Momentum-space collective vortical observables

A. Borissov[1]
E. F. Dlin[1,2]
O. V. Teryaev[1]

[1]Joint Institute for Nuclear Research (JINR), Dubna
[2]A. Alikhanyan National Laboratory (AANL), Yerevan

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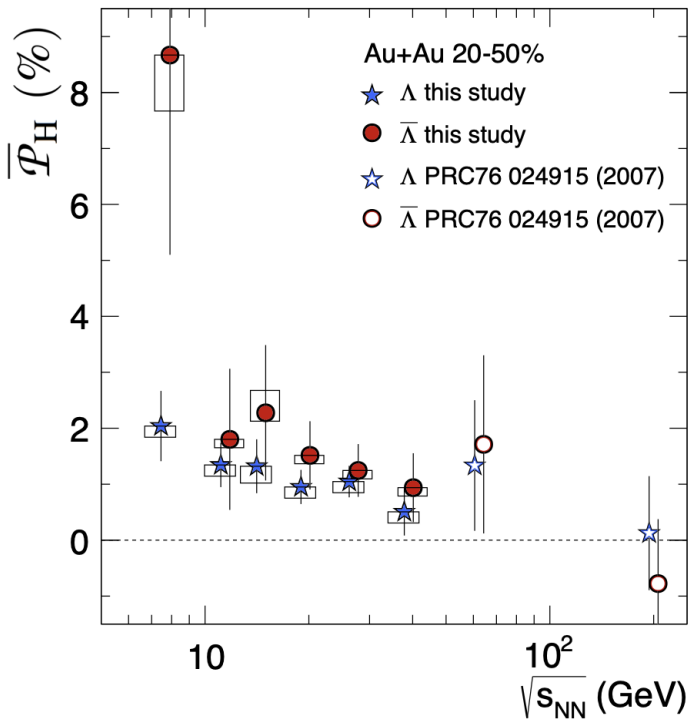
Global hyperon polarization — a trace of vorticity

- STAR: $\Lambda/\bar{\Lambda}$ polarization in Au+Au follows the collision's orbital angular momentum L [Nature 548, 62 (2017)].
- The QGP is **the most vortical fluid** known: $|\boldsymbol{\omega}| \sim 10^{21}$ s^{-1}.
- $P_H \propto \langle \boldsymbol{\omega} \cdot \hat{\mathrm{L}} \rangle$ — vorticity polarizes quarks along $\hat{\mathrm{L}}$.
- Its $\sim 1\%$ scale and decrease with energy were **predicted** at JINR BLTP [Rogachevsky–Sorin–Teryaev, PRC 82 (2010); Baznat et al., PRC 88 (2013)].

Nature 548, 62–65 (2017)



P_H vs $\sqrt{s_{\mathrm{NN}}}$, 20–50% Au+Au.

Building the observable: from curl to cross product

Polarization is the experimental trace of vorticity. Global $P_H \propto \langle \boldsymbol{\omega} \cdot \hat{\mathrm{L}} \rangle$ [STAR Nature 548 (2017)] follows the orbital angular momentum; the longitudinal component P_z follows flow-induced vorticity [STAR PRL 123; ALICE PRL 128]. The same $\boldsymbol{\omega} = \nabla \times \mathrm{v}(\mathrm{x})$ sources anomalous axial currents (CVE, KVE). A detector records no field — only momenta.

The new idea. A curl is an antisymmetrised pair of derivatives. Quantum mechanics would insert i: $\mathrm{p} = -i\hbar\nabla$, so $\partial_i \to (i/\hbar)\, p_i$ and the image would be complex. We **omit** i (and $\hbar$) to keep a **real** result. What survives is the antisymmetric pairing — a rotation, built from tracks.

$$\underbrace{\omega_z = \partial_x v_y - \partial_y v_x}_{\text{not measurable, T-odd}} \xrightarrow[\partial_i \to p_i]{} \underbrace{\boxed{w_{p,z} = (\mathrm{P} \times \mathrm{V})_z}}_{\text{built from tracks, T-even}}$$

$\mathrm{P} = \langle \mathrm{p} \rangle$, $\mathrm{V} = \langle \mathrm{p}/E \rangle$. **T-parity**: $\boldsymbol{\omega}$ is T-odd (∇_x even $\times$ v odd); w_p is T-even (P odd $\times$ V odd).

Handedness on top of the covariance form

Jet handedness (Efremov, Mankiewicz, Törnqvist): the P-odd correlator $(\mathrm{k}_1 \times \mathrm{k}_2) \cdot \hat{\mathrm{n}}$ of two energy-ordered fragments measures the parent-parton polarization [Phys. Lett. B 284, 394 (1992)]. Teryaev's heavy-ion reduction is the same energy-ordered block [Teryaev–Usubov, PRC 92, 014906 (2015)]. It is an exact identity for the cell (verified to 10^{-16}):

$$\boxed{w_{p,z} = \frac{1}{N^2} \sum_{i<j} (\mathrm{p}_i \times \mathrm{p}_j)_z \left(\frac{1}{E_j} - \frac{1}{E_i} \right)}$$

Higher-energy particle first; $1/E$ is the thermal weight. Two-body reduction of jet handedness — a covariance, not a P-odd excess.

Covariance form (the other exact face). Split V:

$$\mathrm{V} = \left\langle \tfrac{1}{E} \right\rangle \mathrm{P} + \mathrm{cov}_{\mathrm{cell}}(1/E,\ \mathrm{p}) \quad \Rightarrow \quad w_{p,z} = \left(\mathrm{P} \times \mathrm{cov}_{\mathrm{cell}}(1/E, \mathrm{p}) \right)_z .$$

Both faces: one cell number = the local **soft–hard azimuthal ordering** of the spectrum. That ordering *is* the Landau–Eckart mismatch, built from tracks.

Two fluxes, one cell: w_p is a heat-flow direction

Bin all final-state particles of many events on a 3D grid of *momentum* cells (p_x, p_y, p_z). In each cell:

$$\underbrace{\mathrm{P} = \langle \mathrm{p} \rangle}_{\text{energy flux } T^{0i}} \qquad \text{and} \qquad \underbrace{\mathrm{V} = \left\langle \frac{\mathrm{p}}{E} \right\rangle}_{\text{particle flux } N^i} \qquad \left(\mathrm{V} \neq \frac{\langle \mathrm{p} \rangle}{\langle E \rangle} \right).$$

Why those identifications. The energy–momentum tensor is symmetric, $T^{0i} = T^{i0}$: the **energy flux is the momentum density**. Each track contributes p^i to T^{0i}, so the cell mean $\langle \mathrm{p} \rangle$ *is* the energy flux. The Eckart current is $N^\mu = n\,u^\mu$; its spatial part is the mean velocity $\langle \mathrm{p}/E \rangle$ — the **particle flux**.

Landau's frame sets $T^{0i} = 0$; Eckart's frame sets $N^i = 0$. They coincide only for an ideal fluid with no heat flow. $w_p = \mathrm{P} \times \mathrm{V}$ **probes the mismatch**: a spin-free, momenta-only image of how energy and particle currents fail to share a rest frame.

Mixed species at the same $|\mathrm{p}|$ already tilt V off P (a pion is faster than a proton); collective flow correlates the tilt with the event plane. $w_p = 0$ exactly when $\mathrm{P} \parallel \mathrm{V}$.

Thermal vorticity — mechanical and thermal branches

Equilibrium polarization is fixed by **thermal vorticity** $\varpi_{\mu\nu} = -\frac{1}{2}\partial_{[\mu}\beta_{\nu]}$, $\beta^\mu = u^\mu/T$ [Becattini et al.]:

$$\varpi \;=\; \underbrace{\beta\,\omega_K}_{\text{mechanical rotation}/T} \;+\; \underbrace{u\wedge\nabla\beta}_{\text{temperature-gradient term}} \;.$$

Exact split of the cell velocity — direction × speed:

$$\mathsf{V} = \underbrace{\langle\hat{\mathsf{p}}\rangle\left\langle\tfrac{|\mathsf{p}|}{E}\right\rangle}_{\mathsf{V}^{\mathrm{mech}}} + \underbrace{\mathrm{cov}\left(\hat{\mathsf{p}},\,\tfrac{|\mathsf{p}|}{E}\right)}_{\mathsf{V}^{\mathrm{therm}}}.$$

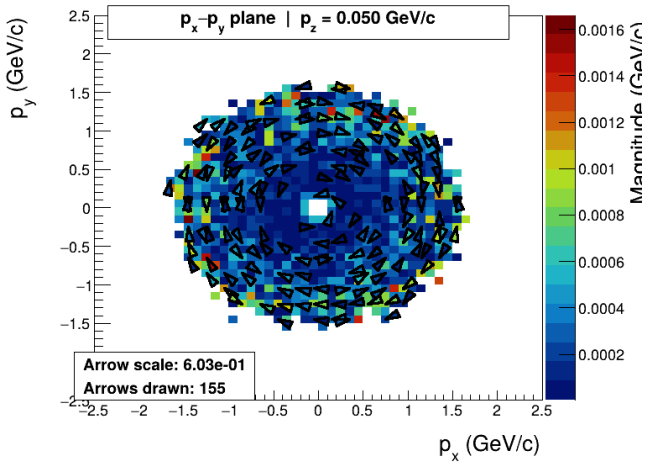
The $1/E$ weight sits in the speed $\frac{|\mathsf{p}|}{E}$. Replacing $1/E \to 1/T$ maps V onto the thermal-vorticity velocity $\beta \sim u/T$:

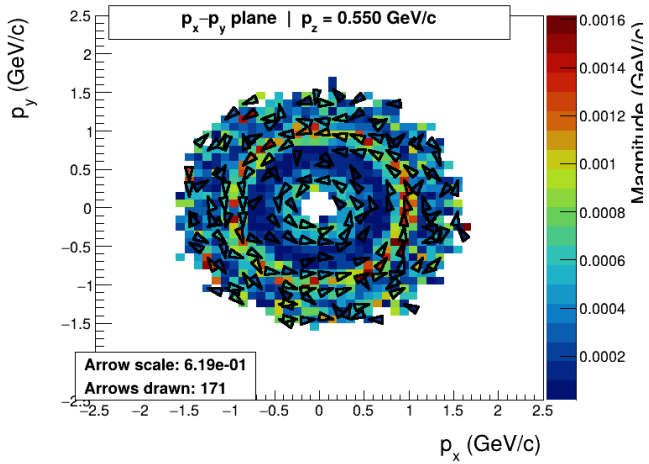
$$\mathsf{V} \;\xrightarrow[1/E\,\to\,1/T]{}\; \tfrac{1}{T}\,\langle\hat{\mathsf{p}}\rangle\,\langle|\mathsf{p}|\rangle \;+\; \mathrm{cov}\left(\hat{\mathsf{p}},\,\tfrac{|\mathsf{p}|}{T}\right),$$

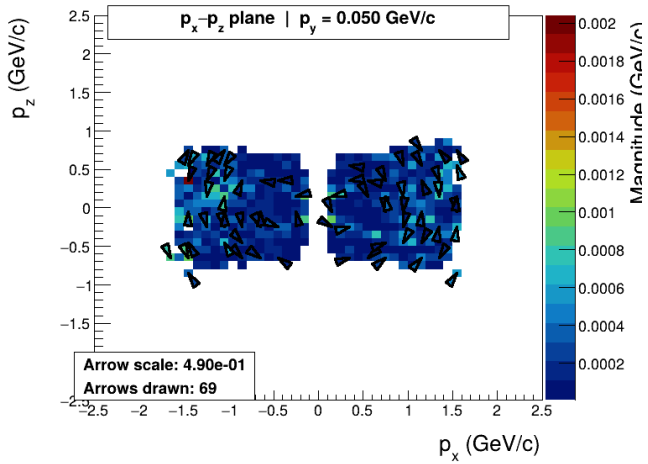
so $\mathsf{w}_p = \mathsf{P}\times\mathsf{V}^{\mathrm{mech}} + \mathsf{P}\times\mathsf{V}^{\mathrm{therm}}$ mirrors $\varpi = \beta\,\omega_K + u\wedge\nabla\beta$.

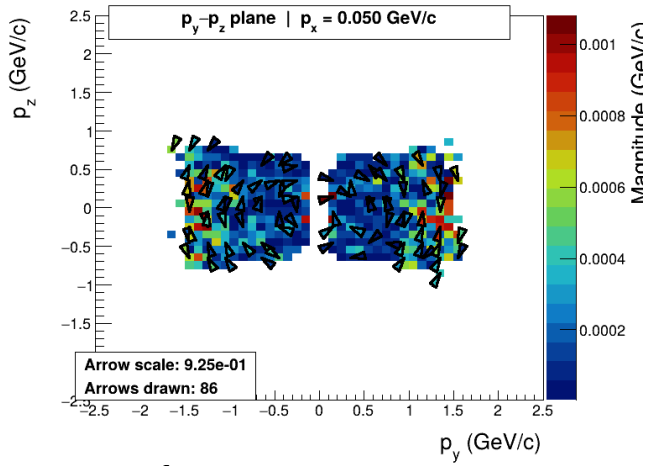
It is **formally consistent to treat energy as temperature**: in a thermal spectrum $E \sim T$, so $1/E$ is the same weight as $1/T$ in β^μ. The substitution is the momentum-space image of thermal vorticity, not an extra assumption.

PHSD Monte Carlo — 2D cuts of w_p



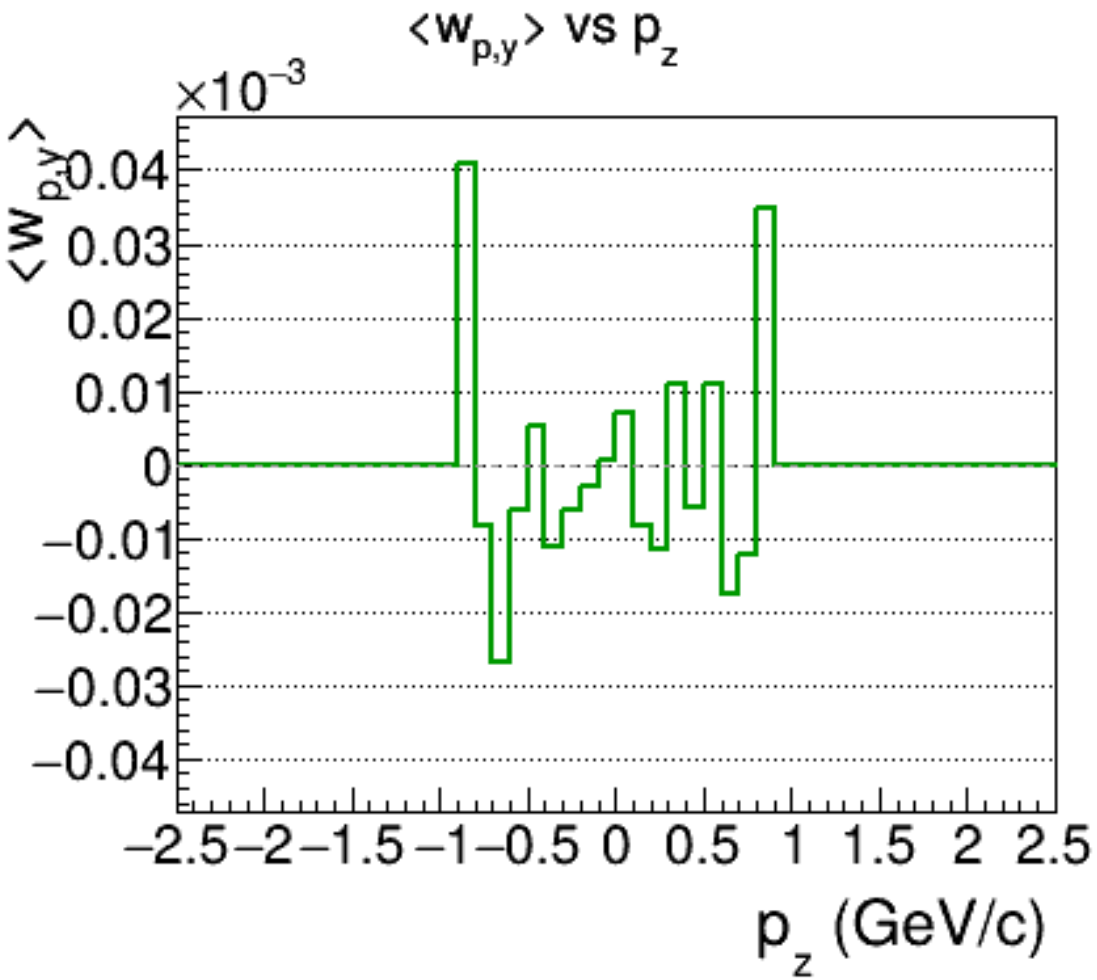






PHSD Au+Au at $\sqrt{s_{\mathrm{NN}}} = 9.2$ GeV, $b \in [4.0, 5.2]$ fm: **10 000 events**, 2.77×10^6 tracks. Aligned to the generated event plane; PID is Monte Carlo truth. Cuts: $p_{\mathrm{T}} > 0.2$ GeV/c, $|y| < 0.5$, cell 0.10 GeV/c. $|w_p|$ maxima: **core–corona / toroidal femto-vortex sheets** [Baznat et al., PRC 93 (2016); Tsegelnik et al., PRC 107 (2023)].

PHSD — $\langle w_{p,y} \rangle$ vs p_z as polarization probe



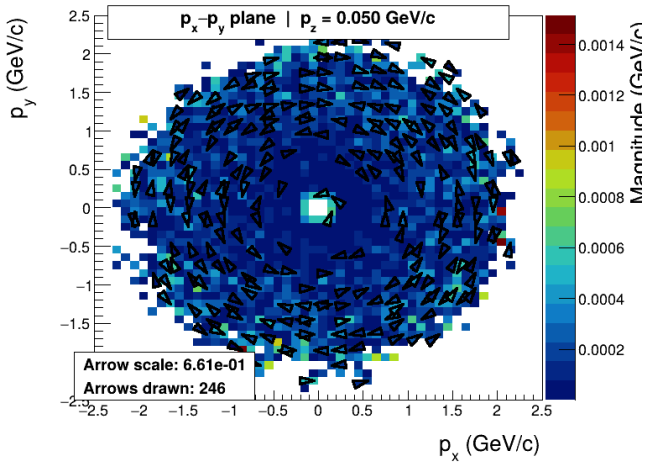
Mean results

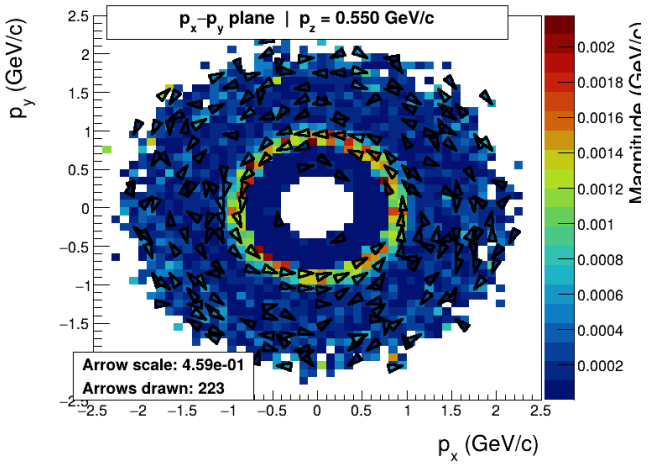
$\langle |w_p| \rangle = 4.7 \times 10^{-4}$ GeV/c
$\simeq 7 \times 10^{20}$ s^{-1}

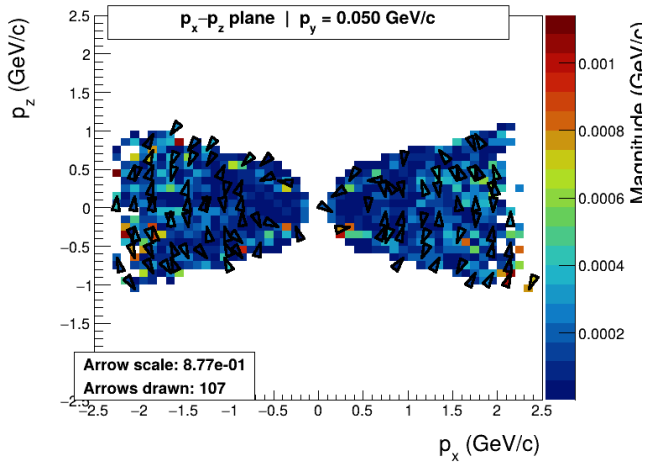
cell max 3.9×10^{-3} GeV/c
$\simeq 6 \times 10^{21}$ s^{-1}

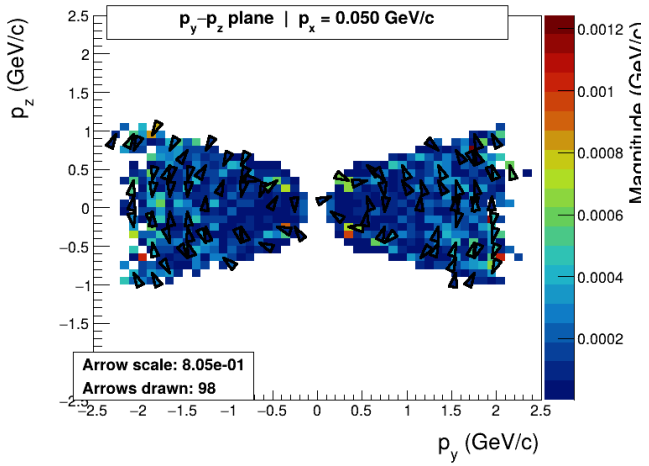
$\langle w_{p,y} \rangle (p_z)$ is the **polarization probe**: an OAM dipole, odd in p_z.

ALICE 2010 — 2D cuts of w_p



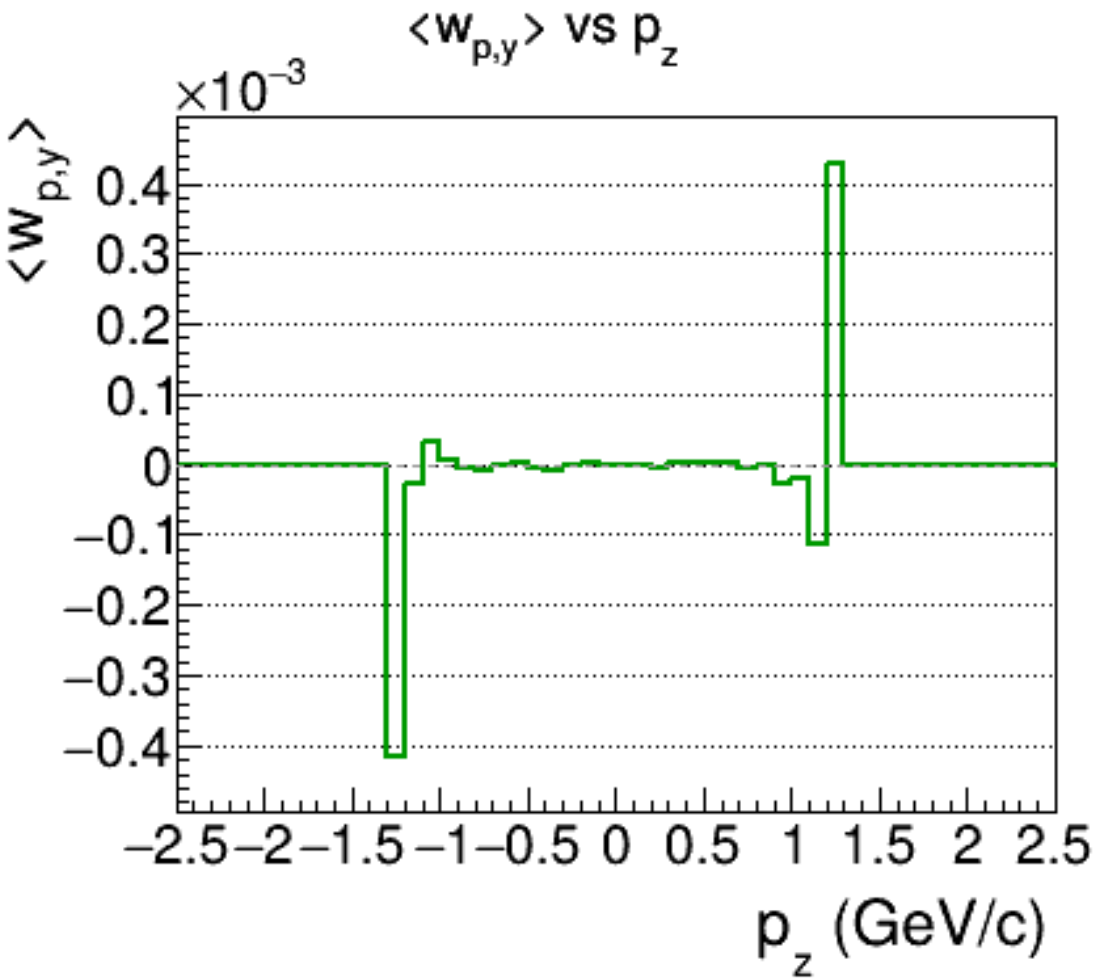






ALICE *real data*: Pb–Pb $\sqrt{s_{\mathrm{NN}}} = 2.76$ TeV, LHC10h ESD run 139038 (CERN Open Data 1102). 13 073 events, 4.89×10^6 tracks. Aligned to the sub-event Ψ_2; PID from the ESD. Same cuts: $p_{\mathrm{T}} > 0.2$ GeV/c, $|y| < 0.5$, cell 0.10 GeV/c. $|w_p|$ maxima: core–corona / toroidal femto-vortex sheets [Baznat et al., PRC 93 (2016); Tsegelnik et al., PRC 107 (2023)].

ALICE — $\langle w_{p,y} \rangle$ vs p_z



Mean results

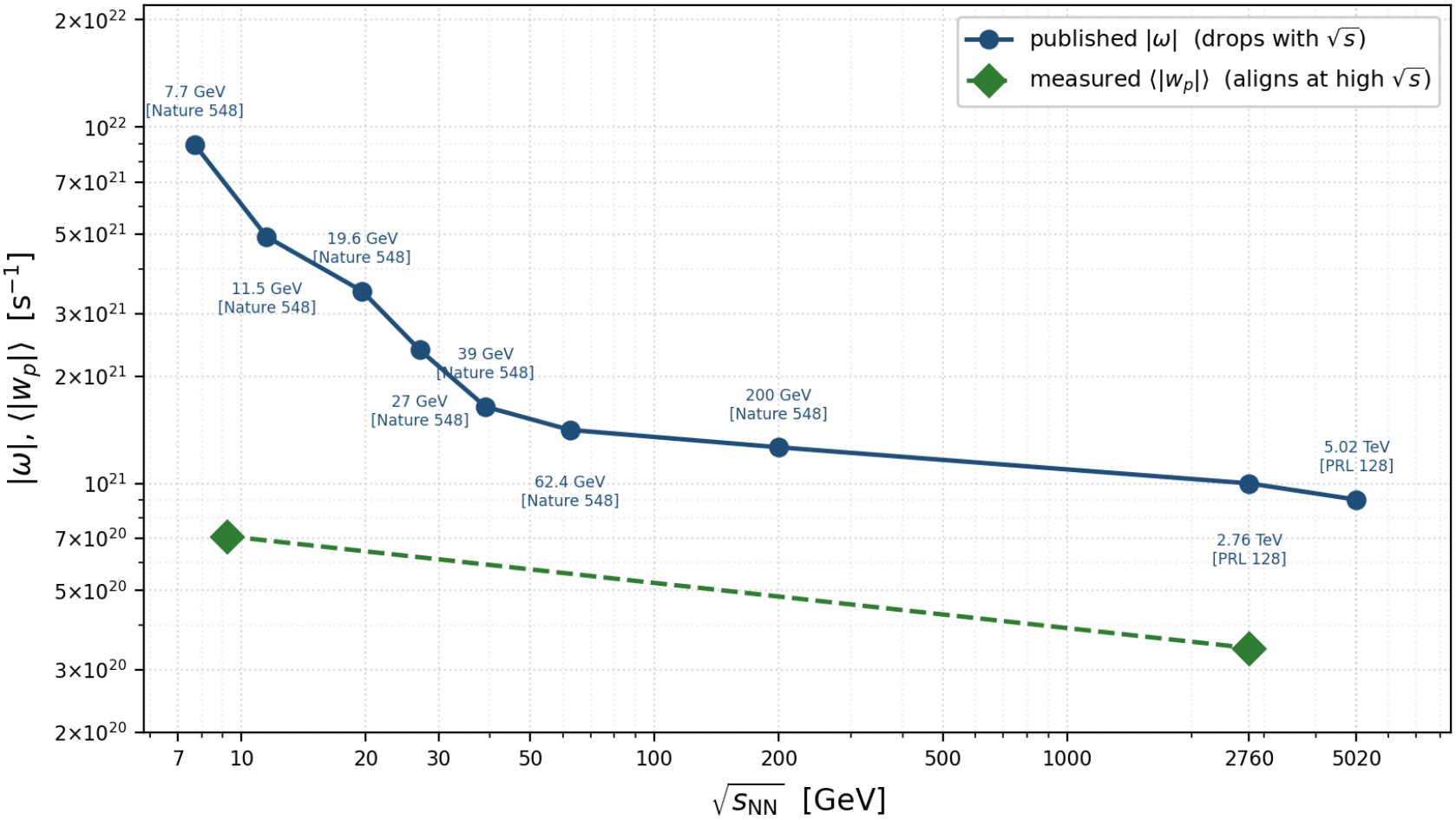
$\langle |w_p| \rangle = 2.3 \times 10^{-4}$ GeV/c
$\simeq 3 \times 10^{20}$ s^{-1}

cell max 5.2×10^{-3} GeV/c
$\simeq 8 \times 10^{21}$ s^{-1}

$\langle w_{p,y} \rangle (p_z)$: odd OAM dipole at $|p_z| \simeq 1.3$ GeV/c.

Compare to ALICE local $|\omega|$ from P_z [PRL 128].

Energy scan — published $|\omega|$ drops; w_p aligns



Published $|\omega|(\sqrt{s_{NN}})$ falls with energy; measured $\langle|w_p|\rangle$ aligns with it as $\sqrt{s_{NN}}$ rises to ALICE.

Conclusions

- w_p is the momentum-space image of vorticity via $\partial_i \to p_i$. Quantum mechanics would insert i; we omit it to keep a real observable. $\boldsymbol{\omega}$ is T-odd, w_p is T-even.
- $\langle p \rangle$ is the cell energy flux ($T^{0i} = T^{i0}$, energy flow = momentum density); $\langle p/E \rangle$ is the particle flux. w_p probes their Landau–Eckart mismatch.
- Jet handedness [Efremov–Mankiewicz–Törnqvist, PLB 284 (1992)] is the exact pair identity of $w_{p,z}$; the covariance form is the same object.
- The exact split $V = V^{\mathrm{mech}} + V^{\mathrm{therm}}$ with $1/E \to 1/T$ mirrors thermal vorticity. Treating energy as temperature is formally consistent.
- Mean $\langle |w_p| \rangle$: 7×10^{20} s^{-1} (PHSD Au+Au 9.2 GeV, 10^4 events, MC-truth plane and PID) and 3×10^{20} s^{-1} (ALICE Pb–Pb 2.76 TeV, 1.3×10^4 events). As energy rises, P_H and $|\boldsymbol{\omega}|$ drop; w_p approaches the ALICE-derived local scale. Peaks are consistent with core–corona vortex sheets.

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Thank you for your attention!