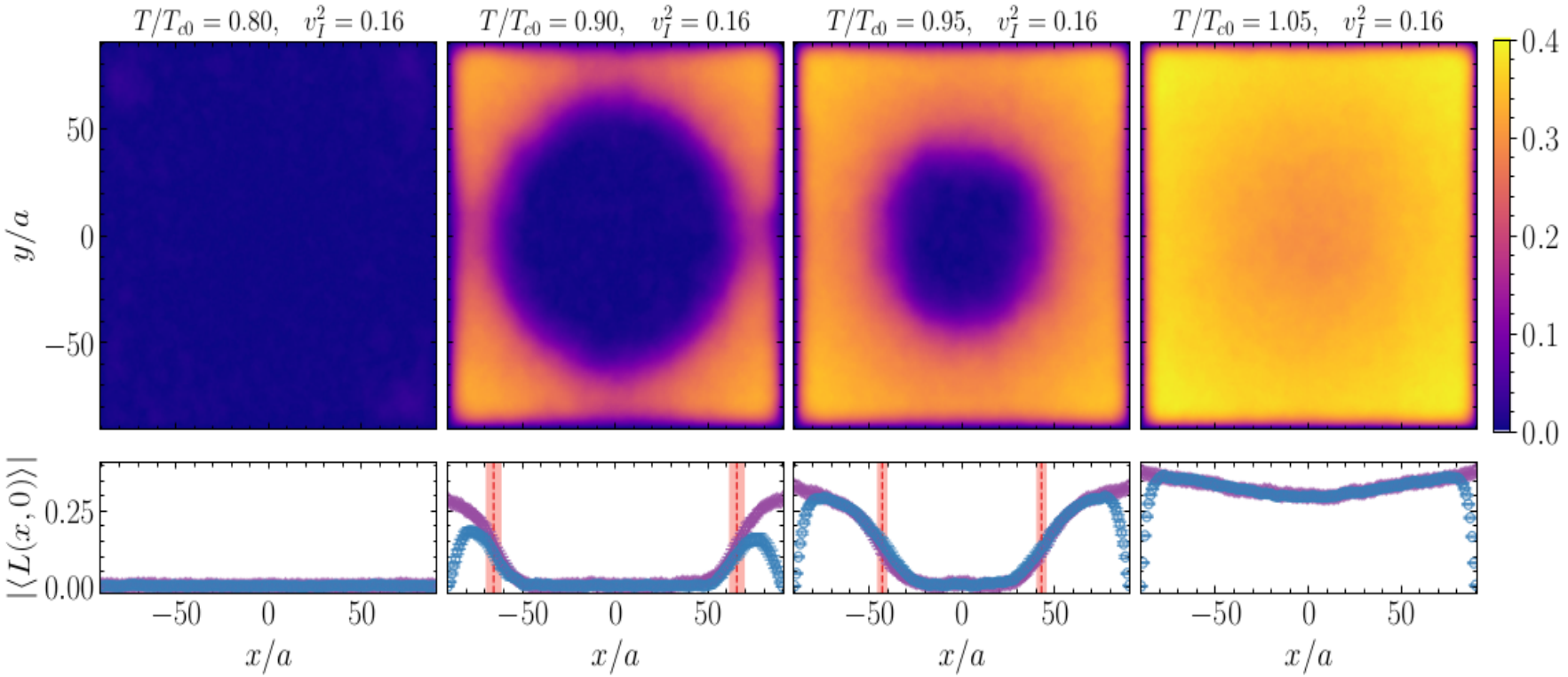
the picture from *doi: 10.3389/fphy.2015.00054*

Negative Barnett effect



- $\vec{J} = I_0 \vec{\Omega} = -\left(\frac{\partial F}{\partial \vec{\Omega}}\right)_T, \quad \vec{J} = \vec{L} + \vec{S}$
- For classical motion $\vec{J} = \vec{L}$ and $I_0 > 0$
- But the spin $\vec{S}$ is quantum
- The term $\langle H^2 \rangle$ is related to spin of gluons
- $\vec{L} \uparrow\uparrow \vec{\Omega}, \quad \vec{S} \uparrow\downarrow \vec{\Omega}$ might lead to $\vec{J} \uparrow\downarrow \vec{\Omega}$ and $I_0 < 0$

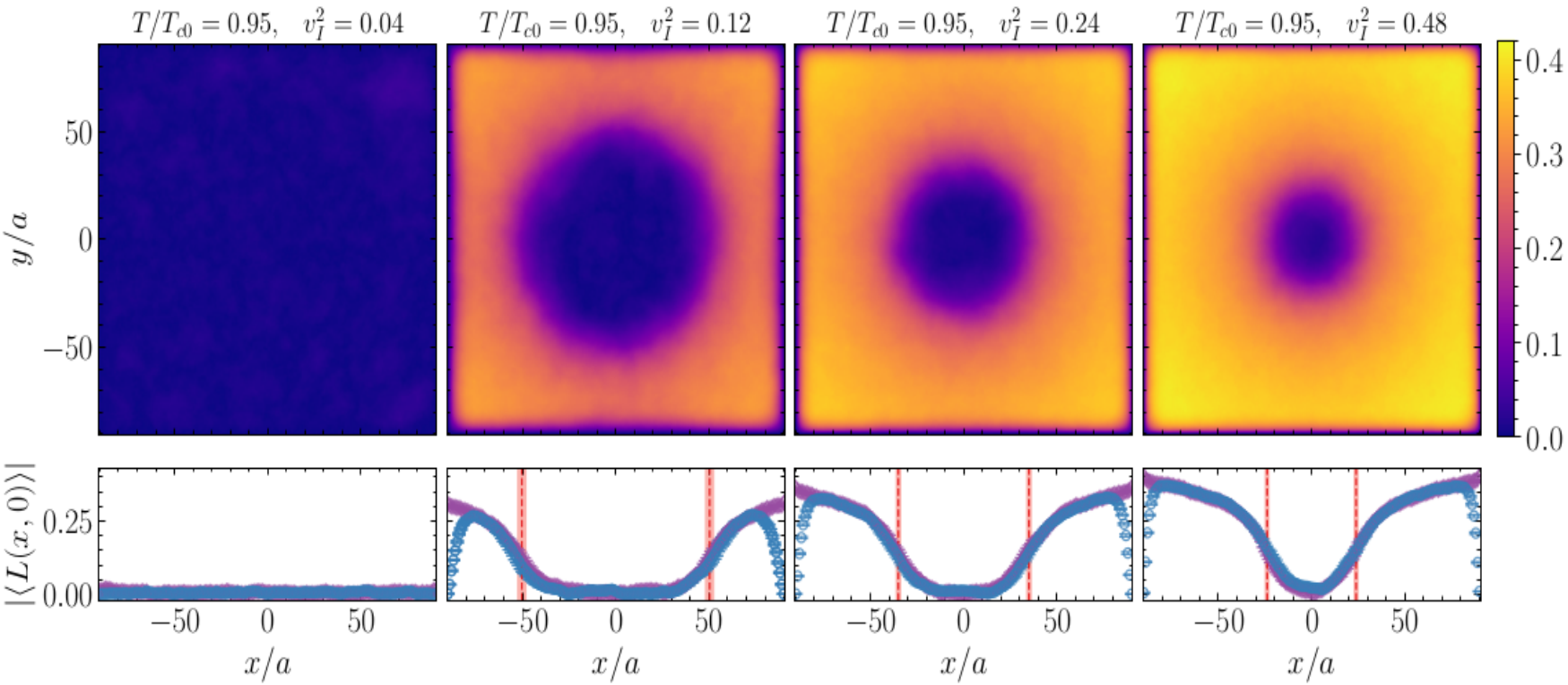
Spatial confinement/deconfinement transition in GP



- Finite temperature confinement/deconfinement becomes spatial transition:
  For a certain parameters spatially separated confinement and deconfinement phases coexist simultaneously
- For $\Omega^2 < 0$ confinement in the center and deconfinement in the periphery
- Confinement/inhomogeneous state/deconfinement
  Inhomogeneous state takes place below T_{c0}
- Theoretical studies

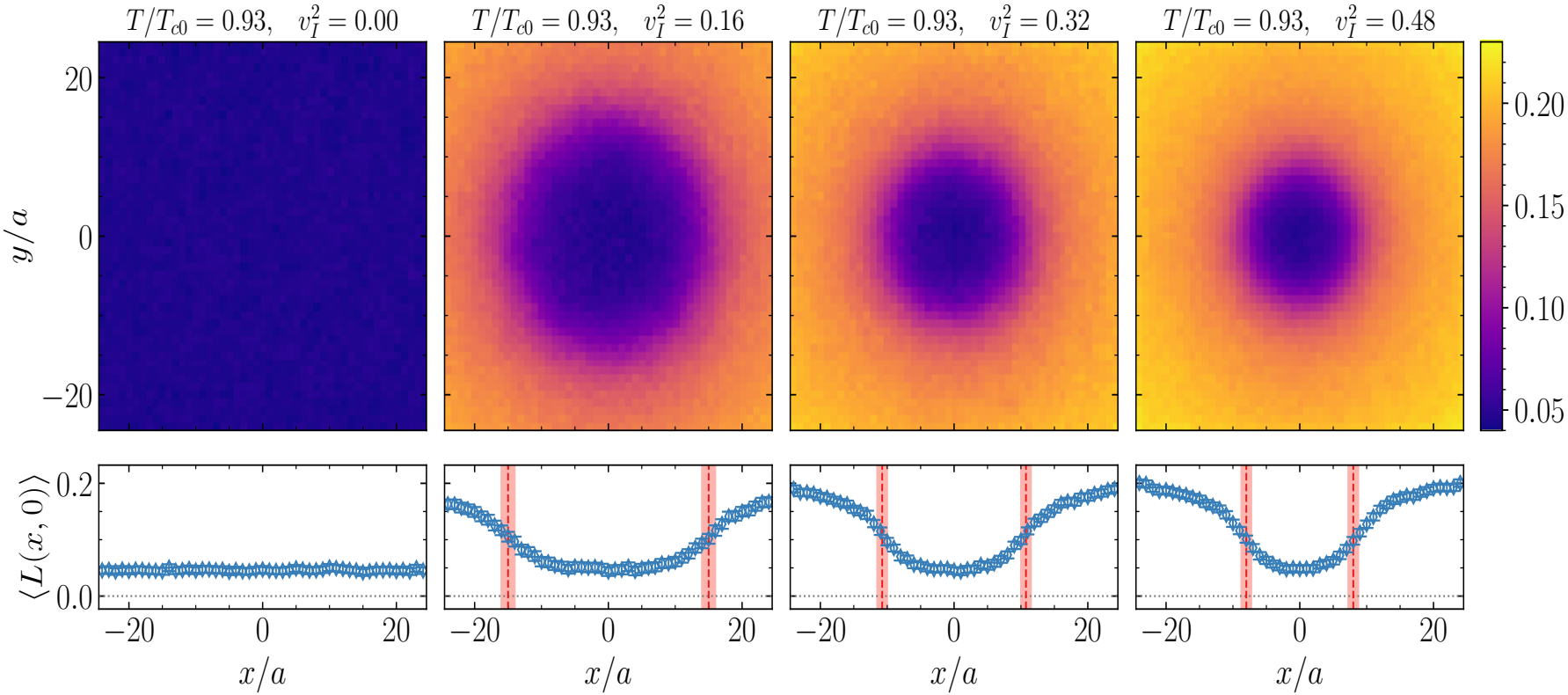
V. Braguta et al., Phys.Lett.B 855 (2024) 138783; JHEP 09 (2025) 079

Spatial confinement/deconfinement transition in GP



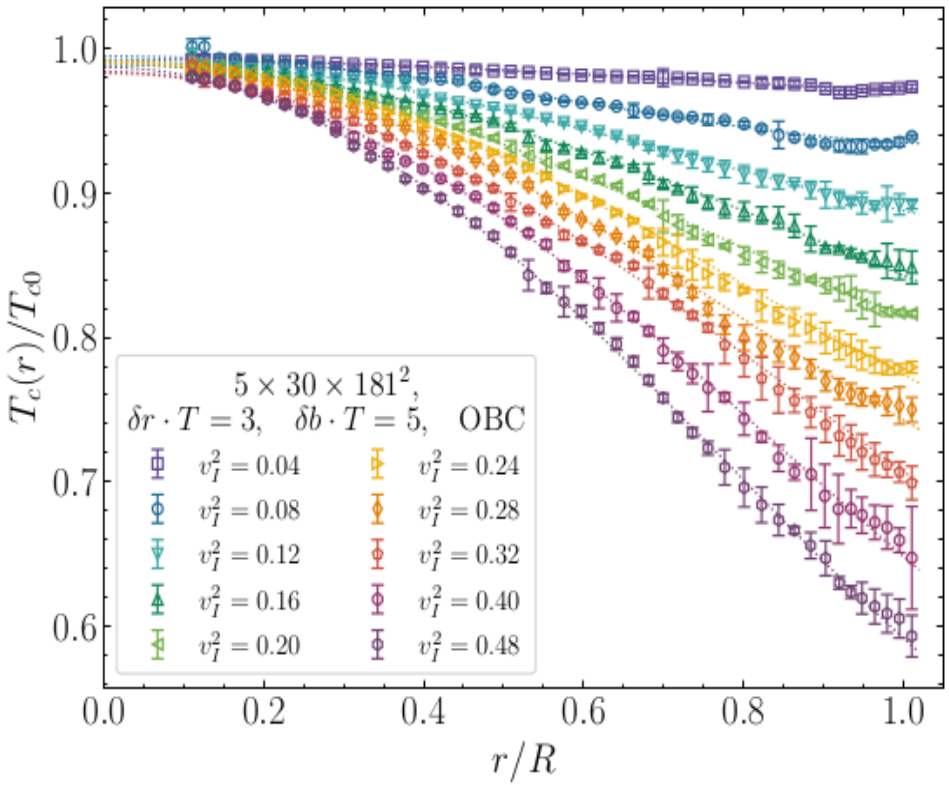
- The phase transition is induced by rotation

Spatial confinement/deconfinement transition in QCD



- It remains to be true for quarks (Preliminary results!)

Local critical temperature $T_c(r, \Omega_I)$



- TE prediction:

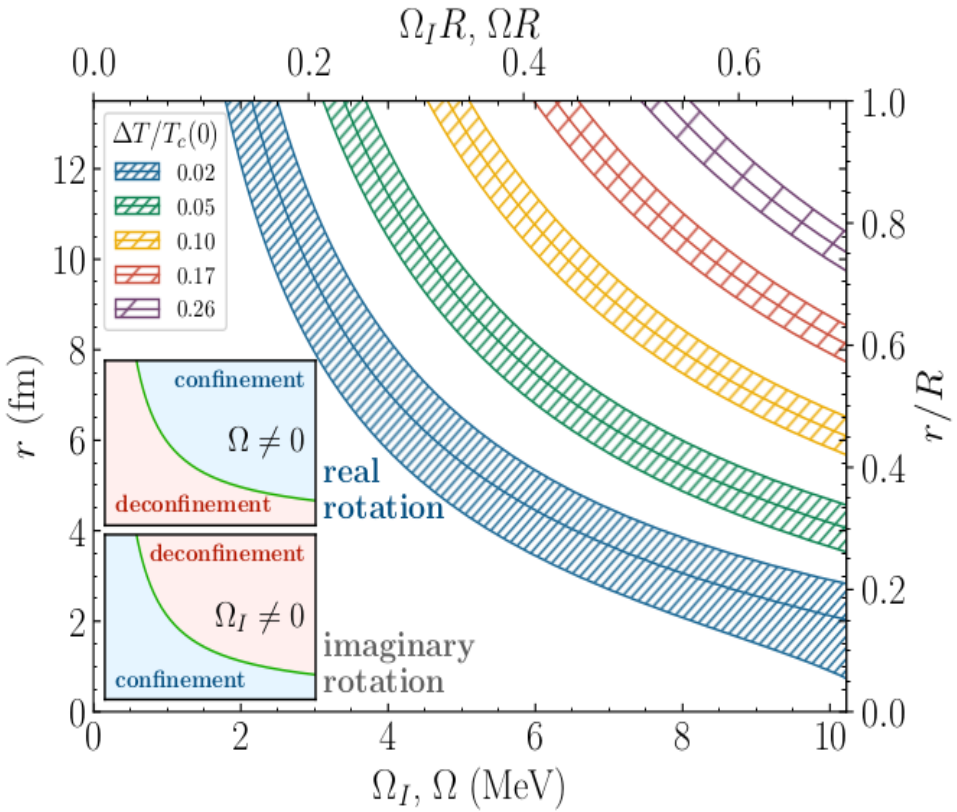
$$T(r) = \frac{T}{1+\Omega_I^2 r^2} = T_{c0} \Rightarrow T_c(r, \Omega_I) = T_{c0}(1 + \Omega_I^2 r^2)$$

- Our results can be well described by the formula

$$\frac{T_c(r,\Omega_I)}{T_{c0}} = 1 - \kappa_2 (\Omega_I r)^2, \quad \kappa_2 > 0$$

- In disagreement with TE law
- Within the uncertainty $\frac{T_c(r=0,\Omega_I)}{T_{c0}} = 1$

Analytical continuation to real rotation



- Analytical continuation $\Omega_I^2 \to -\Omega^2$:

$$\frac{T_c(r,\Omega)}{T_{c0}} = 1 + \kappa_2 (\Omega r)^2$$

- Inhomogeneous state can be realised for $T > T_{c0}$
- Deconfinement in the center and confinement in the periphery

Local thermalization approximation

$$S = \frac{1}{2g^2}\int d^4x \Big[((E_x^a)^2 + (E_y^a)^2 + (E_z^a)^2 + (H_y^a)^2) + \\ + \left(1 - (\Omega r)^2\right)((H_x^a)^2 + (H_z^a)^2) + 2ir\Omega(H_z^a E_x^a - H_x^a E_z^a)\Big]$$

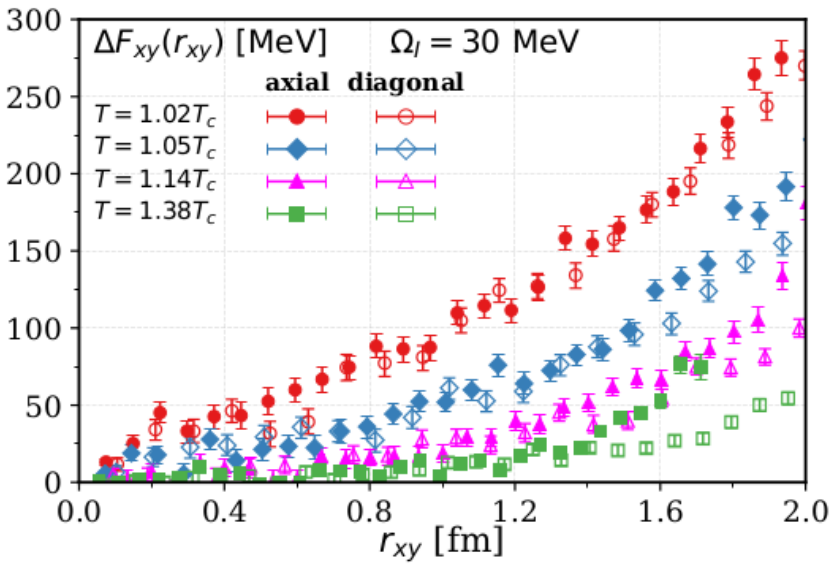
- For slow rotation $\Omega\zeta \ll 1$ the coefficients vary slowly
- Local thermalization approximation: study the action with the coefficients freezed at $r = r_0$
- Linear in Ω term can be neglected
- Local thermalization approximation was checked on the lattice

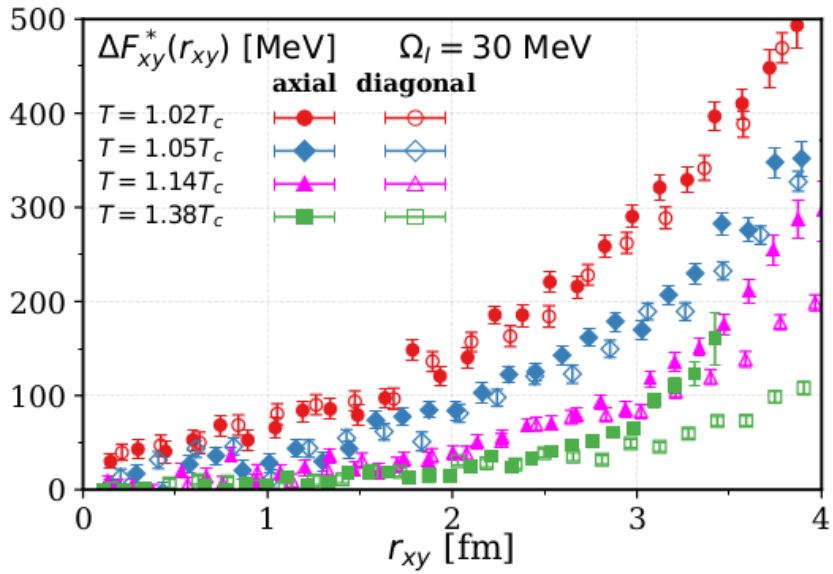
Origin of the inhomogeneous phase transition

$$S_G = \int d^4x \left[\frac{1}{2g^2} \left((E_x^a)^2 + (E_y^a)^2 + (E_z^a)^2 + (H_y^a)^2 \right) + \right.$$
$$\left. + \frac{1}{2\tilde{g}^2} \left((H_x^a)^2 + (H_z^a)^2 \right) \right]$$

- External gravitational field leads to the asymmetric action $\frac{g^2}{\tilde{g}^2} = 1 - (\Omega r)^2$
- The asymmetry $g^2/\tilde{g}^2$ is larger in the periphery region leading to the shift of the local critical temperature
- Different critical temperatures at different points
- GR effect!

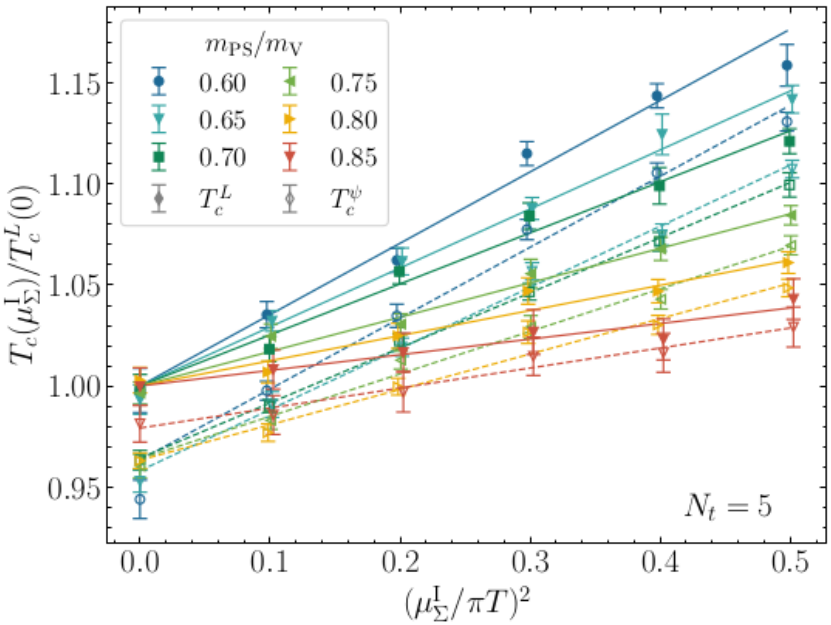
Free energy of rotating static quark

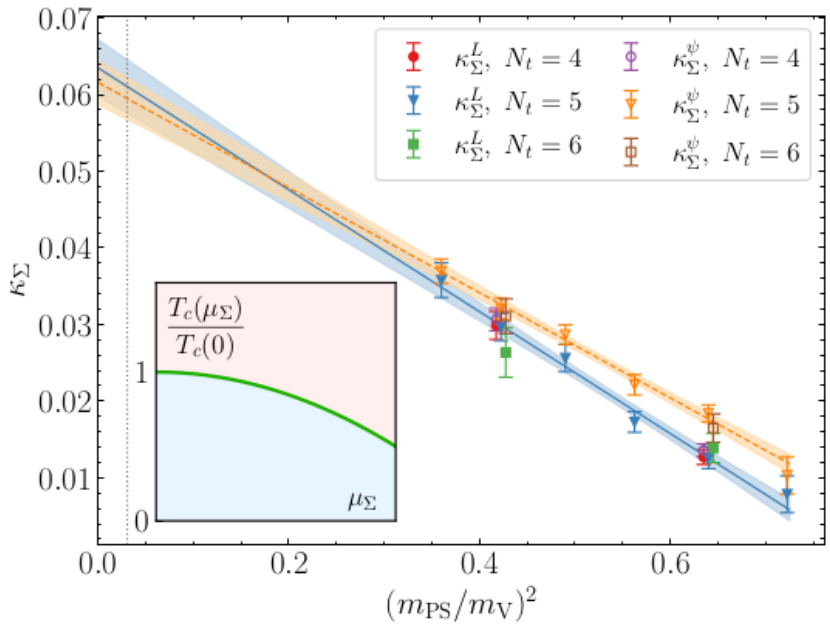




- Interaction potential of rotating static quarks
  Heng-Tong Ding et al., e-Print: 2607.26939
- $\Delta F = a\Omega^2 + b(\Omega r)^2$, $b > 0$ $\Rightarrow$ centripetal force acts on static quark
- For usual rotation $\Delta F = b(\Omega r)^2$, $b < 0$ $\Rightarrow$ centrifugal force
- This phenomenon takes place in the region $T \sim T_c$

Spin potential





- Spin potential μ_Σ: $Z = Tr[e^{-\beta\hat{H}}] \to Tr[e^{-\beta(\hat{H}-\mu_\Sigma\hat{S})}]$
- Lattice simulation at imaginary μ_Σ

  V. Braguta et al., Phys.Rev.D 111 (2025) 11, 114508
- Spin potential decreases the critial temperatures

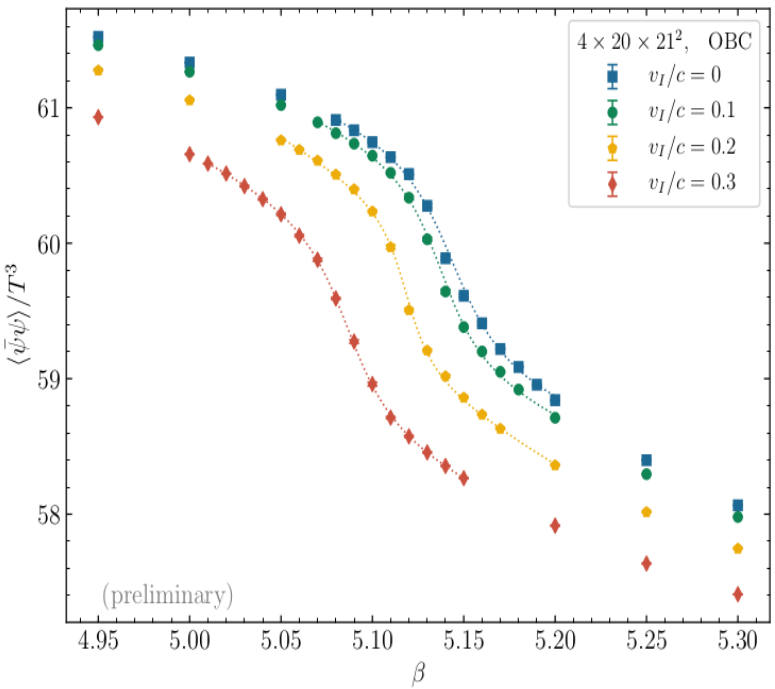
$$\frac{T_c(\mu_\Sigma)}{T_c} = 1 - \kappa_\Sigma \mu_\Sigma^2, \quad \kappa_\Sigma > 0$$

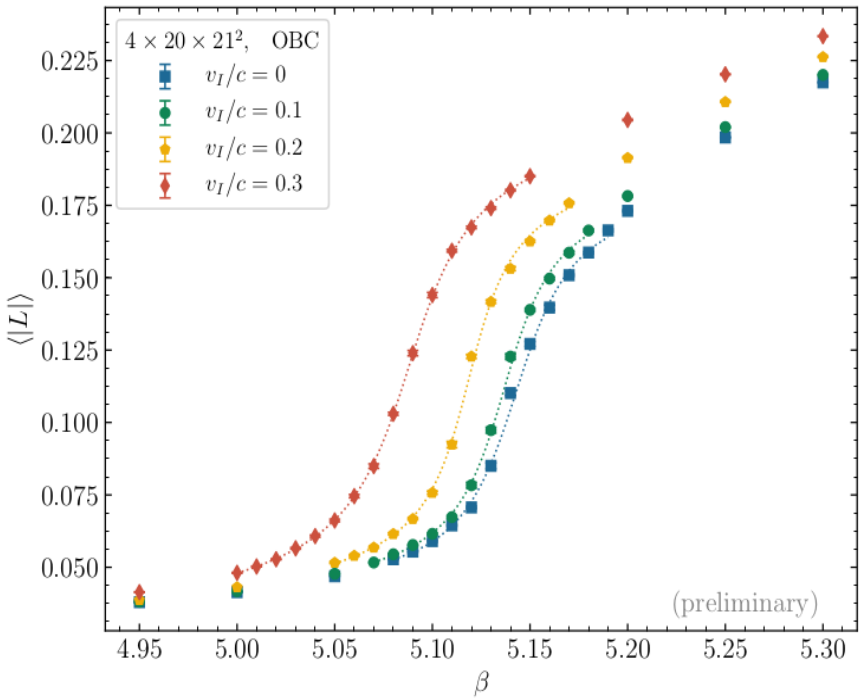
Conclusion

- Lattice studies of rotating gluodynamics and QCD have been reviewed
  - Equation of state and the moment of inertia
  - Spatial confinement/deconfinement transition
  - Free energy of rotating static quark
  - Spin potential
- One observes unusual phenomena in the region $T \sim T_c$: Negative moment of inertia, spatial deconfinement transition, centripetal force for rotating static quark
- These phenomena disappear at larger temperatures

THANK YOU!

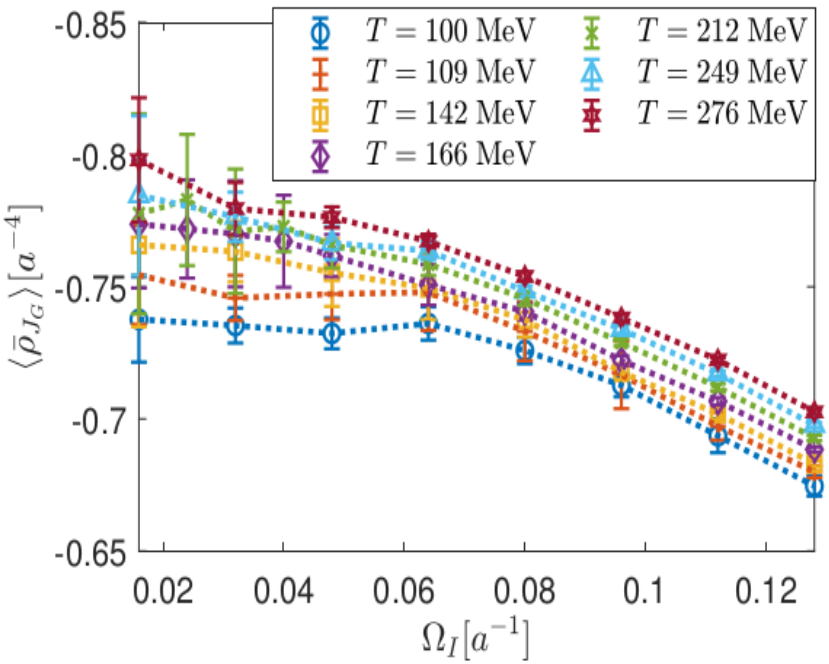
Backup slides: Simulation with fermions

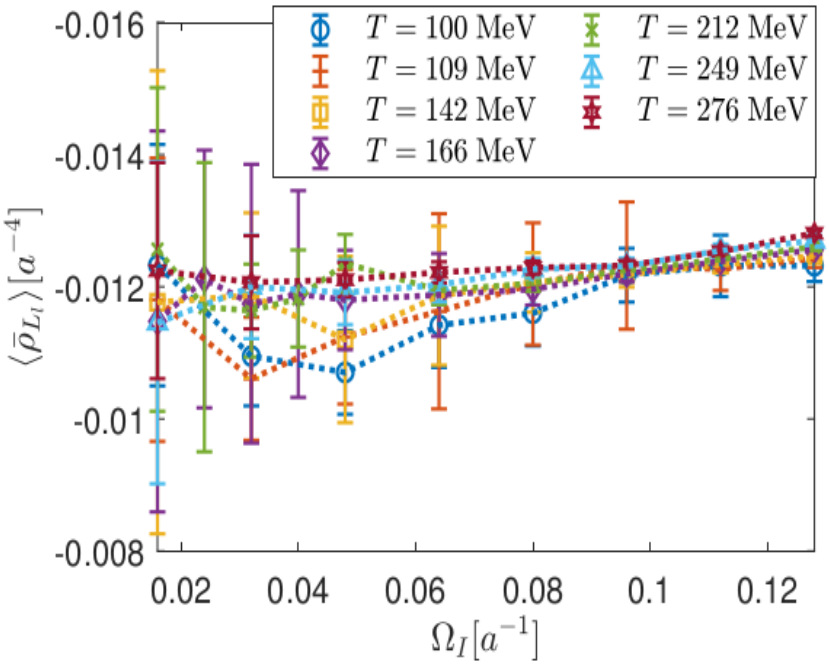




- Lattice simulation with Wilson fermions
- Critical couplings of both transitions coincide
- Critical temperatures are increased

Backup slides:
Simulation with fermions (e-Print: 2307.05755)





- Rotational rigidities: $\rho_{J_G} = \frac{J_G}{\Omega R^2}, \quad \rho_{L_f} = \frac{L_f}{\Omega R^2}$
- Spin susceptibility: $\zeta_f = \frac{s}{\Omega}$
- Negative moment of inertia