ON ELECTRICITY AND MAGNETISM IN THE SPACE-TIME FILM THEORY

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Concept of Unified Field

The concept of unified field theory is known for a long time.

According to this concept we consider a fundamental nonlinear field model with the following two relations to the material world:

In the Concept of Unified Field

- The elementary particles must be represented as space-localized solutions of this model.
- All interactions between the particles must appear naturally as the manifestation of the nonlinearity of the model.

It is known that the creator of relativity theory A. Einstein was a proponent of this concept. Also many famous researchers worked in the framework of this concept.

Within the framework of the unified field theory, the main problems of modern theoretical physics must be solved. Among these problems are the calculation of the mass spectrum of charged leptons, as well as the unification of electromagnetism and gravity.

But we see a relatively weak progress in this direction of investigation. This is connected with the appropriate extraordinary mathematical difficulties.

Choice of the Unified Model

There are two basic questions concerning a choice of the unified field model:

- The tensor character of the field.
- The invariance group of the model equations.

Also we must define a space dimension which will be considered.

Here we consider the generally covariant field models in four-dimensional space-time.

We will discuss the models for two types of fields in the space under consideration:

- The vector field.
- The scalar field.

According to the original plan for the talk, more attention was supposed to be paid to the scalar model. However, during the preparation of the talk, an unfortunate error was discovered in the calculations that had already been included in the published works. In this talk, in particular, this error will be indicated.

Generally Covariant Vector Field Model in Space-Time[1]

Let us consider the action which has the following world volume form:

$$\mathcal{A} = \int\limits_{\mathbb{V}} \sqrt{|\mathfrak{M}|}\,(\mathrm{d}x)^4 = \int\limits_{\mathbb{V}} \pounds\,\mathrm{d}\mathbb{V}\,, \quad \mathfrak{M}_{\mu\nu} = \mathfrak{m}_{\mu\nu} + \chi^2\,F_{\mu\nu}\,, \tag{1a}$$

$$F_{\mu\nu} \doteq \frac{\partial A_\nu}{\partial x^\mu} - \frac{\partial A_\mu}{\partial x^\nu}\,, \quad \pounds \doteq \sqrt{1 - \chi^2\,\mathcal{I} - \chi^4\,\mathcal{J}^2}\,, \tag{1b}$$

$$\mathcal{I} \doteq -\frac{1}{2}\,\mathfrak{m}^{\mu\sigma}\,\mathfrak{m}^{\nu\rho}\,F_{\mu\nu}\,F_{\sigma\rho}\,, \quad \mathcal{J} \doteq -\frac{1}{8}\,\imath^{\mu\nu\sigma\rho}\,F_{\mu\nu}\,F_{\sigma\rho}\,, \tag{1c}$$

where $\mathfrak{M} \doteq \det(\mathfrak{M}_{\mu\nu})$, $(\mathrm{d}x)^4 \doteq \mathrm{d}x^0\mathrm{d}x^1\mathrm{d}x^2\mathrm{d}x^3$, $\mathbb{V}$ is space-time volume, $\mathfrak{m}_{\mu\nu}$ are components of metric tensor for flat four-dimensional space-time, $\{A_\mu\}$ is the vector real field function, χ is dimensional constant. The Greek indices take values $\{0, 1, 2, 3\}$.

The model equation for the vector field function $\{A_\mu\}$ is

$$\frac{1}{\sqrt{|\mathfrak{m}|}}\,\frac{\partial}{\partial x^\mu}\,\sqrt{|\mathfrak{m}|}\,G^{\mu\nu} = 0\,, \qquad G^{\mu\nu} \doteq \frac{1}{\pounds}\left(F^{\mu\nu} + \frac{\chi^2}{2}\,\mathcal{J}\,\imath^{\mu\nu\sigma\rho}\,F_{\sigma\rho}\right)\,. \tag{2}$$

[1] A. A. Chernitskii, *Nonlinear electrodynamics: singular solitons and their interactions*, 360 p., in Russian. Saint Petersburg, ENJEKON, 2012; `http://chernitskii.ru/books/`.

Energy-Momentum Density Tensor

Let's write the components of the energy-momentum density tensor in terms of the components of the quasi-bivector of electromagnetic field inductions $\{\mathbf{D}, \mathbf{B}\}$, as well as approximate expressions for a weak field when $\{\mathbf{D}, \mathbf{B}\} \approx \{\mathbf{E}, \mathbf{H}\}$:

$$\mathcal{T}^{00} = \mathcal{E} = \frac{1}{4\pi\,\chi^2}\left(\sqrt{1+\chi^2\left(\mathbf{D}^2+\mathbf{B}^2\right)+\chi^4\left(\mathbf{D}\times\mathbf{B}\right)^2}-1\right)\,, \tag{3a}$$

$$\approx \frac{1}{8\pi}\left(\mathbf{D}^2+\mathbf{B}^2\right)\,, \tag{3b}$$

$$\mathcal{T}^{0i} = \mathcal{T}^{i0} = \mathcal{P}^i = \left(\mathbf{D}\times\mathbf{B}\right)^i = \left(\mathbf{E}\times\mathbf{H}\right)^i\,, \tag{3c}$$

$$\mathcal{T}^{ij} = \frac{1}{4\pi}\left(\underline{\mathfrak{m}}^{ij}\left(\mathbf{D}\cdot\mathbf{E}+\mathbf{B}\cdot\mathbf{H}\right)-\left(D^i\,E^j+B^i\,H^j\right)\right)-\underline{\mathfrak{m}}^{ij}\,\mathcal{E} \tag{3d}$$

$$\approx \frac{1}{4\pi}\left(\frac{1}{2}\,\underline{\mathfrak{m}}^{ij}\left(\mathbf{D}^2+\mathbf{B}^2\right)-\left(D^i\,D^j+B^i\,B^j\right)\right)\,. \tag{3e}$$

Extremal Space-Time Film Model[2]

Let us consider the action which has the following world volume form:

$$\mathcal{A} = \int\limits_{\mathbb{V}} \sqrt{|\mathfrak{M}|}\,(\mathrm{d}x)^4 = \int\limits_{\mathbb{V}} \pounds\,\mathrm{d}\mathbb{V}\,, \tag{4a}$$

$$\mathfrak{M}_{\mu\nu} = \mathfrak{m}_{\mu\nu} + \chi^2\,\frac{\partial\Phi}{\partial x^\mu}\,\frac{\partial\Phi}{\partial x^\nu}\,, \qquad \pounds \doteq \sqrt{\left|1 + \chi^2\,\mathfrak{m}^{\mu\nu}\,\frac{\partial\Phi}{\partial x^\mu}\,\frac{\partial\Phi}{\partial x^\nu}\right|}\,, \tag{4b}$$

where $\mathfrak{M} \doteq \det(\mathfrak{M}_{\mu\nu})$, $(\mathrm{d}x)^4 \doteq \mathrm{d}x^0\mathrm{d}x^1\mathrm{d}x^2\mathrm{d}x^3$, $\mathbb{V}$ is space-time volume, $\mathfrak{m}_{\mu\nu}$ are components of metric tensor for flat four-dimensional space-time, Φ is scalar real field function, χ is dimensional constant. The Greek indices take values $\{0, 1, 2, 3\}$.

The model equation for the scalar field function Φ is

$$\frac{1}{\sqrt{|\mathfrak{m}|}}\,\frac{\partial}{\partial x^\mu}\sqrt{|\mathfrak{m}|}\,\Upsilon^\mu = 0\,, \qquad \Upsilon^\mu \doteq \frac{\Phi^\mu}{\pounds}\,, \quad \Phi^\mu = \mathfrak{m}^{\mu\nu}\,\Phi_\nu\,, \quad \Phi_\nu \doteq \frac{\partial\Phi}{\partial x^\nu}\,. \tag{5}$$

[2] A. A. Chernitskii, Lightlike shell solitons of extremal space-time film, *J. Phys. Commun.* **2** (2018) 105013; arXiv:1506.09137.

Energy-Momentum Density Tensor

Let's write the components of the energy-momentum density tensor in terms of the components of the quasi-four-vector of the field $\{\Upsilon_0, \Phi_1, \Phi_2, \Phi_3\}$, as well as approximate expressions for a weak field when $\Upsilon_\mu \approx \Phi_\mu$:

$$\mathbb{T}^{00} = \mathcal{E} = \frac{1}{4\pi\,\chi^2}\left(\sqrt{\left(1+\chi^2\,\underline{\mathfrak{m}}^{ij}\,\Phi_i\,\Phi_j\right)\left(1+\chi^2\,\Upsilon_0^2\right)}-1\right)\,, \tag{6a}$$

$$\approx \frac{1}{8\pi}\left(\Phi_0^2+\Phi_1^2+\Phi_2^2+\Phi_3^2\right)\,, \tag{6b}$$

$$\mathbb{T}^{0i} = \mathbb{T}^{i0} = \mathcal{P}^i = \frac{1}{4\pi}\,\Phi^i\,\Upsilon^0 = -\frac{1}{4\pi}\,\Phi_i\,\Upsilon_0\,, \tag{6c}$$

$$\mathbb{T}^{ij} = \frac{1}{4\pi}\left(\frac{\Phi^i\,\Phi^j}{\mathcal{L}}-\frac{1}{\chi^2}\,\underline{\mathfrak{m}}^{ij}\,(\mathcal{L}-1)\right) \tag{6d}$$

$$\approx \frac{1}{4\pi}\left(\Phi_i\,\Phi_j-\frac{1}{2}\,\underline{\mathfrak{m}}_{ij}\left(\Phi_1^2+\Phi_2^2+\Phi_3^2-\Phi_0^2\right)\right) \tag{6e}$$

$$\mathcal{L} = \frac{\sqrt{1+\chi^2\left(\Phi_1^2+\Phi_2^2+\Phi_3^2\right)}}{\sqrt{1+\chi^2\,\Upsilon_0^2}} \approx 1+\frac{\chi^2}{2}\left(\Phi_1^2+\Phi_2^2+\Phi_3^2-\Phi_0^2\right) \tag{6f}$$

Charged Spheroidal Soliton or Spheron

In unified field theory, a point-like spinless charged particle is represented by a spheroidal soliton or spheron.

In its own coordinate system, this soliton is a static solution to the field model, depending only on one radial coordinate:

$$\dot{r} \doteq \sqrt{\left(\dot{x}^1\right)^2 + \left(\dot{x}^2\right)^2 + \left(\dot{x}^3\right)^2} \,. \tag{7}$$

Here, $\dot{x}^\mu$ $(\mu = 0, 1, 2, 3)$ are the rectangular rectilinear coordinates of the intrinsic coordinate system of the spheron.

The Temporal rotation transformation (Lorentz) transformation converts a stationary spheron into one that is moving in the coordinate system $\{x^\nu\}$:

$$\dot{x}^\mu = \dot{L}^\mu_{\cdot\nu}\, x^\nu \,, \quad x^\nu = \dot{L}^{\cdot\nu}_{\mu}\, \dot{x}^\mu \tag{8}$$

Both field models under consideration have the exact spheron solution.

Vector Spheron in Its Own Coordinate System

The vector model has the following spheron solution in its own coordinate system:

$$\dot{A}^0 = -\dot{A}_0 = \phi = \frac{\not{q}}{\dot{r}}\, \not{F}_{\frac{1}{4};\frac{1}{2};\frac{5}{4}}\left(-\frac{\bar{r}^4}{\dot{r}^4}\right) \tag{9a}$$

$$\dot{A}_i = \dot{A}^i = 0\,. \tag{9b}$$

where $\bar{r}^4 \doteqdot \chi^2\, \not{q}^2$, $\not{F}_{\alpha;\beta;\gamma}(z)$ is a hypergeometric function, also denoted as ${}_2F_1(\alpha,\beta;\gamma;z)$. The graph of the function ϕ is shown in Fig. 1.

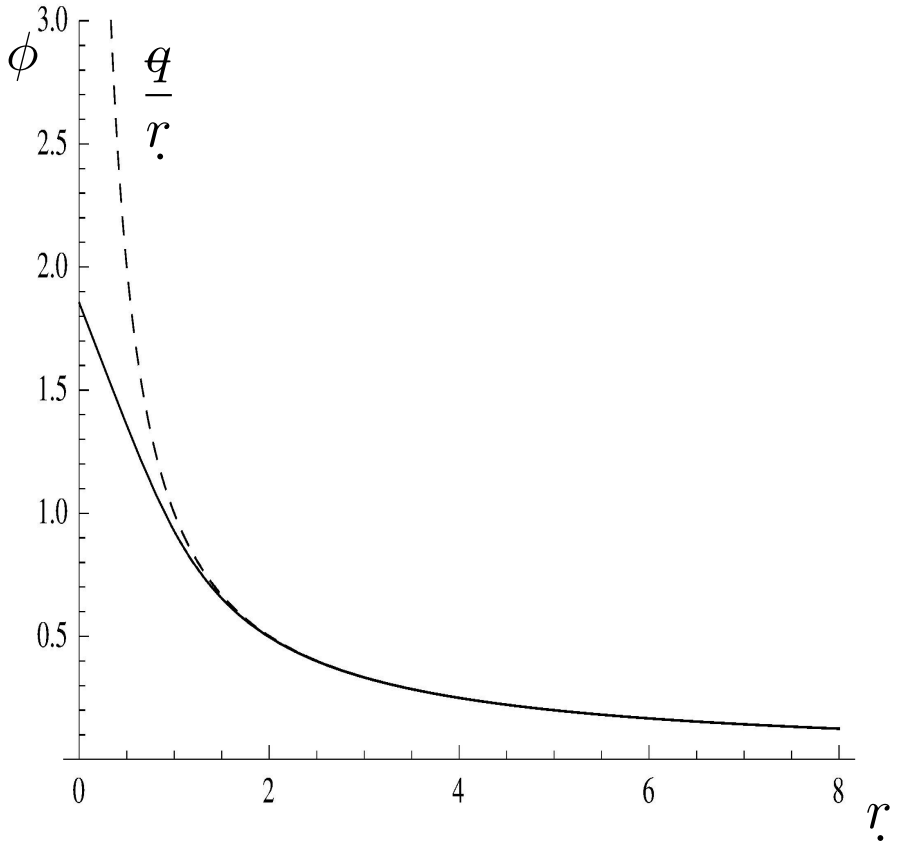


Рис. 1: The function ϕ of the vector spheron (9) for $\bar{r} = 1$ and $\not{q} = 1$.

Moving Vector Spheron

The electric induction of the vector spheron, which at the initial moment of time is located at the point $\{x_1, x_2, x_3\} = \{a_1, a_2, 0\}$ and is moving along the axis x^1 with a velocity v, has the form

$$D_1 = q\,\frac{x_1 - a_1 - v x^0}{\sqrt{1-v^2}\,r^3}\,, \quad D_2 = q\,\frac{x_2 - a_2}{\sqrt{1-v^2}\,r^3}\,, \quad D_3 = q\,\frac{x_3}{\sqrt{1-v^2}\,r^3}\,, \tag{10}$$

where $r = \sqrt{(x^1 - a_1 - v\,x^0)^2/(1-v^2) + (x_2 - a_2)^2 + x_3^2}$

At the initial moment of time ($x^0 = 0$) and at the origin ($x^1 = x^2 = x^3 = 0$), the electric induction

$$D_1 = -q\,\frac{(1-v^2)\,a_1}{\left(a_1^2 + (1-v^2)\,a_2^2\right)^{3/2}}\,, \quad D_2 = -q\,\frac{(1-v^2)\,a_2}{\left(a_1^2 + (1-v^2)\,a_2^2\right)^{3/2}}\,, \quad D_3 = 0\,. \tag{11}$$

If the center of the spheron is located on the axis x_2 ($a_1 = 0$), then

$$D_1 = 0\,, \quad D_2 = -\frac{q\,\mathrm{sign}(a_2)}{\sqrt{1-v^2}\,a_2^2}\,, \quad D_3 = 0\,. \tag{12}$$

Scalar Spheron in Its Own Coordinate System

The scalar model has the following spheron solution in its own coordinate system:

$$\Phi = \frac{q}{r}\, F_{\frac{1}{4};\frac{1}{2};\frac{5}{4}}(1) = \frac{q}{\bar{r}}\,\frac{\sqrt{\pi}\,\Gamma(5/4)}{\Gamma(3/4)} \qquad \text{for} \quad r \leqslant \bar{r}\,, \tag{13a}$$

$$\Phi = \frac{q}{r}\, F_{\frac{1}{4};\frac{1}{2};\frac{5}{4}}\left(\frac{\bar{r}^4}{r^4}\right) \qquad \text{for} \quad r \geqslant \bar{r}\,. \tag{13b}$$

where $\bar{r}^4 \doteq \chi^2 q^2$, $F_{\alpha;\beta;\gamma}(z)$ is a hypergeometric function, also denoted as ${}_2F_1(\alpha,\beta;\gamma;z)$, and $\Gamma(z)$ is the gamma function. The graph of the function (13) is shown in Fig. 2.

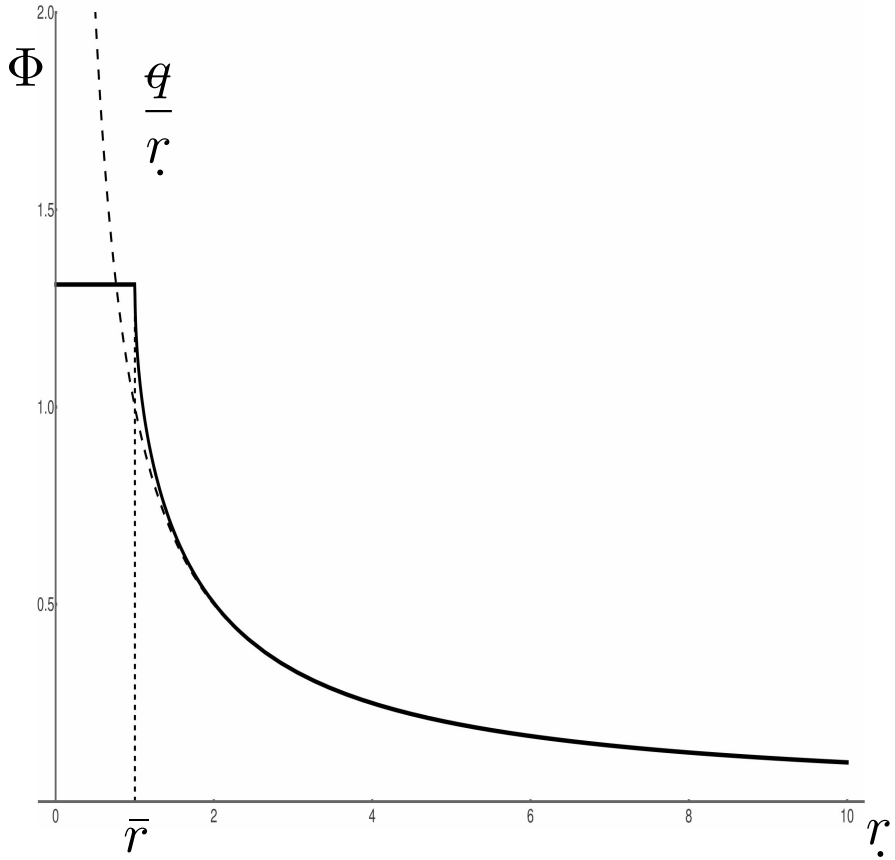


Рис. 2: The field function of the spheron (13) for $\bar{r} = 1$ and $q = 1$.

Moving Scalar Spheron

The spatial components of the dual four-vector of the field of a scalar spheron, which is located at the initial moment of time at the point $\{x_1, x_2, x_3\} = \{a_1, a_2, 0\}$ and is moving along the x^1 axis with velocity v:

$$\Upsilon_1 = -q\,\frac{x_1 - a_1 - v\,x^0}{(1-v^2)\,r^3}\,, \quad \Upsilon_2 = -q\,\frac{x_2 - a_2}{r^3}\,, \quad \Upsilon_3 = -q\,\frac{x_3}{r^3}\,, \tag{14}$$

At the initial moment of time ($x^0 = 0$) and at the origin ($x^1 = x^2 = x^3 = 0$).

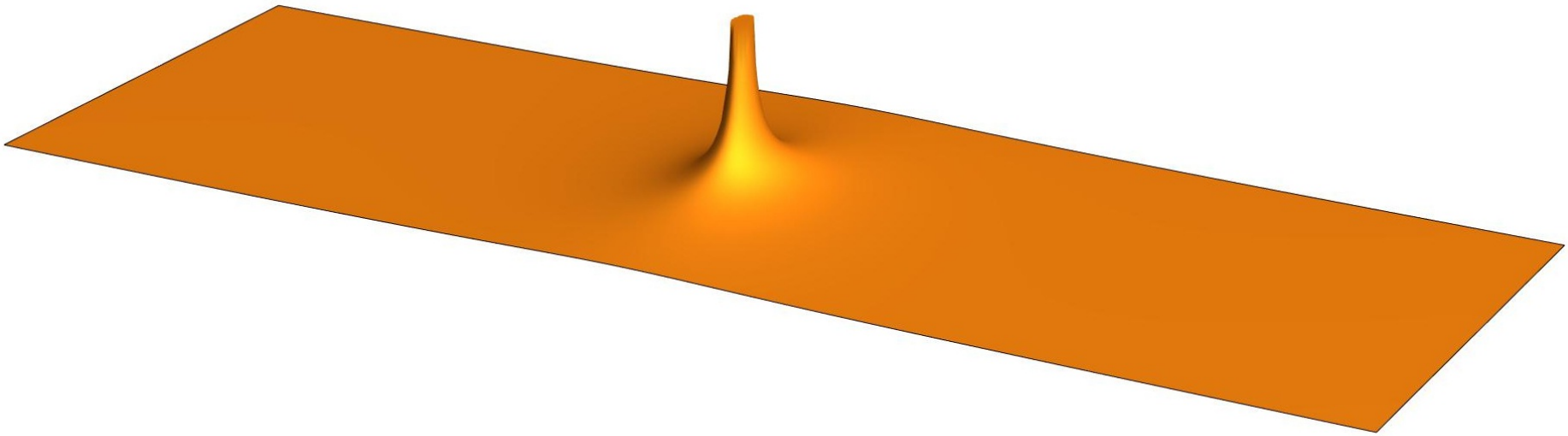
$$\Upsilon_1 = q\,\frac{\sqrt{1-v^2}\,a_1}{\left(a_1^2 + (1-v^2)\,a_2^2\right)^{3/2}}\,, \quad \Upsilon_2 = q\,\frac{(1-v^2)^{3/2}\,a_2}{\left(a_1^2 + (1-v^2)\,a_2^2\right)^{3/2}}\,, \quad \Upsilon_3 = 0\,. \tag{15}$$

If the center of the spheron is located on the axis x_2 ($a_1 = 0$), then

$$\Upsilon_1 = 0\,, \quad \Upsilon_2 = \frac{q\,\mathrm{sign}(a_2)}{a_2^2}\,, \quad \Upsilon_3 = 0\,. \tag{16}$$

Video of Moving Scalar Spheron

VIDEO



Multi-Spheron Solution

We will call a multi-spheron or N-spheron solution of the model an exact multi-soliton solution such that, near N spatial points within a certain time interval, this solution is close to the function of a moving or stationary spheron.

Within the framework of this solution, one can consider, in particular:

- The stability of spherons
- Trejectories of spherons.

The deviation of the motion of the spherons from uniform rectilinear motion is due to their interaction.

Finding a two-spheron solution, let alone a multi-spheron one, is a very complex mathematical problem.

Ininitial Approximation

At the initial stage of research into multi-soliton solutions, approximate methods are used.

In particular, a field configuration with spherons separated by sufficiently large distances is considered. In this case, we are talking about the long-range interaction of spherons.

As an initial approximation, a certain sum of spheron solutions with time-dependent velocities is taken.

The models under consideration allow for various formulations in the form of systems of first-order differential equations. When summing spheron solutions, the choice of field functions for summation is important.

One of the methods for finding the equations of spheron trajectories is to use the integral law of conservation of energy–momentum. In this case, we can talk about the force interaction between spherons.

We will analyze the force acting on the stationary spheron from the distant spherons.

The Integral Law of Conservation from the Differential

Integration by parts of the energy-momentum differential conservation law gives the appropriate integral conservation law

$$\int\limits_{\mathbb{V}} \frac{\partial \mathbb{T}^{\mu\nu}}{\partial x^{\nu}} \, \mathrm{d}\mathbb{V} = 0 \quad \Longrightarrow \quad \int\limits_{\Sigma} \mathbb{T}^{\mu\nu} \, \mathrm{d}\Sigma_{\nu} = 0 \, , \tag{17}$$

where $\mathbb{V}$ is four-dimensional volume including the three-dimensional volume $\mathbb{V}$ and time interval $\left[x^0 - \Delta x^0/2 \, , \; x^0 + \Delta x^0/2\right]$, Σ is its three-dimensional boundary hypersurface, $\mathrm{d}\Sigma_{\nu}$ are components of the outer hypersurface element four-vector:

$$\mathrm{d}\Sigma_0 = \pm \mathrm{d}\mathbb{V} \, , \qquad \mathrm{d}\mathbb{V} = \mathrm{d}x^1 \mathrm{d}x^2 \mathrm{d}x^3 \, ,$$
$$\mathrm{d}\Sigma_i = \mathrm{d}\sigma_i \, \mathrm{d}x^0 \, , \quad \mathrm{d}\sigma_1 = \pm \mathrm{d}x^2 \mathrm{d}x^3 \, , \quad \mathrm{d}\sigma_2 = \pm \mathrm{d}x^1 \mathrm{d}x^3 \, , \quad \mathrm{d}\sigma_3 = \pm \mathrm{d}x^1 \mathrm{d}x^2 \, ,$$

where $\mathrm{d}\sigma_i$ are the outer normal components of the two-dimensional surface σ bounded the three-dimensional volume $\mathbb{V}$.

Momentum and Force Hypersurfaces

Let us divide the three-dimensional closed hypersurface Σ into two unclosed hypersurfaces in some coordinate system such that $\Sigma = \overset{P}{\Sigma} \cup \overset{F}{\Sigma}$:

$$\overset{P}{\Sigma} = \left.\mathbb{V}\right|_{x^0 = x^0 - \Delta x^0/2} \cup \left.\mathbb{V}\right|_{x^0 = x^0 + \Delta x^0/2}, \tag{18a}$$

$$\overset{F}{\Sigma} = \sigma \cup \left[x^0 - \Delta x^0/2\,,\; x^0 + \Delta x^0/2\right]. \tag{18b}$$

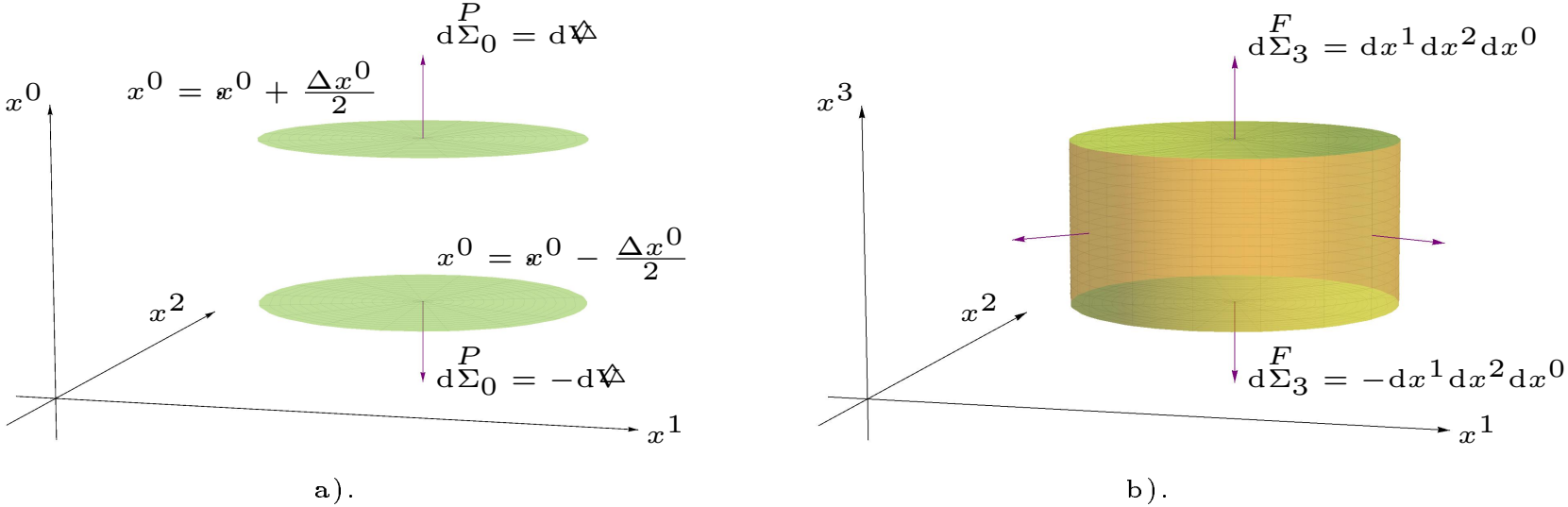


Fig.4. a). Section for $\overset{P}{\Sigma}$ in three-dimensional space $\{x^0, x^1, x^2\}$;
b). Section for $\overset{F}{\Sigma}$ in three-dimensional space $\{x^1, x^2, x^3\}$.

Momentum and Force

Thus we have the integral conservation law in the following form:

$$\Delta\mathbb{P}^{\mu}_{\mathbb{V}} = \bar{\Delta}\mathbb{F}^{\mu}_{\sigma}\,, \qquad \Delta\mathbb{P}^{\mu}_{\mathbb{V}} \doteq \mathbb{P}^{\mu}_{\mathbb{V}}\Big|_{x^0=\mathscr{x}^0-\frac{\Delta x^0}{2}}^{x^0=\mathscr{x}^0+\frac{\Delta x^0}{2}}\,, \quad \bar{\Delta}\mathbb{F}^{\mu}_{\sigma} \doteq \int_{\mathscr{x}^0-\frac{\Delta x^0}{2}}^{\mathscr{x}^0+\frac{\Delta x^0}{2}} \mathbb{F}^{\mu}_{\sigma}\,\mathrm{d}x^0\,, \tag{19}$$

where the momentum of field into the three-dimensional volume $\mathbb{V}$ and the force on the two-dimensional surface σ are defined:

$$\mathbb{P}^{\mu}_{\mathbb{V}} \doteq \int\limits_{\mathbb{V}} \mathbb{T}^{\mu 0}\,\mathrm{d}\mathbb{V}\,, \qquad \mathbb{F}^{\mu}_{\sigma} \doteq -\int\limits_{\sigma} \mathbb{T}^{\mu i}\,\mathrm{d}\sigma_i\,. \tag{20}$$

If the functions $\mathbb{F}^{\mu}_{\sigma}(x^0)$ are continuous, then, in accordance with the mean value theorem, we have

$$\Delta\mathbb{F}^{\mu}_{\sigma} = \mathbb{F}^{\mu}_{\sigma}(\mathscr{x}^{0\prime})\,\Delta x^0\,, \tag{21}$$

where $\mathscr{x}^{0\prime} \in (\mathscr{x}^0 - \frac{\Delta x^0}{2}, \mathscr{x}^0 + \frac{\Delta x^0}{2})$. Substituting (21) into (19), dividing by Δx^0 and letting Δx^0 tend to 0 ($\Delta x^0 \to 0$), we obtain the following dynamic law:

$$\frac{\mathrm{d}\mathbb{P}^{\mu}_{\mathbb{V}}}{\mathrm{d}\mathscr{x}^0} = \mathbb{F}^{\mu}_{\sigma}\,. \tag{22}$$

Configuration of Spherons

The first spheron is at rest at the origin of coordinates $\{0, 0, 0\}$. The second one moves along the axis x_1 with a velocity component v $(-1 < v < 1)$, and at the time $x^0 = 0$ it is located at the point with coordinates $\{a_1, a_2, 0\}$. Spherical radius functions $\overset{1}{r}$ and $\overset{2}{r}$ of the spherons have the form:

$$\overset{1}{r} = r = \sqrt{x_1^2 + x_2^2 + x_3^2}\,, \tag{23a}$$

$$\overset{2}{r} = \sqrt{\frac{\left(x_1 - a_1 - v\,x^0\right)^2}{1 - v^2} + \left(x_2 - a_2\right)^2 + x_3^2}\,. \tag{23b}$$

Figure 3 shows the arrangement of spherons at time $x^0 = 0$.

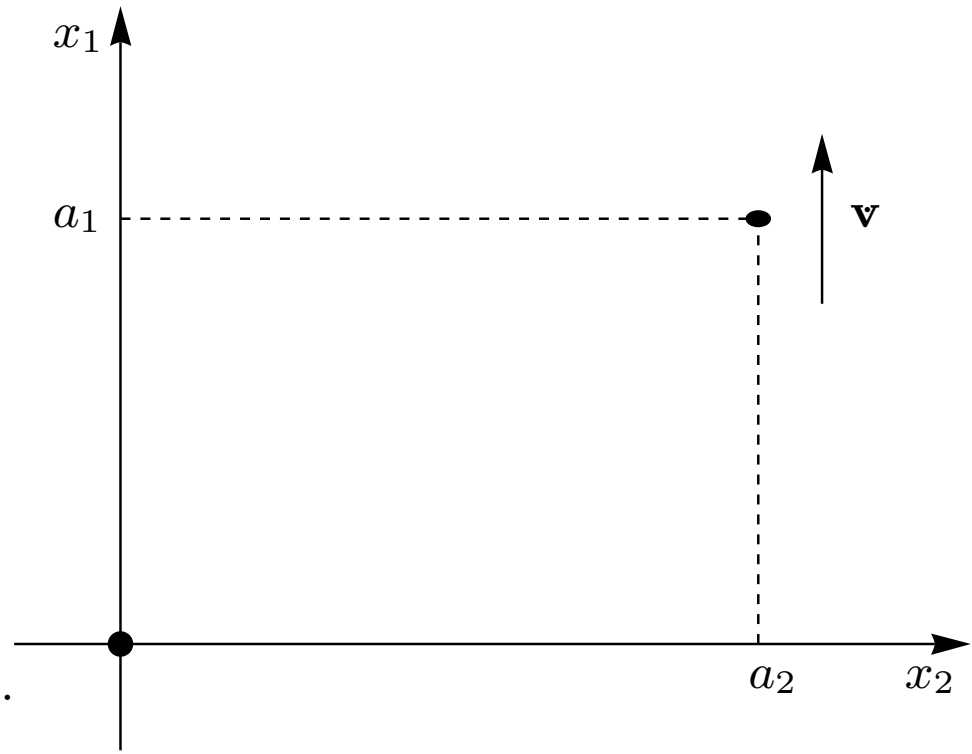


Рис. 3: The arrangement of the interacting spherons at time $x^0 = 0$.

The Sum of Spheron Fields in the Vector Model

The components of electric induction that are included in the force acting on the first spheron:

$$D_1 = \frac{\overset{1}{q}\, x_1}{r^3} - \frac{\overset{2}{q}\,(1-\boldsymbol{v}^2)\, a_1}{\left(a_1^2 + (1-\boldsymbol{v}^2)\, a_2^2\right)^{3/2}} \;, \quad D_2 = \frac{\overset{1}{q}\, x_2}{r^3} - \frac{\overset{2}{q}\,(1-\boldsymbol{v}^2)\, a_2}{\left(a_1^2 + (1-\boldsymbol{v}^2)\, a_2^2\right)^{3/2}} \;, \quad D_3 = \frac{\overset{1}{q}\, x_3}{r^3} \;, \tag{24}$$

Here, the electric induction of the second spheron is considered constant, equal to its value at the origin.

The corresponding cross terms of the components of the energy-momentum density tensor are:

$$\widetilde{T}^{11} = \frac{1}{4\pi}\left(-\overset{1}{D}_1\,\overset{2}{D}_1 + \overset{1}{D}_2\,\overset{2}{D}_2 + \overset{1}{D}_3\,\overset{2}{D}_3\right) = Q\,(x_1\, a_1 - x_2\, a_2) \;, \tag{25a}$$

$$\widetilde{T}^{12} = \frac{1}{4\pi}\left(-\overset{1}{D}_1\,\overset{2}{D}_2 - \overset{1}{D}_2\,\overset{2}{D}_1\right) = Q\,(x_1\, a_2 + x_2\, a_1) \;, \tag{25b}$$

$$\widetilde{T}^{13} = \frac{1}{4\pi}\left(-\overset{1}{D}_1\,\overset{2}{D}_3 - \overset{1}{D}_3\,\overset{2}{D}_1\right) = Q\, x_3\, a_1 \;, \tag{25c}$$

$$\widetilde{T}^{22} = \frac{1}{4\pi}\left(-\overset{1}{D}_2\,\overset{2}{D}_2 + \overset{1}{D}_1\,\overset{2}{D}_1 + \overset{1}{D}_3\,\overset{2}{D}_3\right) = Q\,(x_2\, a_2 - x_1\, a_1) \;, \tag{25d}$$

$$\widetilde{T}^{23} = \frac{1}{4\pi}\left(-\overset{1}{D}_2\,\overset{2}{D}_3 - \overset{1}{D}_3\,\overset{2}{D}_2\right) = Q\, x_3\, a_2 \;, \quad Q \doteqdot \frac{\overset{1}{q}\,\overset{2}{q}\,(1-\boldsymbol{v}^2)}{4\pi\, r^3\left(a_1^2 + (1-\boldsymbol{v}^2)\, a_2^2\right)^{3/2}} \;. \tag{25e}$$

Force in the Vector Model

By performing integration over the surface surrounding the region where the first spheron is localized, we obtain the components of the force acting on the first spheron from the second:

$$\mathbb{F}_1 = -\bar{Q}\, a_1 \,, \quad \mathbb{F}_2 = -\bar{Q}\, a_2 \,, \quad \mathbb{F}_2 = 0 \,, \quad \bar{Q} \doteqdot \frac{\overset{1}{q}\overset{2}{q}\left(1-\boldsymbol{v}^2\right)}{\left(a_1^2+\left(1-\boldsymbol{v}^2\right)a_2^2\right)^{3/2}} \,. \tag{26}$$

As can be seen, the components of the force correspond to the repulsion of charges of the same sign.

If the second spheron is located on the axis x^2 ($a_1 = 0$), then the components of the force

$$\mathbb{F}_1 = \mathbb{F}_3 = 0 \,, \quad \mathbb{F}_2 = -\frac{\overset{1}{q}\overset{2}{q}\,\mathrm{sign}(a_2)}{\sqrt{1-\boldsymbol{v}^2}\,a_2^2} \,. \tag{27}$$

The Sum of Spheron Fields in the Scalar Model

The components of the dual four-vector of the field, which are substituted into the force acting on the first spheron:

$$\Upsilon_1 = -\frac{\overset{1}{q}\, x_1}{r^3} + \frac{\overset{2}{q}\sqrt{1-\boldsymbol{v}^2}\, a_1}{\left(a_1^2 + (1-\boldsymbol{v}^2)\, a_2^2\right)^{3/2}}\,, \quad \Upsilon_2 = -\frac{\overset{1}{q}\, x_2}{r^3} + \frac{\overset{2}{q}\,(1-\boldsymbol{v}^2)^{3/2}\, a_2}{\left(a_1^2 + (1-\boldsymbol{v}^2)\, a_2^2\right)^{3/2}}\,, \quad \Upsilon_3 = -\frac{\overset{1}{q}\, x_3}{r^3}\,, \tag{28}$$

Here, the dual four-vector of the field of the second sferon is considered constant, equal to its value at the origin.

The corresponding cross terms of the components of the energy-momentum density tensor are:

$$\tilde{T}{}^{11} = \frac{1}{4\pi}\left(\overset{1}{\Upsilon}_1\,\overset{2}{\Upsilon}_1 - \overset{1}{\Upsilon}_2\,\overset{2}{\Upsilon}_2 - \overset{1}{\Upsilon}_3\,\overset{2}{\Upsilon}_3\right) = Q\left(-\frac{x_1\, a_1}{\sqrt{1-\boldsymbol{v}^2}} + x_2\, a_2\sqrt{1-\boldsymbol{v}^2}\right)\,, \tag{29a}$$

$$\tilde{T}{}^{12} = \frac{1}{4\pi}\left(\overset{1}{\Upsilon}_1\,\overset{2}{\Upsilon}_2 + \overset{1}{\Upsilon}_2\,\overset{2}{\Upsilon}_1\right) = -Q\left(x_1\, a_2\sqrt{1-\boldsymbol{v}^2} + \frac{x_2\, a_1}{\sqrt{1-\boldsymbol{v}^2}}\right)\,, \tag{29b}$$

$$\tilde{T}{}^{13} = \frac{1}{4\pi}\left(\overset{1}{\Upsilon}_1\,\overset{2}{\Upsilon}_3 + \overset{1}{\Upsilon}_3\,\overset{2}{\Upsilon}_1\right) = -Q\,\frac{x_3\, a_1}{\sqrt{1-\boldsymbol{v}^2}}\,, \tag{29c}$$

$$\tilde{T}{}^{22} = \frac{1}{4\pi}\left(\overset{1}{\Upsilon}_2\,\overset{2}{\Upsilon}_2 - \overset{1}{\Upsilon}_1\,\overset{2}{\Upsilon}_1 - \overset{1}{\Upsilon}_3\,\overset{2}{\Upsilon}_3\right) = Q\left(-x_2\, a_2\sqrt{1-\boldsymbol{v}^2} + \frac{x_2\, a_1}{\sqrt{1-\boldsymbol{v}^2}}\right)\,, \tag{29d}$$

$$\tilde{T}{}^{23} = \frac{1}{4\pi}\left(\overset{1}{\Upsilon}_2\,\overset{2}{\Upsilon}_3 + \overset{1}{\Upsilon}_3\,\overset{2}{\Upsilon}_2\right) = -Q\, x_3\, a_2\sqrt{1-\boldsymbol{v}^2}\,, \quad Q \doteqdot \frac{\overset{1}{q}\,\overset{2}{q}\,(1-\boldsymbol{v}^2)}{4\pi\, r^3\left(a_1^2 + (1-\boldsymbol{v}^2)\, a_2^2\right)^{3/2}}\,. \tag{29e}$$

Force in the Scalar Model

Integration over the surface surrounding the region where the first spheron is located yields the components of the force acting on the first spheron:

$$\mathbb{F}_1 = \frac{\bar{Q}\, a_1}{\sqrt{1-\boldsymbol{v}^2}}\,, \quad \mathbb{F}_2 = \bar{Q}\, a_2 \sqrt{1-\boldsymbol{v}^2}\,, \quad \mathbb{F}_2 = 0\,, \quad \bar{Q} \doteqdot \frac{\overset{1}{q}\overset{2}{q}\left(1-\boldsymbol{v}^2\right)}{\left(a_1^2 + (1-\boldsymbol{v}^2)\, a_2^2\right)^{3/2}}\,. \tag{30}$$

If the second spheron is located on the axis x^2 ($a_1 = 0$), then the components of the force

$$\mathbb{F}_1 = \mathbb{F}_3 = 0\,, \quad \mathbb{F}_2 = \frac{\overset{1}{q}\overset{2}{q}\,\mathrm{sign}(a_2)}{a_2^2}\,. \tag{31}$$

As can be seen, the components of the force correspond to the attraction of charges of the same sign. This result contradicts the conclusion drawn in the article[3], according to which the long-range interaction of two stationary spherons in the scalar model is identical to the Coulomb interaction of point charged particles. It must be acknowledged that this conclusion is based on a computational error.

[3] A. A. Chernitskii, About long-range interaction of spheroidal solitons in scalar field nonlinear model, *Journal of Physics: Conf. Series* **938** (2017) 012029.

Comments on the Obtained Results

It should be noted that the analysis of the interaction of spherons presented is approximate. The conditions for the applicability of this analysis could be assessed if an exact two-spheron solution were available, but such a solution has not yet been found. Alternatively, it would be necessary to substantiate some iterative procedure that leads to this solution. A numerical solution could also be used.

Finding exact solutions to nonlinear field models and justifying iterative procedures for finding them presents significant mathematical challenges and is quite rare in the literature on theoretical physics. The numerical solution of nonlinear field models can also be quite complex and require substantial computational resources.

Thus, strictly speaking, the question of the nature of the long-range interaction of spherons in the scalar model under consideration cannot be considered definitively resolved.

Other published results of research on this scalar model are of independent interest.

Configuration of Spherons

The test spheron rests at the origin of coordinates $\{0, 0, 0\}$. An infinite chain of spherons moves along the x_1 axis with a velocity component $\boldsymbol{v}$ $(-1 < \boldsymbol{v} < 1)$.

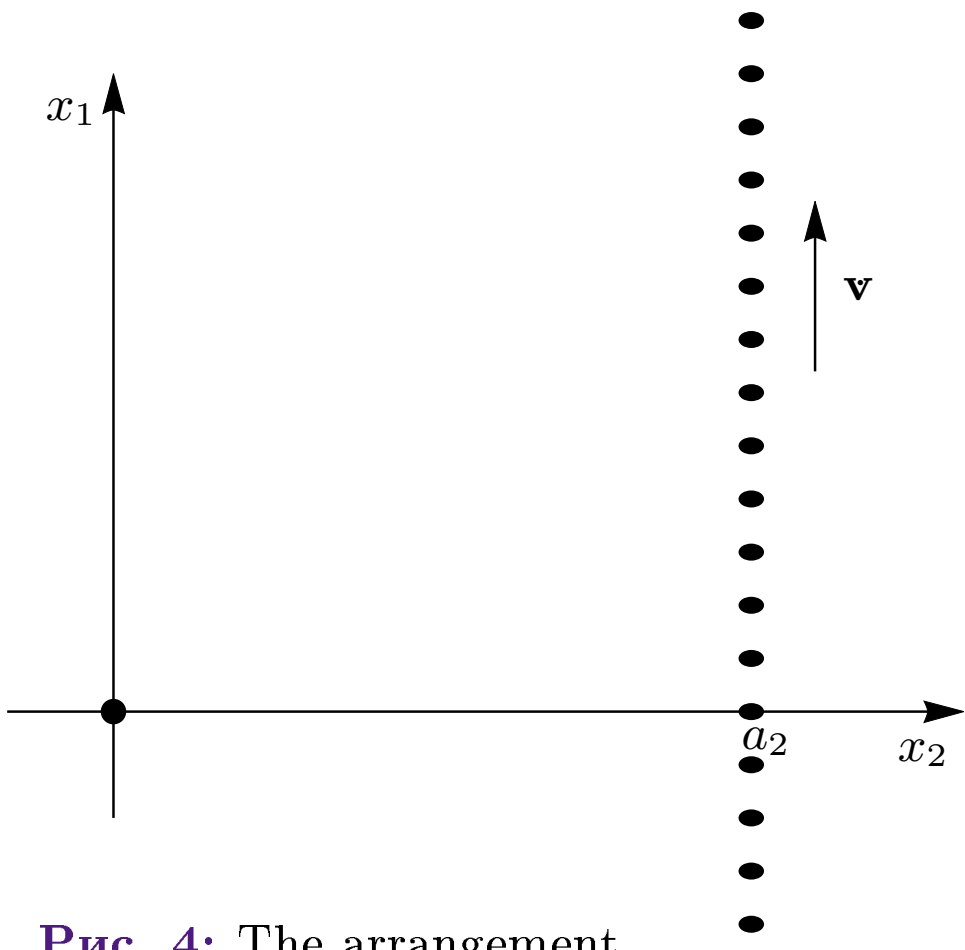


Рис. 4: The arrangement of interacting spheroons at time $x^0 = 0$.

The Force Acting on the Test Spheron

Let us replace the charge of the moving spheron $\overset{2}{q}$ in the obtained expressions for the force in the vector (26) and scalar (30) models with $\bar{q}\,\Delta a_1$, where $\bar{q}$ is the average linear charge density of the chain, and Δa_1 is the distance between the spherons of the chain.

Let us sum up the force acting on the test spheron from all the spherons in the chain. By replacing Δa_1 with $\mathrm{d}a_1$ and the sum with an integral, we obtain the components of the force for the vector and scalar models, respectively.

$$\mathbb{F}_1 = 0\,, \quad \mathbb{F}_2 = -\frac{2\overset{1}{q}\bar{q}}{a_2}\,, \tag{32a}$$

$$\mathbb{F}_1 = 0\,, \quad \mathbb{F}_2 = \frac{2\overset{1}{q}\bar{q}\sqrt{1-\boldsymbol{v}^2}}{a_2}\,. \tag{32b}$$

As can be seen, in the case of the scalar model (32b), the force decreases as the speed increases. However, this decrease is small for the actual drift speeds of charges in current-carrying conductors.

In the case of the vector model (32a), the force does not depend on the velocity. Thus, there is an exact compensation of positive and negative charges in current-carrying conductors.

Conclusions

- Charged spheroidal solitons, or spheroons, are considered in two generally covariant nonlinear field models: vector and scalar. These solitons represent point charged particles without spin.
- An approximate method for finding the equations of the trajectory of interacting spherons is considered.
- Within the framework of the method, the long-range interaction between a stationary spheron and a moving one is considered. It is shown:
  - In the vector model, spherons interact as real point-like charged particles without spin.
  - In the scalar model, the method yields attraction between solitons with charges of the same sign. However, this does not definitively resolve the question of the model's applicability to reality. An unambiguous answer to the question can be provided by an exact two-sphere solution, which has not yet been found.
- Due to the similar characteristic type of the generally covariant action of the two models, it is proposed to use the term "space-time film" to also denote the vector model.