XXth Workshop on High Energy Spin Physics (DSPIN-26)

Angular Coefficients of the Drell-Yan Process from T-even Hadronic Structure

based on
Phys.Rev.D 110, 074028 (2024), Phys.Rev.D 112, 054023 (2025)

Iurii A. Anikin[a,b], Valery E. Lyubovitskij, Alexey S. Zhevlakov[a]

[a] **Joint Institute for Nuclear Research**
Bogoliubov Laboratory of Theoretical Physics

[b] **Moscow Institute of Physics and Technology**
Landau Phystech-School of Physics and Research

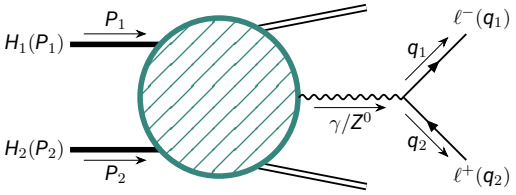
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Introduction

In this study we analyze Drell-Yan angular distributions based on collinear pQCD at NLO(α_s)

In particular, we focus on the small-Q_T limit, which is important due to several reasons:

- for deeper understanding of this limit in the collinear pQCD
- for performing resummation of the hadronic structure functions at small QT proposed in [Phys. Rev. D 76, 074006 (2007)]
- for testing the consistency between TMD and collinear factorization at subleading power (NLP, NNLP)



$$H_1(P_1) + H_2(P_2) \to \gamma/Z + X \to \ell^-(l_1) + \ell^+(l_2) + X$$

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02 Next-to leading order pQCD results

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01

Drell-Yan process fundamentals

Factorization of hadronic and leptonic processes,
collinear factorization and γ/Z^0 interference

Drell-Yan process fundamentals

Using the dilepton center-of-mass frame, the cross section is factorized into leptonic and hadronic tensors

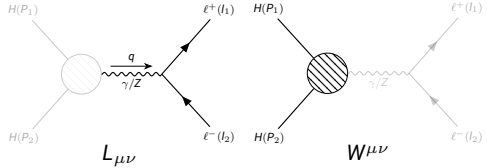
$$\frac{\mathrm{d}\sigma}{\mathrm{d}Q^2\,\mathrm{d}Q_T\,\mathrm{d}y\,\mathrm{d}\Omega} = \frac{\alpha^2}{4(2\pi)^4 Q^4 s^2} L_{\mu\nu}(l_1,q) W^{\mu\nu}(P_1,P_2,q)$$

Hadronic tensor in terms of structure functions

$$\begin{aligned} W^{\mu\nu} &= (X^\mu X^\nu + Y^\mu Y^\nu) W_T + i(X^\mu Y^\nu - X^\nu Y^\mu) W_{T_P} \\ &+ (Y^\mu Y^\nu - X^\mu X^\nu) W_{\Delta\Delta} - (X^\nu Y^\mu + X^\mu Y^\nu) W_{\Delta\Delta_P} \\ &- (X^\nu Z^\mu + X^\mu Z^\nu) W_\Delta - (Y^\nu Z^\mu + Y^\mu Z^\nu) W_{\Delta_P} \\ &+ i(X^\nu Z^\mu - X^\mu Z^\nu) W_\nabla + i(Y^\mu Z^\nu - Y^\nu Z^\mu) W_{\nabla_P} \\ &+ Z^\mu Z^\nu W_L \end{aligned}$$

Collinear factorization formula

$$W_{\mathrm{I}} = \frac{1}{x_1 x_2} \sum_{\mathrm{a,b}} \int_{x_1}^1 \mathrm{d}z_1 \int_{x_2}^1 \mathrm{d}z_2\, f_{\mathrm{a}/1}\left(\frac{x_1}{z_1}\right) f_{\mathrm{b}/2}\left(\frac{x_2}{z_2}\right) \times \tilde{\omega}_{\mathrm{I}}^{\mathrm{ab}}(z_1,z_2)\delta\Big((\hat{s}+\hat{t}+\hat{u}-Q^2)/\hat{s}\Big), \qquad z_i = \frac{x_i}{\xi_i}$$



For consideration of the γ/Z interference, we require the following specific electroweak couplings

$$\begin{aligned} g^{Z\gamma}_{\mathrm{EW};1} &= 1 + 2\, g^V_{Zq}\, g^V_{Zl}\, \mathrm{Re}[D_Z(Q^2)] \\ &+ \Big((g^V_{Zq})^2 + (g^A_{Zq})^2\Big)\Big((g^V_{Z\ell})^2 + (g^A_{Z\ell})^2\Big)\, |D_Z(Q^2)|^2 \end{aligned}$$

$$g^{Z\gamma}_{\mathrm{EW};2} = 4 g^A_{Zq}\Big[2\, g^V_{Zq}\Big(g^A_{Z\ell} g^V_{Z\ell}\Big)|D_Z(Q^2)|^2 + g^A_{Z\ell}\mathrm{Re}[D_Z(Q^2)]\Big]$$

Here, g^V_{Zf} and g^A_{Zf} are the standard vector and axial-vector couplings of the Z^0 boson to fermions. And $D_Z(Q^2)$ is the Breit-Wigner propagator.

Angular coefficients

The lepton angular distribution, which encodes information about polar and azimuthal asymmetries, can be parameterized by seven angular coefficients

$$\begin{aligned}\frac{\mathrm{d}N}{\mathrm{d}\Omega} &= \frac{\mathrm{d}\sigma}{\mathrm{d}Q^2\,\mathrm{d}Q_T\,\mathrm{d}y\,\mathrm{d}\Omega}\left(\frac{\mathrm{d}\sigma}{\mathrm{d}Q^2\,\mathrm{d}Q_T\,\mathrm{d}y}\right)^{-1} \\ &= \frac{3}{16\pi}\Big[1+\cos^2\theta+\frac{1}{2}A_0\left(1-3\cos^2\theta\right)+A_1\sin 2\theta\cos\phi \\ &+\frac{1}{2}A_2\sin^2\theta\cos 2\phi+A_3\sin\theta\cos\phi+A_4\cos\theta \\ &+A_5\sin^2\theta\sin 2\phi+A_6\sin 2\theta\sin\phi+A_7\sin\theta\sin\phi\Big]\end{aligned}$$

Relationship with structure functions

$$A_0=\frac{2W_L}{2W_T+W_L},\quad A_1=\frac{2W_\Delta}{2W_T+W_L},\quad A_2=\frac{4W_{\Delta\Delta}}{2W_T+W_L},\quad A_3=\frac{2W_{\nabla_P}}{2W_T+W_L},\quad A_4=\frac{2W_{T_P}}{2W_T+W_L},$$
$$A_5=\frac{2W_{\Delta\Delta_P}}{2W_T+W_L},\quad A_6=\frac{2W_{\Delta_P}}{2W_T+W_L},\quad A_7=\frac{2W_\nabla}{2W_T+W_L}$$

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Relationship with structure functions

T-even $A_0=\frac{2W_L}{2W_T+W_L}$, $A_1=\frac{2W_\Delta}{2W_T+W_L}$, $A_2=\frac{4W_{\Delta\Delta}}{2W_T+W_L}$, $A_3=\frac{2W_{\nabla_P}}{2W_T+W_L}$, $A_4=\frac{2W_{T_P}}{2W_T+W_L}$,

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Forward-backward asymmetry

$$A_{\mathrm{FB}}=\frac{\int_0^1\frac{\mathrm{d}N}{\mathrm{d}\cos\theta}-\int_{-1}^0\frac{\mathrm{d}N}{\mathrm{d}\cos\theta}}{\int_0^1\frac{\mathrm{d}N}{\mathrm{d}\cos\theta}+\int_{-1}^0\frac{\mathrm{d}N}{\mathrm{d}\cos\theta}}=\frac{3}{8}A_4$$

02

Next-to leading order pQCD results

Next-to leading order perturbative QCD predictions for parton-level subprocesses contributing to the Drell-Yan process. And comparison with experimental data from the ATLAS [JHEP 08 (2016) 159] and CMS [Phys.Lett.B 718 (2013) 752-772, Phys.Lett.B 750 (2015) 154-175] Collaborations

Quark-antiquark annihilation

Notations

$$C_{q\bar{q}} = \frac{C_F}{N_c} = \frac{N_c^2 - 1}{2N_c^2} = \frac{4}{9}, \quad \rho^2 = \frac{Q_T^2}{Q^2}$$

$$g_{q\bar{q};i} = (8\,\pi^2 e_q^2\,\alpha_s)\,C_{q\bar{q}}\,g_{\mathrm{EW};i}^{Z\gamma} = G_i C_{q\bar{q}}$$

Structure functions

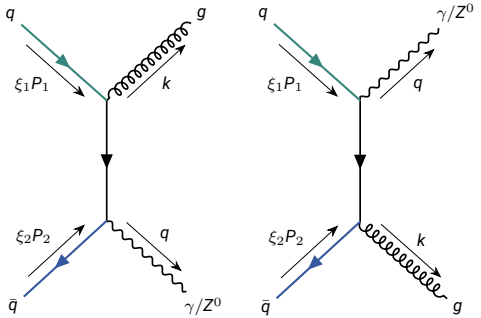
$$\tilde{\omega}_T^{q\bar{q}} = g_{q\bar{q};1}\,\frac{1}{\rho^2}\left(1 + \frac{\rho^2}{2}\right)\frac{z_1^2 + z_2^2}{z_1 z_2}$$

$$\tilde{\omega}_L^{q\bar{q}} = 2\,\tilde{\omega}_{\Delta\Delta}^{q\bar{q}} = g_{q\bar{q};1}\,\frac{z_1^2 + z_2^2}{z_1 z_2}$$

$$\tilde{\omega}_\Delta^{q\bar{q}} = g_{q\bar{q};1}\,\frac{1}{\rho}\,\frac{z_1^2 - z_2^2}{z_1 z_2}$$

$$\tilde{\omega}_{T_P}^{q\bar{q}} = g_{q\bar{q};2}\,\frac{\sqrt{1+\rho^2}}{\rho^2}\,\frac{z_1^2 + z_2^2}{z_1 z_2}$$

$$\tilde{\omega}_{\nabla_P}^{q\bar{q}} = g_{q\bar{q};2}\,\frac{\sqrt{1+\rho^2}}{\rho}\,\frac{z_1^2 - z_2^2}{z_1 z_2}$$



Quark-gluon scattering

Notations

$$C_{qg} = \frac{T_F}{N_c} = \frac{1}{2N_c} = \frac{1}{6}\,, \quad \rho^2 = \frac{Q_T^2}{Q^2}$$

$$g_{qg;i} = (8\,\pi^2 e_q^2\,\alpha_s)\,C_{qg}\,g_{\mathrm{EW};i}^{Z\gamma} = G_i C_{qg}$$

Structure functions

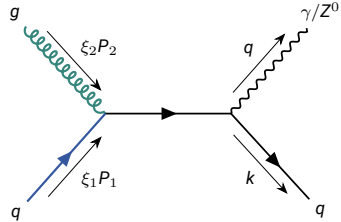
$$\tilde{\omega}_T^{qg} = g_{qg;1}\,\frac{1}{\rho^2}\,\frac{1-z_2}{z_1 z_2}\left(z_2^2 + (1-z_1 z_2)^2 + \rho^2\left(1 - \frac{z_1^2}{2} - z_1 z_2 (z_1+z_2)\right)\right)$$

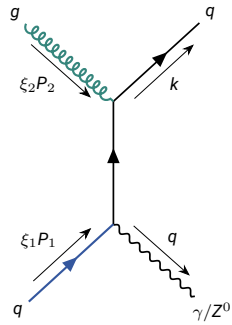
$$\tilde{\omega}_L^{qg} = 2\,\tilde{\omega}_{\Delta\Delta}^{qg} = g_{qg;1}\,\frac{1-z_2}{z_1 z_2}\left(z_2^2 + (z_1+z_2)^2\right)$$

$$\tilde{\omega}_\Delta^{qg} = g_{qg;1}\,\frac{1}{\rho}\,\frac{1-z_2}{z_1 z_2}\left(z_1^2 - 2z_2^2\right)$$

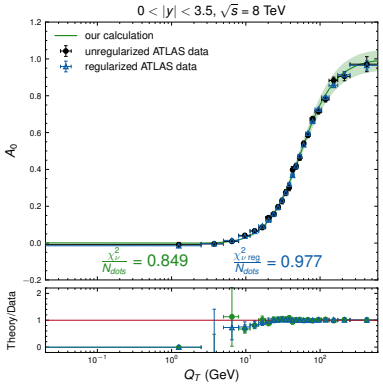
$$\tilde{\omega}_{T_P}^{qg} = g_{qg;2}\,\frac{\sqrt{1+\rho^2}}{\rho^2}\,\frac{1-z_2}{z_1 z_2}\left(z_2^2 + (1-z_2)^2 - (1-z_1)^2\right)$$

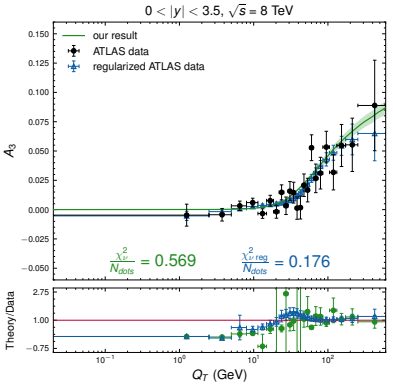
$$\tilde{\omega}_{\nabla_P}^{qg} = g_{qg;2}\,\frac{\sqrt{1+\rho^2}}{\rho}\,\frac{1-z_2}{z_1 z_2}\left(1 - 2z_2^2 - (1-z_1)^2 + 2z_2(1-z_1)\right)$$

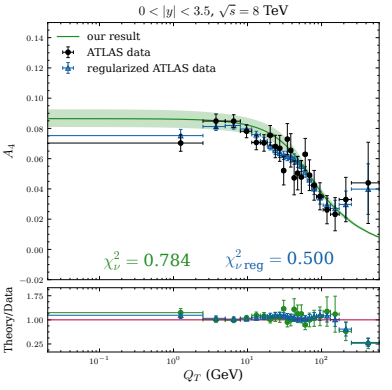




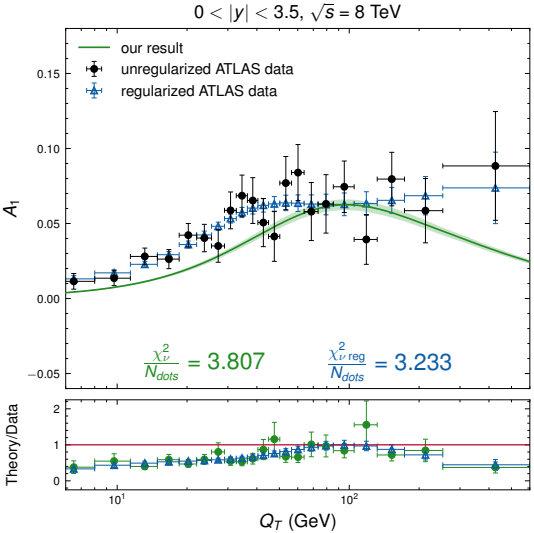
Angular coefficients $A_{0,3,4}$

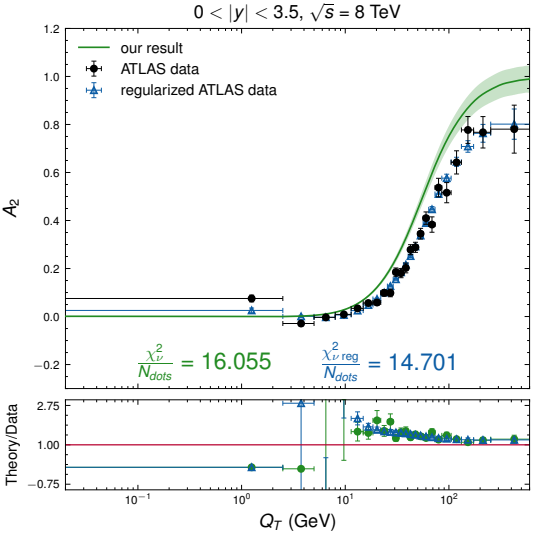




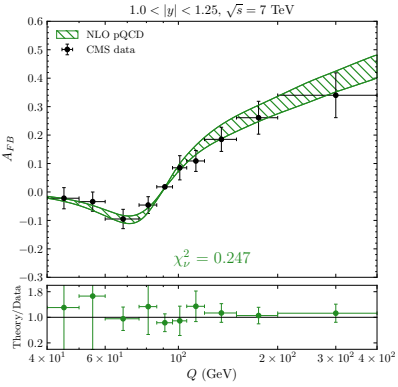


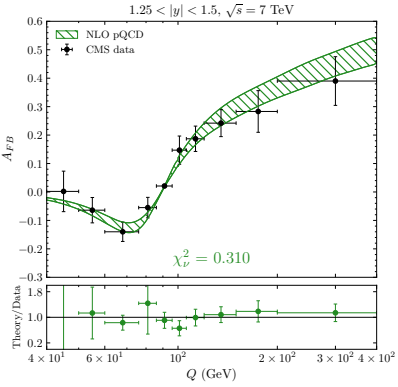
Angular coefficients A_1 and A_2

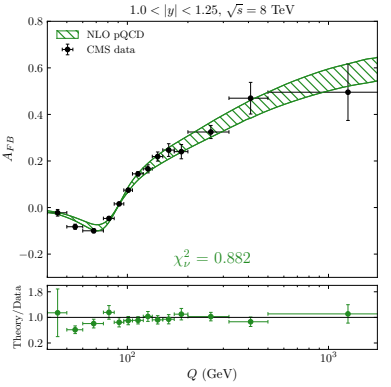




Forward-backward asymmetry







03

Small-Q_T^2/Q^2 expansion results

The study of the transverse momentum Q_T dependence
of the hadronic structure functions and angular coefficients
up to leading power in the Q_T^2/Q^2 expansion

Small-Q_T^2/Q^2 expansion overview

General form of the hadronic structure functions

$$W_{\mathrm{I}} = \frac{1}{x_1 x_2} \sum_{a,b} \int_{x_1}^{1} dz_1 \int_{x_2}^{1} dz_2 \, \tilde{w}_{\mathrm{I}}^{ab}(z_1, z_2, \rho^2) \, \delta\Big((\hat{s} + \hat{t} + \hat{u} - Q^2)/\hat{s} \Big) \, f_{a/H_1}\Big(\frac{x_1}{z_1}\Big) \, f_{b/H_2}\Big(\frac{x_2}{z_2}\Big), \qquad \rho^2 = \frac{Q_T^2}{Q^2}$$

Small-Q_T^2/Q^2 expansion overview

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Sources of Q_T dependence:

- direct dependence of the partonic structure function $\tilde{w}_{ab}$ on Q_T

Small-Q_T^2/Q^2 expansion overview

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Sources of Q_T dependence:

- direct dependence of the partonic structure function $\tilde{w}_{ab}$ on Q_T
- nontrivial Q_T dependence of phase space delta function

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Sources of Q_T dependence:

- direct dependence of the partonic structure function $\tilde{w}_{ab}$ on Q_T
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Small-Q_T^2/Q^2 expansion overview

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Sources of Q_T dependence:

- direct dependence of the partonic structure function $\tilde{w}_{ab}$ on Q_T

$$\tilde{w}_{\rm I}^{ab}(z_1, z_2, \rho^2) = \sum\nolimits_{n=0}^{\infty} (\rho^2)^n \frac{1}{n!} \partial_{\rho^2}^n \tilde{w}_{\rm I}^{ab}(z_1, z_2, \rho^2)\Big|_{\rho^2=0}$$

- nontrivial Q_T dependence of phase space delta function
- implicit Q_T dependence of fraction variables x_1, x_2

Small-Q_T^2/Q^2 expansion overview

General form of the hadronic structure functions

$$W_{\rm I} = \frac{1}{x_1 x_2} \sum\nolimits_{a,b} \int_{x_1}^{1} dz_1 \int_{x_2}^{1} dz_2\, \tilde{w}_{\rm I}^{ab}(z_1, z_2, \rho^2)\, \delta\left((1-z_1)(1-z_2) - \frac{\rho^2 z_1 z_2}{1+\rho^2}\right) f_{a/H_1}\left(\frac{x_1}{z_1}\right) f_{b/H_2}\left(\frac{x_2}{z_2}\right), \qquad \rho^2 = \frac{Q_T^2}{Q^2}$$

Sources of Q_T dependence:

- direct dependence of the partonic structure function $\tilde{w}_{ab}$ on Q_T

$$\tilde{w}_{\rm I}^{ab}(z_1, z_2, \rho^2) = \sum\nolimits_{n=0}^{\infty} (\rho^2)^n \frac{1}{n!} \partial_{\rho^2}^n \tilde{w}_{\rm I}^{ab}(z_1, z_2, \rho^2)\Big|_{\rho^2=0}$$

- nontrivial Q_T dependence of phase space delta function

$$\delta\left((1-z_1)(1-z_2) - \frac{\rho^2 z_1 z_2}{1+\rho^2}\right) = \delta(1-z_2)\, G_1(z_1, z_2) + \delta(1-z_1)\, G_1(z_2, z_1) + \delta(1-z_1)\,\delta(1-z_2)\, G_2(z_1, z_2)\,,$$

$$G_1(z_1, z_2) = \sum\nolimits_{N,M,K=0}^{\infty} \frac{(\rho^2)^{N+M+K}\,(1+\rho^2)^{N+1}}{(N!)^2\, M!\, K!} \frac{(-1)^{N+K}\,(N+K)!}{(1-z_1)_{+,N+K}^{N+K+1}} \partial_{z_2}^{N+M}\,,$$

$$G_2(z_1, z_2) = -\log\frac{\rho^2}{1+\rho^2} \sum\nolimits_{N,M,K=0}^{\infty} \frac{(\rho^2)^{N+M+K}\,(1+\rho^2)^{N+1}}{(N!)^2\, M!\, K!} \partial_{z_1}^{N+K}\, \partial_{z_2}^{N+M}$$

- implicit Q_T dependence of fraction variables x_1, x_2

Small-Q_T^2/Q^2 expansion overview

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- implicit Q_T dependence of fraction variables x_1, x_2

$$W_{\rm I} = \sum\nolimits_{k=0}^{\infty} (\rho^2)^k\, W_{{\rm I};k}^{{\rm direct}+\delta}(x_1,x_2,\log\rho^2)\,,$$

$$W_{\rm I} = \sum_{m=0}^{\infty} (\rho^2)^m\, W_{\rm I}^{{\rm N}^m{\rm LP}}(x_1^0,x_2^0,L_\rho) = \sum_{k=0}^{\infty} \sum_{s_1,s_2=0}^{\infty} (\rho^2)^k (\sqrt{1+\rho^2}-1)^{s_1+s_2}\, \frac{(x_1^0)^{s_1}(x_2^0)^{s_2}}{s_1!\,s_2!}\, \partial_{x_1^0}^{s_1}\partial_{x_2^0}^{s_2} W_{{\rm I};k}^{{\rm direct}+\delta}(x_1^0,x_2^0,\log\rho^2)$$

Small-Q_T^2/Q^2 expansion overview

General form of the hadronic structure functions

$$W_{\rm I} = \frac{1}{x_1 x_2} \sum\nolimits_{a,b} \int_{x_1}^1 dz_1 \int_{x_2}^1 dz_2\, \tilde{w}_{\rm I}^{ab}(z_1,z_2,\rho^2)\, \delta\left((1-z_1)(1-z_2) - \frac{\rho^2 z_1 z_2}{1+\rho^2}\right) f_{a/H_1}\left(\frac{x_1}{z_1}\right) f_{b/H_2}\left(\frac{x_2}{z_2}\right), \qquad \rho^2 = \frac{Q_T^2}{Q^2}$$

Sources of Q_T dependence: **The details can be heard in Zhevlakov A. plenary talk on Wednesday at 12:20**

- direct dependence of the partonic structure function $\tilde{w}_{ab}$ on Q_T

$$\tilde{w}_{\rm I}^{ab}(z_1,z_2,\rho^2) = \sum\nolimits_{n=0}^{\infty} (\rho^2)^n \frac{1}{n!}\, \partial_{\rho^2}^n \tilde{w}_{\rm I}^{ab}(z_1,z_2,\rho^2)\Big|_{\rho^2=0}$$

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$$G_1(z_1,z_2) = \sum\nolimits_{N,M,K=0}^{\infty} \frac{(\rho^2)^{N+M+K}\,(1+\rho^2)^{N+1}}{(N!)^2\, M!\, K!}\, \frac{(-1)^{N+K}\,(N+K)!}{(1-z_1)_{+,N+K}^{N+K+1}}\, \partial_{z_2}^{N+M},$$

$$G_2(z_1,z_2) = -\log\frac{\rho^2}{1+\rho^2} \sum\nolimits_{N,M,K=0}^{\infty} \frac{(\rho^2)^{N+M+K}\,(1+\rho^2)^{N+1}}{(N!)^2\, M!\, K!}\, \partial_{z_1}^{N+K}\, \partial_{z_2}^{N+M}$$

- implicit Q_T dependence of fraction variables x_1, x_2

$$W_{\rm I} = \sum\nolimits_{k=0}^{\infty} (\rho^2)^k\, W_{{\rm I};k}^{{\rm direct}+\delta}(x_1,x_2,\log\rho^2),$$

$$W_{\rm I} = \sum_{m=0}^{\infty} (\rho^2)^m\, W_{\rm I}^{{\rm N}^m{\rm LP}}(x_1^0,x_2^0,L_\rho) = \sum_{k=0}^{\infty} \sum_{s_1,s_2=0}^{\infty} (\rho^2)^k (\sqrt{1+\rho^2}-1)^{s_1+s_2}\, \frac{(x_1^0)^{s_1}(x_2^0)^{s_2}}{s_1!\, s_2!}\, \partial_{x_1^0}^{s_1}\partial_{x_2^0}^{s_2} W_{{\rm I};k}^{{\rm direct}+\delta}(x_1^0,x_2^0,\log\rho^2)$$

Small-Q_T^2/Q^2 expansion at LP

At leading power (LP), we obtain the well-known delta-function (see, e.g., [Phys. Rev. D 74, 014004 (2006)])

$$\delta\left((1-z_1)(1-z_2)-\frac{\rho^2}{1+\rho^2}z_1z_2\right)=\frac{\delta(1-z_1)}{(1-z_2)_+}+\frac{\delta(1-z_2)}{(1-z_1)_+}-\delta(1-z_1)\delta(1-z_2)\log\rho^2\,,\qquad \rho^2=\frac{Q_T^2}{Q^2}$$

Quark-antiquark annihilation hadronic structure functions

$$\begin{aligned}
W_T^{\mathrm{LP};q\bar q}&=\frac{1}{\rho^2}W_L^{\mathrm{LP};q\bar q}=\frac{2}{\rho^2}W_{\Delta\Delta}^{\mathrm{LP};q\bar q}=\frac{g_{q\bar q;1}}{g_{q\bar q;2}}W_{T_P}^{\mathrm{LP};q\bar q}\\
&=\frac{g_{q\bar q;1}}{\rho^2x_1^0x_2^0}\frac{1}{C_F}\Big[-C_F(2\log\rho^2+3)q_1(x_1^0)\bar q_2(x_2^0)\\
&\quad+q_1(x_1^0)\Big(P_{qq}\otimes\bar q_2\Big)(x_2^0)+\Big(P_{qq}\otimes q_1\Big)(x_1^0)\bar q_2(x_2^0)\Big]\\
W_\Delta^{\mathrm{LP};q\bar q}&=\frac{g_{q\bar q;1}}{g_{q\bar q;2}}W_{\nabla_P}^{\mathrm{LP};q\bar q}\\
&=\frac{g_{q\bar q;1}}{\rho\,x_1^0x_2^0}\frac{1}{C_F}\Big[q_1(x_1^0)\Big(\tilde P_{qq}\otimes\bar q_2\Big)(x_2^0)-\Big(\tilde P_{qq}\otimes q_1\Big)(x_1^0)\bar q_2(x_2^0)\Big]
\end{aligned}$$

Quark-gluon scattering hadronic structure functions

$$\begin{aligned}
W_T^{\mathrm{LP};qg}&=\frac{g_{qg;1}}{g_{qg;2}}W_{T_P}^{\mathrm{LP};qg}=\frac{2g_{qg;1}}{\rho^2x_1^0x_2^0}q_1(x_1^0)\Big(P_{qg}^+\otimes g_2\Big)(x_2^0)\,,\\
W_L^{\mathrm{LP};qg}&=2W_{\Delta\Delta}^{\mathrm{LP};qg}=\frac{2g_{qg;1}}{x_1^0x_2^0}q_1(x_1^0)\Big(P_{qg}^-\otimes g_2\Big)(x_2^0)\,,\\
W_\Delta^{\mathrm{LP};qg}&=\frac{g_{qg;1}}{g_{qg;2}}W_{\nabla_P}^{\mathrm{LP};qg}=\frac{2g_{qg;1}}{\rho\,x_1^0x_2^0}\,q_1(x_1^0)\,\Big(\tilde P_{qg}\otimes g_2\Big)(x_2^0)
\end{aligned}$$

where

Convolution

$$(\mathcal{P}\otimes f)(x)=\int_x^1\frac{dz}{z}\,\mathcal{P}(z)\,f\Big(\frac{x}{z}\Big)$$

Partonic splitting functions

$$\begin{aligned}
P_{qq}(z)&=C_F\left[\frac{1+z^2}{(1-z)_+}+\frac{3}{2}\delta(1-z)\right]\\
P_{qg}^{\pm}(z)&=T_F\left[z^2+(1\mp z)^2\right]\\
\tilde P_{qq}(z)&=C_F\left[1+z\right]\\
\tilde P_{qg}(z)&=T_F\left[1-2z^2\right]
\end{aligned}$$

Small-Q_T^2/Q^2 expansion at LP

At LO and NLO, we have the Lam–Tung relation

$$W_L = 2W_{\Delta\Delta}$$

while it is violated at high $\rho^2 = Q_T^2/Q^2$ at NNLO (α_s^2)
[Nucl.Phys.B 387 (1992) 3-85, JHEP 11 (2017) 003]

For T-even structure functions of Drell-Yan process, we obtain new relations that exist at low ρ^2

Quark-antiquark annihilation

$$W_T^{\mathrm{LP};q\bar{q}} = \frac{1}{\rho^2} W_L^{\mathrm{LP};q\bar{q}} = \frac{2}{\rho^2} W_{\Delta\Delta}^{\mathrm{LP};q\bar{q}} = \frac{g_{q\bar{q};1}}{g_{q\bar{q};2}} W_{T_P}^{\mathrm{LP};q\bar{q}},$$

$$W_\Delta^{\mathrm{LP};qg} = \frac{g_{qg;1}}{g_{qg;2}} W_{\nabla_P}^{\mathrm{LP};qg}$$

Quark-gluon scattering

$$W_T^{\mathrm{LP};qg} = \frac{g_{qg;1}}{g_{qg;2}} W_{T_P}^{\mathrm{LP};qg},$$

$$W_L^{\mathrm{LP};qg} = 2W_{\Delta\Delta}^{\mathrm{LP};qg},$$

$$W_\Delta^{\mathrm{LP};qg} = \frac{g_{qg;1}}{g_{qg;2}} W_{\nabla_P}^{\mathrm{LP};qg}$$

Relationship with angular coefficients

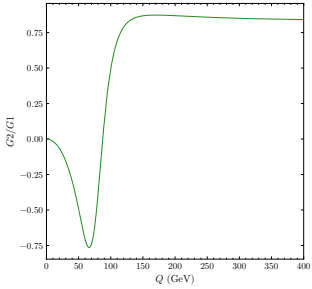
$$A_0 = \frac{2W_L}{2W_T + W_L}, \quad A_1 = \frac{2W_\Delta}{2W_T + W_L}, \quad A_2 = \frac{4W_{\Delta\Delta}}{2W_T + W_L},$$

$$A_3 = \frac{2W_{\nabla_P}}{2W_T + W_L}, \quad A_4 = \frac{2W_{T_P}}{2W_T + W_L}$$

In case $Q_T \to 0$

$$A_0^{\mathrm{LP}} = A_1^{\mathrm{LP}} = A_2^{\mathrm{LP}} = A_3^{\mathrm{LP}} = 0$$

$$A_4^{\mathrm{LP}} = \frac{2W_{T_P}^{\mathrm{LP}}}{2W_T^{\mathrm{LP}} + W_L^{\mathrm{LP}}} \approx \frac{W_{T_P}^{\mathrm{LP}}}{W_T^{\mathrm{LP}}} = \frac{G_2}{G_1}$$



Conclusion

We presented analytical results for the Drell-Yan T-even hadronic structure functions in the framework of the pQCD based on the collinear factorization scheme and at the next-to-leading order in the α_s expansion

We demonstrated that our theoretical predictions for angular coefficients and forward-backward asymmetry are in good agreement with data from ATLAS and CMS Collaborations

We proposed systematic and analytic Q_T^2/Q^2 expansion for structure functions in Drell-Yan process up to desired order

For T-even structure functions, in addition to the Lam-Tung relation, we obtain new relations that exist at low Q_T^2/Q^2

Additionally, we pointed out that the new relations play an important role in fixing the value of the angular coefficient A_4 in the low-Q_T limit

Thanks for your attention!

Iurii A. Anikin
iuanikin@theor.jinr.ru
t.me/yuriyan42

04

Additional slides

Statistical comparison of the angular coefficients with the experimental data from ATLAS [JHEP 08 (2016) 159], CMS [Phys. Lett. B 750 (2015) 154–175], and LHCb [Phys. Rev. Lett. 129 (2022) 9] Collaborations.
Comparison of QCD with the geometric method [Phys. Rev. D 96 (2017) 5]

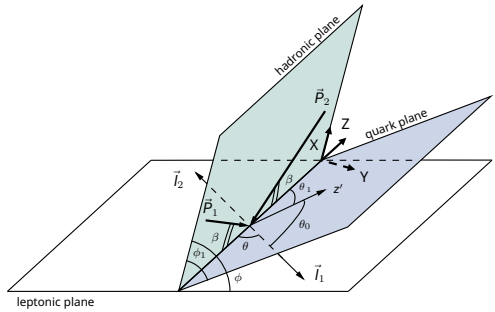
Geometrical method

Angular distribution of leptons with respect to the natural quark–antiquark axis

$$\frac{dN}{d\Omega} = \frac{3}{16\pi}\left[1 + \cos^2\theta_0 + a\cos\theta_0\right]$$

Written in terms of the observed angles ϕ and θ

$$\begin{aligned}\frac{dN}{d\Omega} = \frac{3}{16\pi}\Bigg[&1 + \cos^2\theta + \frac{1}{2}\sin^2\theta_1(1 - 3\cos^2\theta)\\ &+ a\cos\theta_1\cos\theta + \frac{1}{2}\sin 2\theta_1\cos\phi_1\sin 2\theta\cos\phi\\ &+ \frac{1}{2}\sin^2\theta_1\cos 2\phi_1\sin^2\theta\cos 2\phi\\ &+ a\sin\theta_1\cos\phi_1\sin\theta\cos\phi\\ &+ \frac{1}{2}\sin^2\theta_1\sin 2\phi_1\sin^2\theta\sin 2\phi\\ &+ \frac{1}{2}\sin 2\theta_1\sin\phi_1\sin 2\theta\sin\phi\\ &+ a\sin\theta_1\sin\phi_1\sin\theta\sin(\phi)\Bigg]\end{aligned}$$



Angular coefficients

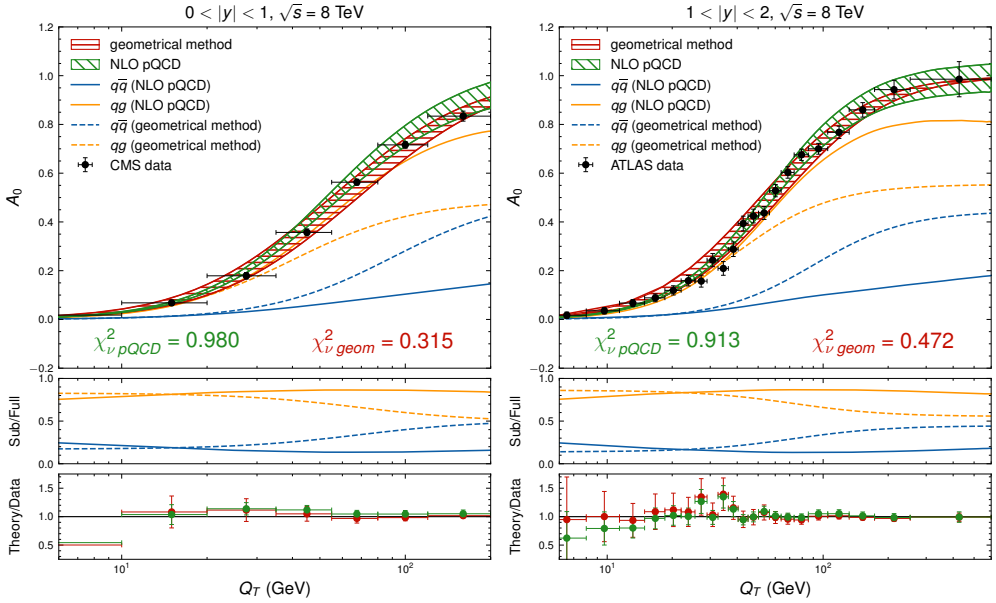
$$A_0^{\text{geom}} = f\frac{\rho^2}{1+\rho^2} + (1-f)\frac{5\rho^2}{1+\rho^2}\,,\quad A_2^{\text{geom}} = r_2 A_0^{\text{geom}}\,,$$

$$A_1^{\text{geom}} = r_1\left[f\frac{\rho}{1+\rho^2} + (1-f)\frac{\sqrt{5}\,\rho}{1+5\,\rho^2}\right],$$

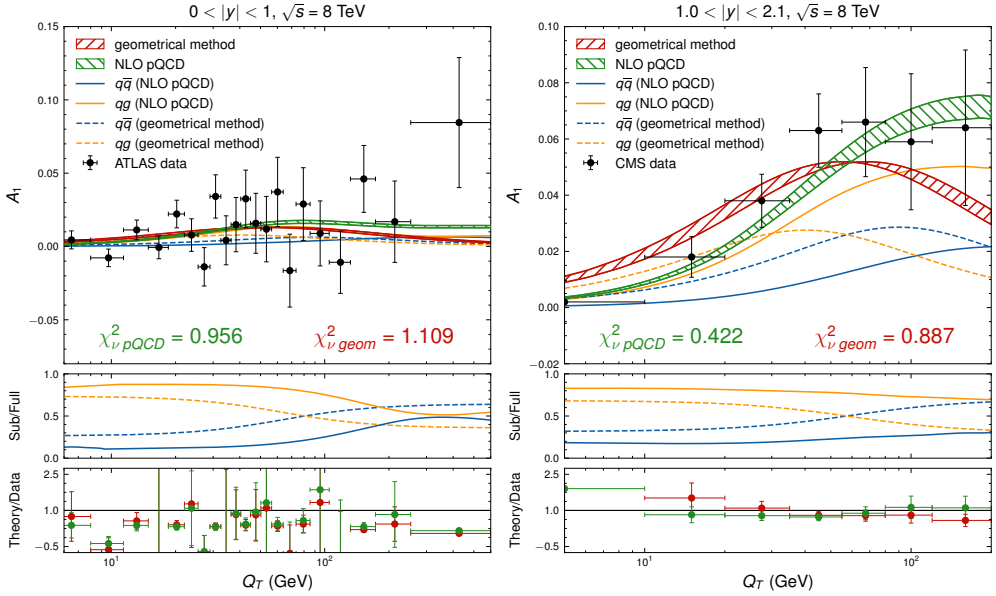
$$A_3^{\text{geom}} = r_3\left[f\frac{\rho}{\sqrt{1+\rho^2}} + (1-f)\frac{\sqrt{5}\,\rho}{\sqrt{1+5\,\rho^2}}\right],$$

$$A_4^{\text{geom}} = r_4\left[f\frac{1}{\sqrt{1+\rho^2}} + (1-f)\frac{1}{\sqrt{1+5\,\rho^2}}\right],$$

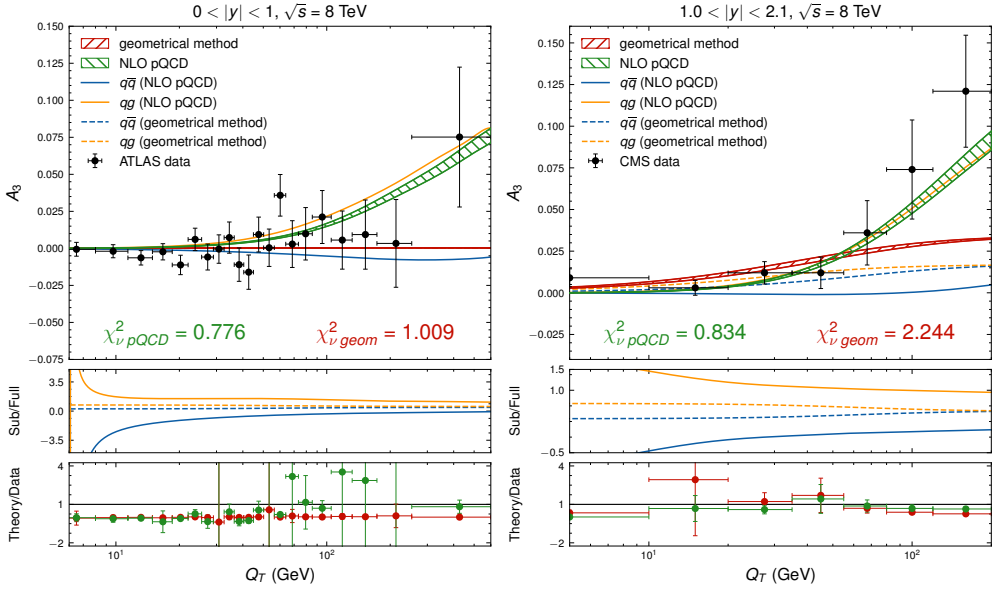
Angular coefficient A_0



Angular coefficient A_1



Angular coefficient A_3



Angular coefficient A_4

