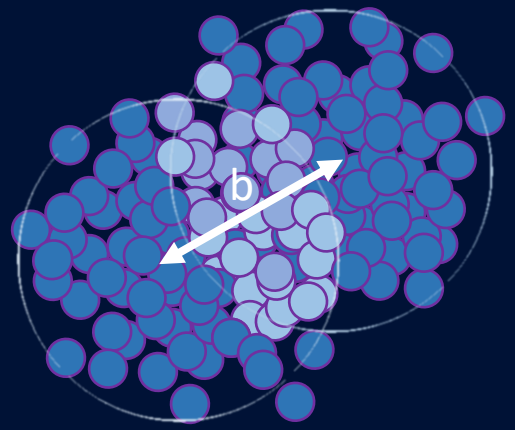


Impact Parameter Reconstruction in Nucleus-Nucleus Collisions Using Two Observables

Idrisov Dim, Fedor Guber, Nikolay Karpushkin, Parfenov Peter

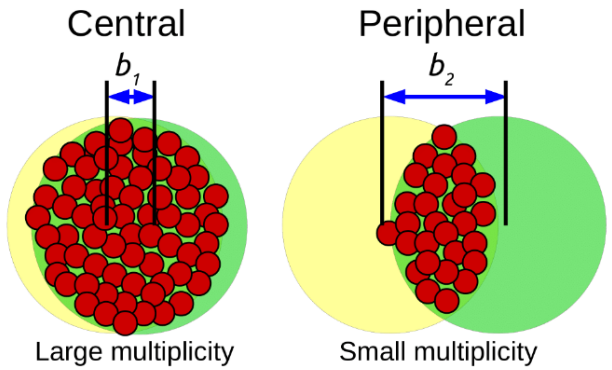
INR RAS, Moscow, Russia

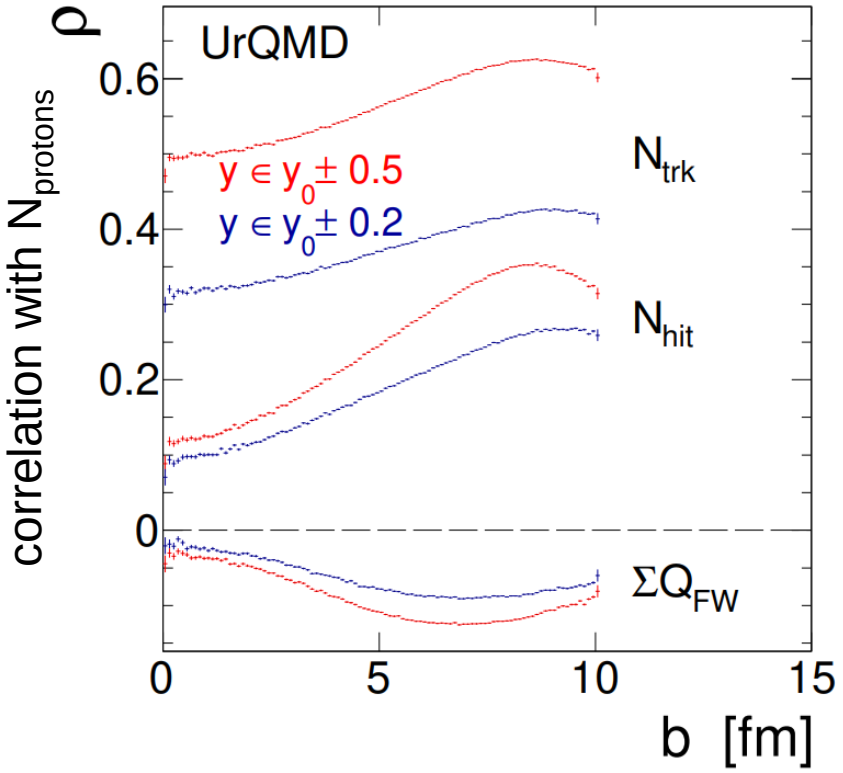
MPD Cross-PWG Meeting
04 April, 2026

Centrality

- Evolution of matter produced in heavy-ion collisions depend on its initial geometry
- Centrality procedure maps initial geometry parameters with measurable quantities (multiplicity or energy of the spectators)
- **This allows comparison of the future BMAN results with the data from other experiments (STAR BES, NA49/NA61 scans) and theoretical models**

$$c(b) = \frac{\int_0^b \frac{d\sigma}{db'} db'}{\int_0^\infty \frac{d\sigma}{db'} db'} = \frac{1}{\sigma_{A-A}} \int_0^b \frac{d\sigma}{db'} db'$$



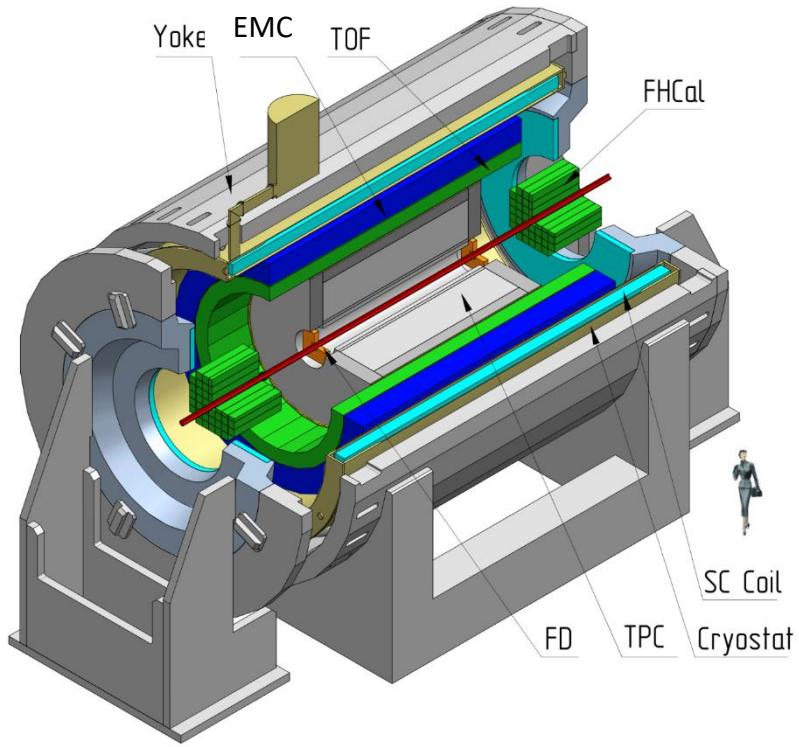


HADES; Phys.Rev.C 102 (2020) 2, 024914

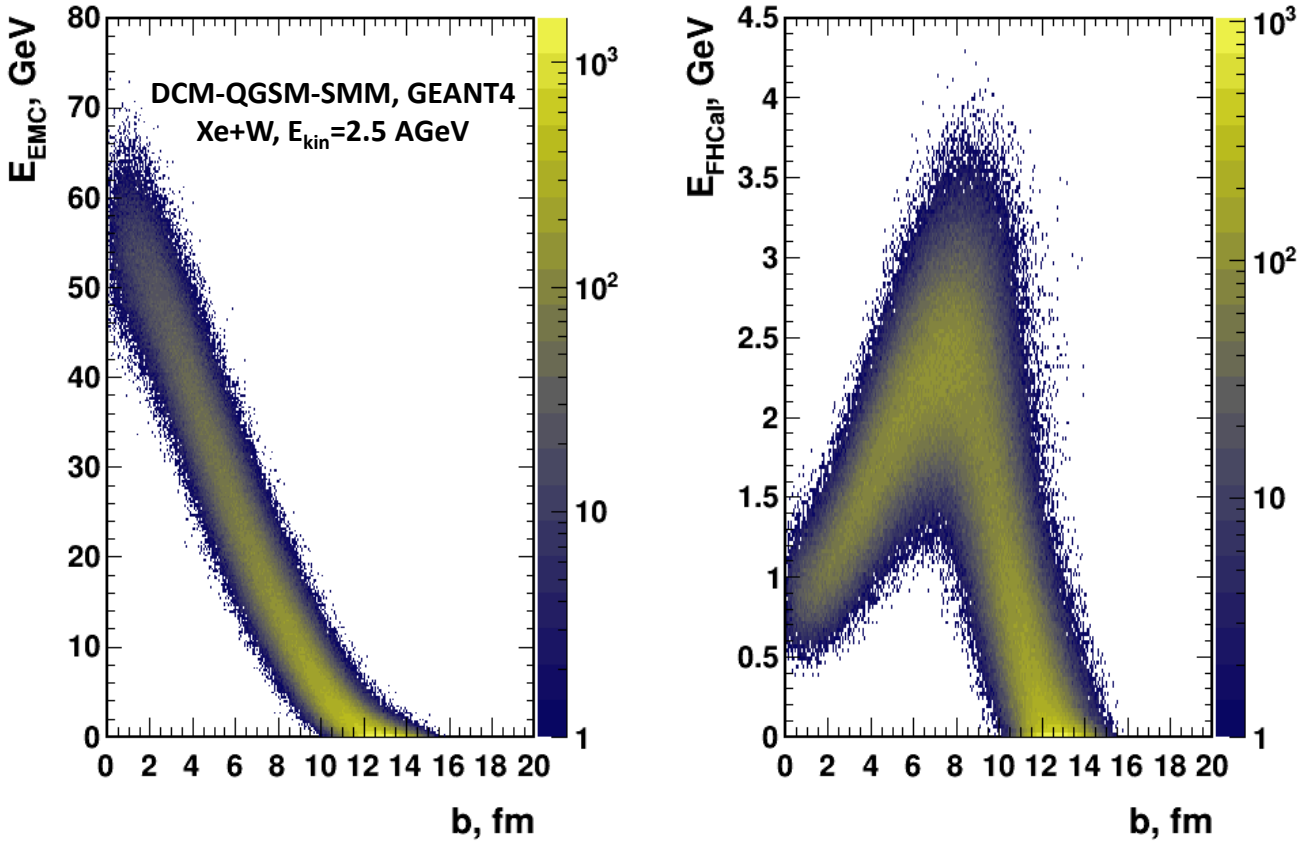
- A number of produced protons is stronger correlated with the number of produced particles (track & RPC+TOF hits) than with the total charge of spectator fragments (FW)
- to suppress self-correlation biases, it is necessary to use spectators fragments for centrality estimation

MPD Experiment at NICA

DCM-QGSM-SMM → GEANT4 → Reconstruction → Centrality determination



Multi-Purpose Detector (MPD) Stage 1



The analysis was performed using the DCM-QGSM-SMM model.

The Bayesian inversion method (Γ-fit): DCM-QSM-SMM based

- The fluctuation kernel for multiplicity at fixed impact parameter is Gamma distr.:

$$P(M \mid c_b) = \frac{1}{\Gamma(k(c_b))\theta^2} M^{k(c_b)-1} e^{-M/\theta}$$

$$c_b = \int_0^b P(b')db' \quad \text{– centrality based on impact parameter}$$

$$\theta = \frac{D(M)}{\langle M \rangle}, \quad k = \frac{\langle M \rangle}{\theta}$$

$\langle M \rangle, D(M)$ – average and variance of Multiplicty

$$P(M) = \int_0^1 P(M \mid c_b)dc_b$$

$$\langle M \rangle = m_1 \cdot \langle M' \rangle$$

$$D(M) = m_1^2 \cdot D(M') + m_1 \cdot m_2 \langle M' \rangle$$

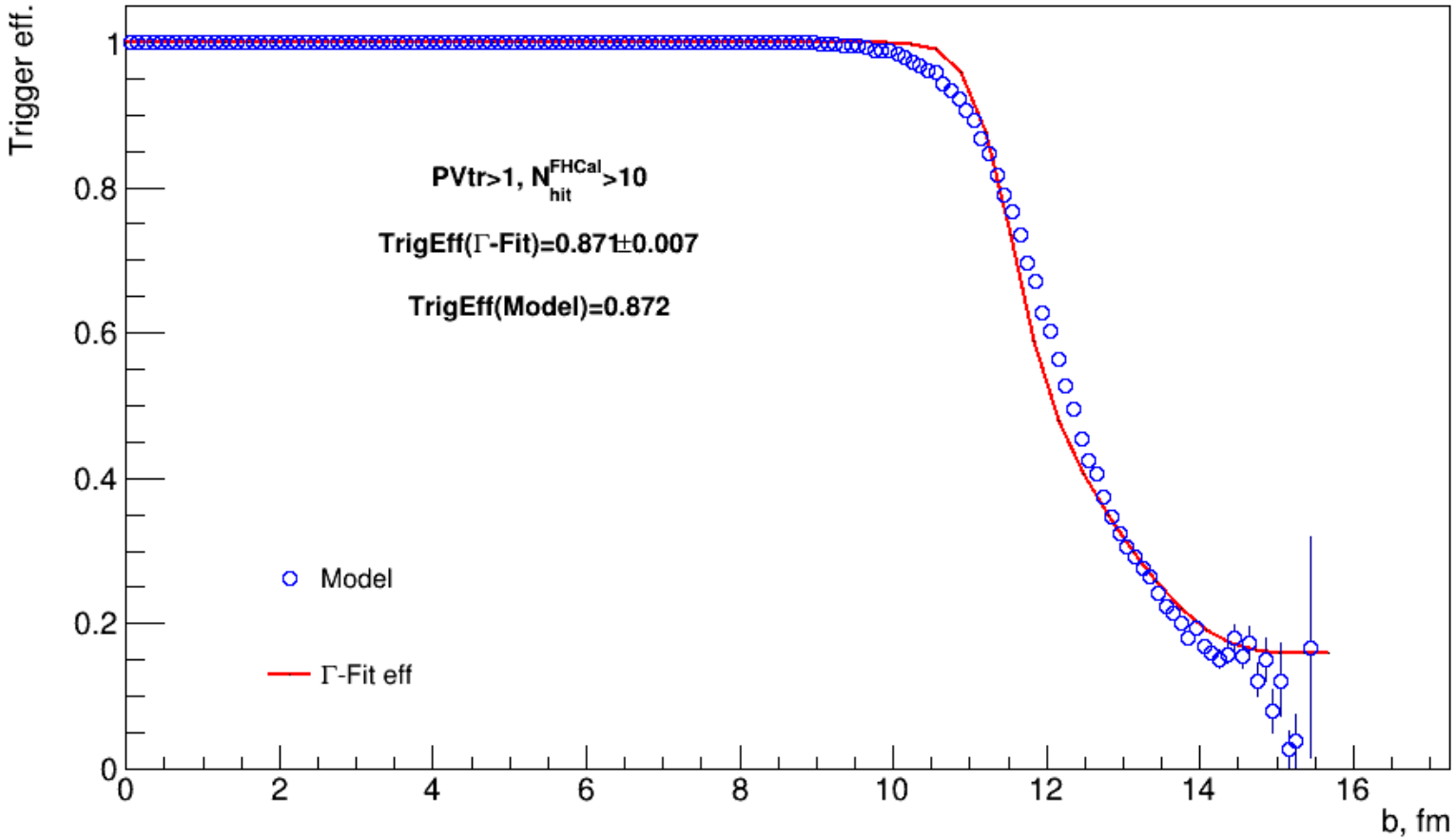
$\langle M'(c_b) \rangle$ – average value and var. of energy/mult.

$D(M'(c_b))$ from the rec. model data

$$P_{eff}(b) = \int_0^{M_{\max}} P_{eff}(M) P(M \mid b) dM$$

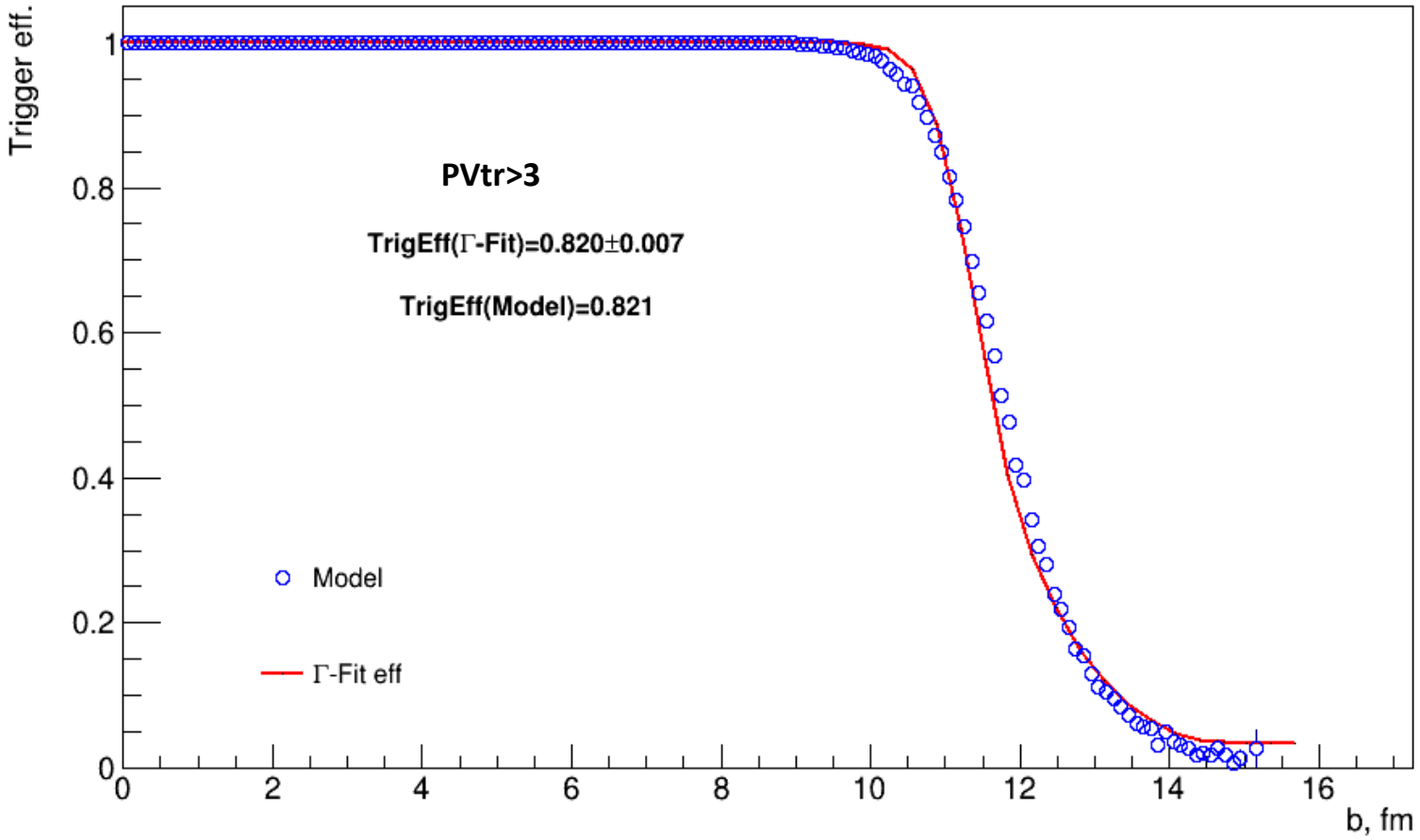
Convoluted trigger efficiency can be calculated using Bayes' theorem

Fit results: trigger efficiency



The calculated efficiency estimate is consistent with the data from the model.

Fit results: trigger efficiency



The calculated efficiency estimate is in agreement with the data from the model.

The Bayesian inversion method (Γ-fit): 2D fit

- The fluctuation kernel for two observables at fixed impact parameter can be describe by 2D Gamma distr.:

$$P(E,E_F \mid c_b) = G_{2D}(E,E_F,\langle E\rangle,\langle E_F\rangle,D(E),D(E_F),R)$$

$\langle E\rangle, D(E)$ – average value and variance of energy in EMC

$R(E,E_F)$ – Pirson correlation coefficient

$$P(E,M) = \int_0^1 P(E,M \mid c_b)dc_b$$

$$R(E,M) = \frac{\varepsilon_1^2 m_1^2}{\varepsilon_2 m_2} R(E',M') \qquad \varepsilon_1,\varepsilon_2,m_1,m_2 \text{ - fit parameters}$$

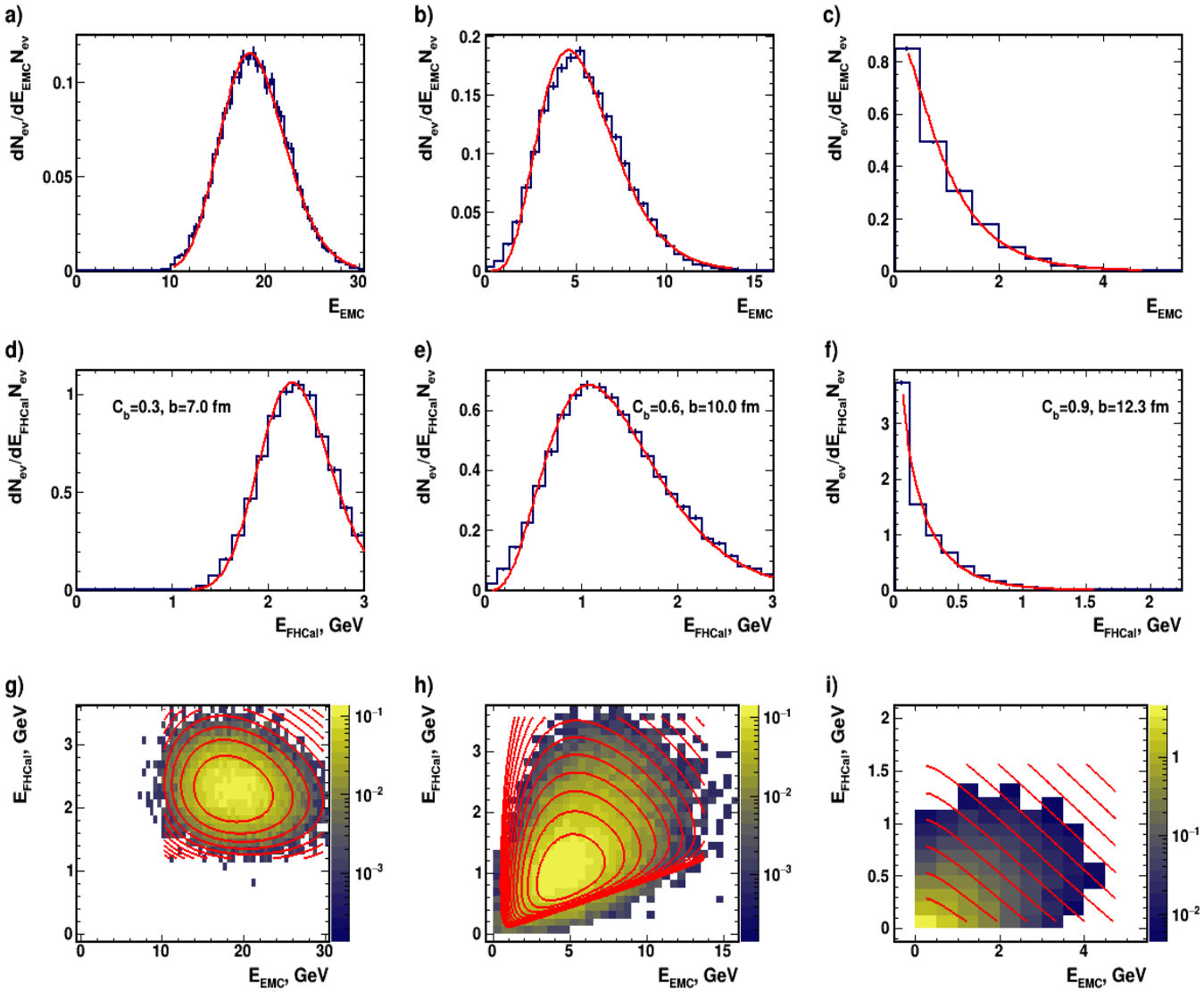
$\langle E'(c_b)\rangle$ – average value and var. of energy/mult.

$D(E'(c_b))$ from the rec. model data

$$\langle E\rangle = \varepsilon_1\langle E'(c_b)\rangle, \quad D(E) = \varepsilon_2 D(E'(c_b))$$
$$\langle M\rangle = m_1\langle M'\rangle, \quad D(M) = m_1^2 D(M') + m_2\langle M'\rangle$$

$\langle E'(c_b)\rangle, \quad D(E'(c_b))$ - can be approximated by polynomials

The fluctuation of energy and multiplicity at fixed impact parameter



It is possible to find such a rotation angle of the system that $\mathrm{cov}(x, y) = 0$

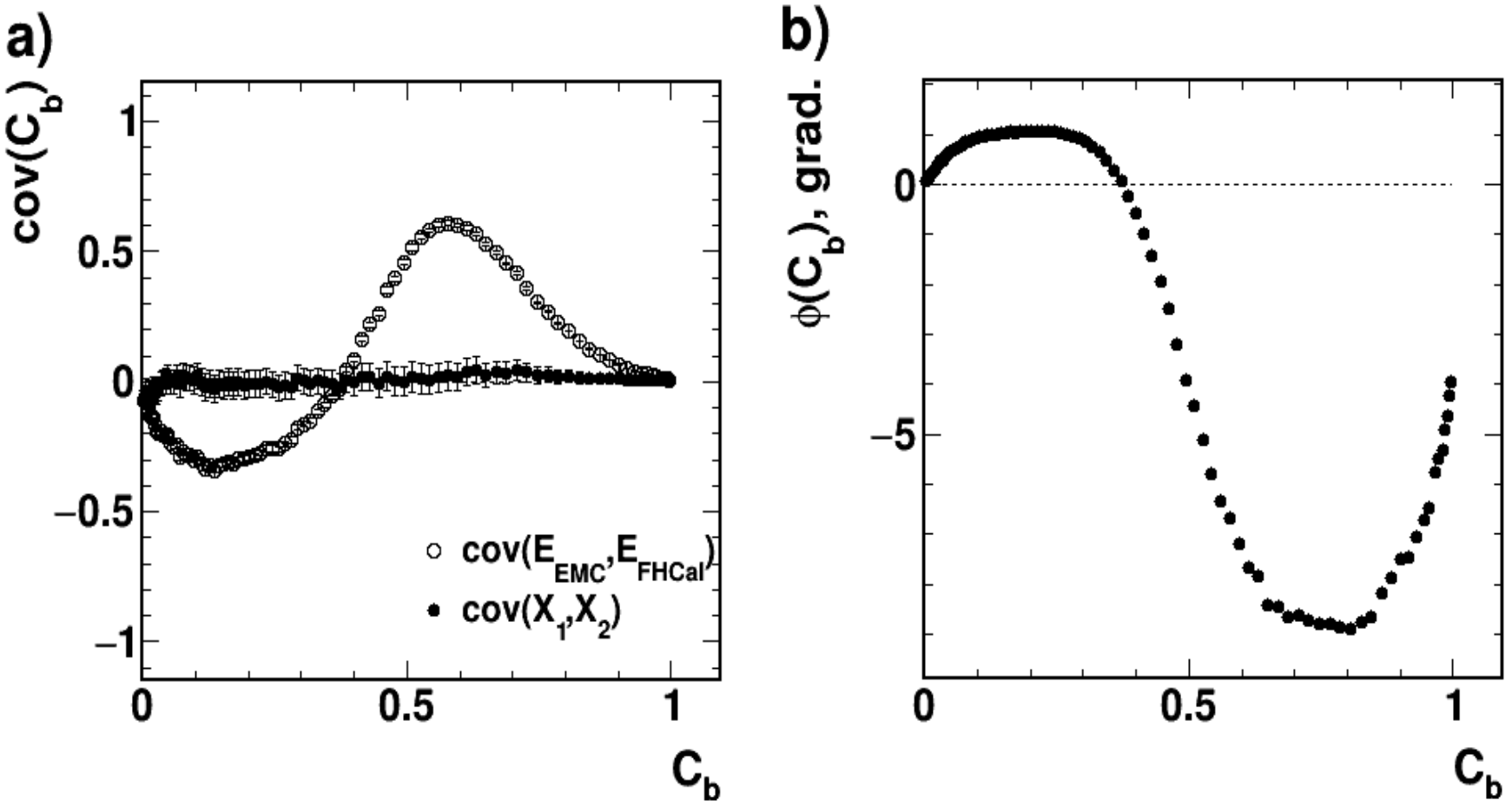
$$\alpha = \arctan\left(\frac{2\sqrt{D(E)D(E_F)}R(E,E_F)}{D(E)-D(E_F)}\right)$$

$$G_{2D} = G(x,\theta_x,k_x)\cdot G(y,\theta_y,k_y)$$

The probability of *b* for fixed range of observables can be find using Bayes' theorem:

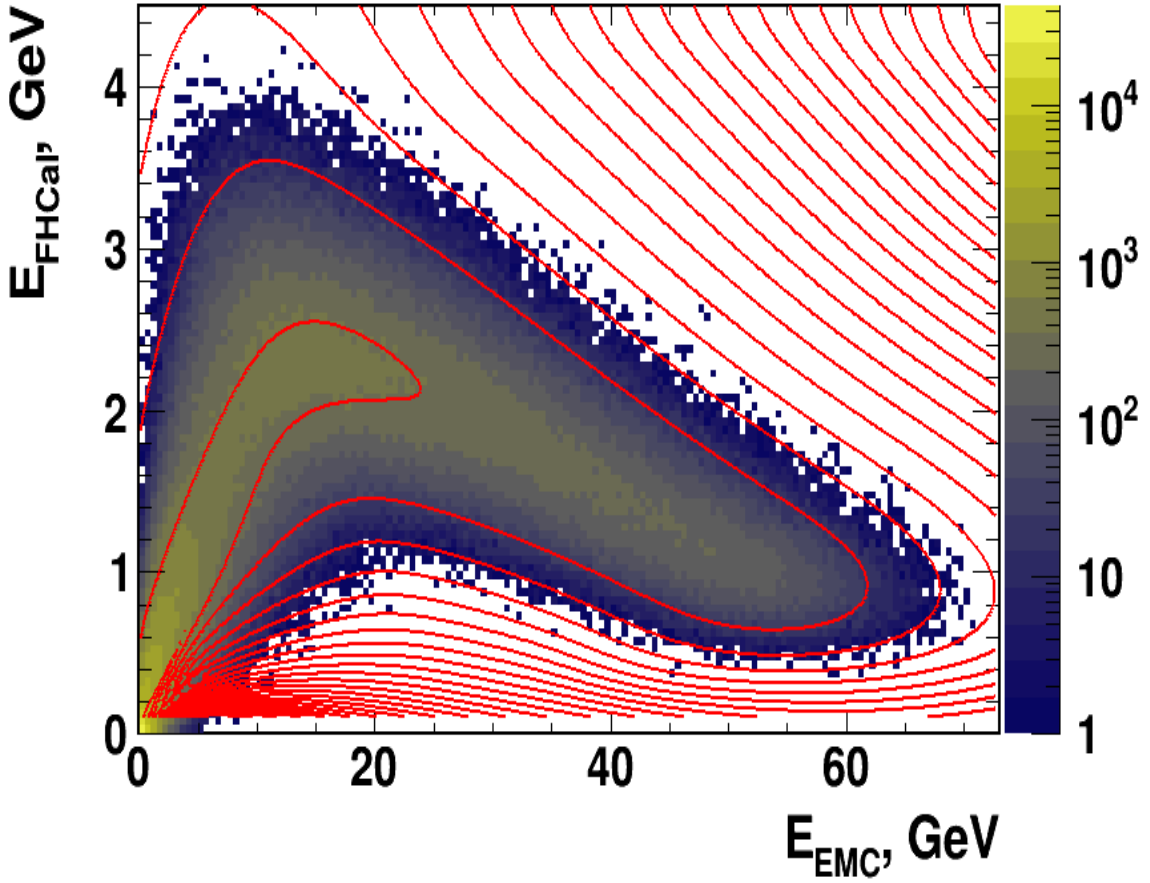
$$P(b\,|\,\mathrm{E}_1,\mathrm{E}_2,E_F^1,E_F^2) = P(b)\frac{\int_{E_1}^{E_2}\int_{E_F^1}^{E_F^2} P(E,E_F\,|\,c_b)dE_F dE}{\int_{E_1}^{E_2}\int_{E_F^1}^{E_F^2}\int_0^1 P(E,E_F\,|\,c_b)dE_F dE dc_b}$$

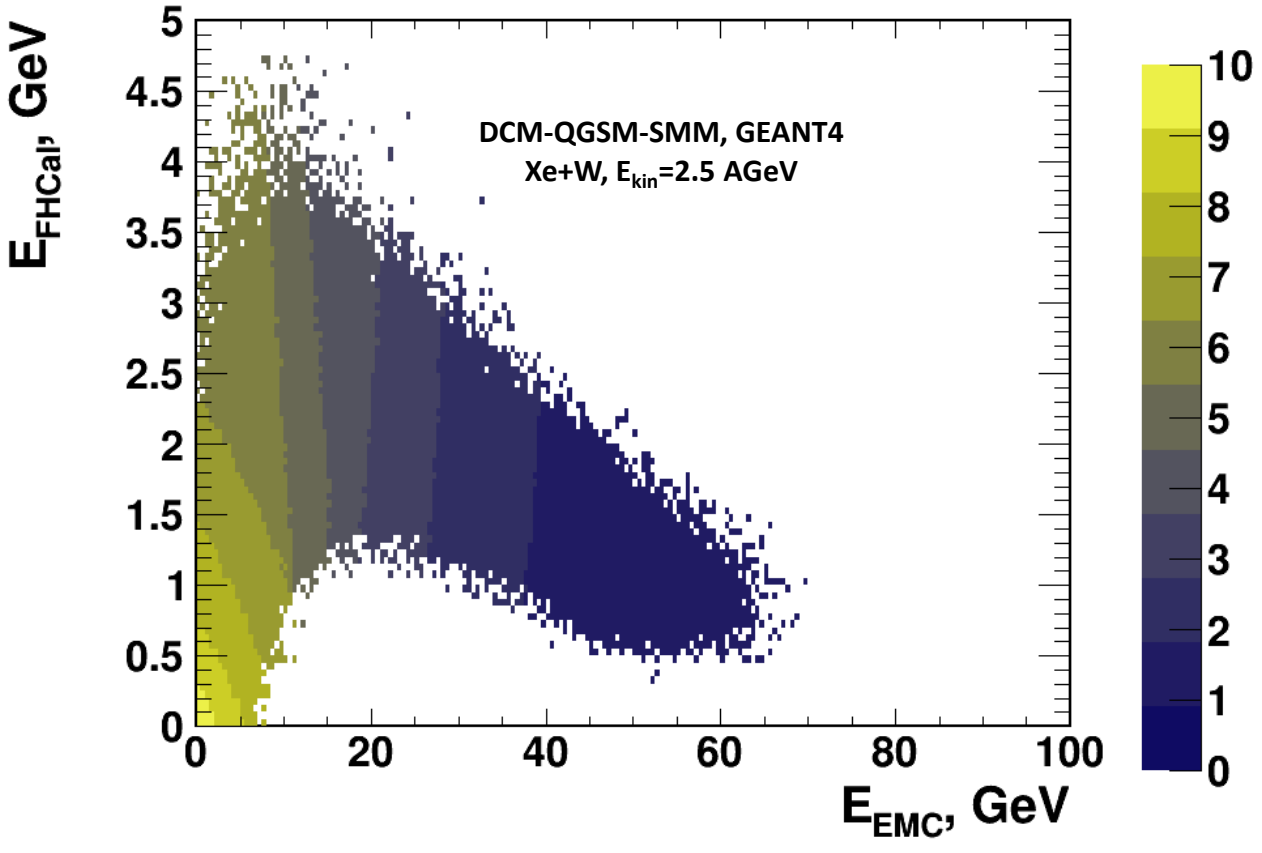
Suppression of the linear component of correlation



Rotating the coordinate system makes it possible to suppress linear correlation.

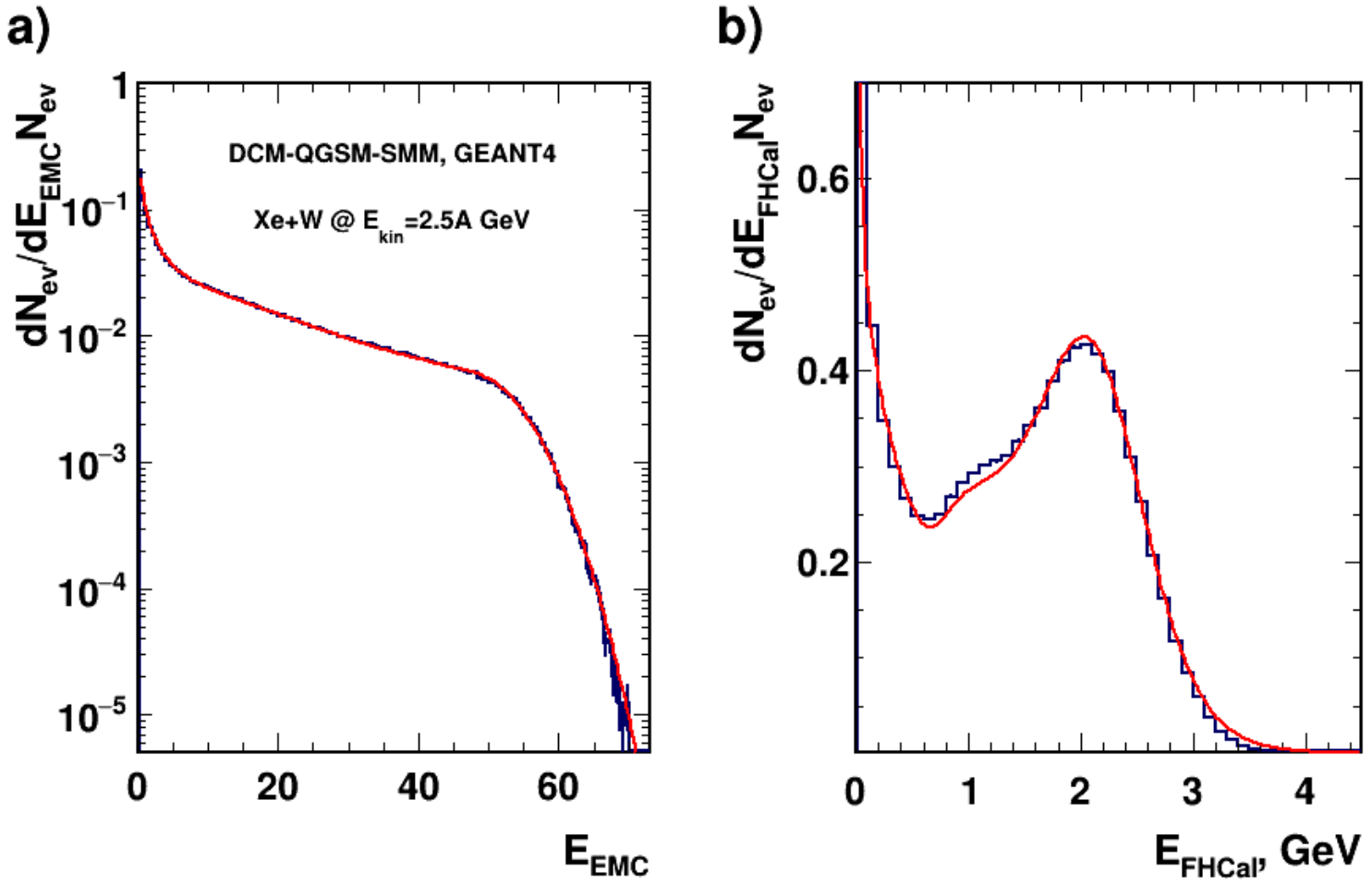
2D Bayesian approach: results





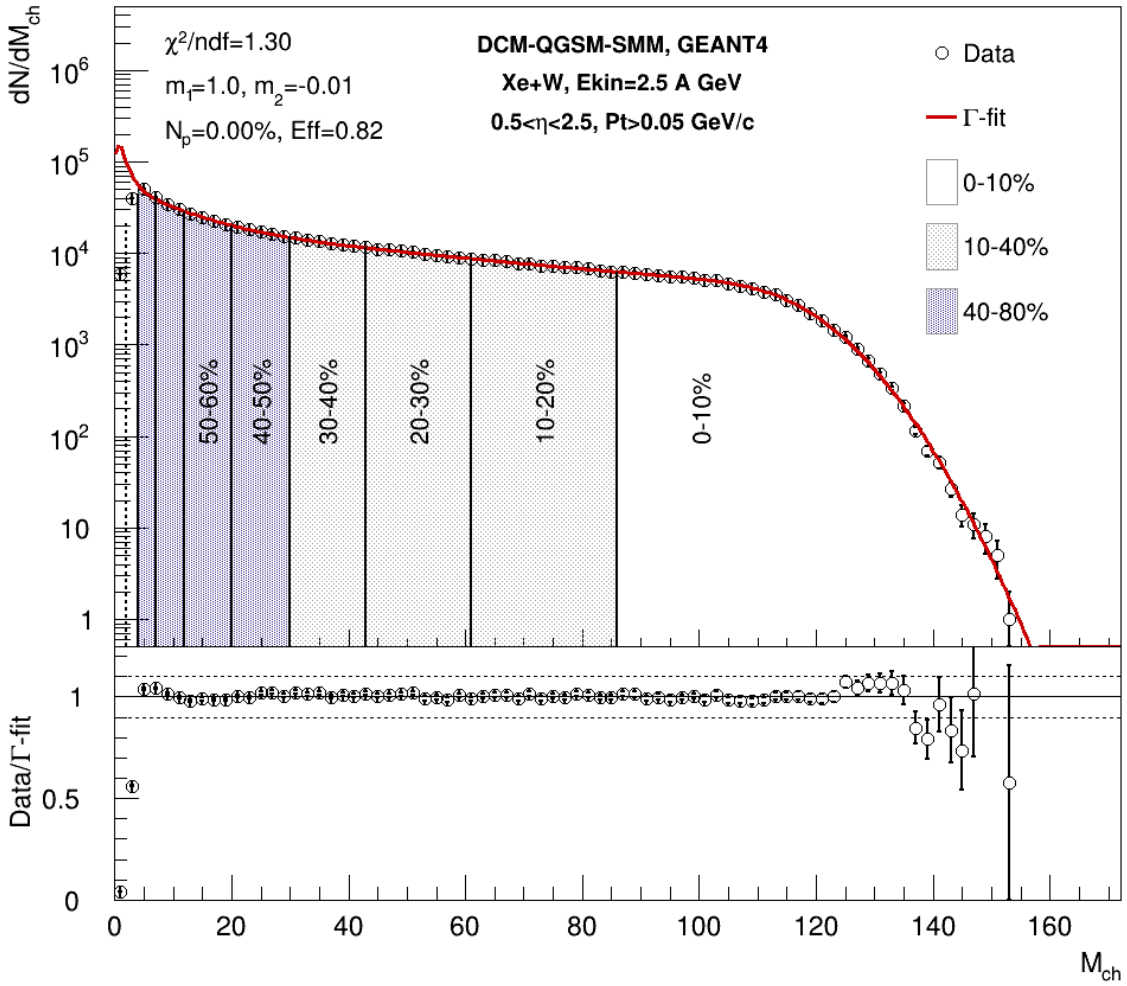
Good agreement between fit and data.

2D Bayesian approach: results

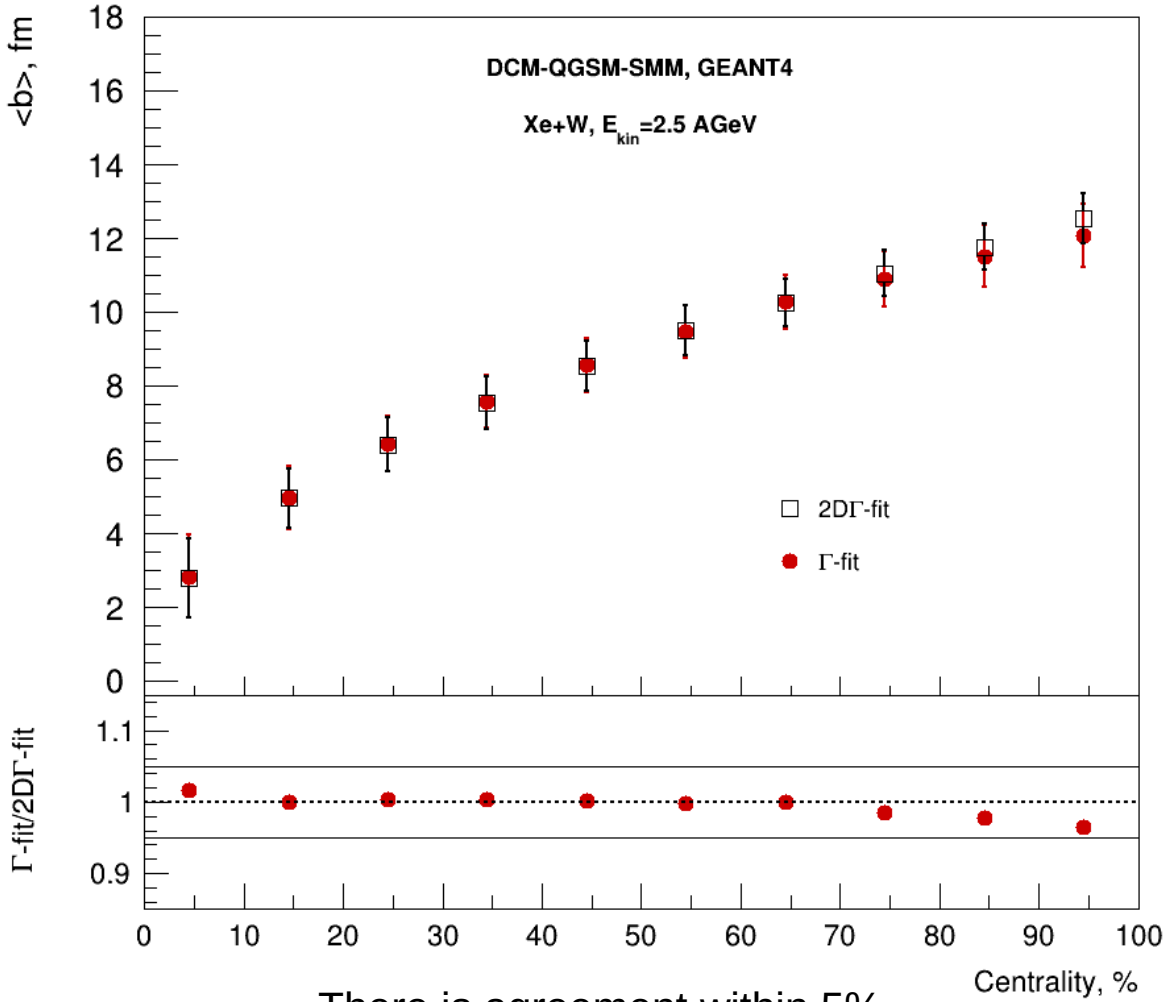


Good agreement between fit and data.

Comparison centrality determination methods



Good agreement with fit



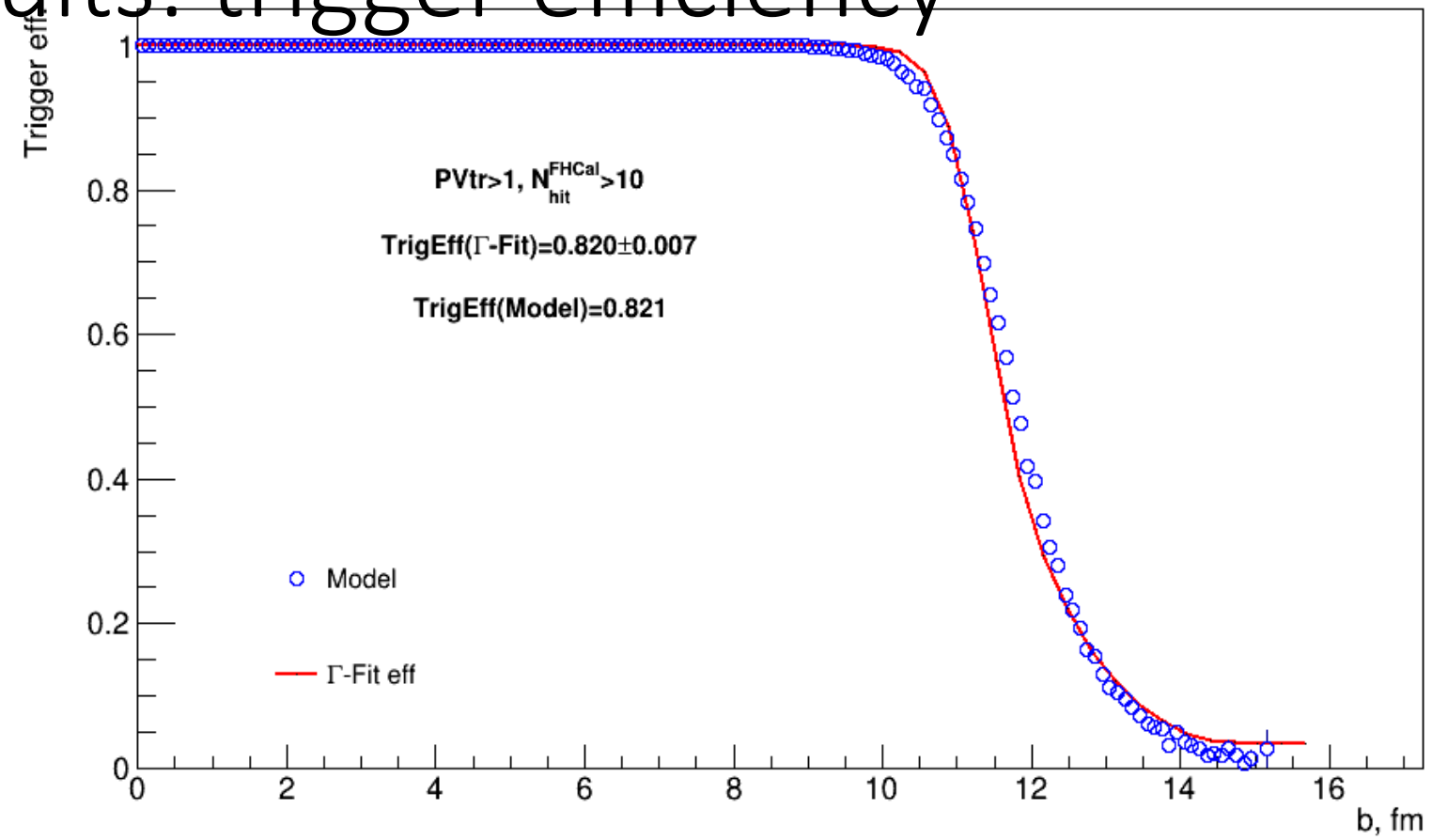
There is agreement within 5%.

Summary and Outlook

- An approach for centrality determination accounting for trigger system efficiency is proposed.
  - The calculated efficiency estimate is consistent with the data from the model.
- The Bayesian inversion method reproduces observables for fixed-target mode at the MPD:
  - Multiplicity-based and 2D approaches using E_{EMC} and E_{FHCal} show results consistent with model data
- The obtained results will be published in a journal.

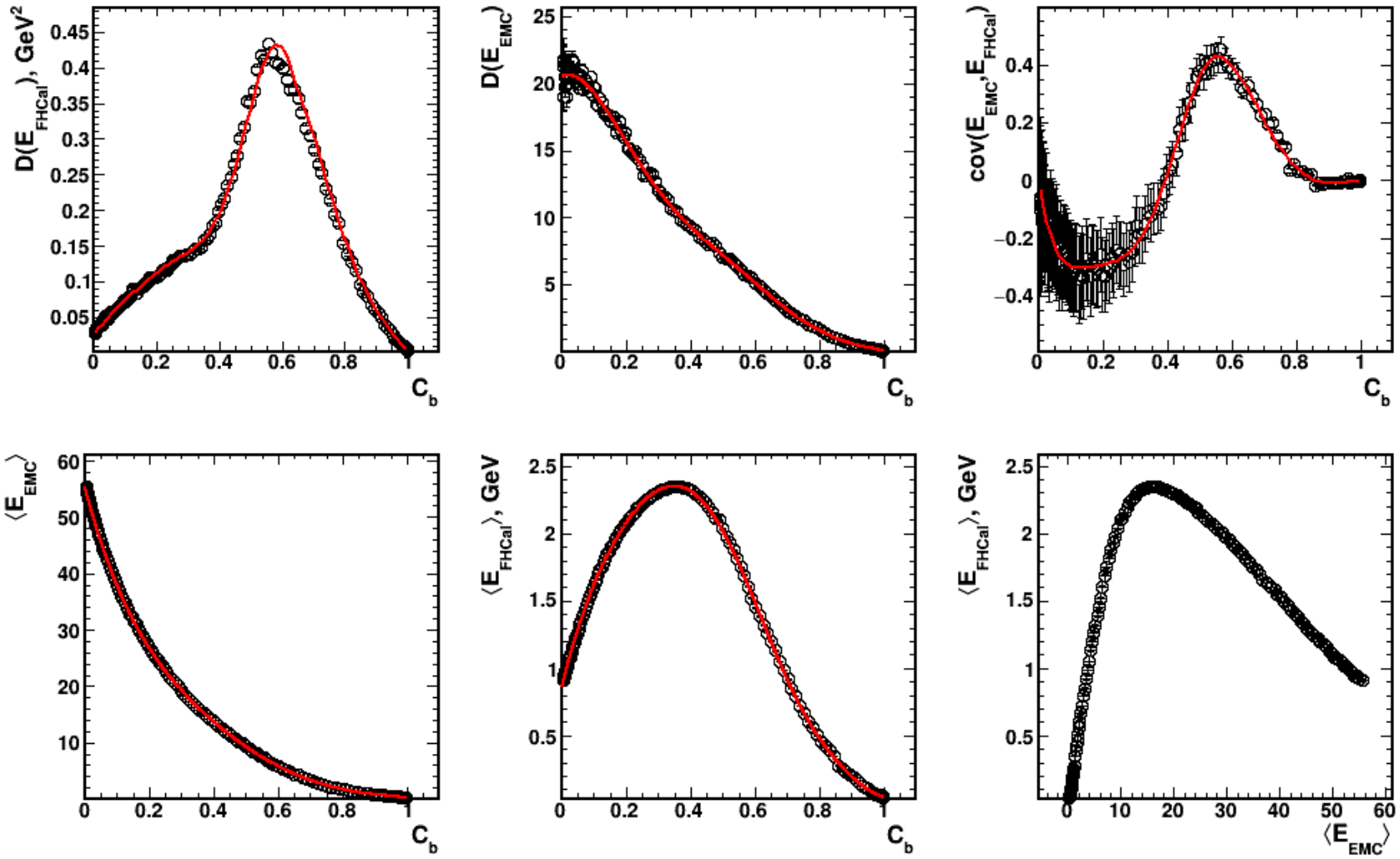
Thank you for your attention!

Fit results: trigger efficiency



Lower efficiency improves differential efficiency estimation by reducing peripheral errors in mean and variance fitting, while total efficiency accuracy remains unchanged

Dependence of the average value and variance on centrality



Averages and variances can be fited by a smooth function.

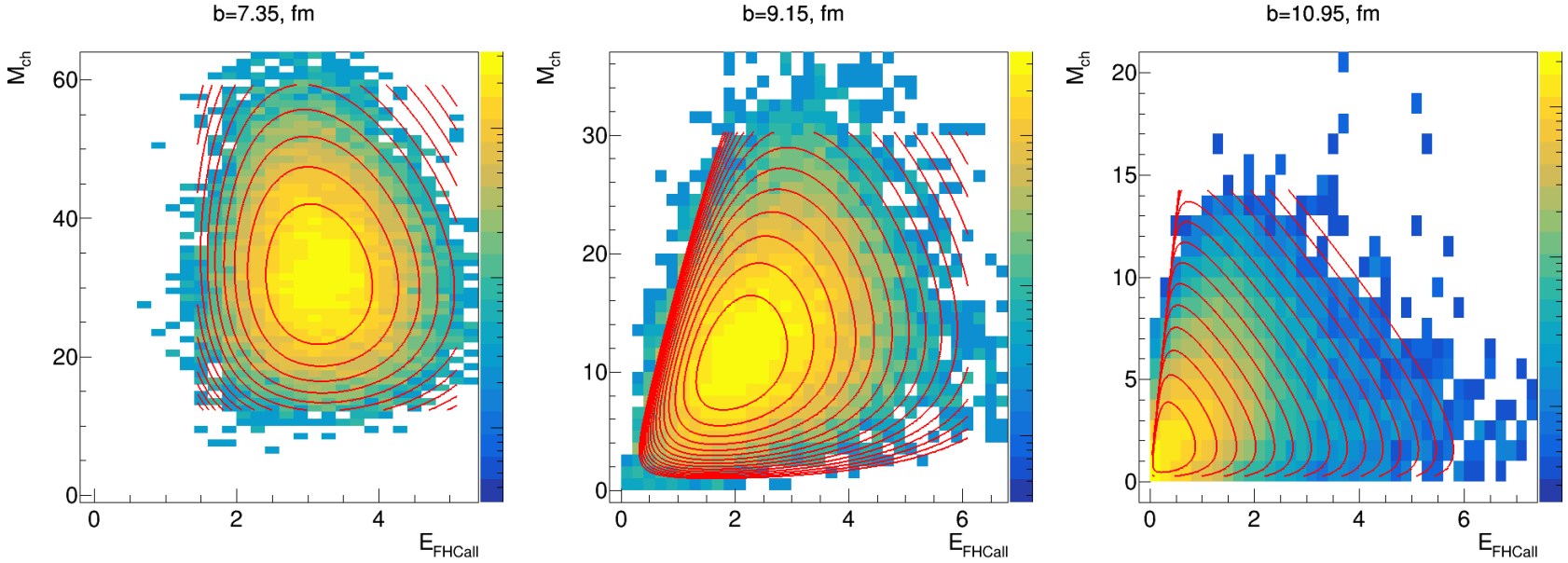
The fluctuation of energy and multiplicity at fixed impact parameter

It is possible to find such a rotation angle of the system that $\mathrm{cov}(x, y) = 0$

$$x = \cos(\alpha)E + \sin(\alpha)M,$$
$$y = -\sin(\alpha)E + \cos(\alpha)M$$

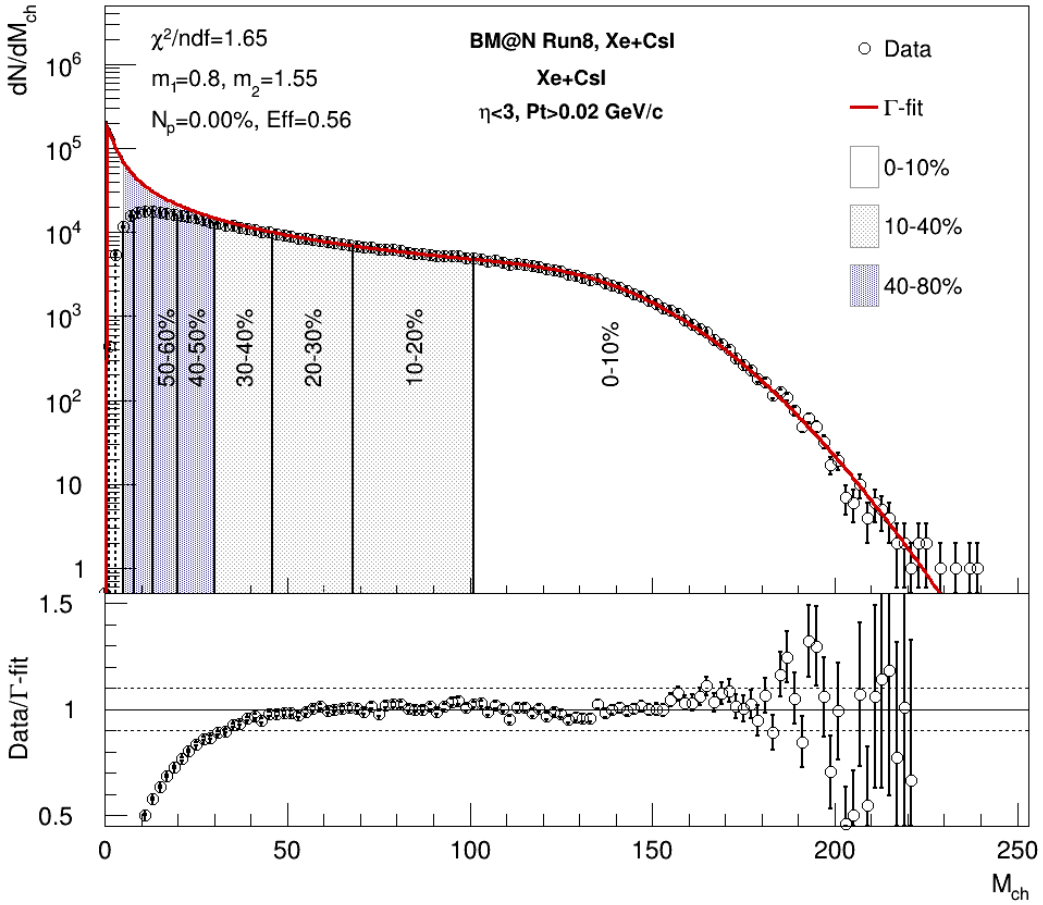
$$\alpha = \arctan\left(\frac{2\sqrt{D(E)D(M)}R(E,M)}{D(E) - D(M)}\right)$$

$$G_{2D}(E_{FH}, M_{ch}, \langle E \rangle, \langle M \rangle, D(E), D(M), R) = G(x, \theta_x, k_x) \cdot G(y, \theta_y, k_y)$$



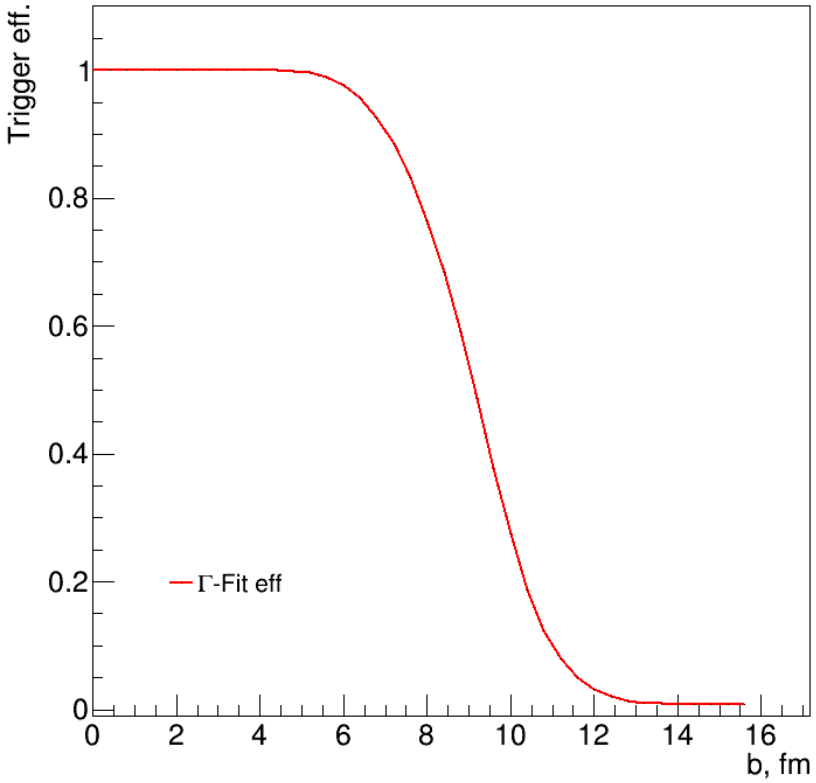
The distribution of energy and multiplicity at a fixed impact parameter is well described by the gamma distribution

Fit results: experimental data



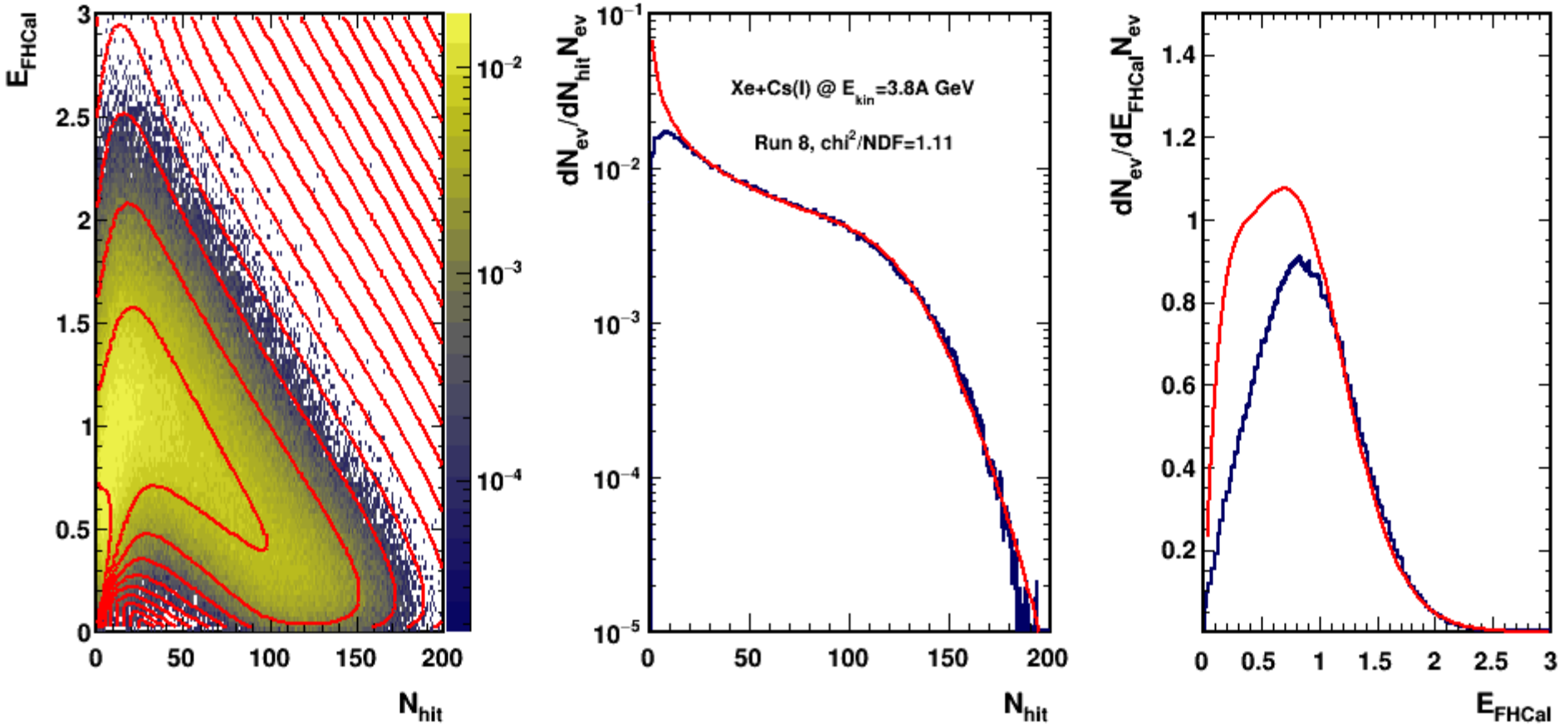
Good agreement with fit

$$P_{eff}(b) = \int_0^{M_{\max}} P_{eff}(M) P(M \mid b) dM$$



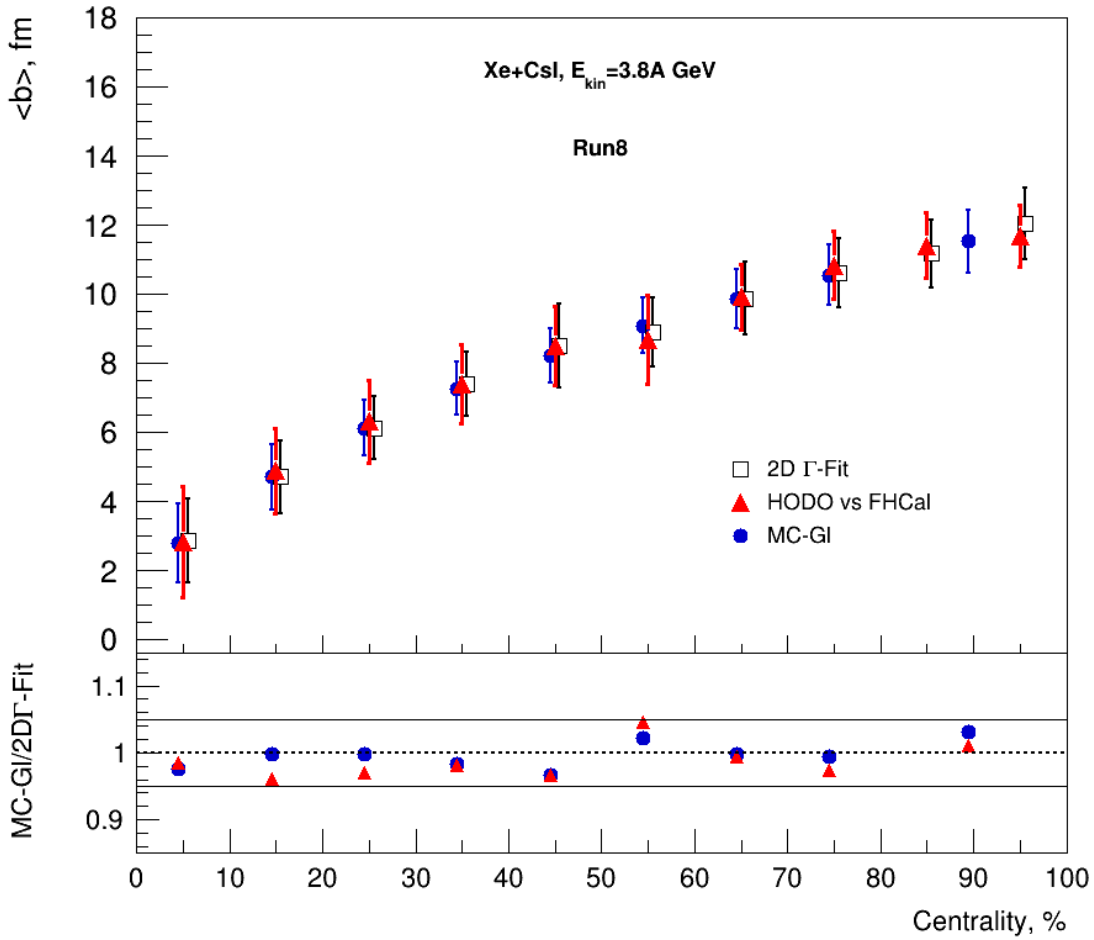
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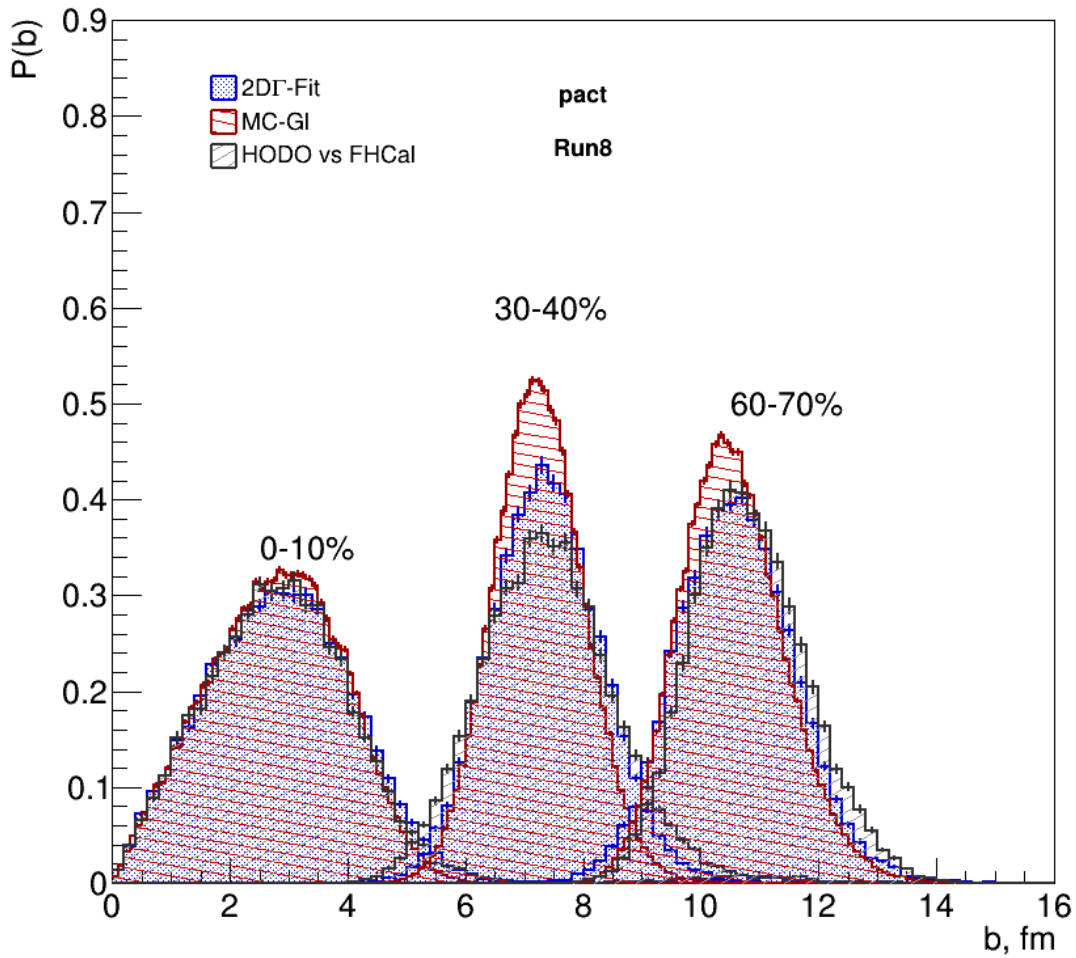
2D fit results



The fit function qualitatively reproduces the multiplicity-energy correlation from FHCal

Comparison with Glauber MC fit





There is agreement within 5%.